

ESCUELA TÉCNICA SUPERIOR DE INGENIERÍA (ICAI)

## Proyecto Fin de Carrera

# Modeling time-dependent demand elasticity in a Probabilistic Production Costing model Application to the Spanish electricity market

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## 1. INTRODUCTION, MOTIVATION & OBJECTIVES

The energy industry, and in particular the electricity sector, has been subject to major reforms over the recent decades. Until these processes started, the activity that we now call generation supply was part of the chain of activities of vertically integrated utilities and was thus performed as either a public service or as a regulated monopoly.

Although these reforms have taken very different forms, all of them have shared a common approach that consists of taking steps towards introducing competition at any of the activities in which it was considered feasible. These reforms have been traditionally designated as "liberalization" or "deregulation" processes.

From the regulatory perspective, the fact is that in the case of the energy industry, the reform has entailed exactly the opposite. Rather than a "deregulatory" process, the industry has experienced an intensely "re-regulatory" one, see Borenstein & Bushnell (2000) or Ruff (2003).

In that context, the objective of regulation is, broadly speaking, to prevent from occurring inefficient outcomes that would otherwise occur (or inversely, to produce efficient outcomes that would otherwise not occur). In particular, the regulator must guarantee a minimum required level of security of supply.

## **1.1** Security of Generation Supply

Modern society depends critically on the availability of electricity. It is widely recognised that the lack of supply has dramatic social, economic and political consequences. Hence, avoiding emergency situations and ensuring that electric energy is supplied according to the desired quality standards represents a chief concern for regulators worldwide.

With the advance of electricity markets, regulation is increasingly required in order to supervise that the market is able to guarantee an adequate level of security of supply. This is particularly relevant at the generation activity, where the liberalization process has been far more intense.

## **1.1.1** The 4 Dimensions of the Security of Supply Problem

The problem of security of supply at the generation level can be split up into four major dimensions according to their time horizon (Rodilla, 2010). This classification not only facilitates a better understanding of the problem but also the design of the regulatory mechanisms if required. The four dimensions are the following:

- Security: the very short-term dimension. Defined by the North American Electric Reliability Council as the "ability of the electrical system to support unexpected disturbances such as electrical short circuits or unexpected loss of components of the system" (NERC, 1997).
- Firmness: the short to medium-term dimension. Defined in Batlle & Pérez-Arriaga (2008) as the ability of the already installed facilities to supply electricity efficiently.

This dimension is determined by the characteristics of the existing generation portfolio and the medium-term resource-management carried out by generators (fuel provision, water reservoir management, maintenance scheduling, etc.).

- Adequacy: the long-term dimension. Defined as the existence of enough available generation capacity (installed or expected to be installed) to efficiently meet demand in the long term.
- Strategic expansion policy, which concerns the very long-term availability of energy resources and infrastructures. This dimension usually entails the diversification of primary sources of energy through a balanced generation portfolio.



Figure 1.1-I The 4 dimensions of security of supply

## 1.2 The Need for Regulatory Tools & Support Models

Rodilla (2010) showed that in the "deregulated" and "liberalized" scheme that governs electricity business, the intervention of the regulator is vital to ensure that an appropriate level of security of supply is achieved.

This regulatory intervention, which is carried out in order to drive the market towards an efficient outcome, is in practice implemented by introducing additional mechanisms, that is to say, additional rules. These mechanisms are regulatory tools aimed at providing additional (and optimal) signals to market agents.

In order to perform this role adequately, the regulator needs another kind of tools: Assessment tools (models of different nature) that can be used either to evaluate the market performance and thus detect the need of introducing additional mechanisms, or to assist in the design of these mechanisms, by evaluating ex-ante the impact of the regulatory solutions that could be considered.

## 1.3 The New Active Role of Demand

The active participation of the demand side in power systems has always been considered an important requirement for their well-functioning, particularly for that of the liberalized ones. Nonetheless, it has not played this desired key role so far due to several reasons (demand immaturity and lack of technical solutions).

This situation is highly expected to change in the near future with the upcoming technical and regulatory developments of smart-grids and demand response tools. In this new context, the ability to properly consider this new active role of demand into electric power system analysis tools turns to be essential.

#### 1.3.1 Reliability Measures in the Presence of Demand Elasticity

As highlighted by Rodilla & Batlle (2010), classical reliability measures such as the Loss Of Load Probability (LOLP), and the Expected Non-Served Energy (ENSE) leave aside very meaningful information when used to assess the performance of a system where elastic demand plays a significant role. As a matter of fact and as it can be checked in Figure 1.3.1-I, in a fully elastic demand scenario, the values of the LOLP and of the ENSE are, according to their traditional definition, always equal to zero. Consequently, even a permanent scenario of severe scarcity could be obscured by the use of the aforementioned metrics.



Figure 1.3.1-I Reliability measures in the presence of demand elasticity

In order to reflect the elastic consumption in the performance measure, the evaluation of the distribution function of what was termed as the Value of the Non-Purchased Energy (VNPE) was proposed.

## 1.4 PPC Models as a Tool to Assess System Performance

One of the tools that has been intensively used to support electric power systems long-term planning and reliability analysis are Probabilistic Production Costing models (PPC models).



These models have traditionally been used in electric power systems as a support tool in the centralized long-term decision-making process.

This approach has attracted considerable efforts from academia since the late 60's. However, although some approaches have been developed to model load shifting programs, as for example Malik (2001), the literature has lacked until recently of efficient algorithms to explicitly introduce demand response to prices in PPC models.

In Rodilla & Batlle (2010) an algorithm that allows extending the classic PPC models design to explicitly represent demand elasticity was proposed.

#### 1.5 Scope and Objectives

This dissertation addresses the problem of developing an adequacy analysis model for electricity markets in which elastic demand plays a significant role.

Taking as a starting point the PPC model approach proposed in Rodilla & Batlle (2010), we will formulate a PPC model to carry out reliability assessments of power generation systems where elastic consumption is substantial. The model proposed will be aimed at coping with a variety of peculiarities of electricity wholesale markets.

In addition to that or, we will face the challenge of developing a methodology that captures the time-dependent nature of elastic demand. The tool developed must allow for finding patterns and realistically describe the elastic demand of a given electricity market.

Eventually we will prove the capabilities of the model by analyzing a real-size case example based on the Spanish electricity market.

To summarize, the objectives of the dissertation are the following:

- I. Develop a Probabilistic Production Costing model to estimate the adequacy of a power generation system. The following modelling challenges will have to be met.
  - I.A. Develop model that allows for analysis in a conventional thermal generation system with fully inelastic load.
  - I.B. Adapt the conventional PPC model for loading energy limited units.
  - I.C. Extend the PPC model with regard to taking into account intermittent renewable generation.
  - I.D. Construct an equivalent load and generation model to integrate price sensitive demand into the conventional PPC model as in Rodilla & Batlle (2010).
  - I.E. Adjust the PPC model in order to properly reflect the effect of the interruptible load described in MITyC (2007).
  - I.F. Develop an algorithm to compute the distribution function of the Value of the Non-Purchased Energy within the PPC frame-work.

- II. Address the time-dependent nature of demand elasticity in an electricity market. In order to do so, a robust and flexible clustering tool will be developed.
- III. Carry out a simulation of the Spanish electricity market for 2016.

## **1.6** Organization of this Document

The document proceeds as follows. Chapter 2 describes the Probabilistic Production Costing model developed. In Chapter 3, a methodology to characterize the elastic demand of any given electricity market is formulated. Chapter 4 is devoted to the analysis of a real-size electricity market. Chapter 5 concludes and summarizes the results obtained.

## 1.7 References

Borenstein, S. & J. Bushnell (2000). "Electricity restructuring: Deregulation or reregulation. Regulation". The Cato Review of Business and Government 23(2), pp.46-52, 2000.

Batlle, C. & I.J. Pérez-Arriaga (2008). "Design criteria for implementing a capacity mechanism in deregulated electricity markets". Utilities Policy, 16(3):184-193, 2008.

Malik, A. S. (2001). "Modelling and Economic Analysis of DSM Programs in Generation Planning", International Journal of Electrical Power & Energy Systems, Volume 23, Issue 5, June 2001.

MITyC, (2007). Ministerio de Industria y Comercio, Orden ITC/2370/2007, July 2007.

NERC (1997). "NERC Planning Standards". North American Electric Reliability Council, September 1997.

