

HYDRO RESOURCE MANAGEMENT, RISK AVERSION AND EQUILIBRIUM IN AN INCOMPLETE ELECTRICITY MARKET SETTING

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Abstract

Since the outset of power system reform, one of the main objectives of regulation has been to assess whether the market, of its own accord, can induce agents to adopt decisions that maximise social welfare.

This paper analyses the effect of generating companies' risk aversion on their medium-term (typically 1 year) hydroelectric resource planning, along with its possible inducement of system operation that deviates from the centralised (maximum social welfare) solution.

Forward markets may play a key role by making hedging instruments available to risk-averse agents. A stylised mathematical model is used in this study to prove the equivalence of centralised planning and market equilibrium in the presence of such agents under the following assumptions: 1) Both the spot and forward markets are perfectly competitive 2) it has at least one risk-neutral consumer or arbitrageur; 3) all agents share the same beliefs about uncertain parameters; 4) only one price is in place in each trading period (which can be perfectly hedged with a forward contract); and 5) a solution for the resulting market equilibrium problem exists.

The findings show that such equivalence vanishes when forward markets are missing or inaccessible (attributable in some electricity markets to the absence of demand-side participation). This article consequently suggests that requiring demand-side agents to sign forward contracts with generators might constitute an effective regulatory measure where no fully functional forward market is already in place.

Keywords: medium-term planning, electricity markets, regulatory intervention

1 INTRODUCTION

Free markets and efficiency

Since the outset of power system reform, *one of the main objectives of regulation has been to assess whether the market, of its own accord, can induce agents to adopt decisions that maximise social welfare.* Where they deem that this may not be the case, but rather that the market deviates significantly from the social optimum, regulators may contemplate introducing mechanisms that would guide the market toward such an ideal. Real-time (in the US) and balancing (in the EU) markets, as well as other types of reserve markets run by system operators, are good examples of regulatory measures that aim to remedy market agents' potential inability to guarantee very short-term system security. Another obvious example of such measures, capacity mechanisms (Batlle & Rodilla, 2012) designed to guarantee long-term system adequacy, have been implemented since the advent of the market in North and South America (Batlle et al., 2014). Their institution in Europe is now being debated in depth (EU Commission, 2012 & 2013).

The consensus opinion around the adoption of regulatory measures in general is that it should be preceded by accurate problem identification to be able to effectively tackle the specific market failure at issue (the actual ailment)². Market failures (including non-participation by agents or externalities) and how to deal

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² Insofar as regulation is not perfect, before intervening, regulators should ensure that any harm that might be caused by introducing an "imperfect mechanism" does not outweigh the potential benefits.

with them in the short³ (security) and very long⁴ (adequacy) term have long been debated. The medium-term dimension of the problem, and more specifically efficient medium-term resource management of existing facilities, has received scant attention in the academic literature, however.

The medium-term dimension of the problem

The medium term is typically defined to mean a 1-year period. Generators must manage their fuel stocks and hydro reserves in this time frame and establish optimal generating unit maintenance schedules. Such medium-term decisions largely condition economically efficient dispatching given, for instance, their direct impact on the availability of resources when most needed. In electricity markets, these medium-term decisions are driven exclusively by market signals.

The importance of satisfactory medium-term resource management has been indirectly acknowledged in the practical regulation of electricity systems worldwide. Many regulatory mechanisms geared to enhancing adequacy (to attract new capacity) also include powerful incentives for both new and existing generators to increase their availability in the medium term, particularly in the presence of system scarcity. The penalties for non-compliance explicitly laid down in New England forward capacity market contracts are a case in point (see Batlle & Rodilla, 2010). This issue has become increasingly relevant with the growing penetration of variable energy resources such as wind and PV solar facilities.

Hydro resource management and risk aversion

Of the various types of medium-term resource management involved in electric power generation, the focus here is on hydro reserves. In light of their flexible implementation, efficiently managed hydro resources can be used to deal effectively with potentially high prices, particularly in many of today's markets where the deployment of intermittent generation technologies is growing fast.

The complexity involved in the operation of real hydroelectric systems⁵ is simplified in the stylised formulation of hydro generation used here for readier calculation and subsequent interpretation of the optimality conditions. More specifically, the hydro system is modelled as an energy-constrained resource in which a certain amount of available energy has to be allocated over time, irrespective of inflow chronology during the year, limits to power output or non-linear dependencies.

The uncertainty around inflows and the many constraints on reservoirs renders management of this resource and its inherent risks particularly complex. More specifically, the efficiency of medium-term resource management, which is associated with risk management, may be affected by a significant market failure, namely medium- and long-term electricity market incompleteness, as described below.

This paper analyses the combined effect of risk aversion among hydro (or hydro-thermal) generators and their inability to efficiently hedge medium- to long-term positions and shows that it may compromise efficient medium-term resource management⁶. In other words, incomplete markets are found to be able to steer hydro resource management away from the optimisation of social welfare. Assuming that market prices reflect true marginal costs, the study also shows that the model delivers the same solution for central planning, i.e., maximisation of expected social welfare, as for market equilibrium when the participants are risk neutral. When generators with hydro capacity are risk averse, however, they may use that capacity to hedge their exposure to risk, inducing dispatching that deviates from the maximum social welfare solution.

³ See for instance Batlle et al. (2007) and Vandezande et al. (2009).

⁴ See for instance Hogan (2005), Joskow (2007) and Pérez-Arriaga & Linares (2008).

⁵ Actual hydro system planning and operation are subject to many constraints that span all the decision variables involved. In cascaded multi-reservoir systems, account must be taken of the time- and space-related links that interconnect all the hydro plants operating on the same river basin. Natural inflows at hydro network nodes and losses due to evaporation or seepage must also be carefully modelled. Inflow chronology is one of the main sources of uncertainty. Further complexity is introduced by the non-linear dependence among net head, water flow and power output, along with other constraints such as water rights for consumption or irrigation. See Labadie (2004) for a comprehensive review of the state-of-the-art optimisation of reservoir system management and operations.

⁶ As a general rule, incomplete markets are not Pareto efficient (see for instance Magill and Quinzii, 2002).

A number of analyses have been published on the effect of risk aversion on hydro resource planning by generating companies: see, for instance, Unger (2002) and Fleten et al. (2002). The scope of those analyses is enlarged here to include the social consequences of risk-averse behaviour and the respective regulatory implications.

Incomplete markets

Long-term financial markets are categorised in the literature as complete or incomplete (Duffie, 1996). Long-term markets are defined to be incomplete when perfect inter-agent risk transfer does not take place. One of the major causes of that obstruction is what is known as the missing markets problem. This situation and its consequences for electricity markets have been analysed in connection with efficiency in several contexts, including regional markets (Smeers, 2004) and long-term investment (Willems and Morbee, 2010).

The latter authors noted that the first and foremost conclusion drawn from the literature on the effects of the pricing of additional assets is that welfare is lower in incomplete than complete markets because risk is imperfectly allocated in the former⁷. That is the effect analysed here in the context of medium-term hydro planning.

Risk, equilibrium models and incomplete markets

Most equilibrium models that incorporate risk and long-term contracts are designed for long-term investment decision-making. As a general rule, the conclusions drawn from such models is that in the absence of long-term markets, agents' decisions differ from the ones they would have adopted if they were risk neutral. In this vein, the impact of uncertain CO₂ permit policies (Fan et al., 2009) and price uncertainty (Ehremann et al., 2011) have been shown to have an impact that may call for compensatory regulatory measures.

Electricity market volatility is known to constitute an incentive for risk-averse producers and consumers to hedge their exposure to electricity prices by buying and selling derivatives. Studies have been conducted in which long-term contracting is built into the models to calculate equilibrium when risk-averse agents can hedge their risk. The overall conclusion is that investment in power plants rises under such circumstances because the more effective hedging afforded by additional derivatives makes them more appealing to investors. Ehremann et al. (2013) and Willems et al. (2010) address this issue in some depth.

Other studies analyse the effects on welfare of including long-term contracting in equilibrium calculations. Research by Willem et al. (2010) showed that when forward contracting was introduced in the model, aggregate social welfare (calculated as the sum of individual firms' utilities) rose with the number of options (which generated a more complete market).

To the best of the present authors' knowledge, no prior equilibrium model studies have been conducted on the effects of risk aversion on medium-term hydro resource management. The findings of the present study of such effects are wholly in line with the earlier long-term approaches cited above.

Objectives and roadmap

The present theoretical analysis addresses hydro resource management in a context characterised by (i) perfect competition, (ii) risk-averse agents and (iii) incomplete long-term markets. It explores the impact of incomplete long-term markets, which translate into less socially efficient hydro resource management. Two scenarios are envisaged in the study: (i) a market where no financial instruments are available to generators, and (ii) a market where forward contracts are available for the following period (a more complete long-term market scenario).

Four multi-stage stylised models are developed to illustrate the discussion: centralised, risk-neutral welfare maximisation (used as benchmark and labelled **Cen**); a perfectly competitive market with risk-neutral generators (**MrNe**); a perfectly competitive market with risk-averse generators and no hedging

⁷ Market completion does not necessarily benefit all agents. Complete markets are Pareto efficient, but not necessarily Pareto dominant with respect to all possible incomplete market allocations.

instruments (**MrAv**); and the same perfectly competitive market with risk-averse agents in which forward contracting is available (**MrAvF**).

The paper is structured as follows.

- 1) The optimal solution assuming central planning (**Cen**) is formulated and found in section 2 for use as the benchmark against which to compare the results computed for the other three market settings.
- 2) The impact of generator risk aversion is analysed in detail. The findings for the **MrNe** and **MrAv** settings are compared to the benchmark and the respective conclusions are drawn. In the absence of instruments with which generators can hedge their exposure (i.e., an effective long-term market), risk aversion is deemed to alter efficient management of generating resources (section 3).
- 3) Forward contracting is then factored into the model in setting **MrAvF** and the findings are compared to the results observed in section 3. The existence of effective risk hedging, represented here in the form of forward markets, is shown to enhance hydro resource management efficiency (section 4).
- 4) These theoretical considerations are illustrated in a case study (section 5)
- 5) Lastly, the conclusions set out in section 6 suggest that if no effective long-term market arises of its own accord, regulator intervention may be considered as a potential alternative to ensure suitable medium-term hydro resource management.

2 THE BENCHMARK PROBLEM

This section formulates and finds the optimal conditions for the setting used as a benchmark in the analysis, namely risk-neutral central planning. A simplified version of the traditional formulation discussed by Pérez-Arriaga & Meseguer (1997) is used.

2.1 General modelling assumptions

The power system model applied includes only the components essential to the purpose of the study, to analyse the possible effect of risk aversion on medium-term planning. Unnecessary details that might mask the regulatory analysis are intentionally excluded⁸.

The setting assumed is described below.

- For the sake of simplicity, all demand is aggregated in a single demand-side agent with no bargaining power. The amount of power consumed, q_t , in each time period t , results in a certain degree of demand satisfaction or utility $U_t^D(q_t)$. Demand utility functions are assumed to be strictly increasing ($dU_t^D / dq_t > 0$) and concave ($d^2U_t^D / dq_t^2 \leq 0$).
- The generation side is deemed to comprise a large number of generation companies unable to affect the spot price of electricity (no market power). Likewise to simplify the model, all generators are assumed to be aggregated into a single perfectly competitive agent. The amount of power produced, q_t , in each time period t is defined as the sum of the output of the thermal (T) and hydro (H) units. The total amount of electricity produced in each period of time is therefore represented as follows:

$$q_t = q_t^T + q_t^H \quad (1)$$

- Thermal generation costs, $C_t(q_t^T)$, are assumed to be strictly increasing and convex.

⁸ As noted earlier, regulatory decisions should be informed by much more detailed and sophisticated models. For reasons of simplicity and clarity, here the regulator is assumed to be free of any risk aversion and to act on behalf of demand.

2.2 Problem formulation

The benchmark problem is formulated in terms of maximising expected social welfare, in turn defined as the difference between expected demand utility and expected generation costs, subject to the constraints imposed by the hydro reserve balance equation.

Optimisation problems in which uncertainty is a factor are generally solved by assuming that probability is distributed discretely (see Birge & Louveaux (1997)). The configuration consequently adopted here is a multistage scenario tree such as shown in Figure 1.

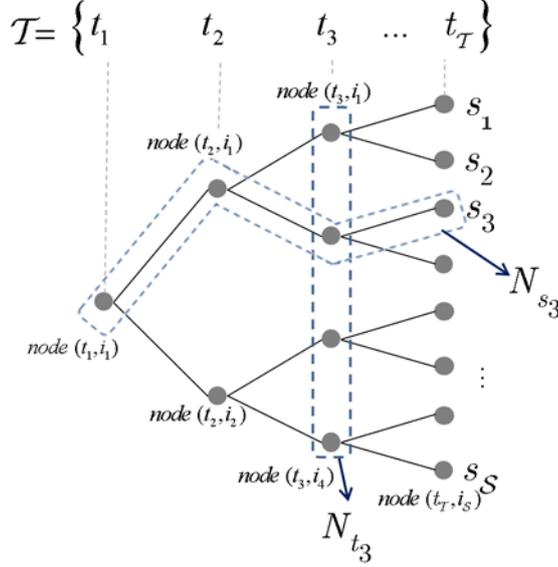


Figure 1. Scenario tree

The notation used throughout for random parameters and decision-making variables is explained below.

- Each node on the multistage tree is represented as (t, i) , i.e., the combination of two indexes: the time period (t), and the node within each period (i). For instance, the root node in period one is represented as (t_1, i_1) . N represents the set of all the nodes considered in the problem and N_t the set of nodes at each time period t .
- S represents the set of scenarios. Each scenario s is characterised by a probability p_s , with $\sum_{s \in S} p_s = 1$. N_s represents the set of nodes in scenario s .
- \mathcal{T} denotes the set of time periods.
- Terminal nodes are given as $(t_{\mathcal{T}}, i)$.
- The terminal node for the last scenario is referred to as $(t_{\mathcal{T}}, i_S)$.
- The probability that decision-making will reach node (t, i) is $p_{t,i}$ (with all probabilities assumed to be non-null). The sum of the probabilities for all the nodes in any given time period is one, i.e.: $\sum_{(t,i) \in N_t} p_{t,i} = 1$.
- Each node (t, i) has a set of descendant nodes $D_{(t,i)}$. To simplify the notation, the transition probability from (t, i) to any descendant (the probability of transition from (t, i) to $(t+1, j) \in D_{(t,i)}$) is represented as $p_{(t,i),j}$.

The objective function for the central planner can be expressed as the maximisation of expected social welfare, i.e., of the net benefits of power production and consumption, subject to the maximum hydro generation available in each scenario and provided that the demand balance equation holds at each node:

$$\begin{aligned}
 & \max_{q^T, q^H, q} \sum_{(t,i) \in N} p_{t,i} \cdot \left(U_{t,i}^D(q_{t,i}) - C_{t,i}(q_{t,i}^T) \right) \\
 & \text{s.t.}: \\
 & \quad q_{t,i}^T + q_{t,i}^H = q_{t,i} \quad : \lambda_{t,i}, \quad \forall (t,i) \in N \\
 & \quad \sum_{(t,i) \in N_s} q_{t,i}^H \leq Q_s^H \quad : \mu_s, \quad \forall s \in \mathcal{S}
 \end{aligned} \tag{2}$$

To simplify the resulting optimality conditions, no limits on thermal or hydro capacity are built into the model nor are network effects contemplated in the analysis. A stylised formulation is adopted for the hydro reserve balance equation in each scenario s . In this simplified model, hydro generation is deemed to be a limited energy resource, with total available energy for scenario s represented as Q_s^H . Decision-makers must determine how to allocate such uncertain energy optimally across all the time periods in the scenario, which entails adopting a single “here and now” decision in the first period. Lagrange multiplier μ_s is used to represent the additional welfare that can be obtained if Q_s^H were to rise by one unit in the scenario at issue. Lagrange multiplier $\lambda_{t,i}$, in turn, represents the system’s marginal cost for each node.

For notational simplicity, q^T is used to denote the vector containing all the variables representing thermal generation in all the nodes on the stochastic tree: $q^T = (q_{t_1, i_1}^T, \dots, q_{t_T, i_S}^T)$. A similar criterion is applied for $q^H = (q_{t_1, i_1}^H, \dots, q_{t_T, i_S}^H)$, $q = (q_{t_1, i_1}, \dots, q_{t_T, i_S})$, and the Lagrange multipliers.

2.2.1 Optimal conditions

The Lagrangian function $\mathcal{L}(q^T, q^H, q, \lambda, \mu)$ is formulated to obtain the necessary first-order conditions and compute the first partial derivatives with respect to the decision variables:

$$\begin{aligned}
 \mathcal{L}(q^T, q^H, q, \lambda, \mu) = & \sum_{(t,i) \in N} p_{t,i} \cdot \left(U_{t,i}^D(q_{t,i}) - C_{t,i}(q_{t,i}^T) \right) \\
 & + \sum_{(t,i) \in N} \lambda_{t,i} \cdot \left(q_{t,i} - (q_{t,i}^T + q_{t,i}^H) \right) + \sum_{s \in \mathcal{S}} \mu_s \cdot \left(Q_s^H - \sum_{(t,i) \in N_s} q_{t,i}^H \right)
 \end{aligned} \tag{3}$$

The optimality conditions found with the partial derivatives of the Lagrangian function with respect to the thermal unit output and the total demand at each node lead to a generally accepted conclusion: the function is optimised when the thermal cost at each node (t, i) is equal to marginal demand-side utility at the same node:

$$\left. \begin{aligned}
 \frac{\partial \mathcal{L}(\cdot)}{\partial q_{t,i}^T} = 0 & \rightarrow p_{t,i} \cdot \frac{dC_{t,i}(q_{t,i}^T)}{dq_{t,i}^T} = -\lambda_{t,i} \\
 \frac{\partial \mathcal{L}(\cdot)}{\partial q_{t,i}} = 0 & \rightarrow p_{t,i} \cdot \frac{dU_{t,i}^D(q_{t,i})}{dq_{t,i}} = -\lambda_{t,i}
 \end{aligned} \right\} \rightarrow \frac{dC_{t,i}(q_{t,i}^T)}{dq_{t,i}^T} = \frac{dU_{t,i}^D(q_{t,i})}{dq_{t,i}}, \quad \forall (t,i) \in N \tag{4}$$

Equating the partial derivative with respect to hydro generation to zero yields the following equation:

$$\frac{\partial \mathcal{L}(\cdot)}{\partial q_{t,i}^H} = 0 \rightarrow \sum_{s|(t,i) \in N_s} \mu_s = -\lambda_{t,i}, \quad \forall (t,i) \in N \quad (5)$$

Since according to (4), $-\lambda_{t,i} = p_{t,i} \frac{dU_{t,i}^D(q_{t,i})}{dq_{t,i}}$, it follows that:

$$p_{t,i} \cdot \frac{dU_{t,i}^D}{dq_{t,i}} = \sum_{s|(t,i) \in N_s} \mu_s, \quad (t,i) \in N \quad (6)$$

Note that equation (6) links marginal demand-side utility at each node (t,i) to the expected marginal demand-side utilities at its descendant nodes $D_{(t,i)}$. For instance, since at time period $t_{\mathcal{T}}$, terminal nodes are “visited” by one possible scenario only, the expression for the upper node at the last stage would be:

$$p_{t_{\mathcal{T}},i_1} \cdot \frac{dU_{t_{\mathcal{T}},i_1}^D}{dq_{t_{\mathcal{T}},i_1}} = \mu_{s_1} \quad (7)$$

At the preceding node $(t_{\mathcal{T}-1},i_1)$, the Lagrange multipliers for the other descendants would have to be added to scenario s_1 . Where it had three descendants, the expression for this node would be:

$$p_{t_{\mathcal{T}-1},i_1} \cdot \frac{dU_{t_{\mathcal{T}-1},i_1}^D}{dq_{t_{\mathcal{T}-1},i_1}} = \mu_{s_1} + \mu_{s_2} + \mu_{s_3} \quad (8)$$

Substituting (7) in (8) and applying analogous relationships to scenarios s_2 and s_3 yields:

$$p_{t_{\mathcal{T}-1},i_1} \cdot \frac{dU_{t_{\mathcal{T}-1},i_1}^D}{dq_{t_{\mathcal{T}-1},i_1}} = p_{t_{\mathcal{T}},i_1} \cdot \frac{dU_{t_{\mathcal{T}},i_1}^D}{dq_{t_{\mathcal{T}},i_1}} + p_{t_{\mathcal{T}},i_2} \cdot \frac{dU_{t_{\mathcal{T}},i_2}^D}{dq_{t_{\mathcal{T}},i_2}} + p_{t_{\mathcal{T}},i_3} \cdot \frac{dU_{t_{\mathcal{T}},i_3}^D}{dq_{t_{\mathcal{T}},i_3}} \quad (9)$$

When marginal utility is isolated at the predecessor node, the result is:

$$\frac{dU_{t_{\mathcal{T}-1},i_1}^D}{dq_{t_{\mathcal{T}-1},i_1}} = \left(\frac{p_{t_{\mathcal{T}},i_1}}{p_{t_{\mathcal{T}-1},i_1}} \right) \cdot \frac{dU_{t_{\mathcal{T}},i_1}^D}{dq_{t_{\mathcal{T}},i_1}} + \left(\frac{p_{t_{\mathcal{T}},i_2}}{p_{t_{\mathcal{T}-1},i_1}} \right) \cdot \frac{dU_{t_{\mathcal{T}},i_2}^D}{dq_{t_{\mathcal{T}},i_2}} + \left(\frac{p_{t_{\mathcal{T}},i_3}}{p_{t_{\mathcal{T}-1},i_1}} \right) \cdot \frac{dU_{t_{\mathcal{T}},i_3}^D}{dq_{t_{\mathcal{T}},i_3}} \quad (10)$$

Normalising descendant to predecessor node probability is equivalent to applying the transition probability that links the parent node to its children. General expression (11) can be readily deduced by applying the same idea recursively (the general proof for generating the following expression is given in item 7.2.1 of the annex):

$$\frac{dU_{t,i}^D}{dq_{t,i}} = \sum_{(t+1,j) \in D_{(t,i)}} p_{(t,i),j} \frac{dU_{t+1,j}^D}{dq_{t+1,j}}, \quad (t,i) \in N, t \in \mathcal{T} - \{t_{\mathcal{T}}\}, \quad (11)$$

Note that according to (4), this condition can also be expressed in terms of marginal costs:

$$\frac{dC_{t,i}^T}{dq_{t,i}^T} = \sum_{(t+1,j) \in D_{(t,i)}} p_{(t,i),j} \frac{dC_{t+1,j}^T}{dq_{t+1,j}^T}, \quad (t,i) \in N, t \in \mathcal{T} - \{t_{\mathcal{T}}\}, \quad (12)$$

In centralised planning, then, hydro reserve management involves balancing marginal thermal costs between periods. At each node (t, i) , the marginal cost equals the expected marginal cost (observed from (t, i)) in the descendant nodes.

3 MARKET EQUILIBRIUM: GENERATION-SIDE RISK NEUTRALITY AND RISK AVERSION

Electric power industry deregulation has led to the creation of a variety of wholesale electricity markets where this commodity is traded by buyers (demand side) and sellers (generation companies). The spot market, on which all generators are paid and all consumers are charged the same price, is the framework chosen for this analysis. In the stylised model used, $\pi_{t,i}$ stands for the spot market price at node (t, i) .

Generation company and consumer behaviours are modelled using a market equilibrium approach, as explained the sections below.

3.1 The demand side

The demand side is assumed to have no influence on the spot market price ($d\pi_{t,i} / dq_{t,i} = 0$). Consumers are also assumed to be risk neutral with respect to electricity prices⁹.

The assumption of demand-side risk neutrality is a key hypothesis around the forward market described in subsequent sections. Nonetheless, as shown in the annex, for the objectives addressed here, the same results would be obtained if demand were risk averse, assuming the presence of a risk-neutral arbitrageur.

For the intents and purposes of modelling, consumers' joint decisions can be likened to an optimisation problem, as follows:

$$\max_q \sum_{s \in \mathcal{S}} p_s \cdot \left(\sum_{(t,i) \in N_s} \left(U_{t,i}^D(q_{t,i}) - \pi_{t,i} \cdot q_{t,i} \right) \right) \quad (13)$$

Note that here the objective function is expressed in terms of scenarios. This approach is more suitable for the discussion that follows.

Let $\mathcal{L}^D(q)$ be the Lagrangian function for the demand side. The optimality conditions are given by:

$$\frac{d\mathcal{L}^D(\cdot)}{dq_{t,i}} = \frac{dU_{t,i}^D}{dq_{t,i}} - \pi_{t,i} = 0, \quad \forall (t, i) \in N \quad (14)$$

which implies that (in each node) the demand side consumes electricity until the marginal utility obtained is equal to its price.

3.2 The generation side

As generators are expected to behave competitively, market prices are assumed to mirror actual marginal costs (absence of market power). This entails introducing market price as a constant exogenous variable in the generator problem.

Generators' (G) risk aversion is modelled as a (concave) utility function U^G that evaluates total profit utility in a given scenario. Utility is consequently expressed in terms of total profit earned during the entire time horizon in that scenario. The use of such utility functions is equivalent to assigning more weight to very low and less to very high profit scenarios.

⁹ For example, demand preference for a \$10 payment is the same as tossing a coin to pay \$5 or \$15 (=50 % probability).

3.2.1 Use of the generator utility function to model risk version

From a generator’s perspective, the medium-term planning and management of its generation plants in a market environment is subject to many sources of uncertainty.

Where generators are risk neutral, their objective is to maximise the expected value of their bare profit (defined as the difference between market revenues and production costs). Where they are risk averse, however, profit distribution across all scenarios might lead to unacceptably low profits in some. Several techniques can be applied in risk-based planning of generation resources with a view to reducing the impact of such low profits (left-hand tail of the profit probability distribution). Fleten et al. (2002), for instance, proposed minimising negative deviations from a pre-set target profit in each scenario. García-González et al. (2007) introduced conditional value at risk (CVaR) into the optimisation model, capitalising on its tractability in linear programming. The utility function approach is adopted in the present study, for it embodies a compact and differentiable representation of generators’ preferences¹⁰, facilitating conceptual model formulation and interpretation of the results. This utility function, U^G , is defined for each generator G in terms of the profit earned in scenario s , (b_s^G). As illustrated in Figure 2, the risk-averse utility function is assumed to be strictly monotonic ($dU^G / db > 0$) and concave ($d^2U^G / db^2 < 0$).

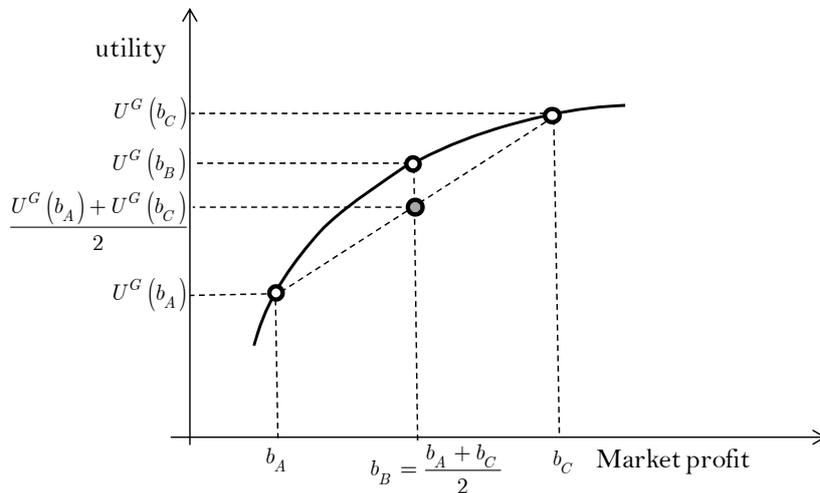


Figure 2. Generator utility function

Figure 2 shows that a concave utility function penalises low-profit scenarios. Note that this function also penalizes uncertainty, albeit indirectly. Take two scenarios, A and C , with the same probability of occurrence and characterised by profits b_A and b_C , respectively. The expected utility under such an uncertain situation would be $U^G(b_A) / 2 + U^G(b_C) / 2$. That delivers lower utility than would be obtained if the probability of earning the average profit, $b_B = b_A / 2 + b_C / 2$, were one.

If a single generation company is assumed to engage in both thermal and hydro generation, these functions can be used to compute such a company’s maximum utility as set out below, where discounting is excluded for simplicity:

¹⁰ Similar approaches can be found in the literature. See, for instance, Fan et al. (2009), who use the generator’s utility function in a conceptual analysis of investment in generation, particularly in a context of regulatory uncertainty around CO₂ policies.

$$\begin{aligned}
 & \max_{q_{t,i}^T, q_{t,i}^H, s \in \mathcal{S}} \sum p_s \cdot U^G \left(\sum_{(t,i) \in N_s} \left(\pi_{t,i} \cdot (q_{t,i}^T + q_{t,i}^H) - C_{t,i}(q_{t,i}^T) \right) \right) \\
 & \text{s.t.: } \sum_{(t,i) \in N_s} q_{t,i}^H \leq Q_s^H \quad : \mu_s, \quad \forall s \in \mathcal{S}
 \end{aligned} \tag{15}$$

Note that the argument of the utility function is the generation company's profit in scenario s . Defining that profit as in (16), the expression used hereafter is $U^G(b_s^G)$ and the chain rule is applied as necessary to differentiate when $U^G(b_s^G)$ refers to $q_{t,i}^T$ and when to $q_{t,i}^H$.

$$b_s^G = \sum_{(t,i) \in N_s} \left(\pi_{t,i} \cdot (q_{t,i}^T + q_{t,i}^H) - C_{t,i}(q_{t,i}^T) \right) \tag{16}$$

In this market environment, the Lagrange water constraint multiplier, μ_s , may be interpreted as the product of the value of water in scenario s for the generator that owns the hydro resource times its respective probability.