Rodilla, P., Baíllo, A., Cerisola, S. & C. Batlle (2010). "Regulatory Intervention to Ensure an Efficient Medium-Term Planning in Electricity Markets". Working Paper IIT-10-008A, submitted to Energy Economics.

Rodilla, P. (2010). "Regulatory Tools to Enhance Security of Supply at the Generation Level in Electricity Markets". PhD Thesis. IIT, Universidad Pontificia Comillas, September 2010.

Rodilla, P. & C. Batlle (2010). "Redesigning Probabilistic Production Costing Models and Reliability Measures in the Presence of Market Demand Elasticity". IIT Working Paper IIT-10-028A, June 2010. Submitted to IEEE Transactions on Power Systems. Available at <u>www.iit.upcomillas.es/batlle</u>.

Ruff, L. E. (2003). "UnReDeregulating Electricity: Hard Times for a True Believer. Seminar on New Directions in Regulation". Kennedy School of Government, Harvard University, Cambridge, MA, May 2003.

# 2. A PROBABILISTIC PRODUCTION COSTING MODEL TO ESTIMATE THE RELIABILITY OF A POWER GENERATION SYSTEM

## 2.1 Introduction

After having introduced, in Chapter 1, the general problem of Security-of-Supply at the generation level and, more specifically, the upcoming problem of appraising the reliability of a power system in a context where demand plays an increasingly active role; we proceed now to introduce the model which constitutes the main theme of the present study.

This chapter can be considered to be the conceptual core of this dissertation. It describes in detail the formulation proposed to carry out reliability assessments of power generation systems in electricity markets where elastic demand consumption is substantial. The model design is aimed towards being a valid tool to cope with a variety of peculiarities of electricity wholesale markets.

#### 2.1.1 Structure of the Chapter

The Chapter proceeds as follows. Section 2.2 reviews the concept of Probabilistic Production Costing model and the relevant literature. Then, in Section2.3, we describe the conventional thermal model, its assumptions, the dispatching algorithm and the results provided. Next, in Section 2.4, we explain how energy-limited units (typically hydro units) can be modelled, how their dispatch can be carried out and discuss some of its implications concerning reliability assessments. In Section 2.5 we show how intermittent renewable generation can be included in the model. In Section 2.6 we show how elastic demand can be modeled within the Probabilistic Production Costing model methodology. In Section 2.7 we provide a general formulation of the VNPE calculation algorithm. Section 2.8 concludes and discusses some applications and extensions.

## 2.2 Introduction to Probabilistic Production Costing Models

As described in Rodilla (2010), Probabilistic Production Costing models (in the following PPC models) have traditionally been used as a support tool in electric power systems, where they are particularly applicable to the centralized long-term decision-making process. These models are characterized by being focused in representing the random nature of some of the most relevant variables involved in the long-term planning problem (typically the demand values and the available capacity of each generating unit).

This approach allows for reliability assessments of real-size electric power systems with little computational effort. However, achieving that requires making severe simplifications regarding mainly short and medium-term operational and planning constraints of the generation plants.

The PPC framework has attracted considerable efforts from academia since the late 60's. The basic model was applied to a non-constrained thermal system; see the pioneering works of Baleriaux et al. (1967) and Booth (1972).

The major outputs that were first calculated by means of these models were:

• Reliability measures: the loss of load probability (LOLP), the loss of load expectancy (LOLE) and the expected value of the non-served energy (ENSE) among others.

• Expected production schedules, that is, the expected energy generated by each generating unit (in average).

• Expected production costs (idem).

It is important to note that there are a considerable number of papers addressing several variations on the conventional approach. For instance, there are remarkable works focused on introducing simplified alternatives to include either hydraulic units as in Finger (1979), Ramos et al. (1991) or Malik (2004); storage units as in Conejo (1987) or Invernizzi et al. (1988); or time-dependent units as in Conejo et al. (1985).

One of the most popular extensions to the basic model is the so-called frequency and duration method. This extension takes into account information on both the frequency and the duration of the different states (demand interval, outage rates, etc), and allows calculating additional results as, for example, the mean time existing between two consecutive critical events (most typically scarcities). This frequency and duration methodology embraces a whole set of different approaches. See for instance the works of Halperin & Adler (1958), Ringlee & Wood (1969), Ayoub & Patton (1976) or Finger (1979).

Additional relevant research has been conducted within the PPC framework, for example in Leite da Silva et al. (1988) or in Lee et al. (1990) a means to calculate the underlying variance of the results is provided. A description on how to estimate derivatives (e.g. marginal values), can be found in Ramos et al. (1994) and also in Maceira & Pereira (1996).

These PPC models have been extensively applied to determine the marginal contribution of each generating unit to the regulator's reliability objectives. One of the first works on this topic is the one carried out by Garver (1966). A more recent work that addresses the problem of determining this contribution to reliability objectives (in this particular case, the contribution of wind energy) applying a PPC model can be found in Kahn (2004). This sort of calculations have been used, for example, to set the compensation for each generating unit in some real systems in which a capacity payment mechanism had been implemented (this was the case of the former Chilean mechanism or the Panamanian case among others).

## 2.3 The Conventional Thermal System Model

In this section, we describe the conventional thermal model, its underlying assumptions, the dispatching algorithm (which is, in fact, the core of the model) and the basic results it provides.

## 2.3.1 Modeling Assumptions

The simplest conventional PPC model is built upon two central assumptions. On the one hand, it is assumed that all generation plants can produce at full capacity at any time unless



when they are out-of-order due to a forced outage. On the other hand, hourly load is considered to be inelastic and stochastic.

These models were conceived to check a fairly simple reliability condition: whenever the system's inelastic demand exceeds the available generating capacity a loss of load takes place. The probability of such an event happening (the Loss of Load Probability or LOLP) and the corresponding expected value of the non-served energy (ENSE), have been the most noteworthy results obtained from these models regarding reliability.

In such a context, the loss of load probability distribution can be evaluated by means of the distribution of the difference between two random variables: the demand and the total generation available. This difference is usually evaluated in a generic random hour. Longer term results (e.g. the expected non-served energy in a whole year) can be calculated by directly extending the results obtained when computing this generic hour.

If all variables (demand and available generation capacity in the most simple case) are statistically independent, then the computation of the former difference considerably simplifies, since the sum (or difference) of two independent random variables is equal to the convolution of their probability distribution functions.

We next present how the demand and the thermal generating units are modeled and the order followed in the convolution operation to simulate the generating units' scheduling. Then we explain how to interpret the results obtained when performing this operation.

## The Demand Curve

The empirical (de-cumulative) distribution function of the load is commonly accepted as a well suited proxy for the distribution function of the electricity demand in a generic random hour.

The empirical de-cumulative distribution function of the Load, or empirical DDF, is the DDF associated with the empirical measure of a sample of observations (i.e. a set of k hourly load values): It is usually denoted by  $\hat{S}_k(L)$  and estimates the true underlying DDF of the sampled hourly load, referred to as S(L). More formally, the estimator  $\hat{S}_k(L)$  is said to converge almost surely to S(L) as  $k \to \infty$ , for every value of L, thus the estimator  $\hat{S}_k(L)$  is also said to be consistent.

$$\hat{S}_k(L) \xrightarrow{a.s.} S(L)$$

We will now proceed to outline how  $\hat{S}_k(L)$  can be obtained from historical hourly load data. Let  $(l_1,...,l_k)$  be a set of independent and identically distributed random variables with the common DDF S(L) (i.e. observed values of hourly load). Then the empirical DDF is defined as:

$$\hat{S}_{k}(L) = \frac{NumberOfElementsInTheSample \ge L}{k} = \frac{1}{k} \sum_{i=1}^{k} 1 \cdot \{l_{i} \ge L\}$$

Therefore, for a given set of historical data, the percentage of time that the load level is greater than or equal to a given load level will be interpreted as a probability. Thus, at any given time (hour) there will be a probability of 1 that the load will be higher than the minimum load in the sample being considered.

Under those assumptions, and as illustrated in Figure 2.3.1-I, we can estimate the DDF of the Load S(L) by just rotating the axes of the load-duration curve corresponding to the historical horizon considered, and then normalizing the time period so that the vertical axis gives the percentage of time (the probability) that a certain value of demand level is equalled or exceeded. The DDF of the Load is often referred to as either the hourly Load Complementary Distribution Function (LCDF) or as the Inverted Load Duration Curve (ILDC).



Figure 2.3.1-IChronologic load (red curve), monotone load curve (blue curve in the left-hand graph) and estimated distribution function of the load (right-hand graph).