The Lagrangian function for modelling generation is:

$$\mathcal{L}^G(q^T, q^H) = \sum_{s \in \mathcal{S}} \left(p_s \cdot U^G(b_s^G) + \mu_s \left(Q_s^H - \sum_{(t,i) \in N_s} q_{t,i}^H \right) \right) \tag{17}$$

The first set of optimality conditions is:

$$\frac{\partial \mathcal{L}^G(\cdot)}{\partial q_{t,i}^T} = \left(\sum_{s|(t,i) \in N_s} p_s \frac{dU^G(b_s^G)}{db_s^G} \cdot \frac{\partial P^G}{\partial q_{t,i}^T} \right) = \left(\sum_{s|(t,i) \in N_s} p_s \frac{dU^G(b_s^G)}{db_s^G} \right) \cdot \left(\pi_{t,i} - \frac{dC_{t,i}}{dq_{t,i}^T} \right) = 0, \tag{18}$$

with $t \in \mathcal{T}, (t,i) \in N_t$. Given that $U^G(\cdot)$ is defined as an increasing function and assuming non-nullity

for all the probabilities, the above condition is only fulfilled where $\pi_{t,i} = \frac{dC_{t,i}}{dq_{t,i}^T} \quad \forall t \in \mathcal{T}, (t,i) \in N_t$. In

other words, generators increase their thermal unit output until their marginal costs equal the price of electricity.

The second set of optimality conditions is:

$$\begin{aligned}
 \frac{\partial \mathcal{L}^G(\cdot)}{\partial q_{t,i}^H} &= \left(\sum_{s|(t,i) \in N_s} p_s \frac{dU^G(b_s^G)}{db_s^G} \frac{db_s^G}{dq_{t,i}^H} \right) - \sum_{s|(t,i) \in N_s} \mu_s = \\
 & \pi_{t,i} \left(\sum_{s|(t,i) \in N_s} p_s \frac{dU^G(b_s^G)}{db_s^G} \right) - \sum_{s|(t,i) \in N_s} \mu_s = 0,
 \end{aligned} \tag{19}$$

with $t \in \mathcal{T}, (t,i) \in N_t$. Optimality condition (19) is assumed to hold in the event of both risk neutrality and risk aversion.

3.2.2 Risk-neutral generation companies

Where a generation company is risk neutral (i.e., $U^G(b_s^G) = b_s^G$), the equation $\pi_{t_{\mathcal{T}},i} = \mu_s / p_s$, $s | (t_{\mathcal{T}}, i) \in N_s$ holds at any of the terminal nodes of the tree, which means that the value of water for scenario s is equal to the price of electricity at the final node of that scenario. Moving backward node by node, each node fulfils the following:

$$\pi_{t_{\mathcal{T}}-1,i} \left(\sum_{s|(t_{\mathcal{T}}-1,i) \in N_s} p_s \right) - \sum_{s|(t_{\mathcal{T}}-1,i) \in N_s} \mu_s = \pi_{t_{\mathcal{T}}-1,i} \left(\sum_{s|(t_{\mathcal{T}}-1,i) \in N_s} p_s \right) - \sum_{s|(t_{\mathcal{T}}-1,i) \in N_s} p_s \pi_{t_{\mathcal{T}},i} = 0 \quad (20)$$

which yields:

$$\pi_{t_{\mathcal{T}}-1,i} = \sum_{s|(t_{\mathcal{T}}-1,i) \in N_s} \left(\frac{p_s}{\sum_{s|(t_{\mathcal{T}}-1,i) \in N_s} p_s} \right) \pi_{t_{\mathcal{T}},i} \Rightarrow \pi_{t_{\mathcal{T}}-1,i} = \sum_{(t_{\mathcal{T}},j) \in D_{(t_{\mathcal{T}}-1,i)}} p_{(t_{\mathcal{T}}-1,i),j} \cdot \pi_{t_{\mathcal{T}},i} \quad (21)$$

Expression (21) can be readily generalised to:

$$\pi_{t,i} = \sum_{(t+1,j) \in D_{(t,i)}} p_{(t,i),j} \cdot \pi_{t+1,j}, \quad t \in \mathcal{T} - \{t_{\mathcal{T}}\}, (t,i) \in N, \quad (22)$$

Equation (22) means that water resources are managed in a manner such that the spot price at each node equals the expected future spot price as “seen” from that node. Given the relationship between price and marginal demand-side utility, this is equivalent to the condition set out in (11). Where generators are risk neutral, then, their medium-term planning decisions are the same as the decisions adopted in the benchmark setting.

3.2.3 Risk-averse generation companies

Applying the approach described in connection with expressions (20), (21) and (22) to general expression (22), which includes generator utility, yields the following equation:

$$\pi_{t,i} = \sum_{(t+1,j) \in D_{(t,i)}} \tilde{p}_{(t,i),j} \cdot \pi_{t+1,j}, \quad (t,i) \in N | t \in \mathcal{T} - \{t_{\mathcal{T}}\}, \quad (23)$$

where transition probabilities $\tilde{p}_{(t,i),j}$ adopt the form of a risk-modified probability:

$$\tilde{p}_{(t,i),j} = \sum_{s|(t+1,j) \in N_s} \left(\frac{p_s \cdot \frac{dU_s^G(b_s^G)}{db_s^G}}{\sum_{s|(t,i) \in N_s} p_s \cdot \frac{dU_s^G(b_s^G)}{db_s^G}} \right) \quad (24)$$

An analogous interpretation can be formulated from these new probabilities: water resources are used to equate the price at each node (t,i) to the expected price in the descendant nodes $(t+1,j) \in D_{(t,i)}$, but here the risk-modified probabilities $\tilde{p}_{(t,i),j}$ are applied.

Inasmuch as the utility function is defined as concave, the lower the income in a certain scenario, the higher the value of the respective derivative dU_s^G / db_s^G and consequently the higher the value of the associated risk-modified probability. Therefore, the prices at any of the descendant nodes that translate into low-profit scenarios carry greater weight in the objective function.

Where risk-averse generation companies with hydro facilities are deprived of hedging instruments, they may use their hydro resources to hedge their risk exposure. Resource management under such conditions differs from the approach adopted in the benchmark setting.

3.3 Computing perfectly competitive market equilibrium

Consequently, market equilibrium can be computed by simultaneously solving demand-side (14) and generation-side (18) and (19).

Taken together, equations (14) and (18) are equivalent to equation (4) and can be interpreted to conform to the definition of a perfectly competitive market: at equilibrium, the marginal cost of electricity is equal to marginal demand-side utility (at each node), which determines the price paid by consumers and received by generators.

As noted earlier, where generation companies are assumed to be risk neutral, optimality equation (22) is equivalent to expression (11): i.e., market equilibrium under those circumstances and centralised risk-neutral welfare maximisation settings yield the same outcome.

In contrast, where generation companies are risk averse, the outcomes differ. As noted, hydro production would be expected to shift away from the benchmark approach to hedge against low-profit scenarios and maximise the expected value of companies' utility function. This conclusion is illustrated in the case study (see section 5).

4 MARKET EQUILIBRIUM WITH RISK-AVERSE GENERATION AND A FORWARD MARKET

This section addresses the impact of the existence of a forward market on medium-term hydro reserve management in the presence of risk-averse generation. To simplify the problem, the generator's only counterparty is assumed to be risk-neutral electricity demand. This setting is broadened in the annex to accommodate a more realistic market structure in which a risk-neutral arbitrageur balances its spot and forward market positions.

4.1 Forward markets

In real systems, agents may hedge their risk by trading a variety of instruments. The tool analysed here is the forward market.

A single-level equilibrium problem as defined in Cabero et al. (2010), in which agents make simultaneous (open-loop) forward- and spot-market decisions, is adopted for greater simplicity.¹¹

On the forward market defined here, agents may sign energy sale contracts in a given period for delivery in the following period. Where periods are quarterly, for instance (as is often the case in real market trading), generators are assumed to be able to sign a contract in period t to sell quantity q^F at price π^F in period $t + 1$. Sellers are typically generation-side and buyers demand-side agents. However, since these roles may be reversed when agents need to correct their market positions, the model should impose no constraints on trading negative quantities in the forward market.

In forward markets such as described above, contracts may be signed at each node of the stochastic tree depicted in section 2.2 with maturity at any of its descendant nodes. Contracts are signed between a generator (party) and a demand-side agent (counterparty). On the demand side, all consumers are assumed to be aggregated into a single perfectly competitive agent. Where the generation side comprises several companies, all are assumed to be free to sign independent contracts with the aggregate demand. Consequently, the number of possible contracts at each node is the same as the number of generators.

¹¹ The open-loop approach forfeits gaming strategy details stemming from the fact that participation is sequential in the forward (first) and spot (second) markets. These strategies are of interest when analysing market power: see for instance Allaz & Villa (1993). No strategic behaviour is envisaged in this paper, however, in which both supply and demand are assumed to behave competitively.

Nonetheless, as in section 3, to simplify the discussion below, the generation side is assumed to consist in a single perfectly competitive generator.

4.2 Joint forward-spot equilibrium

Computation of the optimality conditions defining joint forward-spot equilibrium is a pre-requisite to reproducing market participant behaviour where both spot and forward markets are in place. That, in turn, entails defining the following variables:

- $q_{(t,i)}^F$: quantity specified in the forward contract signed at node (t, i) for delivery by the generator to the demand agent at descendant nodes $(t + 1, j) \in D_{(t,i)}$.
- $\pi_{(t,i)}^F$: electricity price specified in the forward contract signed at node (t, i) for performance at descendant nodes $(t + 1, j) \in D_{(t,i)}$.

4.2.1 The demand side

Demand-side behaviour can be modelled in terms of an optimisation problem, formulated as follows:

$$\begin{aligned} \max_{q_{t,i}} \sum_{t \in \mathcal{T}} \left[\sum_{(t,i) \in N_t} p_{t,i} \left(U_{t,i}^D(q_{t,i}) - \pi_{t,i} q_{t,i} \right) \right] + \\ + \sum_{t \in \mathcal{T} - \{t_T\}} \left[\sum_{(t,i) \in N_t} \left[p_{t,i} (-\pi_{t,i}^F q_{t,i}^F) + \sum_{(t+1,j) \in D_{(t,i)}} p_{(t+1,j)} \pi_{(t+1,j)} q_{t,i}^F \right] \right] \end{aligned} \quad (25)$$

The first member is the same as in (13), while the second, which does not include terminal nodes, means that conclusion of a contract by the demand agent at time t entails a payment of $\pi_t^F q_t^F$ to reduce exposure to spot price π_{t+1} to the net quantity $(q_{t+1} - q_t^F)$.

Let $\mathcal{L}^D(q_{t,i}, q^F)$ be the Lagrangian function for the demand side. The first optimality condition

$\frac{d\mathcal{L}^D(\cdot)}{dq_{t,i}} = 0$ is analogous to equation (14). The second optimality condition is:

$$\frac{d\mathcal{L}^D(\cdot)}{dq_{(t,i)}^F} = p_{t,i} (-\pi_{t,i}^F) + \sum_{(t+1,j) \in D_{(t,i)}} p_{(t+1,j)} \pi_{(t+1,j)} = 0 \quad (26)$$

which yields:

$$\pi_{(t,i)}^F = \frac{\sum_{(t+1,j) \in D_{(t,i)}} p_{(t+1,j)} \pi_{(t+1,j)}}{p_{t,i}}, \quad \forall (t, i) \in N_s \mid t \in \mathcal{T} - \{t_T\} \quad (27)$$

This equation can be re-written in terms of the probability transition as follows:

$$\pi_{t,i}^F = \sum_{(t+1,j) \in D_{(t,i)}} p_{(t,i),j} \cdot \pi_{t+1,j}, \quad (t, i) \in N \mid t \in \mathcal{T} - \{t_T\} \quad (28)$$

According to equation (28), the demand-side agent trades electricity on the forward market until the forward price specified in a contract signed in one node equals the expected spot price in its descendant nodes. Note that this condition always holds, regardless of the generator's risk profile.

4.3 The generation side

The optimisation problem that models generation, formulated in equation (15), is modified: the former definition of profit in each scenario s (16) is replaced by (29) and forward contracts are introduced as new variables $q_{(t,i)}^F$ and $\pi_{(t,i)}^F$.

$$b_s^G = \sum_{(t,i) \in N_s} \left(\pi_{t,i} (q_{t,i}^T + q_{t,i}^H) - C_{t,i}(q_{t,i}^T) \right) + \sum_{(t,i) \in N_s | t \in \mathcal{T} - \{t_{\mathcal{T}}\}} \left((\pi_{t,i}^F - \pi_{(t+1,j(i,s))}) \cdot q_{t,i}^F \right), \forall s \in S \quad (29)$$

Note that for given any node (t, i) in scenario s , there is only one descendant node in scenario s , symbolised as $(t+1, j(i, s))$. The first member in (29) is the same as (16), while the second includes all the nodes in the scenario except the terminal node.

A further optimality condition arises when the Lagrangian function defined in (17) is derived for new variable $q_{t,i}^F$ and generator profit as re-defined in (29) is added:

$$\frac{\partial \mathcal{L}^G(\cdot)}{\partial q_{t,i}^F} = \sum_{s | (t,i) \in N_s} p_s \frac{dU^G(b_s^G)}{db_s^G} \left(\pi_{t,i}^F - \pi_{(t+1,j(i,s))} \right) = 0, \forall (t, i) \in N \quad (30)$$

Equation (30) establishes a significant link between the forward price at node (t, i) and the spot prices at its descendant nodes:

$$\pi_{t,i}^F = \frac{\sum_{s | (t,i) \in N_s} p_s \frac{dU^G(b_s^G)}{db_s^G} \left(\pi_{(t+1,j(i,s))} \right)}{\sum_{s | (t,i) \in N_s} p_s \frac{dU^G(b_s^G)}{db_s^G}}, \quad (t, i) \in N | t \in \mathcal{T} - \{t_{\mathcal{T}}\} \quad (31)$$

Much as in (23), expression (31) can be re-written with the risk-modified probabilities defined in (24):

$$\pi_{t,i}^F = \sum_{(t+1,j) \in D_{(t,i)}} \tilde{p}_{(t,i),j} \cdot \pi_{t+1,j}, \quad (t, i) \in N | t \in \mathcal{T} - \{t_{\mathcal{T}}\} \quad (32)$$

The immediate consequence of establishing (23) and (32) as requisite conditions is that:

$$\pi_{t,i}^F = \pi_{t,i}, \quad (t, i) \in N | t \in \mathcal{T} - \{t_{\mathcal{T}}\} \quad (33)$$

As a result, the price of the forward contract at equilibrium must equal the spot price at the node where the contract is signed. Since the participation of the demand side as a counterparty in the forward market necessitates the inclusion of equation (27), substituting (33) into that equation yields:

$$\pi_{t,i} = \frac{\sum_{(t+1,j) \in D_{(t,i)}} p_{(t+1,j)} \pi_{(t+1,j)}}{p_{t,i}}, \quad \forall (t, i) \in N | t \in \mathcal{T} - \{t_{\mathcal{T}}\} \quad (34)$$

Therefore, even where generators are risk averse, where a forward market exists, the spot price equals the expected values of the spot prices at the descendant nodes based on the original, rather than the risk-modified, probabilities. This finding shows that under the present hypotheses, the existence of a functional long-term market restores the optimal resource management solution.

Nonetheless, the impact of the details of generators' technical constraints is not addressed in this analysis of optimality. The optimisation models are intentionally formulated without such realistic constraints to obtain meaningful and reasonably simple optimality conditions. If the solution were limited by one or several such constraints, the optimality conditions would adopt different forms. The mixed

complementarity problem (MCP) formulations given in the annex may constitute a baseline case into which the effect of additional constraints could be built to find numerical solutions with specific MCP-solving software.

5 NUMERICAL EXAMPLE

The numerical example given below illustrates the ideas introduced here. The analysis is based on the comparison of the results obtained for the next four settings implemented in GAMS:

- centralized planning (labelled **Cen**).
- market with risk-neutral agents (labelled **MrNe**).
- market with risk-averse agents and a spot market only (labelled **MrAv**).
- market with risk-averse agents and forward markets (labelled **MrAvF**).

The centralised planning approach, which can be likened to a multistage stochastic optimisation problem, was solved using the CONOPT3 platform, inasmuch as the resulting model was non-linear.

Market equilibrium was found for a two-agent hypothesis, as follows:

1. aggregation of all electricity consumption in a risk-neutral, perfectly competitive demand agent (*Esco1*)
2. generation pooled in a single, perfectly competitive company (*Genco1*) with a hydrothermal portfolio.

The simultaneous maximisation of all agents' utility functions, together with the spot and forward market clearing conditions, yielded a set of equations with a mixed complementary problem (MCP) structure. For a full formulation of such problems, solved with the PATH platform, see sections 7.2-7.4 in the annex.

5.1 Input data

The time horizon defined was 1 year, divided into 4 quarters, in which the sources of uncertainty were demand-side utility and the amount of hydro energy available for each scenario. Demand-side utility at each node (t, i) was assumed to constitute a quadratic function:

$$U^D(q_{t,i}) = \alpha_{t,i} \cdot q_{t,i} - 0.5q_{t,i}^2 \quad (35)$$

where linear coefficients $\alpha_{t,i}$ were unknown and node-dependent. Marginal demand-side utility was therefore defined as:

$$\frac{dU^D(q_{t,i})}{dq_{t,i}} = \alpha_{t,i} - q_{t,i} \quad (36)$$

As the unit chosen for power was GW, demand-side utility was expressed in k\$/h and marginal demand-side utility in \$/MWh.

The stochastic tree used in this example had 24 equally probable scenarios, structured as shown in Figure 5, which gives the values for $\alpha_{t,i}$. The probability for each scenario was assumed to be constant and equal to 1/24. The nodes were labelled with subscripts comprising two numerals: the first specifying the time period or stage, followed by a decimal point, and the second the order of the node in the stage (the number of nodes per stage was: 1 in the first, 4 in the second, 12 in the third and 24 in the fourth).

The numerical parameters for thermal and hydro generators were chosen as proposed by Bushnell (2003)¹². In this case study, Southern California Edison (SCE) and Bonneville Power Administration (BPA) respectively represented thermal and hydro generation.

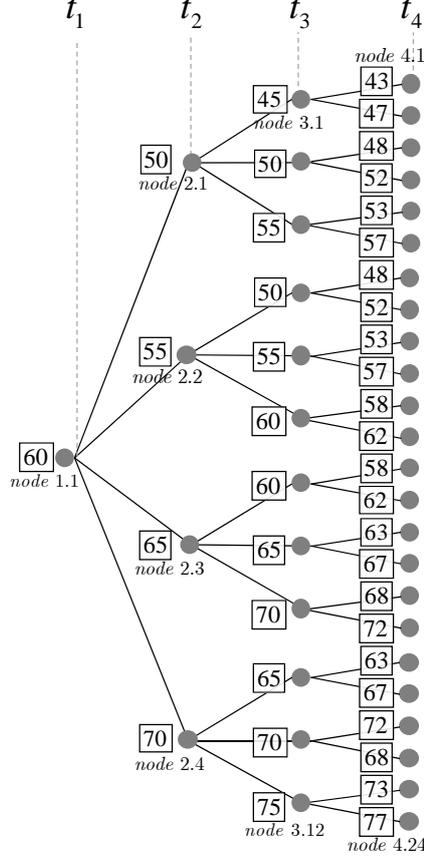


Figure 3. Stochastic tree data: linear term of the demand function at each node (coefficient $\alpha_{t,i}$)

The marginal cost function given in Bushnell (2003, Figure 1) was used as input data to estimate SCE's marginal thermal cost, applying a quadratic function to approximate the piecewise linear function. Given that thermal power, q^T , was expressed in GW, the approximate marginal cost function found was:

$$\frac{dC(q^T)}{dq^T} = 0.2881 \cdot (q^T)^2 - 0.0448 \cdot q^T + 20.3737, \quad [$/MWh] \quad (37)$$

which yielded the following cost function:

$$C(q^T) = 0.09602 \cdot (q^T)^3 - 0.02242 \cdot (q^T)^2 + 20.3737 \cdot q^T, \quad [k$/h]. \quad (38)$$

Note that although this cost function might adopt the form of stochastic data or made time-dependent if necessary, it was regarded as constant for reader reproduction of the results; that assumption had no effect on its general applicability. Moreover, since this cost function was defined as being convex only for positive generation values, in the GAMS implementation q^T had necessarily to be greater than or equal to 0.

¹² That pioneering study, which introduced the modelling framework for analysing competition among several firms with both hydro and thermal generating capacity, focused in particular on the role of hydro generation in market equilibrium. Bushnell's (2003) study of the electricity market in the western United States analysed three major utilities: the Bonneville Power Administration, Pacific Gas & Electric, and Southern California Edison.

Each firm's hydro generation parameters were taken from Bushnell's Table 2 (2003). Further to those data, the annual hydro energy available to BPA was estimated at 95 188 GWh. Since the decision variables in the present study were the average hydro power at each stage (quarter), the maximum energy available for the entire year would be:

$$q_{Q1}^H \cdot dur_{Q1} + q_{Q2}^H \cdot dur_{Q2} + q_{Q3}^H \cdot dur_{Q3} + q_{Q4}^H \cdot dur_{Q4} = 95\,188 \text{ GWh} \quad (39)$$

Assuming the same duration for each quarter ($dur_{Q_i} = dur_Q = 2\,190 \text{ h}$), the limit of the energy resources available as given in equation (2) would be $Q^H \simeq 43.5 \text{ GWh/h}$. The following distribution of hydro energy for each scenario was established to model the effect of hydro flow uncertainty: $Q_{s_1}^H = 55$, $Q_{s_2}^H = 54$, $Q_{s_3}^H = 53$, $Q_{s_4}^H = 52$, ..., $Q_{s_{22}}^H = 34$, $Q_{s_{23}}^H = 33$, and $Q_{s_{24}}^H = 32 \text{ (GWh/h)}$, for an average of 43.5 GW/h.

Any of a number of utility functions could be used to model market participants' risk aversion (Kalleberg & Ziemba, 1983). The exponential function was chosen here for the convenience afforded by some of its properties, such as a constant Arrow-Pratt absolute risk aversion coefficient (R_A). Nonetheless, the quadratic and logarithmic utility functions were likewise deployed with the default parameters listed in Table i to show that other functions can also be used.

Table i. Utility functions

Exponential	$U(w) = 1 - e^{-\beta \cdot w}$	$R_A = \beta$	$\beta = 4$
Quadratic	$U(w) = w - \beta \cdot w^2$	$R_A = \frac{2\beta}{1 - 2\beta w}$	$\beta = 0.4$
Logarithmic	$U(w) = \log(\beta + w)$	$R_A = \frac{1}{\beta + w}$	$\beta = 0.5$

Note that in the problem posed the argument in the utility function would be the agent's total profit obtained in a given scenario (or total demand-side surplus assuming a risk-averse demand agent). However, with a view to facilitating interpretation of the results when comparing different types of utility functions and ensuring a similar range of variations, profit was normalised for each scenario (w_s) as an auxiliary variable by dividing profit b_s by the maximum profit obtained by the agent in setting

MrNe (b_{MrNe}^{\max}).

$$w_s = \frac{b_s}{b_{MrNe}^{\max}} \quad (40)$$

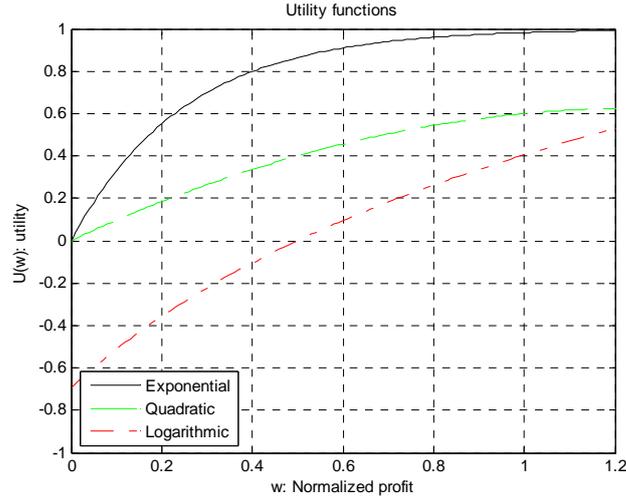


Figure 4. Utility functions.

5.2 Two agent (generation and demand) hypothesis results

The thermal and hydro generator scheduling obtained (using the exponential utility function) is given in Tables ii and iii, respectively. The tables show the power output at each time stage and setting (columns) for all 24 scenarios (rows). The average is given in the bottom row. For reader identification of the cells that contain data for the same node on the stochastic tree, the respective series of scenarios are shown as blocks with grey or white backgrounds. In stage t_1 , for instance, the power output was the same for all the scenarios, inasmuch as this is the stage associated with the root node on the stochastic tree. A similar situation is observed in stage t_2 for scenarios $\{s_1, \dots, s_6\}$, $\{s_7, \dots, s_{12}\}$, \dots , $\{s_{19}, \dots, s_{24}\}$.

Table ii. Thermal generation, q^T (GW)

	t1				t2				t3				t4			
	Cen	MrNe	MrAv	MrAvF	Cen	MrNe	MrAv	MrAvF	Cen	MrNe	MrAv	MrAvF	Cen	MrNe	MrAv	MrAvF
s1	8.499	8.499	8.045	8.499	5.927	5.927	6.007	5.927	4.371	4.371	4.674	4.371	3.582	3.582	3.956	3.582
s2	8.499	8.499	8.045	8.499	5.927	5.927	6.007	5.927	4.371	4.371	4.674	4.371	5.037	5.037	5.333	5.037
s3	8.499	8.499	8.045	8.499	5.927	5.927	6.007	5.927	5.900	5.900	6.144	5.900	5.295	5.295	5.580	5.295
s4	8.499	8.499	8.045	8.499	5.927	5.927	6.007	5.927	5.900	5.900	6.144	5.900	6.448	6.448	6.694	6.448
s5	8.499	8.499	8.045	8.499	5.927	5.927	6.007	5.927	7.170	7.170	7.380	7.170	6.660	6.660	6.900	6.660
s6	8.499	8.499	8.045	8.499	5.927	5.927	6.007	5.927	7.170	7.170	7.380	7.170	7.645	7.645	7.860	7.645
s7	8.499	8.499	8.045	8.499	7.380	7.380	7.456	7.380	6.131	6.131	6.364	6.131	5.546	5.546	5.816	5.546
s8	8.499	8.499	8.045	8.499	7.380	7.380	7.456	7.380	6.131	6.131	6.364	6.131	6.664	6.664	6.899	6.664
s9	8.499	8.499	8.045	8.499	7.380	7.380	7.456	7.380	7.368	7.368	7.571	7.368	6.871	6.871	7.100	6.871
s10	8.499	8.499	8.045	8.499	7.380	7.380	7.456	7.380	7.368	7.368	7.571	7.368	7.834	7.834	8.040	7.834
s11	8.499	8.499	8.045	8.499	7.380	7.380	7.456	7.380	8.456	8.456	8.637	8.456	8.015	8.015	8.217	8.015
s12	8.499	8.499	8.045	8.499	7.380	7.380	7.456	7.380	8.456	8.456	8.637	8.456	8.874	8.874	9.060	8.874
s13	8.499	8.499	8.045	8.499	9.445	9.445	9.514	9.445	8.458	8.458	8.635	8.458	8.018	8.018	8.215	8.018
s14	8.499	8.499	8.045	8.499	9.445	9.445	9.514	9.445	8.458	8.458	8.635	8.458	8.876	8.876	9.058	8.876
s15	8.499	8.499	8.045	8.499	9.445	9.445	9.514	9.445	9.440	9.440	9.602	9.440	9.040	9.040	9.218	9.040
s16	8.499	8.499	8.045	8.499	9.445	9.445	9.514	9.445	9.440	9.440	9.602	9.440	9.823	9.823	9.989	9.823
s17	8.499	8.499	8.045	8.499	9.445	9.445	9.514	9.445	10.341	10.341	10.492	10.341	9.973	9.973	10.136	9.973
s18	8.499	8.499	8.045	8.499	9.445	9.445	9.514	9.445	10.341	10.341	10.492	10.341	10.697	10.697	10.851	10.697
s19	8.499	8.499	8.045	8.499	10.489	10.489	10.563	10.489	9.595	9.595	9.750	9.595	9.202	9.202	9.371	9.202
s20	8.499	8.499	8.045	8.499	10.489	10.489	10.563	10.489	9.595	9.595	9.750	9.595	9.973	9.973	10.131	9.973
s21	8.499	8.499	8.045	8.499	10.489	10.489	10.563	10.489	10.485	10.485	10.624	10.485	10.697	10.697	10.850	10.697
s22	8.499	8.499	8.045	8.499	10.489	10.489	10.563	10.489	10.485	10.485	10.624	10.485	10.269	10.269	10.427	10.269
s23	8.499	8.499	8.045	8.499	10.489	10.489	10.563	10.489	11.315	11.315	11.450	11.315	10.975	10.975	11.119	10.975
s24	8.499	8.499	8.045	8.499	10.489	10.489	10.563	10.489	11.315	11.315	11.450	11.315	11.644	11.644	11.781	11.644
ave	8.499	8.499	8.045	8.499	8.310	8.310	8.385	8.310	8.253	8.253	8.444	8.253	8.236	8.236	8.442	8.236