Once again, it is important to bear in mind that S(L) does not represent a demand monotone curve anymore, but the de-cumulative distribution function of the demand in a generic hour. In other words, hourly load is supposed to be behaved as a random variable which is distributed according to S(L).

For further details on the empirical distribution function and its properties, see Shorack & Wellner (1986) and Van der Vaart (1998).

## Thermal Generators Modeling

Let us denote each thermal generator's available capacity by  $Q_t$ . Within the PPC framework, this parameter is modeled as an independent discrete random variable. In the model proposed, it is actually represented through a Bernoulli random variable or, to be more precise, by means of a Bernoulli random variable (that models only the availability) multiplied by the generator's maximum capacity  $\overline{q}_t$ .

A Bernoulli distribution is a discrete probability distribution, which takes value one with success probability p and value zero with failure probability q = 1 - p. Being the latter, in



this case, equal to the forced outage rate (*FOR*) of the generator. Therefore, we are considering a two-state model, where the plant is either able to produce at full capacity (with probability p) or unable to produce at all due to a forced outage (with probability q = 1 - p = FOR). Accordingly, the probability mass function (PMF) of the available capacity Q, is:

$$m_{\underline{Q}_{t}}(\underline{Q}_{t} = q_{t}^{*}) = \begin{cases} 1 - FOR & q_{t}^{*} = \overline{q}_{t} \\ FOR & if & q_{t}^{*} = 0 \\ 0 & Otherwise \end{cases}$$

Under this modeling assumption the de-cumulative distribution function of the available capacity takes the form represented in Figure 2.3.1-II.



Figure 2.3.1-II Distribution function of the available capacity of a thermal unit.

The other parameter that is used to characterize the behaviour of a thermal generator is its marginal cost, denoted by  $MC_t$ . When operating and network constraints are dismissed, the dispatch that yields the minimum operating cost is the one in which generators are loaded in order of increasing marginal cost. Hence, knowing the marginal cost of the generators or at least their relative value is of utmost importance. This ranking is commonly known as the merit order or the loading order. The loading algorithm that is going to be described in the next section does, in fact, follow this merit order.

## 2.3.2 The Loading Algorithm

In this section we describe the loading algorithm that is the core of the conventional Probabilistic Production Costing model. As it has been outlined in Section 2.3.1, the loss of load probability distribution can be evaluated by means of the distribution of the difference between two random variables: the demand and the total generation available. If all variables happen to be statistically independent, then the computation of the former difference is equal to the convolution of their probability distribution functions.

In the first part of this section we will introduce the concept of convolution which is the foundation of the conventional PPC model. The second part will be devoted to explain how

it can be applied to analyse the reliability of electric power systems with exclusively thermal generation.

#### Mathematical Concept of Convolution

We turn now to the important question of determining the distribution of a sum (or difference) of independent random variables in terms of the distributions of the individual constituents.

#### a. Sum of Discrete Random Variables

Suppose *X* and *Y* are two independent discrete random variables with probability mass functions  $m_X(x)$  and  $m_Y(y)$ . Let Z = X + Y. We would like to determine the distribution function  $m_Z(z)$  of *Z*. To do this, it is enough to determine the probability that *Z* takes on the value *z*, where *z* is an arbitrary integer. Suppose that X = k, where *k* is some integer. Then Z = z if, and only if, Y = z - k. So the event Z = z is the union of the pair-wise disjoint events:

$$(X = k)$$
 and  $(Y = z - k)$ 

Where *k* runs over the integers. Since these events are pair-wise disjoint, we have:

$$P(Z=z) = \sum_{k=-\infty}^{\infty} P(X=k) \cdot P(Y=z-k)$$

Thus, we have found the distribution function of the random variable Z. This leads to the following definition.

Let *X* and *Y* be two independent integer-valued random variables, with distribution functions  $m_X(x)$  and  $m_Y(y)$  respectively. Then the *convolution* of  $m_X(x)$  and  $m_Y(y)$  is the distribution function  $m_Z(z) = m_X(x) * m_Y(y)$  given by the next equation for every integer *z*.

$$m_Z(z) = \sum_k m_X(k) \cdot m_Y(z-k)$$

The function  $m_Z(z)$  is the probability mass function of the random variable Z = X + Y.

It is easy to prove that the convolution operation is commutative, and it is straightforward to show that it is also associative.

Although the described procedure applies to random integer variables it is not difficult to extend it to any discrete random variable, as long as it is defined for a countably infinite number of discrete values.



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#### b. Sum of Continuous Random Variables

Let *X* and *Y* be two continuous random variables with density functions  $f_X(x)$  and  $f_Y(y)$ , respectively. Assume that both  $f_X(x)$  and  $f_Y(y)$  are defined for all real numbers. Then the *convolution*  $f_X * f_Y$  of  $f_X$  and  $f_Y$  is the function given by

$$(f_X * f_Y)(z) = \int_{-\infty}^{+\infty} f_X(z - y) \cdot f_Y(y) \cdot dy = \int_{-\infty}^{+\infty} f_Y(z - x) \cdot f_X(x) \cdot dx$$

This definition is analogous to the definition of the convolution of two probability mass functions. Thus it should not be surprising that if X and Y are independent, the density of their sum is the convolution of their densities. That is to say, the sum Z = X + Y is a random variable with density function  $f_Z(z)$ , where  $f_Z$  is the convolution of  $f_X$  and  $f_Y$ .

For further details on the sum of random variables and its properties, see Grinstead & Snell (2003).

#### Application to Power Systems

As described in Booth (1972), let us introduce the concept of "equivalent load after loading the first *n* generating units", denoted by  $EqL_n$ . This  $EqL_n$ , which is a random variable, represents the load that remains un-served after having loaded the first *n* groups in the merit order. For instance, the equivalent load after having loaded the first unit in the merit order will be equal to the difference between two random variables, on the one hand the load *L* of the system and, on the other hand, the available capacity of the first unit denoted by  $Q_1$ . Explicitly:

$$EqL_1 = L - Q_1$$

As shown in the previous section, the probability mass function of the load after having loaded the first unit can be computed as follows:

$$m_{EqL_1}(EqL_1 = l) = \sum_k m_{EqL_1}(k) \cdot m_{Q_1}(l-k)$$

Generally, the equivalent load after having loaded the first n units can be expressed as follows:

$$EqL_n = L - \sum_{t=1}^n Q_t$$

Or, alternatively:

$$EqL_n = EqL_{n-1} - Q_n$$

And its PMF could be computed by carrying the convolution of the variables involved, as expressed next:

$$m_{EqL_n}(EqL_n = l) = \sum_k m_{EqL_{n-1}}(k) \cdot m_{Q_n}(l-k)$$

By successively applying the former expression, the distribution functions of the different equivalent loads can be obtained. Obviously, the first equivalent load (n = 0) represents the load of the system, that is to say  $EqL_0 = L$ . The subsequent equivalent loads represent the load yet to be covered after dispatching each one of the generators in the system. The last equivalent load  $EqL_n$  represents the un-served demand once all of the generators have been loaded (i.e. the amount of non-served energy).

However, as the available capacity of the thermal units is modeled by means of a binary random variable, the computation of the successive distribution functions of the equivalent load considerably simplifies.

Let us assume that  $S(EqL_{n-1})$  is the DDF of the un-served load after having loaded the first n-1 groups in the merit order, and that the  $n^{th}$  thermal unit can be represented by means of a forced outage rate  $FOR_n$  and a maximum output  $\overline{q}_n$ . Thus, the DDF of the equivalent load after having loaded the first n groups could be simply computed by applying the following expression:

$$S(EqL_n = l) = FOR_n \cdot S(EqL_{n-1} = l) + (1 - FOR_n) \cdot S(EqL_{n-1} = l + \overline{q}_n)$$

As it can be easily noticed, the DDF of  $EqL_n$  is equal to the sum of two distinct and rather meaningful terms:

- On the one hand, the term  $FOR_n \cdot S(EqL_{n-1} = l)$ , which is equal to the distribution function of the un-served load before loading the  $n^{th}$  thermal unit multiplied by its forced outage rate. In other words, the  $n^{th}$  generator will be not available with a probability equal to  $FOR_n$  and in such an event the un-served load will remain to be distributed according to  $S(EqL_{n-1} = l)$ .
- On the other hand, the term  $(1 FOR_n) \cdot S(EqL_{n-1} = l + \overline{q}_n)$ , which is equal to the distribution function of the un-served load that would result of the loading of the  $n^{th}$  thermal unit if it were fully available, multiplied by its availability rate. That is to say, the  $n^{th}$  generator will be available with a probability equal to  $1 FOR_n$  and in such an event the un-served load will be distributed according to  $S(EqL_{n-1} = l + \overline{q}_n)$ .