Table iii. Hydro generation, q^H (GW)

	t1				t2				t3				t4			
	Cen	MrNe	MrAv	MrAvF												
s1	10.700	10.700	13.299	10.700	13.846	13.846	13.492	13.846	14.946	14.946	13.869	14.946	15.508	15.508	14.341	15.508
s2	10.700	10.700	13.299	10.700	13.846	13.846	13.492	13.846	14.946	14.946	13.869	14.946	14.508	14.508	13.341	14.508
s3	10.700	10.700	13.299	10.700	13.846	13.846	13.492	13.846	13.963	13.963	12.884	13.963	14.491	14.491	13.325	14.491
s4	10.700	10.700	13.299	10.700	13.846	13.846	13.492	13.846	13.963	13.963	12.884	13.963	13.491	13.491	12.325	13.491
s5	10.700	10.700	13.299	10.700	13.846	13.846	13.492	13.846	12.968	12.968	11.888	12.968	13.485	13.485	12.321	13.485
s6	10.700	10.700	13.299	10.700	13.846	13.846	13.492	13.846	12.968	12.968	11.888	12.968	12.485	12.485	11.321	12.485
s7	10.700	10.700	13.299	10.700	11.886	11.886	11.492	11.886	12.944	12.944	11.882	12.944	13.470	13.470	12.327	13.470
s8	10.700	10.700	13.299	10.700	11.886	11.886	11.492	11.886	12.944	12.944	11.882	12.944	12.470	12.470	11.327	12.470
s9	10.700	10.700	13.299	10.700	11.886	11.886	11.492	11.886	11.949	11.949	10.885	11.949	12.465	12.465	11.324	12.465
s10	10.700	10.700	13.299	10.700	11.886	11.886	11.492	11.886	11.949	11.949	10.885	11.949	11.465	11.465	10.324	11.465
s11	10.700	10.700	13.299	10.700	11.886	11.886	11.492	11.886	10.951	10.951	9.884	10.951	11.462	11.462	10.325	11.462
s12	10.700	10.700	13.299	10.700	11.886	11.886	11.492	11.886	10.951	10.951	9.884	10.951	10.462	10.462	9.325	10.462
s13	10.700	10.700	13.299	10.700	9.909	9.909	9.462	9.909	10.940	10.940	9.900	10.940	11.451	11.451	10.340	11.451
s14	10.700	10.700	13.299	10.700	9.909	9.909	9.462	9.909	10.940	10.940	9.900	10.940	10.451	10.451	9.340	10.451
s15	10.700	10.700	13.299	10.700	9.909	9.909	9.462	9.909	9.941	9.941	8.897	9.941	10.449	10.449	9.343	10.449
s16	10.700	10.700	13.299	10.700	9.909	9.909	9.462	9.909	9.941	9.941	8.897	9.941	9.449	9.449	8.343	9.449
s17	10.700	10.700	13.299	10.700	9.909	9.909	9.462	9.909	8.942	8.942	7.893	8.942	9.448	9.448	8.346	9.448
s18	10.700	10.700	13.299	10.700	9.909	9.909	9.462	9.909	8.942	8.942	7.893	8.942	8.448	8.448	7.346	8.448
s19	10.700	10.700	13.299	10.700	7.916	7.916	7.392	7.916	8.938	8.938	7.928	8.938	9.446	9.446	8.381	9.446
s20	10.700	10.700	13.299	10.700	7.916	7.916	7.392	7.916	8.938	8.938	7.928	8.938	8.446	8.446	7.381	8.446
s21	10.700	10.700	13.299	10.700	7.916	7.916	7.392	7.916	7.941	7.941	6.961	7.941	8.443	8.443	7.348	8.443
s22	10.700	10.700	13.299	10.700	7.916	7.916	7.392	7.916	7.941	7.941	6.961	7.941	7.443	7.443	6.348	7.443
s23	10.700	10.700	13.299	10.700	7.916	7.916	7.392	7.916	6.939	6.939	5.920	6.939	7.444	7.444	6.390	7.444
s24	10.700	10.700	13.299	10.700	7.916	7.916	7.392	7.916	6.939	6.939	5.920	6.939	6.444	6.444	5.390	6.444
avg	10.700	10.700	13.299	10.700	10.889	10.889	10.459	10.889	10.947	10.947	9.899	10.947	10.964	10.964	9.843	10.964

The first conclusion to be drawn from these results is that centralised operator scheduling, i.e., the setting that maximises social welfare, was the same as competitive market scheduling when the agents were risk neutral. Where generators were risk averse, however, scheduling deviated from that optimal solution. More specifically, in this example the hydro generator clearly preferred to raise its hydro output in stage t_1 to 13.290 GW, compared to the 10.700 GW found for the risk-neutral setting.

This was corroborated by the spot prices π for each scenario. Since hydro-thermal scheduling was the same for settings Cen and MrNe, their respective prices were identical but different from the values found for setting MrAv (see Table iv).

Table iv. Spot prices π (€/MWh)

	Cen, MrNe, MrAvF				MrAv			
	t1	t2	t3	t4	t1	t2	t3	t4
s1	40.8007	30.227	25.683	23.910	38.6564	30.501	26.457	24.704
s2	40.8007	30.227	25.683	27.456	38.6564	30.501	26.457	28.327
s3	40.8007	30.227	30.137	28.214	38.6564	30.501	30.972	29.094
s4	40.8007	30.227	30.137	32.061	38.6564	30.501	30.972	32.981
s5	40.8007	30.227	34.862	32.854	38.6564	30.501	35.732	33.779
s6	40.8007	30.227	34.862	36.869	38.6564	30.501	35.732	37.819
s7	40.8007	35.734	30.926	28.985	38.6564	36.053	31.754	29.857
s8	40.8007	35.734	30.926	32.867	38.6564	36.053	31.754	33.774
s9	40.8007	35.734	35.683	33.665	38.6564	36.053	36.544	34.576
s10	40.8007	35.734	35.683	37.701	38.6564	36.053	36.544	38.635
s11	40.8007	35.734	40.593	38.522	38.6564	36.053	41.478	39.458
s12	40.8007	35.734	40.593	42.663	38.6564	36.053	41.478	43.615
s13	40.8007	45.646	40.602	38.532	38.6564	46.024	41.466	39.445
s14	40.8007	45.646	40.602	42.673	38.6564	46.024	41.466	43.602
s15	40.8007	45.646	45.619	43.511	38.6564	46.024	46.501	44.439
s16	40.8007	45.646	45.619	47.728	38.6564	46.024	46.501	48.669
s17	40.8007	45.646	50.717	48.579	38.6564	46.024	51.615	49.517
s18	40.8007	45.646	50.717	52.855	38.6564	46.024	51.615	53.803
s19	40.8007	51.595	46.467	44.352	38.6564	52.045	47.322	45.248
s20	40.8007	51.595	46.467	48.581	38.6564	52.045	47.322	49.487
s21	40.8007	51.595	51.574	52.860	38.6564	52.045	52.414	53.802
s22	40.8007	51.595	51.574	50.288	38.6564	52.045	52.414	51.225
s23	40.8007	51.595	56.746	54.580	38.6564	52.045	57.630	55.491
s24	40.8007	51.595	56.746	58.911	38.6564	52.045	57.630	59.829
avg	40.8007	40.8007	40.8007	40.8007	38.6564	41.1556	41.6571	41.7157

As explained in the discussion of equations (14) and (18), in the market equilibrium solution the spot price equalled both marginal demand utility and marginal cost.

In node 2.2 (scenarios s_7 - s_{12} in stage t_2), for instance, the spot price for the risk-neutral setting was 35.734 €/MWh, while total output was $q = 19.266$ GW ($q^T = 7.380$ and $q^H = 11.886$).

According to (37), the marginal cost was:

$$\left. \frac{dC(q^T)}{dq^T} \right|_{q^T=7.373} = 0.2881 \cdot (7.380)^2 - 0.0448 \cdot 7.380 + 20.3737 = 35.734 \quad [€/MWh] \quad (41)$$

According to (36), in turn, the marginal demand-side utility at node 2.2 was likewise 35.734 €/MWh.

$$\left. \frac{dU^D(q)}{dq} \right|_{q=18.443} = 55 - 19.266 = 35.734 \quad [€/MWh]$$

Moreover, in the risk-neutral setting, the spot price at each node was the expected spot price at its descendant nodes, which for node 2.2 would be 3.4, 3.5 and 3.6. Since probability was the same for all the scenarios in this example, the following condition would have to hold:

$$35.734 = \frac{30.926+35.683+40.593}{3}$$

In the risk-averse setting (absence of a forward market) the foregoing would not apply, however, for the expected values would have to be computed using the risk-modified probabilities shown in equation (24). The spot prices for the **Cen**, **MRNe** and **MrAvF** settings are shown in Figure 5. The spot prices for setting **MrAv** are depicted in Figure 6. The average price is shown as a dashed line in both figures.

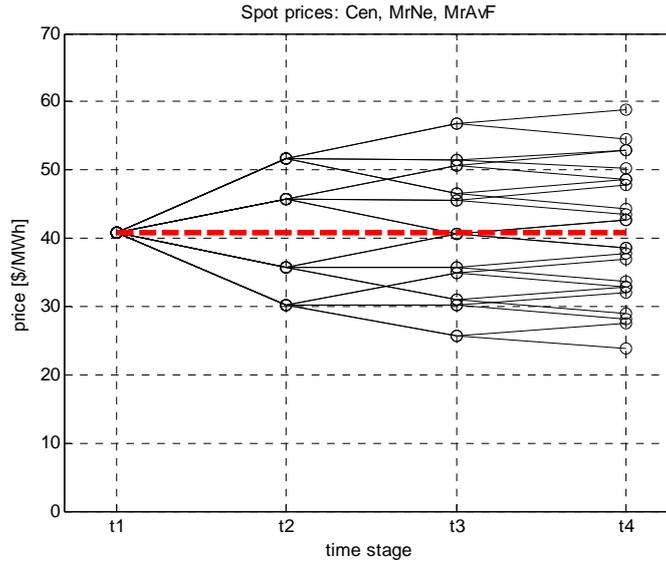


Figure 5. Spot prices (π) for settings Cen, MrNe, and MrAvF.

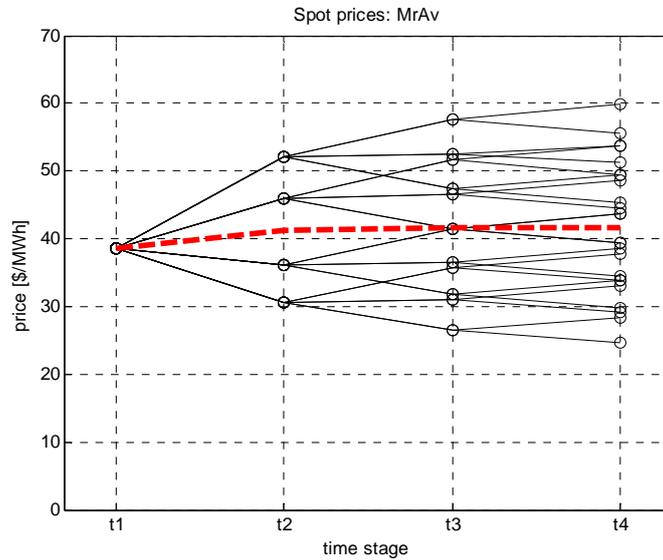


Figure 6. Spot prices (π) for setting MrAv

Where risk-averse market agents were able to hedge their risk by buying or selling forward contracts, the outcome was the same as for the central planning and risk-neutral settings. Tables ii, iii and iv clearly show that hydro-thermal scheduling and the spot prices were the same in setting **MrAvF** as in **MrNe** and **Cen**.

The information on the contracts signed (forward prices and quantities) to attain these results is summarised in Table v. Inasmuch as the generator was able to sign a forward contract at any except the terminal node on the tree, the node at issue is shown in the table. Note that in the present stylised formulation, the three types of utility functions mentioned above delivered very similar model values for the quantities contracted and listed in Table v. Further to the discussion in section 4, the maturity date of a contract signed at time t was assumed to be $t + 1$. In the quadratic utility function, for instance, the generator signed a forward contract at stage t_1 to sell 58.001 GW at a price of 40.750 \$/MWh in all the nodes in stage t_2 . In stage t_2 , the same agent sold 34.960 GW at \$32.561 \$/MWh. The latter contract would be applicable to nodes 3.1, 3.2 and 3.3 (scenarios s_1 to s_6), node 2.1 descendants. Note that the quantities demanded in forward contracts totalled in some cases up to twice the total power output in a given stage. As noted in the discussion of equation (33), the forward contract prices were the same as the

spot market price at the node where the contract was signed. That explains why forward prices depend neither on the type of utility function deployed nor the level of risk aversion.

Table v. Forward market contracts

node	t	π^F (€/MWh)	Exponential utility q^F (GW)	Quadratic utility q^F (GW)	Logarithmic utility q^F (GW)
1.1	t1	40.801	25.137	25.236	25.198
2.1	t2	30.227	26.502	26.502	26.500
2.2	t2	35.734	23.809	23.809	23.808
2.3	t2	45.646	20.684	20.684	20.683
2.4	t2	51.595	16.750	16.750	16.749
3.1	t3	25.683	12.116	12.116	12.114
3.2	t3	30.137	12.049	12.049	12.048
3.3	t3	34.862	11.467	11.467	11.466
3.4	t3	30.926	11.125	11.125	11.124
3.5	t3	35.683	10.488	10.488	10.488
3.6	t3	40.593	9.612	9.612	9.611
3.7	t3	40.602	9.600	9.600	9.600
3.8	t3	45.619	8.569	8.569	8.568
3.9	t3	50.717	7.428	7.428	7.427
3.10	t3	46.467	7.549	7.549	7.549
3.11	t3	51.574	38.486	38.486	38.482
3.12	t3	56.746	5.154	5.154	5.154

Table vi, in turn, gives the risk-modified probabilities obtained for each market setting from the standpoint of the generation company.

Table vi. Original (MrNe) and risk-modified (MrAv, MrAvF) probabilities for *Genco 1*

	MrNe	MrAv	MrAvF
s1	0.041666667	0.071697411	0.041961546
s2	0.041666667	0.067240906	0.041961546
s3	0.041666667	0.060130423	0.041892595
s4	0.041666667	0.05620146	0.041892595
s5	0.041666667	0.050062831	0.041960209
s6	0.041666667	0.046859554	0.041960209
s7	0.041666667	0.059357834	0.042893007
s8	0.041666667	0.055765218	0.042893007
s9	0.041666667	0.050037358	0.042693193
s10	0.041666667	0.047110984	0.042693193
s11	0.041666667	0.042345422	0.042890917
s12	0.041666667	0.040049435	0.042890917
s13	0.041666667	0.035756919	0.037693948
s14	0.041666667	0.033814916	0.037693948
s15	0.041666667	0.030522571	0.037418161
s16	0.041666667	0.029040097	0.037418161
s17	0.041666667	0.02637367	0.037692489
s18	0.041666667	0.025272946	0.037692489
s19	0.041666667	0.032997033	0.044458137
s20	0.041666667	0.03159767	0.044458137
s21	0.041666667	0.026262587	0.043989619
s22	0.041666667	0.030644377	0.043989619
s23	0.041666667	0.025782516	0.04445618
s24	0.041666667	0.025075859	0.04445618

Lastly, analysing the generator's and the demand agent's profit distribution by scenario for settings **MrNe**, **MrAv** and **MrAvF** (Table vii) yielded relevant findings.

Table vii. *Genco1* and *Esco1* profit by setting and scenario

	<i>Genco1</i>			<i>Esco1</i>		
	MrNe	MrAv	MrAvF	MrNe	MrAv	MrAvF
s1	1789.38	1816.81	2197.08	748.59	757.18	340.88
s2	1832.33	1857.39	2197.08	757.36	764.15	392.62
s3	1906.77	1928.06	2198.12	772.80	777.63	481.46
s4	1953.13	1970.79	2198.12	775.83	779.78	530.84
s5	2031.12	2043.93	2197.10	785.49	788.24	619.51
s6	2077.15	2085.74	2197.10	785.19	787.48	665.24
s7	1919.76	1936.24	2183.20	732.61	738.32	469.17
s8	1962.95	1975.72	2183.20	734.86	739.82	514.61
s9	2036.40	2044.25	2186.15	743.41	747.30	593.66
s10	2078.74	2082.36	2186.15	742.70	746.21	635.29
s11	2151.65	2149.80	2183.23	747.92	750.72	716.33
s12	2191.45	2185.05	2183.23	745.18	747.82	753.40
s13	2262.50	2256.74	2264.90	749.23	751.72	746.83
s14	2302.25	2292.05	2264.90	746.49	748.82	783.85
s15	2372.72	2356.82	2269.54	749.32	751.17	852.49
s16	2408.86	2388.30	2269.54	745.10	746.94	884.42
s17	2475.73	2449.20	2264.92	746.11	747.63	956.91
s18	2507.49	2476.16	2264.92	740.78	742.39	983.35
s19	2330.02	2307.53	2160.53	699.28	702.79	868.76
s20	2361.94	2334.93	2160.53	695.05	698.57	896.45
s21	2487.70	2451.87	2167.23	706.61	709.19	1027.08
s22	2388.75	2354.30	2167.23	680.28	684.30	901.79
s23	2507.02	2463.54	2160.56	689.92	693.11	1036.38
s24	2529.34	2481.11	2160.56	683.88	687.25	1052.66
avg	2202.71	2195.36	2202.71	737.67	741.19	737.67

Table viii lists the average profit for both agents (computed according to the original probabilities) and the sum of the two.

Table viii. Average *Genco1* and *Esco1* and total profit

	MrNe	MrAv	MrAvF
Genco1	2202.71	2195.36	2202.71
Esco1	737.67	741.19	737.67
Total	2940.38	2936.55	2940.38

Note that settings **MrNe** and **MrAvF** exhibited the same total, which was higher than for **MrAv**. Profit was much more widely scattered under setting **MrNe** than **MrAvF** for the generator and vice-versa for the demand agent.

6 CONCLUSIONS

With the advent of electricity markets, regulation has become necessary to ensure market efficiency and a suitable supply of electricity. This paper focuses on the medium-term efficiency associated with hydro resource management.

The findings show that when generation is risk averse and no functional forward market is in place, market equilibrium may be reached under circumstances other than would have been envisaged by a central planner (whose solution maximises total welfare). The reason is the (socially) inefficient hydro plant planning that results when generation companies manage their hydro resources to hedge against low-profit scenarios.

The study also analyses the ability of a functional long-term market to remedy such inefficiencies. Specifically, in a simplified setting characterised by: a perfectly competitive market, at least one risk-neutral consumer or arbitrageur, common market-wide beliefs about uncertain parameters and a single price per stage, the existence of a forward market can deliver the same scheduling as the centralised solution. The findings likewise show that a perfectly competitive demand side, even where risk neutral, would have incentives to sign forward contracts.

To date, however, for a number of reasons, demand has not yet played that vital role in most electricity markets (see, for example, Neuhoff and De Vries, 2004; Rodilla and Batlle, 2012). The end result,

irrespective of the reason, is incomplete and hence dysfunctional long-term markets, which detracts from the efficiency of electricity supply.

The inference is that if demand were required to engage in long-term contracting, this hydro scheduling-related market failure might be combatted. Nonetheless, on the understanding that regulation is never perfect, before introducing such a mechanism, regulators should evaluate the harm that could potentially be caused by "imperfect intervention".

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7 ANNEX

This annex presents the complete mathematical formulation of the problems **Cen**, **MrNe**, **MrAv** and **MrAvF**, the corresponding Karush-Kuhn-Tucker (KKT) conditions that can be used to implement the MCP problems, and the mathematical proofs about the equivalences among them.

See section 2.2 for the details about the problem formulation used.

7.1 Optimality conditions of Cen

The centralized problem (**Cen**) is stated as follows:

$$\begin{aligned} \max_{q^T, q^H, q} \quad & \sum_{t \in \mathcal{T}} \sum_{(t,i) \in N_t} p_{t,i} \cdot \left(U_{t,i}^D(q_{t,i}) - C_{t,i}(q_{t,i}^T) \right) \\ \text{s.t.} \quad & \\ & q_{t,i}^T + q_{t,i}^H = q_{t,i} \quad : \lambda_{t,i}, \quad \forall (t,i) \in N \\ & \sum_{(t,i) \in N_s} q_{t,i}^H \leq Q_s^H \quad : \mu_s, \quad \forall s \in \mathcal{S} \end{aligned} \quad (42)$$

The corresponding Lagrangian function can be formulated as:

$$\begin{aligned} \mathcal{L}(q^T, q^H, q, \lambda, \mu) = \quad & \sum_{t \in \mathcal{T}} \sum_{(t,i) \in N_t} p_{t,i} \cdot \left(U_{t,i}^D(q_{t,i}) - C_{t,i}(q_{t,i}^T) \right) \\ & + \sum_{(t,i) \in N} \lambda_{t,i} \cdot \left(q_{t,i} - (q_{t,i}^T + q_{t,i}^H) \right) + \sum_{s \in \mathcal{S}} \mu_s \cdot \left(Q_s^H - \sum_{(t,i) \in N_s} q_{t,i}^H \right) \end{aligned} \quad (43)$$

The KKT conditions are the next ones:

$$p_{t,i} \cdot \frac{dC_{t,i}(q_{t,i}^T)}{dq_{t,i}^T} = -\lambda_{t,i}, \quad \forall (t,i) \in N \quad (44)$$

$$\sum_{s/(t,i) \in N_s} \mu_s = -\lambda_{t,i}, \quad \forall (t,i) \in N \quad (45)$$

$$p_{t,i} \cdot \frac{dU_{t,i}^D(q_{t,i})}{dq_{t,i}} = -\lambda_{t,i}, \quad \forall (t,i) \in N \quad (46)$$

$$q_{t,i}^T + q_{t,i}^H = q_{t,i}, \quad \forall (t,i) \in N \quad (47)$$

$$0 \leq \mu_s \perp \left(\sum_{(t,i) \in N_s} q_{t,i}^H - Q_s^H \right) \leq 0, \quad \forall s \in \mathcal{S} \quad (48)$$

Equations (44), (5), and (46) are obtained after applying $\partial \mathcal{L}(\cdot) / \partial q_{t,i}^T = 0$, $\partial \mathcal{L}(\cdot) / \partial q_{t,i}^H = 0$, and $\partial \mathcal{L}(\cdot) / \partial q_{t,i} = 0$ respectively. Equations (47) and (48) define the primal feasibility, the dual feasibility, and the complementary slackness conditions.

7.2 Equilibrium conditions of MrNe

The problem **MrNe** is formulated as the simultaneous maximization of the expected profit of the market agents, and the market clearing conditions (Gabriel et al., 2010). Assuming two agents (a demand and a generator), the **MrNe** can be stated as:

$$\begin{aligned} \text{Demand :} & \quad \left\{ \max_q \sum_{s \in \mathcal{S}} p_s \cdot \left(\sum_{(t,i) \in N_s} \left(U_{t,i}^D(q_{t,i}) - \pi_{t,i} \cdot q_{t,i} \right) \right) \right\} \\ \text{Generation :} & \quad \left\{ \max_{q^T, q^H} \sum_{s \in \mathcal{S}} p_s \cdot \left(\sum_{(t,i) \in N_s} \left(\pi_{t,i} \cdot (q_{t,i}^T + q_{t,i}^H) - C_{t,i}(q_{t,i}^T) \right) \right) \right\} \\ & \quad \left\{ \text{s.t.: } \sum_{(t,i) \in N_s} q_{t,i}^H \leq Q_s^H \quad : \mu_s, \quad \forall s \in \mathcal{S} \right\} \\ \text{Clearing} & \quad \left\{ q_{t,i}^T + q_{t,i}^H = q_{t,i} : \pi_{t,i}, \quad \forall (t,i) \in N \right\} \\ \text{(spot market) :} & \end{aligned} \quad (49)$$

Turning the maximization problems into their corresponding KKT conditions, it is possible to represent the **MrNe** equilibrium by a set of equations that have a Mixed Complementarity Problem (MCP) structure. This requires formulating the corresponding Lagrangian functions for both the demand (D), and the generation (G):

$$\mathcal{L}^D(q, \pi) = \sum_{s \in \mathcal{S}} p_s \cdot \left(\sum_{(t,i) \in N_s} \left(U_{t,i}^D(q_{t,i}) - \pi_{t,i} \cdot q_{t,i} \right) \right) \quad (50)$$

$$\mathcal{L}^G(q^T, q^H, \pi, \mu) = \sum_{s \in \mathcal{S}} p_s \cdot \left(\sum_{(t,i) \in N_s} \left(\pi_{t,i} \cdot (q_{t,i}^T + q_{t,i}^H) - C_{t,i}(q_{t,i}^T) \right) \right) + \sum_{s \in \mathcal{S}} \mu_s \cdot \left(Q_s^H - \sum_{(t,i) \in N_s} q_{t,i}^H \right) \quad (51)$$

Introducing the conditions $\partial \mathcal{L}^D(\cdot) / \partial q_{t,i} = 0$, $\partial \mathcal{L}^G(\cdot) / \partial q_{t,i}^T = 0$, $\partial \mathcal{L}^G(\cdot) / \partial q_{t,i}^H = 0$, the primal/dual feasibility, the complementary slackness conditions, and the market clearing equation, next conditions can be obtained:

$$\pi_{t,i} = \frac{dU_{t,i}^D(q_{t,i})}{dq_{t,i}}, \quad \forall (t,i) \in N \quad (52)$$

$$\pi_{t,i} = \frac{dC_{t,i}(q_{t,i}^T)}{dq_{t,i}^T}, \quad \forall (t,i) \in N \quad (53)$$

$$\sum_{s|(t,i) \in N_s} p_s \cdot \pi_{t,i} + \sum_{s|(t,i) \in N_s} -\mu_s = 0, \quad \forall (t,i) \in N \quad (54)$$

$$0 \leq \mu_s \perp \left(\sum_{(t,i) \in N_s} q_{t,i}^H - Q_s^H \right) \leq 0, \quad \forall s \in \mathcal{S} \quad (55)$$

$$q_{t,i}^T + q_{t,i}^H = q_{t,i}, \quad \forall (t,i) \in N \quad (56)$$

7.2.1 The effect of $\partial \mathcal{L}^G(\cdot) / \partial q_{t,i}^H = 0$ in MrNe

Due to its crucial role in this work, let analyse the effect of applying $\partial \mathcal{L}^G(\cdot) / \partial q_{t,i}^H = 0$ to the **MrNe** problem.

Proposition 1:

The effect of the condition $\partial \mathcal{L}^G(\cdot) / \partial q_{t,i}^H = 0$ in the **MrNe** problem is that the price of a given node (t, i) can be computed as the expected price of its descendant nodes according to the original probabilities¹³:

$$\pi_{t,i} = \mathbb{E}_{\mathbb{P}}(\pi_{(t+1,j) \in D(t,i)}) \quad \forall (t,i) \in N \mid t \in \mathcal{T} - \{t_{\mathcal{T}}\} \quad (57)$$

Proof: For a given node (t, i) of the stochastic tree, equation (54) can be written as:

$$\pi_{t,i} \cdot \sum_{s|(t,i) \in N_s} p_s = \sum_{s|(t,i) \in N_s} \mu_s \quad (58)$$

As the probability of the node (t, i) can be computed as the sum of the probabilities of all scenarios that include it in their pathways, $p_{t,i} = \sum_{s|(t,i) \in N_s} p_s$, previous expression yields:

$$\pi_{t,i} \cdot p_{t,i} = \sum_{s|(t,i) \in N_s} \mu_s \quad (59)$$

Assuming that (t, i) is not a terminal node, for each one of its descendant nodes $(t+1, j) \in D_{(t,i)}$, it is satisfied that:

$$\pi_{(t+1,j)} \cdot p_{(t+1,j)} = \sum_{s|(t+1,j) \in N_s} \mu_s \quad (60)$$

The set of scenarios that pass through (t, i) can be bundled in as many terms as the number of descendant nodes. Thus, (59) can be expressed as:

$$\pi_{t,i} \cdot p_{t,i} = \sum_{j|(t+1,j) \in D(t,i)} \left(\sum_{s|(t+1,j) \in N_s} \mu_s \right) \quad (61)$$

Taking into account (60), equation (59) we can be written as:

¹³. This paper addresses a scenario-based representation of uncertainty in which the set of random variables is defined in probability space $(\mathcal{S}, F, \mathbb{P})$, where \mathcal{S} is the set of possible scenarios, F the set of all events and \mathbb{P} the measure of probability (a function that delivers the probability of each event: $\mathbb{P}: F \rightarrow [0, 1]$). The set of scenarios is assumed to be finite $|\mathcal{S}| = N_s$ and all market participants to share the same beliefs.

$$\pi_{t,i} \cdot p_{t,i} = \sum_{j|(t+1,j) \in D(t,i)} \left(\pi_{(t+1,j)} \cdot p_{(t+1,j)} \right) \quad (62)$$

Therefore:

$$\pi_{t,i} = \frac{\sum_{j|(t+1,j) \in D(t,i)} \left(\pi_{(t+1,j)} \cdot p_{(t+1,j)} \right)}{p_{t,i}} \quad (63)$$

As the transition probability $p_{(t,i),j}$ between the node (t, i) and its descendant $(t+1, j) \in D_{(t,i)}$ can be computed as $p_{(t,i),j} = p_{(t+1,j)} / p_{t,i}$, then (63) can be expressed as:

$$\pi_{t,i} = \sum_{j|(t+1,j) \in D(t,i)} \left(p_{(t,i),j} \cdot \pi_{(t+1,j)} \right) \quad (64)$$

which can be written in a compact form as:

$$\pi_{t,i} = \mathbf{E}_{\mathbb{P}}(\pi_{(t+1,j) \in D(t,i)}) \quad (65)$$

This proposition is also satisfied in the opposite direction, and as it will be shown later, this fact will play an important role in the demonstration of the equivalence between **MrNe** and **MrAvF**.