The following discrete example gives a further insight on how the probabilistic dispatch is carried out. Let us consider the following equivalent load distribution after having loaded the first n-1 units:





Figure 2.3.2-I DDF of the demand. Case Example

And let us assume that the available capacity of the  $n^{th}$  thermal unit can be modelled by means of the following distribution function:



Figure 2.3.2-II DDF of a thermal unit's available capacity. Case example

In this case the equivalent load after having loaded the first n generating units can be effortlessly obtained as follows:

The thermal unit will be not available with a probability equal to  $FOR_n$  and in such an event the un-served load will remain to be distributed according to  $S(EqL_{n-1} = l)$ . Therefore the DDF of the un-served demand will be the one shown in Figure 2.3.2-III.



Figure 2.3.2-III DDF of the demand when generation is not available. Case example

The thermal unit will be available with a probability equal to  $1 - FOR_n$  and in such an event the un-served load will be distributed according to  $S(EqL_{n-1} = l + \overline{q}_n)$ . Thus the DDF of the un-served demand will be the one shown in Figure 2.3.2.IV.





Figure 2.3.2-IV DDF of the demand when generation is available. Case example

Taking into account both scenarios and their probabilities, the DDF of the un-served load after having loaded the first n units can be obtained straightforwardly. The procedure is depicted in the following figure:



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Figure 2.3.2-V. Probabilistic dispatch of a thermal unit. Case example

Finally, Figure 2.3.2-VI illustrates the result of the sequential probabilistic loading of a set of thermal units for a given load distribution, that is to say, the de-cumulative distribution functions of the successive equivalent loads, denoted by  $S(EqL_n)$ .



Figure 2.3.2-VI Successive Equivalent Load curves

#### 2.3.3 Basic Results Provided

The major outputs that can be calculated by means of this model include reliability measures, expected production schedules, expected production costs and marginal price probabilistic distributions.

The equivalent load curve after having dispatched every single generator represents the distribution function of the non-served energy, denoted by S(NSE). Two valuable pieces of information (see Figure 2.3.3) concerning reliability can be easily extracted from it:



Figure 2.3.3 Distribution function of the non-served energy

#### LOLP

The loss of load probability (LOLP) represents the probability that there will still be demand left to be met (un-served demand) after all the generators have been loaded (in the generic random hour being represented). It is the point where distribution curve of the non-served energy cuts the vertical axis (probability). That is to say:

$$LOLP = S(NSE = 0)$$

#### ENSE

The expected non-served energy (ENSE) is the area below this curve. It represents the expected amount of energy (in MWh) that is left unsupplied (in the generic random hour being represented) after all of the generators have been loaded. More formally it can be defined as:

$$ENSE = E[NSE] = \int_{l=0}^{l=+\infty} S(NSE = l) \cdot dl$$



## LOLE

The loss of load expectancy (LOLE) expresses the expected number of hours within a certain period in which the system load is expected to exceed the available electricity generation capacity; see Van Wijck (1990). Once the LOLP has been determined, calculating the LOLE is not difficult at all.

Let  $A^{\Psi}$  be the total number of hours in the time scope analyzed. The number of hours in which the load will exceed the generation capacity will be given by a Binomial Distribution of parameters  $n = A^{\Psi}$  and p = LOLP. The LOLE will be equal to the expected value of that distribution. That is to say:

$$LOLE = LOLP \cdot A^{\Psi}$$

Traditionally, a reliability standard of one day in ten years has been established in electric systems; see FERC (2010).

#### Expected energy supplied by a unit

It can be easily checked that the expected value of the energy produced by a given unit (in a generic hour) can be computed as the difference between the NSEE before and after dispatching that unit, as shown in the next equation:

$$E[E_n] = \int_{l=0}^{l=+\infty} S(EqL_{n-1} = l) \cdot dl - \int_{l=0}^{l=+\infty} S(EqL_n = l) \cdot dl$$

#### Probability of being marginal

As it will be shown further in this chapter, in Section 2.7, the probability of a plant being marginal can be directly obtained within the PPC framework as follows:

$$p_{M \text{ arg inal}}^{n} = S(EqL_{n-1} = 0) - S(EqL_{n} = 0)$$

#### The Hydro-Thermal System Model 2.4

As put across by Batlle (2002), modelling hydro plants is clearly one of the most challenging tasks that can be thought of in the electricity sector. In fact, even the purely theoretical design of a model covering all of its peculiarities is hard to conceive.

For instance, the function that relates, for each plant, the amount of energy that can be generated to the volume of stored water is tremendously complex (i.e. it is non-linear and depends on the shape of the reservoir and even on the water of flow released). A further handicap is the fact that hydro units are commonly connected with other up and downstream storage units (both in series and in parallel) what introduces spatial and time-linking constraints.

Hence, it is clear that any representation of hydro production must contain strong modelling simplifications that, nonetheless, allow for considering the fact that hydro plants are energy

constrained (i.e. it is not possible to produce at maximum power whenever desired). Section 2.4.1 starts describing the assumptions undertaken by the modelling approach. We next show, in section 2.4.2, how hydro units can be incorporated to the PPC frame work. We conclude discussing the validity of the model proposed.

## 2.4.1 Hydro Units Modelling

As it has been previously noted, the remarkable complexity of the hydroelectric system imposes the need for reduced representations of the hydro units. The modelling assumptions carried out in order to make possible the medium and long term analysis proposed in this dissertation are based on the work of Batlle (2002).

Following the approach proposed there, a group of hydro plants set in the same river basin and operated by the same firm are synthesized into a unique plant. From now on, we will refer to these composite representations as hydro plants.

Each one of those hydro plants is characterized by three parameters assumed to be constant for each time period  $\psi$  in which the time scope  $\Psi$  is divided:

- $\overline{q}_h^{\psi} \equiv Maximum$  capacity constraint of hydro plant *h* in period  $\psi$ .
- $\underline{q}_{h}^{\psi} \equiv$  Minimum output (run-of-the-river power) of hydro *h* in period  $\psi$ .
- $e_h^{\psi} \equiv \text{Available energy at hydro plant } h \text{ in period } \psi$ .

As opposed to thermal plants, the three parameters that define the behaviour of each hydro unit are different for each period of time  $\psi$ . While the operating constraints of the thermal plants can be considered to be constant through the whole time scope of analysis, the ones that characterize the production of hydro plants are greatly time-dependent.

Another fundamental issue regarding hydro units modelling is the fact that no forced outage rate is considered. Given that it is not significant compared their energy restriction, it is neglected.

The model inputs are derived from historical values using the GEHA (*Generador de Escenarios Hidráulicos Aleatorios,* hydro production random scenario generator) developed by Batlle (2002). The GEHA consists basically of:

• A time series model that generates a series of hydro energy produced in each period in the whole system.

• A module that distributes that energy production among all the hydro units according to randomly sampled historical data.

• A regression model that relates the energy produced by each hydro unit in a period with its maximum and minimum capacity values for that period, as shown in Figure 2.4.1-I and in Figure 2.4.1-II respectively.

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Figure 2.4.1-I Maximum capacity Vs Available energy



Figure 2.4.1-II Minimum output Vs Available energy

For their further use within the model, two out of the three parameters that characterize a hydro unit are slightly modified. On the one hand, the minimum energy generated by the unit (that associated to the minimum output) is subtracted from the total available energy. That is to say:

$$e_h^{\psi} = \overline{e}_h^{\psi} - \underline{e}_h^{\psi} = \overline{e}_h^{\psi} - \underline{q}_h^{\psi} \cdot a^{\psi}$$

On the other hand, instead of the maximum output, the value of maximum output over minimum is used. Obviously:

$$q_h^{\psi} = \overline{q}_h^{\psi} - \underline{q}_h^{\psi}$$

#### 2.4.2 Loading of Energy-Limited Units

The loading of hydro units is carried out in two steps. Firstly, the run-of-the-river power is deducted from the load of the system in that period, denoted by  $L^{\psi}$ . The total amount of run-of-the-river power is:

$$\underline{Q}_{H}^{\psi} = \sum_{h=1}^{H} \underline{q}_{h}^{\psi}$$

And therefore, the distribution of the load after having loaded the minimum output of every hydro unit will be computed as follows:

$$S(L_{\underline{Q}_{H}^{\psi}}^{\psi}=l)=S(L^{\psi}=l+\underline{Q}_{H}^{\psi})$$

The loading of the run-of-the-river power is illustrated in Figure 2.4.2-I.



Figure 2.4.2-I Loading of the run-of-the river output

Secondly, the remaining available energy of each hydro unit  $e_h^{\psi}$  is loaded. Since plants are loaded sequentially (one after another), as a first step, they have to be sort according to the merit order that is described later in this section. Hydro plants are then scheduled attempting to make the most of their available energy. The loading algorithm proceeds as follows:

- Step 0. Let us denote  $L_{Q_{\mu}^{\psi}}^{\psi}$  by  $EqL_{n-1}^{\psi}$
- Step 1. The hydro unit is supposed to produce at full capacity (over minimum) if the equivalent load is equal to or higher than it. On the other hand, if the load is lower than the capacity the production is supposed to be equal to the load. Accordingly, the distribution function of the load after having loaded the hydro unit would be:

$$S(EqL_{n}^{\psi} = l) = \begin{cases} S(EqL_{n-1}^{\psi} = l) & i < 0\\ S(EqL_{n-1}^{\psi} = l + q_{h}^{\psi}) & 0 \le l \end{cases}$$

Step 2. The expected value of the energy that the hydro unit would produce if it were dispatched in that position is computed as follows:

$$E[E_{h}] = \int_{l=0}^{l=+\infty} S(EqL_{n-1}^{\psi} = l) \cdot dl - \int_{l=0}^{l=+\infty} S(EqL_{n}^{\psi} = l) \cdot dl$$

Step 3. If  $E[E_h] \le e_h^{\psi}$ , that is to say, if the expected energy production does not exceed the available energy, the hydro unit is scheduled in that position. Therefore, the equivalent load to be met by the remaining generating units will be the one calculated in Step 2. Additionally, the values of the indexes *n* and *h* are updated (they are increased in one unit) and the dispatch algorithm goes back to Step 1.

If  $E[E_h] > e_h^{\psi}$ , that is to say, if there is not enough available energy, the equivalent load remains to be  $EqL_{n-1}^{\psi}$  and the algorithm proceeds to Step 4

Step 4. The next thermal unit in the merit order is loaded, the equivalent load is modified accordingly and the algorithm goes back Step 1



Figure 2.4.2-II Loading of Thermal and Hydro Units

#### The merit order of hydro generators

The loading order can considerably modify the distribution of the NSE of the system, as shown in Marcos (2007). Of all the possible loading sequences, the one in which the energy limited units are loaded at their maximum power output is the alternative that provides the highest level of reliability – i.e. the higher the power output the lower the values of NSEE and LOLP–.

Consequently, hydro units are ranked according to the total number of hours that they can produce at full capacity. For each unit, that parameter is calculated as:

$$\tau^{\psi}_{h} = \frac{e^{\psi}_{h}}{q^{\psi}_{h}}$$

#### 2.4.3 Validity of the Expected Energy Convolution Model

As highlighted by Marcos (2007), loading energy limited generators using all the energy available in each unit as an expected value (as is the case of most probabilistic models applied to power systems) leads to unfeasible scenarios (due to the lack of available energy) that must be taken into account.

#### Energy supplied by a hydro unit in a given period

Let  $Q_h^{\psi}$  be a random variable that represents the output of hydro unit *h* in any given hour of period  $\psi$  (that is composed of  $a^{\psi}$  hours). Obviously, the output of that unit can be obtained as the difference of two random variables as:

$$Q_h^{\psi} = EqL_{n-1} - EqL_n$$

Therefore it is possible to determine the PMF of the hourly output  $Q_h^{\psi}$  by convolution recalling to the following expression:

$$m_{O_{L}^{\psi}}(Q_{h}^{\psi}) = m_{EqL_{n-1}}(EqL_{n-1}) * m_{-EqL_{n}}(-EqL_{n})$$

Thus, the energy supplied by that hydro unit in period  $\psi$  could be represented by means of another random variable, denoted by  $E_h^{\psi}$ . Since the outputs of different hours are supposed to be independent, the energy produced by the hydro unit could be obtained as follows:

$$E_{h}^{\psi} = \underbrace{Q_{h}^{\psi}\Big|_{1} + Q_{h}^{\psi}\Big|_{2} + \dots + Q_{h}^{\psi}\Big|_{a^{\psi}}}_{a^{\psi} times}$$

Or, in terms of PMF as:

$$m_{E_h^{\psi}}(E_h^{\psi}) = \underbrace{m(Q_h^{\psi}\big|_1) * m(Q_h^{\psi}\big|_2) * \dots * m(Q_h^{\psi}\big|_{a^{\psi}})}_{a^{\psi} times}$$

The former expression requires a remarkable computational effort. However, given that the number of hours in a period is usually large enough ( $a^{\psi} > 30$ ), and that the hourly outputs  $Q_h^{\psi}$  are supposed to be behaved as independent and identically distributed random variables, the computation of the distribution function of the energy supplied by a hydro unit considerably simplifies.



Let us denote the expected value and the variance of  $Q_h^{\psi}$  by  $\mu_h^{\psi}$  and  $(\sigma_h^{\psi})^2$  respectively. Recalling to the Central Limit Theorem (see, for instance, Grinstead and Snell (2003)) we can assert that  $E_h^{\psi}$  converges in distribution to a normal. Formally:

$$E_h^{\psi} \sim \mathbf{N} \left\langle a_h^{\psi} \cdot \boldsymbol{\mu}_h^{\psi}, a_h^{\psi} \cdot (\boldsymbol{\sigma}_h^{\psi})^2 \right\rangle$$

Figure 2.4.3-I shows the distribution function of the energy supplied by a hydro unit in a given period and the unfeasible scenarios.



Figure 2.4.3-I Distribution function of the energy supplied by a hydro unit

#### The Value-at-Risk approach

The convolution model can be improved by limiting the percentage of unfeasible scenarios that are taken into account. This can be done by means of a risk metric as, for instance the Value-at-Risk. Instead of checking the condition  $EE_h \leq e_h^{\psi}$ , an enhanced model would check the condition  $VaR_{\alpha} \leq e_h^{\psi}$  as it is shown in Figure 2.4.3-II.



Figure 2.4.3-II Value-at-Risk approach

However, although it is important to bear in mind these ideas, the model proposed follows the conventional approach given that the overall results do not change significantly.

## 2.5 Introducing Renewable Generation in the Model

As put across by Milligan and Porter (2008), the most straightforward way to represent wind (or, more broadly, intermittent renewable generation) in reliability models is as an hourly modification to the load, using data from the same year, month, day and hour.

Let  $(l_1,...,l_k)$  be a set of observed values of hourly load and  $(w_1,...,w_k)$  a set of observed values of hourly renewable production. It is important to note that the sub-indexes are consistent, that is to say  $l_i$  and  $w_i$  represent the load and the wind production, respectively, of the same hour, day and year.

We can now introduce the concept of hydro-thermal load (load that must be met by both hydro and thermal generators). The set  $(l_1^{HT}, ..., l_k^{HT})$ , whose elements are computed as difference between the load and the wind production, namely  $l_i^{HT} = l_i - w_i$ , is obviously a set of observed values of hydro-thermal load. This set can be then directly used to obtain a proxy for the distribution of the load as shown in Section 2.3.1.

Figure 2.5 illustrates this procedure. Firstly, hourly renewable production (green curve) is subtracted from hourly load (red curve) in order to calculate the hourly hydro-thermal demand (blue curve). Secondly, the hydro-thermal load-duration curve is built. Eventually, the empirical DDF of the hydro thermal load is calculated.

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Figure 2.5 Introducing RES in the PPC model

It should be noted that a detailed representation of renewable generation is far beyond the purpose of this dissertation. If the objective were to investigate the potential impact of future wind development scenarios on system reliability, operations or economics, the approach should be other. In those cases, as discussed in Smith et al. (2007), it would be better to use wind data from a numerical weather prediction (NWP) model that produced hourly or sub-hourly wind speed estimates that could be converted to realistic representations of large-scale renewable power production.

## 2.6 Introducing Demand Elasticity in the Model

In this dissertation we take as a starting point the PPC model approach suggested by Rodilla and Batlle (2010). They propose a very simple and appealing way to integrate demand elasticity into the conventional probabilistic production costing frame work. In Section 2.6.1, that algorithm is described.