Proposition 2:

If the spot price in every node can be computed as the expected value of the prices at its descendant nodes according to the original probabilities, i.e. $\pi_{t,i} = \mathbf{E}_{\mathbb{P}}(\pi_{(t+1,j) \in D(t,i)}) \quad \forall (t, i) \in N \mid t \in \mathcal{T} - \{t_{\mathcal{T}}\}$, then there exists a set of $\mu_s, \forall s \in \mathcal{S}$ such that $\pi_{t,i} \cdot p_{t,i} = \sum_{s|(t,i) \in N_s} \mu_s, \quad \forall (t, i) \in N$.

Proof: The first step is to define the values $\mu_s = \pi_{t_{\mathcal{T}}, i_s} \cdot p_{t_{\mathcal{T}}, i_s}, \forall s \in \mathcal{S}$ at every terminal node. Then, for every father of the terminal nodes it is possible to define the price as the expected value of the prices of its descendants. Each one of these descendants belongs to a single scenario. This leads to $p_{t_{\mathcal{T}-1}, i} \cdot \pi_{t_{\mathcal{T}-1}, i} = \mu_{s_j} + \dots + \mu_{s_l}$. Moving backwards in a recursive way, it is obtained the general expression $\pi_{t,i} \cdot p_{t,i} = \sum_{s|(t,i) \in N_s} \mu_s, \quad \forall (t, i) \in N$

7.3 Equilibrium conditions of MrAv

Assuming a risk averse generator and a risk neutral demand, the **MrAv** can be formulated as the maximization of the expected profit of the demand, the maximization of the expected utility of the generator, and the market clearing conditions:

$$\begin{aligned}
 \text{Demand :} & \quad \left\{ \max_q \sum_{s \in \mathcal{S}} p_s \cdot \left(\sum_{(t,i) \in N_s} \left(U_{t,i}^D(q_{t,i}) - \pi_{t,i} \cdot q_{t,i} \right) \right) \right. \\
 \text{Generation :} & \quad \left\{ \begin{aligned} & \max_{q^T, q^H} \sum_{s \in \mathcal{S}} p_s \cdot U^G(b_s^G) \\ & \text{s.t.: } \sum_{(t,i) \in N_s} q_{t,i}^H \leq Q_s^H \quad : \rho_s, \quad \forall s \in \mathcal{S} \\ & \quad b_s^G = \sum_{(t,i) \in N_s} (\pi_{t,i} \cdot (q_{t,i}^T + q_{t,i}^H) - C_{t,i}(q_{t,i}^T)) \quad : \theta_s, \quad \forall s \in \mathcal{S} \end{aligned} \right. \\
 \text{Clearing} & \quad \left\{ q_{t,i}^T + q_{t,i}^H = q_{t,i} \quad : \pi_{t,i}, \quad \forall (t,i) \in N \right. \\
 \text{(spot market) :} &
 \end{aligned} \tag{66}$$

Notice that we have renamed the dual variable of the maximum available hydro generation. The Lagrangian functions for the demand and the generation are respectively:

$$\mathcal{L}^D(q, \pi) = \sum_{s \in \mathcal{S}} p_s \cdot \left(\sum_{(t,i) \in N_s} \left(U_{t,i}^D(q_{t,i}) - \pi_{t,i} \cdot q_{t,i} \right) \right) \tag{67}$$

$$\begin{aligned}
 \mathcal{L}^G(q^T, q^H, b^G, \pi, \rho, \theta) &= \sum_{s \in \mathcal{S}} p_s \cdot U^G(b_s^G) + \sum_{s \in \mathcal{S}} \rho_s \cdot \left(Q_s^H - \sum_{(t,i) \in N_s} q_{t,i}^H \right) + \\
 &+ \sum_{s \in \mathcal{S}} \theta_s \cdot \left(\sum_{(t,i) \in N_s} \left(\pi_{t,i} \cdot (q_{t,i}^T + q_{t,i}^H) - C_{t,i}(q_{t,i}^T) \right) - b_s^G \right)
 \end{aligned} \tag{68}$$

Introducing the conditions $\partial \mathcal{L}^D(\cdot) / \partial q_{t,i} = 0$, $\partial \mathcal{L}^G(\cdot) / \partial q_{t,i}^T = 0$, $\partial \mathcal{L}^G(\cdot) / \partial q_{t,i}^H = 0$, $\partial \mathcal{L}^G(\cdot) / \partial b_s^G = 0$, the primal/dual feasibility, the complementary slackness conditions, and the market clearing equation, yields:

$$\frac{dU_{t,i}^D(q_{t,i})}{dq_{t,i}} = \pi_{t,i}, \quad \forall (t,i) \in N \tag{69}$$

$$\sum_{s/(t,i) \in N_s} \theta_s \cdot \left(\pi_{t,i} - \frac{dC_{t,i}(q_{t,i}^T)}{dq_{t,i}^T} \right) = 0, \quad \forall (t,i) \in N \tag{70}$$

$$\sum_{s/(t,i) \in N_s} (-\rho_s + \theta_s \cdot \pi_{t,i}) = 0, \quad \forall (t,i) \in N \tag{71}$$

$$p_s \cdot \frac{dU^G(b_s^G)}{db_s^G} - \theta_s = 0 \tag{72}$$

$$0 \leq \rho_s \perp \left(\sum_{(t,i) \in N_s} q_{t,i}^H - Q_s^H \right) \leq 0, \quad \forall s \in \mathcal{S} \tag{73}$$

$$q_{t,i}^T + q_{t,i}^H = q_{t,i}, \quad \forall (t,i) \in N \tag{74}$$

7.3.1 Risk-modified probabilities

The risk-modified probabilities are defined as follows, where the scenarios with a higher impact on how the utility of the generator changes are given a higher weight (the index ω has been introduced as an alternative index for the scenarios to avoid ambiguities):

$$\tilde{p}_s = \frac{p_s \cdot \frac{dU^G(b_s^G)}{db_s^G}}{\sum_{\omega \in \mathcal{S}} p_\omega \cdot \frac{dU^G(b_\omega^G)}{db_\omega^G}}, \quad \forall s \in \mathcal{S} \quad (75)$$

Notice that the denominator of (75) allows ensuring that $\sum_{s \in \mathcal{S}} \tilde{p}_s = 1$. Taken into account (72), it is satisfied that:

$$\tilde{p}_s = \frac{\theta_s}{\sum_{\omega \in \mathcal{S}} \theta_\omega}, \quad \forall s \in \mathcal{S} \quad (76)$$

As \mathbb{P} denoted the probability measure for the original probabilities $p_s, \forall s \in \mathcal{S}$, $\tilde{\mathbb{P}}$ will denote the probability measure that correspond to the risk-adapted probabilities $\tilde{p}_s, \forall s \in \mathcal{S}$.

7.3.2 The effect of $\partial \mathcal{L}^G(\cdot) / \partial q_{t,i}^H = 0$ in **MrAv**

Similarly to 7.2.1, now the effect of applying $\partial \mathcal{L}^G(\cdot) / \partial q_{t,i}^H = 0$ to the **MrAv** problem is analysed.

Proposition 3:

The effect of the condition $\partial \mathcal{L}^G(\cdot) / \partial q_{t,i}^H = 0$ in the **MrAv** problem is that the price of a given node (t, i) can be computed as the expected price of its descendant nodes according to the risk-modified probabilities.

Proof: For a given node (t, i) of the stochastic tree, equation (71) can be written as:

$$\pi_{t,i} \cdot \sum_{s|(t,i) \in N_s} \theta_s = \sum_{s|(t,i) \in N_s} \rho_s \quad (77)$$

Dividing by $\sum_{s \in \mathcal{S}} \theta_s$ both sides of the equality, we obtain:

$$\pi_{t,i} \cdot \sum_{s|(t,i) \in N_s} \frac{\theta_s}{\sum_{s \in \mathcal{S}} \theta_s} = \sum_{s|(t,i) \in N_s} \frac{\rho_s}{\sum_{s \in \mathcal{S}} \theta_s} \quad (78)$$

According to (76), and denoting $\tilde{\mu}_s = \rho_s / \sum_{s \in \mathcal{S}} \theta_s$, yields:

$$\pi_{t,i} \cdot \sum_{s|(t,i) \in N_s} \tilde{p}_s = \sum_{s|(t,i) \in N_s} \tilde{\mu}_s \quad (79)$$

Notice that the expression (79) is completely analogous to (58). Therefore, by applying the same reasoning as in 7.2.1, but using now the risk-modified probabilities, the next expression can be derived:

$$\pi_{t,i} = \sum_{j|(t+1,j) \in D(i,j)} \left(\tilde{p}_{(t,i),j} \cdot \pi_{(t+1,j)} \right), \quad (80)$$

which can be written in a more compact form as follows:

$$\pi_{t,i} = \mathbb{E}_{\tilde{\mathbb{P}}}(\pi_{(t+1,j) \in D(t,i)}) \quad (81)$$

where $\tilde{\mathbb{P}}$ represents the risk-modified probability. Transition probabilities between one node and one of its descendants can be computed as:

$$\tilde{p}_{(t,i),j} = \frac{\sum_{s|(t+1,j) \in N_s} p_s \cdot \frac{dU^G(b_s^G)}{db_s^G}}{\sum_{s|(t,i) \in N_s} p_s \cdot \frac{dU^G(b_s^G)}{db_s^G}} \quad (82)$$

7.4 Equilibrium conditions of MrAvF

The existence of the forward market makes it necessary to include the settlement of such contracts in the definition of the profit for both the demand and the generator. Additionally, it is necessary to add the forward market clearing. Therefore, the **MrAvF** can be stated as follows:

$$\begin{cases} \text{Demand :} & \left\{ \begin{array}{l} \max_q \sum_{s \in \mathcal{S}} p_s \cdot \left(\sum_{(t,i) \in N_s} (U_{t,i}^D(q_{t,i}) - \pi_{t,i} \cdot q_{t,i}) + \sum_{\substack{(t,i) \in N_s \\ | t \in \mathcal{T} - \{t_{\mathcal{T}}\}}} (\pi_{(t+1,j(i,s))} - \pi_{t,i}^F) \cdot q_{t,i}^{F,D} \right) \\ \max_{q^T, q^H} \sum_{s \in \mathcal{S}} p_s \cdot U^G(b_s^G) \\ \text{s.t.:} \sum_{(t,i) \in N_s} q_{t,i}^H \leq Q_s^H \quad : \rho_s, \quad \forall s \in \mathcal{S} \end{array} \right. \\ \text{Generation :} & \left\{ \begin{array}{l} b_s^G = \sum_{(t,i) \in N_s} (\pi_{t,i} \cdot (q_{t,i}^T + q_{t,i}^H) - C_{t,i}(q_{t,i}^T)) \\ \quad + \sum_{\substack{(t,i) \in N_s \\ | t \in \mathcal{T} - \{t_{\mathcal{T}}\}}} (\pi_{t,i}^F - \pi_{(t+1,j(i,s))}) \cdot q_{t,i}^{F,G} \quad : \theta_s, \quad \forall s \in \mathcal{S} \end{array} \right. \\ \text{Clearing (spot) :} & \{q_{t,i}^T + q_{t,i}^H = q_{t,i} \quad : \pi_{t,i}, \quad \forall (t,i) \in N\} \\ \text{Clearing (forward) :} & \{q_{t,i}^{F,G} = q_{t,i}^{F,D} \quad : \pi_{t,i}^F, \quad (t,i) \in N \mid t \in \mathcal{T} - \{t_{\mathcal{T}}\}\} \end{cases} \quad (83)$$

The updated Lagrangian functions for the demand and the generation are the following ones:

$$\mathcal{L}^D(q, q^{F,D}, \pi, \pi^F) = \sum_{s \in \mathcal{S}} p_s \cdot \left(\sum_{(t,i) \in N_s} (U_{t,i}^D(q_{t,i}) - \pi_{t,i} \cdot q_{t,i}) + \sum_{\substack{(t,i) \in N_s \\ | t \in \mathcal{T} - \{t_{\mathcal{T}}\}}} (\pi_{(t+1,j(i,s))} - \pi_{t,i}^F) \cdot q_{t,i}^{F,D} \right) \quad (84)$$

$$\begin{aligned} \mathcal{L}^G(q^T, q^H, b^G, q^{F,G}, \pi, \rho, \theta) &= \sum_{s \in \mathcal{S}} p_s \cdot U^G(b_s^G) + \sum_{s \in \mathcal{S}} \rho_s \cdot \left(Q_s^H - \sum_{(t,i) \in N_s} q_{t,i}^H \right) \\ &+ \sum_{s \in \mathcal{S}} \theta_s \cdot \left(\sum_{(t,i) \in N_s} (\pi_{t,i} \cdot (q_{t,i}^T + q_{t,i}^H) - C_{t,i}(q_{t,i}^T)) + \sum_{\substack{(t,i) \in N_s \\ | t \in \mathcal{T} - \{t_{\mathcal{T}}\}}} (\pi_{t,i}^F - \pi_{(t+1,j(i,s))}) \cdot q_{t,i}^{F,G} - b_s^G \right) \end{aligned} \quad (85)$$

Introducing the conditions $\partial \mathcal{L}^D(\cdot) / \partial q_{t,i} = 0$, $\partial \mathcal{L}^D(\cdot) / \partial q_{t,i}^{F,D} = 0$, $\partial \mathcal{L}^G(\cdot) / \partial q_{t,i}^T = 0$, $\partial \mathcal{L}^G(\cdot) / \partial q_{t,i}^H = 0$, $\partial \mathcal{L}^G(\cdot) / \partial b_s^G = 0$, $\partial \mathcal{L}^G(\cdot) / \partial q_{t,i}^{F,G} = 0$, the primal/dual feasibility, the complementary slackness conditions, and the spot and forward markets clearing equations, next conditions can be obtained:

$$\frac{dU_{t,i}^D(q_{t,i})}{dq_{t,i}} = \pi_{t,i}, \quad \forall (t,i) \in N \quad (86)$$

$$\sum_{s|(t,i) \in N_s} p_s \cdot \left(\pi_{(t+1,j(i,s))} - \pi_{t,i}^F \right) = 0, \quad \forall (t,i) \in N_s \mid t \in \mathcal{T} - \{t_T\} \quad (87)$$

$$\sum_{s|(t,i) \in N_s} \theta_s \cdot \left(\pi_{t,i} - \frac{dC_{t,i}(q_{t,i}^T)}{dq_{t,i}^T} \right) = 0, \quad \forall (t,i) \in N \quad (88)$$

$$\sum_{s|(t,i) \in N_s} \left(-\rho_s + \theta_s \cdot \pi_{t,i} \right) = 0, \quad \forall (t,i) \in N \quad (89)$$

$$p_s \cdot \frac{dU^G(b_s^G)}{db_s^G} - \theta_s = 0, \quad \forall s \in \mathcal{S} \quad (90)$$

$$\sum_{s|(t,i) \in N_s} \theta_s \cdot \left(\pi_{t,i}^F - \pi_{(t+1,j(i,s))} \right) = 0, \quad \forall (t,i) \in N_s \mid t \in \mathcal{T} - \{t_T\} \quad (91)$$

$$0 \leq \mu_s \perp \left(\sum_{(t,i) \in N_s} q_{t,i}^H - Q_s^H \right) \leq 0, \quad \forall s \in \mathcal{S} \quad (92)$$

$$q_{t,i}^T + q_{t,i}^H = q_{t,i}, \quad \forall (t,i) \in N \quad (93)$$

$$q_{t,i}^{F,G} = q_{t,i}^{F,D}, \quad (t,i) \in N \mid t \in \mathcal{T} - \{t_T\}, \quad (94)$$

7.4.1 The effect of $\partial \mathcal{L}^G(\cdot) / \partial q_{t,i}^H = 0$ in **MrAvF**

Proposition 4:

The effect of the condition $\partial \mathcal{L}^G(\cdot) / \partial q_{t,i}^H = 0$ in the **MrAvF** problem is that the price of a given node (t, i) can be computed as the expected price of its descendent nodes according to the risk-modified probabilities.