We consider two different sources of demand elasticity, the one reflected in the demand curves offered in the power pool (Price-Sensitive Demand) and the interruptible load bilaterally contracted by the Transmission System Operator (TSO) to the largest industrial consumers in the system (Reserve-Sensitive Demand). As the approach proposed in Rodilla and Batlle (2010) does only consider Price-Sensitive Demand, it does not allow to explicitly modeling that demand which may be interrupted in the event of a scarce reserve margin. Therefore, the PPC model will be extended to properly reflect the effect of the Reserve-Sensitive Demand. Section 2.6.2 is devoted to that purpose

#### 2.6.1 Modeling Price-Sensitive Demand

We will now describe how demand offer bids can be modelled within the PPC framework resorting to the example used by Rodilla (2010). The underlying idea of the algorithm proposed is illustrated making use of both a deterministic demand and set of thermal units that are always able to produce, that is to say, a set of thermal units whose forced outage rates are equal to zero. Once the main idea is presented, introducing it in the PPC methodology will be straightforward.

#### Modeling demand bids as equivalent generators

Let us consider that demand marginal utility (the demand offer curve) is given by the red step-wise curve presented in Figure 2.6.1-I. The available generators' capacities (MW) and

their corresponding marginal costs (€/MWh) have been represented as an aggregated stepwise curve (dotted in blue) in the same chart.



Original problem

Figure 2.6.1-I Supply and Demand Curves

It is important to note that the demand offer curve comprises two clearly differentiated parts:

- The Inelastic Load, denoted here by  $L_{In}$ . Ideally, the bid price corresponding to it should represent the value of loss of load (VOLL) of the system. Anyway, it is assumed to be much higher than the variable cost of any of the generators.
- The Elastic Load or Price-Sensitive Demand. A set of price-quantity pairs  $(\pi_i, q_i)$  denoted by  $L_i$ .

The algorithm proposed tackles the problem by solving an equivalent one (see Figure 2.6.1-II), in which the elastic demand is substituted by:

• A set of fictitious generators. Each one of them represents an elastic demand bid. This way, each  $L_i$ , becomes a fictitious generator  $G_{Li}^F$ . The maximum output and the marginal cost of the generator will be the quantity and price that define the original demand bid.

• A new fictitious inelastic demand. It will be equal to the total amount of energy demanded (i.e. including both the inelastic and elastic consumption). This new fictitious inelastic demand has been denoted in Figure 2.6.1-II by  $L_{ln}^{F}$ .



Figure 2.6.1-II Equivalent problem formulation

As it can be easily checked, the market outcome remains unchanged, since the market clearing price, and the committed generators are exactly the same in both cases.

It is important to bear in mind that, in the equivalent problem, the production of the fictitious generators represents the elastic demand that was not accepted in the auction. In other words, it represents the energy that was not purchased because the bid price was below the market price.

## The PPC model with elastic demand

As described in the previous section, the approach proposed is based on defining an equivalent problem, where the price-sensitive demand is substituted by an inelastic demand and a set of fictitious generators.

Although it would be possible to represent stochastic demand elasticity by means of the forced outage rate, in this dissertation we consider that for each period  $\psi$  elastic demand is fully determined. As a consequence, for each period, elastic demand will be represented by a set of fictitious generators, defined by the following parameters:

- $\overline{q}_{f}^{\psi} \equiv \text{Maximum capacity constraint of fictitious plant } f$  in period  $\psi$  (quantity bidded).
- $MC_f^{\psi} \equiv Marginal \operatorname{cost} of fictitious generator f in period \psi$  (bid price).
- $FOR_f^{\psi} \equiv$  Forced outage rate of fictitious generator *f* in period  $\psi$  (always equal to zero).

### 2.6.2 .Modeling Reserve-Sensitive Demand

In order to consider the load shedding capability bilaterally contracted by the system operator we propose the following methodology.

For each period  $\psi$  in which the time scope of analysis  $\Psi$  is divided, the interruptible load (*IL*) is placed in the supply function merit order allowing a certain level of reserve margin (*RM*) specified by the Transmission System Operator. That is to say, *IL* is modelled as a fictitious generator characterized by the following parameters:

•  $\overline{q}_{IL}^{\psi} \equiv Maximum$  capacity constraint of fictitious plant *IL*. It is equal to the total interruptible load (in MW).

•  $FOR_{IL}^{\psi} \equiv$  Forced outage rate of fictitious generator IL (always equal to zero).

•  $MC_{IL}^{\psi} \equiv Marginal \operatorname{cost} of fictitious generator IL in period \psi$ . It is set in such away that the position of the fictitious generator IL allows for the desired level of reserve margin.

The reserve margin for each period, namely  $RM^{\psi}$  (in MW), is calculated as a given percentage ( $\mathcal{E}_{RM}$ ) of the maximum load of that period, denoted by  $l_{Max}^{\psi}$ . Formally:

$$RM^{\psi} = \varepsilon_{RM} \cdot l_{Max}^{\psi}$$

Figure 2.6.2 illustrates how the interruptible load is included in the supply function as a fictitious generator.



Figure 2.6.2 Reserve Sensitive Demand



## 2.7 Measuring the Value of Non-Purchased Energy

In this section, we present a methodology for constructing the distribution function of the so called Value of Non-Purchased Energy within the PPC framework. First, we recall the background of the VNPE and its importance. Next, we define it in a precise and concise way. And, eventually we provide a general formulation of the VNPE calculation algorithm.

## 2.7.1 The Background of the Concept

As highlighted in (Rodilla, 2010), classical reliability measures such as the Loss of Load Probability (LOLP), and the Expected Non-Served Energy (ENSE) leave aside very meaningful information when used to assess the performance of a system where elastic demand plays a significant role. As a matter of fact, in a fully elastic demand scenario, the value of the ENSE is, according to its traditional definition, always equal to zero. Consequently, a permanent scenario of severe scarcity could be obscured by the use of the aforementioned metrics.

In order to reflect the elastic consumption in the performance measure, the evaluation of the distribution function of what was termed as the value of the non-purchased energy (VNPE) was proposed. The purpose of this section is to illustrate how it can be calculated in a real-size case.

#### 2.7.2 Brief Overview of the VNPE Concept

Let us characterize the demand by means of a set  $\Omega$  of price-quantity pairs  $(\pi_i, q_i)$  where:

- $\pi_i$  stands for the bid price (i.e. the highest price the buyer is willing to pay).
- $q_i$  denotes the quantity bidded.

We can define the term "value of a bid" as the quantity demanded multiplied by the bid price. Thus, for a given demand and supply deterministic scenario, the value of the non-purchased energy will be the total sum of the values of the bids that are not accepted.



Figure 2.7.2 VNPE for a given deterministic scenario

Otherwise, resorting to the aggregate demand curve  $\pi = D(q)$ , the VNPE could be defined as the area below it in the interval  $(q^*, \overline{q})$ :

$$VNPE = \int_{q^*}^{\overline{q}} D(q) \cdot dq$$

Where:

 $q^*$  stands for the quantity cleared in the market.

 $\overline{q}$  represents the total energy demanded.

#### 2.7.3 Computation Model Description

The whole procedure for building the VNPE distribution function is based on a fairly intuitive idea: whenever the price of a bid is lower than the marginal cost of the system (market clearing price), the bid will not be accepted. Provided that we consider that each elastic demand scenario is fully determined (i.e. the amount of energy demanded at each price level is not a random variable), the previous lemma becomes the guiding principle of a robust and simple way of computing the value of the non-purchased energy.

The plan of this sub-section is as follows. We first present how the VNPE can be calculated for any possible scenario. Subsequently, we show how the probability of each of the scenarios can be estimated. And eventually, we bring in some considerations about the non-served energy.



#### Value of Non-Purchased Energy Calculation

#### a. The marginal plant is a thermal or hydro generator

Let us firstly assume that: the marginal unit is either thermal or hydro (in other words, not fictitious); that additionally, it will be marginal with a probability equal to  $p_{Marginal}$ ; and that the market clearing price of the system is  $\lambda$ . In such an event, the VNPE could be easily computed as follows:

$$VNPE = \sum_{(\pi_i, q_i) \in \Gamma} \pi_i \cdot q_i \qquad \Gamma = \{(\pi_i, q_i) \in \Omega / \pi_i < \lambda\}$$

Therefore, the value of the non-purchased energy will be the one calculated above with a probability equal to  $p_{Marginal}$ .

#### b. The marginal plant is a fictitious generator

If a fictitious generator happens to be the marginal one, that is to say, if a demand bid represented by a so-called fictitious generator is partially accepted, the calculation of the VNPE differs slightly. In addition to the bids that were fully rejected, it is necessary to take into account the value of the bid that was partially turned down.

In order to do so, we make use of the probability mass function (PMF) of the production of the fictitious generator. Assuming that the production of a fictitious generator can be represented by means of a discrete random variable denoted by  $Q_f$ , that function gives the probability that the former is exactly equal to some value.

Once we have obtained the PMF  $m_{Q_f}(Q_f)$ , computing the VNPE and probability of each scenario is straightforward. Formally:

$$VNPE = \pi_f \cdot q_f^* + \sum_{(p_i, q_i) \in \Gamma} \pi_i \cdot q_i \qquad \Gamma = \{(\pi_i, q_i) \in \Omega / \pi_i < \lambda\}$$

Where:

 $\pi_f$  stands for the bid price of the fictitious marginal generator.

 $q_f^*$  denotes the quantity not purchased (production of the fictitious generator).

The value of the non-purchased energy will be the one calculated above with a probability equal to  $m_{Q_f}(Q_f = q_f^*)$  for every value of  $Q_f$  so that the fictitious generator is marginal. That is to say, for every  $Q_f = q_f^*$  such that  $0 < q_f^* < \overline{q}_f$ , where  $\overline{q}_f$  stands for the maximum output of the fictitious generator (quantity demanded).
#### Scenario Probability Estimation

#### a. Probability of being marginal

A generator is said to be marginal if its production is at any point between its minimum and maximum output. As outlined before, evaluating which units are more likely to be marginal (and hence, set the marginal price) is vital to building the VNPE distribution function. For this reason, we explicitly show now how each unit's probability of being marginal can be effortlessly obtained from the PPC model proposed in this dissertation.

Let us recall the concept of the equivalent load after dispatching the first n groups, denoted by  $EqL_n$ . In Figure 2.7.3-I, the de-cumulative distribution functions of the non-served load, both before (blue curve) and after (red curve) having dispatched the n<sup>th</sup> generator, are plotted.



Figure 2.7.3-I Probability of being marginal

By merely having a look at it, it can be noted that: with a probability equal to  $p_1$ , the n<sup>th</sup> group (regardless of its availability), will not produce at all, as the system load before dispatching it is zero (negative values of load may represent the reserve margin). Besides, with a probability equal to  $p_3$ , the load after dispatching the n<sup>th</sup> generator will be strictly positive, thus meaning that it cannot be marginal as additional generating units will be required to meet the demand. That could mean that the generator is either unavailable or incapable of solely satisfying the demand (the load is higher than the plant's capacity).

By the same token, we can deduce that if the plant is neither unavailable, nor producing its maximum or minimum output it must be necessarily marginal. Actually, the probability  $p_2$ 



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represents the likelihood of that happening. Said in a different and more concise way, the n<sup>th</sup> group will be marginal with a probability equal to  $p_2$ .

In conclusion, measuring each generator's probability of being marginal can be reduced to nothing more than plainly subtracting two numbers: the intercepts of both the de-cumulative distribution function of the equivalent load before dispatching the generator (denoted by  $S(EqL_{n-1})$ ) and the de-cumulative distribution function of the equivalent load after dispatching it (denoted by  $S(EqL_n)$ ) with the vertical axis. Explicitly, the n<sup>th</sup> generator will be marginal with a probability equal to:

$$p_{M \, \text{arginal}}^{n} = S(EqL_{n-1} = 0) - S(EqL_{n} = 0)$$

#### b. Non-Purchased Energy Probability Mass Function

In order to be treated within the model, the inelastic load is quantized according to a specified resolution, and so are the supply offers and the elastic bids. As a result, discrete random variables can be proxy for both the production of the generators and the n equivalent loads.

As pointed out above, determining the probability mass function of the production of the fictitious generators is key to building the VNPE distribution function. We will now try to shortly describe how it is obtained from the PPC model by recourse to an example.

Let  $m_{EqL_n}(EqL_n)$  be the probability mass function of the equivalent load after dispatching the first n groups.



Figure 2.7.3-II Probability mass function of the Equivalent Load

And let us assume now that the following generator in the merit order is a fictitious one (a demand bid) of capacity  $\overline{q}_f$ . Obviously, if the equivalent load is either zero or negative, the output of the fictitious generator will be zero. In addition to that, if the load is higher or

equal to the capacity, the output of the generator will be its capacity. And, needless to say, in any other case, the production will be equal to the load and, by the way, the fictitious generator will be marginal.

More formally, the PMF of the production of a fictitious generator that is dispatched after the first n units can be defined as a piecewise function:

$$m_{\mathcal{Q}_f}(\mathcal{Q}_f = q_f^*) = \begin{cases} \sum_{-\infty < EqL_n \le 0} m_{EqL_n}(EqL_n) & q_f^* = 0\\ m_{EqL_n}(EqL_n) & \text{if } 0 < q_f < \overline{q}_f \\ \sum_{\overline{q}_f \le EqL_n < \infty} m_{EqL_n}(EqL_n) & q_f^* = \overline{q}_f \end{cases}$$

Then, it can be easily checked that the PMF of the production of the fictitious generator of the example is the following:



Figure 2.7.3-IIIPMF of the production of a fictitious generator

#### Non-Served Energy

The evaluation of the impact of the non-served energy in the value of the non-purchased energy does not involve any major additional change in either the PPC model or the VNPE computation model. Actually, it can be easily done by just introducing a further fictitious generator whose capacity is equal to the maximum inelastic load, whose offer price is equal to the value of loss of load (VOLL) and is always available.

Besides, this approach grants some other advantages, as the ENSE can be directly obtained as the expected output of this generating unit and the LOLP is nothing but its probability of being marginal, and hence, there is no need for an additional module that calculates this classical reliability measures.

# 2.8 Conclusions

In this chapter we have formulated a Probabilistic Production Costing model to carry out reliability assessments of power generation systems in electricity markets where elastic demand plays a significant role. The model described is able to cope with a variety of peculiarities of electricity wholesale markets.

Moreover, the proposed formulation meets the following modelling challenges:

- Firstly, the model allows for analysis in a conventional thermal generation system with fully inelastic load.
- Secondly, the conventional PPC model has been adapted for loading energy limited units.
- Thirdly, the conventional PPC model has been extended with regard to taking into account intermittent renewable generation.
- Fourthly, an equivalent load and generation model has been constructed to integrate price sensitive demand into the conventional PPC model.
- Fifthly, the PPC model has been adjusted to properly reflect the effect of the reservesensitive demand.
- Sixthly, an algorithm to compute the distribution function of the Value of the Non-Purchased Energy has been implemented within the PPC frame-work.

This PPC model will be used in Chapter 4 to carry out a reliability assessment of the Spanish electricity market.

# 2.9 References

Ayoub, A. K. & A. D. Patton (1976). "A Frequency and Duration Method for Generating System Reliability Evaluation". IEEE Transactions on Power Apparatus and Systems, vol. 95, iss. 6, part 1, pp. 1929-1933, 1976.

Baleriaux, H., E. Jamoulle and F. Linard de Guertechin (1967). "Simulation de l'Explotation d'un Parc de Machines Thermiques de Production d'Electricité Couplé à des Stations de Pompage," Revue E, Vol. V, No. 7, pp. 225-245, 1967.

Batlle, C. (2002). "A Model for Electricity Generation Risk Analysis". PhD Thesis. IIT, Universidad Pontificia Comillas, 2002.

Booth, R. R. (1962). "Power System Simulation Model Based on Probability Analysis," IEEE Transactions on Power Apparatus and Systems, PAS-91, pp. 62-69, January/February 1972.

Conejo, A. J. (1987) "Optimal Utilization of Electricity Storage Reservoirs: Efficient Algorithms Embedded in Probabilistic Production Costing Models". M. S. Thesis. Massachusetts Institute of Technology. August 1987.

Conejo, A. J., I. J. Pérez-Arriaga, A. Ramos and A. Santamaría (1985). "Evaluation of the Impact of Solar Thermal Generation on the Reliability and Economics of an Electrical Utility System", IEEE Mediterranean Electrotechnical Conference. MELECON '85 4: 167-173 A. Luque, A.R. Figueiras Vidal, D. Nobili (eds.) Elsevier Science Publishers B.V. Madrid, Spain October 1985.