Proof: As (89) is the same as (71), the same reasoning developed in 7.3.2 for the **MrAv** case could be repeated for the **MrAvF** setting.

$$\pi_{t,i} = \sum_{j|(t+1,j) \in D(i,j)} \left(\tilde{p}_{(t,i),j} \cdot \pi_{(t+1,j)} \right) \quad (95)$$

7.5 Equivalence between **Cen** and **MrNe**

Proposition 5:

The equations that define the solution of **Cen** can be arranged to be exactly the same as the equations that define the solution of **MrNe**. Then, the values of the primal and dual variables of the solution of **Cen** are also solution of the problem **MrNe** and viceversa so we can write $\mathbf{Cen} \equiv \mathbf{MrNe}$.

Proof: Let introduce the next relationship:

$$\lambda_{t,i} = -p_{t,i} \cdot \pi_{t,i}, \quad \forall (t,i) \in N \quad (96)$$

Taking into account that the probability of a given node (t,i) can be computed as the sum of the probabilities of all scenarios that include it in their pathways: $p_{t,i} = \sum_{s|(t,i) \in N_s} p_s$, $\forall (t,i) \in N$, it can be

seen that the set of equations that define the optimality condition of **Cen**, (44)–(48), is exactly the same as the one of the equilibrium conditions of **MrNe**, (52)–(56). Therefore, (44) is equivalent to (53), (5) is equivalent to (54), (46) is equivalent to (52), (47) is the same as (56), and (48) is the same as (55).

Therefore, it can be stated that:

$$\mathbf{Cen} \equiv \mathbf{MrNe} \quad (97)$$

7.6 Existence and uniqueness of the solution of MrNe

Proposition 6:

Assuming that the problem **Cen** satisfies the required conditions about convexity and smoothness in order to have a unique solution. Then, the solution of **MrNe** exists and it is unique.

Proof: By Proposition 5:, denoting by $\{q^*, q^{T*}, q^{H*}, \lambda^*, \mu^*\}_{\mathbf{Cen}}$ the primal and dual solution of **Cen**, we can write:

$$\{q^*, q^{T*}, q^{H*}, \pi^*, \mu^*\}_{\mathbf{MrNe}} = \left\{ q^*, q^{T*}, q^{H*}, \left(-\frac{\lambda_{t_1, i_1}^*}{p_{t_1, i_1}}, \dots, -\frac{\lambda_{t_T, i_S}^*}{p_{t_T, i_S}} \right)^t, \mu^* \right\}_{\mathbf{Cen}} \quad (98)$$

As the solution of **Cen** exists and it is unique, then the solution **MrNe** will also exist and will also be unique.

7.7 Equivalence between MrNe and MrAvF

Proposition 7:

Assuming that the problem **Cen** satisfies the required conditions about convexity and smoothness in order to have a unique solution, and also assuming as a strong hypothesis that the problem **MrAvF** can be solved. Then, the value of the subset of the primal and dual variables that are common in **MrAvF** and in **MrNe** will be the same, so it can be stated that $\mathbf{MrAvF} \equiv \mathbf{MrNe} \equiv \mathbf{Cen}$.

Proof: The set of primal and dual variables of **MrNe** are $\{q, q^T, q^H, \pi, \mu\}_{\mathbf{MrNe}}$. The set of primal and

dual variables of **MrAvF** are $\{q, q^T, q^H, \pi, q^{F,D}, q^{F,G}, b^G, \rho, \pi^F, \theta\}_{\mathbf{MrAvF}}$. Let x be the subset of variables

which are common in both problems, (the total demand, the thermal generation, the hydro generation and the spot prices):

$$x = \{q, q^T, q^H, \pi\} \quad (99)$$

MrNe and **MrAvF** will be equivalent ($\mathbf{MrAvF} \equiv \mathbf{MrNe}$) if it is satisfied the next condition:

$$x_{\mathbf{MrNe}}^* = x_{\mathbf{MrAvF}}^* \quad (100)$$

To prove it is enough to check that the equations that define the solution of **MrNe**, (52)–(56), are embedded or can be derived from the ones that define the solution of **MrAvF**, (86)–(94):

- Equation (52) is exactly the same as (86).
- Equation (53). Assuming that the utility of the generator is strictly increasing and concave in the domain under study, and that the probability of all the considered scenarios is different to 0. Therefore it is obtained:

$$p_s \cdot \frac{dU^G(b_s^G)}{db_s^G} > 0, \quad \forall s \in \mathcal{S} \quad (101)$$

By (101) and (90), it can be stated that $\theta_s > 0$, $\forall s \in \mathcal{S}$. This implies that (53) is equivalent to (88).

- Equation (54). Isolating the forward price $\pi_{t,i}^F$ from (87) yields:

$$\pi_{t,i}^F = \frac{\sum_{s|(t,i) \in N_s} p_s \cdot \pi_{(t+1,j(i,s))}}{\sum_{s|(t,i) \in N_s} p_s}, \quad \forall (t,i) \in N_s \mid t \in \mathcal{T} - \{t_T\} \quad (102)$$

Therefore:

$$\pi_{t,i}^F = \mathbb{E}_{\mathbb{P}}(\pi_{(t+1,j) \in D(t,i)}) \quad (103)$$

On the other hand, dividing by $\sum_{s \in \mathcal{S}} \theta_s$ both sides of the equality (91), yields:

$$\sum_{s|(t,i) \in N_s} \frac{\theta_s}{\sum_{s \in \mathcal{S}} \theta_s} \cdot (\pi_{t,i}^F - \pi_{(t+1,j(i,s))}) = 0, \quad \forall (t,i) \in N_s \mid t \in \mathcal{T} - \{t_T\} \quad (104)$$

Thus, according to (76), it can be written:

$$\sum_{s|(t,i) \in N_s} \tilde{p}_s \cdot (\pi_{t,i}^F - \pi_{(t+1,j(i,s))}) = 0, \quad \forall (t,i) \in N_s \mid t \in \mathcal{T} - \{t_T\} \quad (105)$$

Forward price from (105) can be computed as follows:

$$\pi_{t,i}^F = \frac{\sum_{s|(t,i) \in N_s} \tilde{p}_s \cdot \pi_{(t+1,j(i,s))}}{\sum_{s|(t,i) \in N_s} \tilde{p}_s}, \quad \forall (t,i) \in N_s \mid t \in \mathcal{T} - \{t_T\} \quad (106)$$

Thus:

$$\pi_{t,i}^F = \mathbb{E}_{\tilde{\mathbb{P}}}(\pi_{(t+1,j) \in D(t,i)}) \quad (107)$$

Notice that satisfying simultaneously (103) and (107) does not imply that the original probabilities \mathbb{P} are the same as the risk-modified ones $\tilde{\mathbb{P}}$. Therefore it is necessary to find an alternative way to draw a condition equivalent to condition (54) of the **MrNe** case. In this sense, the effect of constraint (89), as shown in 7.4.1, is that $\pi_{t,i} = \mathbb{E}_{\tilde{\mathbb{P}}}(\pi_{(t+1,j) \in D(t,i)})$. By (107) this implies that $\pi_{t,i}^F = \pi_{t,i}$ for all non-terminal nodes. Then, by (103), it can be written:

$$\pi_{t,i} = \mathbf{E}_{\mathbb{P}}(\pi_{(t+1,j) \in D(t,i)}) \quad (108)$$

In summary,

$$\left. \begin{aligned} \pi_{t,i}^F &= \mathbf{E}_{\mathbb{P}}(\pi_{(t+1,j) \in D(t,i)}) \\ \pi_{t,i}^F &= \mathbf{E}_{\tilde{\mathbb{P}}}(\pi_{(t+1,j) \in D(t,i)}) \\ \pi_{t,i}^F &= \mathbf{E}_{\tilde{\mathbb{P}}}(\pi_{(t+1,j) \in D(t,i)}) \end{aligned} \right\} \rightarrow \pi_{t,i} = \mathbf{E}_{\mathbb{P}}(\pi_{(t+1,j) \in D(t,i)}) \quad (109)$$

Proposition 2, if the spot price can be computed as the expected value of the price at its descendant nodes according to the original probabilities, there exists a set of $\mu_s, \forall s \in \mathcal{S}$ such as $\pi_{t,i} \cdot p_{t,i} = \sum_{s/(t,i) \in N_s} \mu_s$,

$\forall (t,i) \in N$. Therefore, the joint effect of considering (89), (87) and (91), results in the same condition as (54).

- Equation (55) is exactly the same as (92).
- Equation (56) is exactly the same as (93).

As it was assumed that **MrAvF** can be solved, then conditions (86)–(94) are satisfied for a given set of values of the primal and dual variables. As satisfying (86)–(94) implies that conditions of **MrNe** (52)–(56) are also being satisfied, it can be stated that the solution of **MrAvF** is equivalent to the solution of **MrNe**. Additionally, taken into account the previous relationship between **MrNe** and **Cen** proved in Proposition 5, it can be also stated:

$$\mathbf{MrAvF} \equiv \mathbf{MrNe} \equiv \mathbf{Cen} \quad (110)$$

7.8 Consideration of a risk-neutral arbitrager with risk-averse demand

Let consider that the demand is risk averse. In order to achieve the equivalence between the **MrAvF** and **MrNe** it would be enough to consider that there is a risk-neutral arbitrager operating in the forward market as shown hereafter.

The problem setting in this case would be the following one. Notice that to avoid confusion, the function $U^D(b_s^D)$ denotes the utility function of the demand with respect to the profits obtained, whereas the demand utility with respect the consumed quantity has been denoted as $UD_{t,i}(q_{t,i})$

$$\begin{aligned}
 \text{Demand :} & \left\{ \begin{array}{l} \max_q \sum_{s \in \mathcal{S}} p_s \cdot U^D(b_s^D) \\ \text{s.t.: } b_s^D = \sum_{(t,i) \in N_s} \left(UD_{t,i}(q_{t,i}) - \pi_{t,i} \cdot q_{t,i} \right) + \\ \sum_{\substack{(t,i) \in N_s \\ | t \in \mathcal{T} - \{t_{\mathcal{T}}\}}} \left(\pi_{(t+1,j(i,s))} - \pi_{t,i}^F \right) \cdot q_{t,i}^{F,D} : \theta_s^D, \quad \forall s \in \mathcal{S} \end{array} \right. \\
 \text{Generation :} & \left\{ \begin{array}{l} \max_{q^T, q^H} \sum_{s \in \mathcal{S}} p_s \cdot U^G(b_s^G) \\ \text{s.t.: } \sum_{(t,i) \in N_s} q_{t,i}^H \leq Q_s^H : \rho_s, \quad \forall s \in \mathcal{S} \\ b_s^G = \sum_{(t,i) \in N_s} \left(\pi_{t,i} \cdot (q_{t,i}^T + q_{t,i}^H) - C_{t,i}(q_{t,i}^T) \right) \\ + \sum_{\substack{(t,i) \in N_s \\ | t \in \mathcal{T} - \{t_{\mathcal{T}}\}}} \left(\pi_{t,i}^F - \pi_{(t+1,j(i,s))} \right) \cdot q_{t,i}^{F,G} : \theta_s^G, \quad \forall s \in \mathcal{S} \end{array} \right. \\
 \text{Arbitrager :} & \left\{ \max_q \sum_{s \in \mathcal{S}} p_s \cdot \left(\sum_{\substack{(t,i) \in N_s \\ | t \in \mathcal{T} - \{t_{\mathcal{T}}\}}} \left(\pi_{(t+1,j(i,s))} - \pi_{t,i}^F \right) \cdot q_{t,i}^{F,A} \right) \right. \\
 \text{Clearing (spot market) :} & \{ q_{t,i}^T + q_{t,i}^H = q_{t,i} : \pi_{t,i}, \quad \forall (t,i) \in N \\
 \text{Clearing (forward market) :} & \{ q_{t,i}^{F,G} = q_{t,i}^{F,D} + q_{t,i}^{F,A} : \pi_{t,i}^F, \quad (t,i) \in N \mid t \in \mathcal{T} - \{t_{\mathcal{T}}\} \end{aligned} \tag{111}$$

The Lagrangian function for the arbitrager is:

$$\mathcal{L}^A(q^{F,A}, \pi, \pi^F) = \sum_{s \in \mathcal{S}} p_s \cdot \left(\sum_{\substack{(t,i) \in N_s \\ | t \in \mathcal{T} - \{t_{\mathcal{T}}\}}} \left(\pi_{(t+1,j(i,s))} - \pi_{t,i}^F \right) \cdot q_{t,i}^{F,A} \right) \tag{112}$$

Introducing the condition $\partial \mathcal{L}^D(\cdot) / \partial q_{t,i}^{F,A} = 0$, it yields:

$$\sum_{s \mid (t,i) \in N_s} p_s \cdot \left(\pi_{(t+1,j(i,s))} - \pi_{t,i}^F \right) = 0, \quad \forall (t,i) \in N_s \mid t \in \mathcal{T} - \{t_{\mathcal{T}}\} \tag{113}$$

Equation (113) is the same condition as the one derived in the 2-agents case with a risk-neutral demand (87): the forward price in one node is equal to the expected spot price at its descendants, $\pi_{t,i}^F = \mathbb{E}_{\mathbb{P}}(\pi_{(t+1,j) \in D(t,i)})$. The new condition (113), together with (89), and (91), results in the same condition as (54), establishing the equivalence between **MrNe** and **MrAvF** in case the solution of **MrAvF** exists.

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