FERC, (2010). "Planning Resource Adequacy Assessment Reliability Standard". Federal Energy Regulatory Commission, Notice of Proposed Rulemaking, October 2010.

Finger, S. (1979). "Electric power system production costing and reliability analysis including hydro-electric, storage, and time dependent power plants". MIT Energy Laboratory Technical Report #MIT-EL-79-006, February 1979

Garver, L. L. (1966). "Effective Load Carrying Capability of Generating Units" IEEE Transactions on Power Apparatus and Systems, Vol. PAS-85, no. 8, pp. 910-919, August 1966.

Grinstead, C.M. and J.L. Snell (2003). "Introduction to Probability, 2<sup>nd</sup> Edition". American Mathematical Society, pp. 285-304, 2003.

Halperin, H. & H. A. Adler (1958). "Determination of reserve generating capability". Trans. AIEE, vol. 77, pt. III, pp. 530-544, August 1958.

Invernizzi, A., G. Manzoni and A. Rivoiro (1988). "Probabilistic simulation of generating system operation including seasonal hydro reservoirs and pumped-storage plants". Electrical Power and Energy Systems. Vol. 10, no. 1, pp. 25-35. 1988.

Kahn, E.P., (2004). "Effective Load Carrying Capability of Wind Generation: Initial Results with Public Data", Electricity Journal, December 2004.

Lee, F. N., M., Lin, and A. M., Breipohl (1990). "Evaluation of the variance of production cost using a stochastic outage capacity state model", IEEE Transactions on Power Systems, Vol. 5., No. 4, November 1990.

Leite da Silva, A.M., F. A. F. Pazo Blanco, and J. Coelho (1988). "Discrete Convolution in Generating Capacity Reliability Evaluation- LOLE Calculations and Uncertainty Aspects". Paper 88 WM 186-9. IEEE PES Winter Meeting New York, USA. January 1988.

Maceira, M. E. P and M-V. F. Pereira (1996). "Analytical modeling of chronological reservoir operation in probabilistic production costing [of hydrothermal power systems]", IEEE Transactions on Power System, Vol. 11, No. 1, February 1996

Malik, A. S. (2001). "Modelling and Economic Analysis of DSM Programs in Generation Planning", International Journal of Electrical Power & Energy Systems, Volume 23, Issue 5, June 2001.

Malik, A. S. (2004). "Simulating limited energy units within the framework of ELDC and FD methods", International Journal of Electrical Power & Energy Systems, Volume 26, Issue 8, October 2004.



Marcos, D. (2007). "Criteria for Valuating the Contribution of Power Plants to Long-Term Reliability". Proyecto Fin de Carrera. IIT, Universidad Pontificia Comillas, June 2007.

Milligan, M and K. Porter (2008). "Determining the Capacity Value of Wind: An Update Survey of Methods and Implementation". National Renewable Energy Laboratory, Conference Paper NREL/CP-500-43433. To be presented at WindPower, Houston, Texas. June 2008

MITyC, (2007). Ministerio de Industria y Comercio, Orden ITC/2370/2007, July 2007.

Ramos, A. & J. Arrojo (1991). Storage Plants Energy Optimization in Probabilistic Production Cost Models. IIT Working Paper IIT-91-008. January 1991.

Ramos, A., L. Muñoz-Moro & I. J. Pérez-Arriaga (1994). "Direct computation of derivatives in production cost models based on probabilistic simulation". Conference on Probabilistic Methods Applications in Power Systems, PMAPS-94. pp. 355-360. Rio de Janeiro, Brazil. September 1994.

Ringlee, R. J. and A. J. Wood (1969). "Frequency and Duration Methods for Power System Reliability Calculations. Part II. Demand Model and Capacity Reserve Model". IEEE Transactions, PAS-38, No. 4, April 1969.

Rodilla, P. and C. Batlle (2010). "Redesigning Probabilistic Production Costing Models and Reliability Measures in the Presence of Market Demand Elasticity". IIT Working Paper IIT-10-028A, June 2010. Submitted to IEEE Transactions on Power Systems. Available at www.iit.upcomillas.es/batlle.

Rodilla, P. (2010). "Regulatory Tools to Enhance Security of Supply at the Generation Level in Electricity Markets". PhD Thesis. IIT, Universidad Pontificia Comillas, September 2010.

Shorack, G.R. and J.A. Wellner (1986). "Empirical processes with applications to statistics". Wiley, New York, 1986.

Smith, J., M. Milligan, E. DeMeo, and B. Parsons (2007). "Utility Wind Integration and Operating Impact State of the Art", IEEE Transactions on Power Systems, Vol. 22, No. 3, August 2007.

Van Wijck, A. (1990). "Wind Energy and Electricity Production". Doctoral Thesis, Rijksuniversiteit, Utrecht, 1990.

Van der Vaart, A.W (1998). "Asymptotic statistics". Cambridge University Press, 1998.

# 3. ADDRESSING THE TIME-DEPENDENT NATURE OF DEMAND ELASTICITY IN AN ELECTRICITY MARKET

# 3.1 Introduction

As shown in Chapter 2 the incorporation of demand elasticity in the PPC framework requires that the elasticity be the same for every hour in the simulation period. Nevertheless, a major aspect that can not be neglected is that demand is strongly time-dependent. In this chapter we turn to the important problem of developing a methodology to characterize the elastic demand of a given electricity market.

The main objective of the following sections is to present a tool that captures the timedependent nature of demand (i.e. dependence on the calendar) and allows determining the elastic demand curves that will be used as an input to the model described in Chapter 2. At the same time, the tool will help to divide the time scope of the analysis in an adequate number of periods. In order to do so, historical elastic demand curves are classified and clustered.

# 3.1.1 Structure of the Chapter

The Chapter proceeds as follows. Section 3.2 reviews the concept of a Clustering Algorithm. Then, in Section 3.3, we describe the Neural-Gas Algorithm. In Section 3.4 we provide an insight on how the optimal number of clusters is determined and on how the time scope of analysis is divided. Section 3.5 concludes and discusses some applications and extensions.

# 3.2 Basic Concepts

Cluster analysis or clustering is a method of unsupervised learning by which a set of observations is assigned into subsets (called *clusters*) so that observations in the same cluster are similar in some sense; see Aldenderfer & Blashfield (1984).



Figure 3.2 Result of a cluster analysis shown as the colouring of the squares into three clusters.



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An important step in most clustering algorithms is to select a distance measure, which will determine how the *similarity* of two elements is calculated. This will influence the shape of the clusters, as some elements may be close to one another according to one distance and farther away according to another. Common distance functions include the Euclidean distance, the Manhattan distance, the maximum norm, the Mahalanobis distance or the Hamming distance.

As it has already been mentioned, clustering algorithms are unsupervised, that is to say, they try to find a hidden structure in unlabeled data (data that is not classified a priori).

Representing certain data using the clusters obtained necessarily entails a loss of detail, but facilitates the interpretation of the whole. In our case, the classification of historical elastic demand curves not only facilitates the interpretation of past data but also provides the possibility of determining the presence or absence of patterns on which to infer future trends.

For a thorough classification of clustering algorithms see, for instance, Romesburg (2004).

# 3.3 The Neural-Gas

The Neural Gas is an artificial neural network introduced by Martinetz & Schulten (1991). It is a simple algorithm for finding optimal data representations based on feature vectors. The algorithm was named "Neural Gas" because of the dynamics of the feature vectors during the adaptation process.

#### Distance Measure

Before digging into the details of the algorithm it is important to choose a distance measure, as the calculation process will be based on it. In this case, the most suitable distance measure between to curves is the area (positive definite) between them.



Figure 3.3 Distance between two demand curves

In Figure 3.3 two different elastic demands, namely  $\pi = D_a(q)$  and  $\pi = D_b(q)$ , are depicted. The distance between them (denoted by  $\delta$ ) is, according to the definition proposed, equal to the orange shaded area. Formally:

$$\delta(D_a, D_b) = \int_{\underline{\pi}}^{\overline{\pi}} \left| D_a^{-1}(\pi) - D_b^{-1}(\pi) \right| \cdot d\pi$$

Note that both elastic demand curves are defined within the same range of prices, denoted by  $[\underline{\pi}, \overline{\pi}]$ . The lower bound  $\underline{\pi}$  of this range is equal to zero, and the upper bound  $\overline{\pi}$  is lower than the Value of Lost Load (which ideally is the offer price of the inelastic demand).

As it will be explained in the next section, the clustering process is aimed to calculate, for each cluster, the amount of energy offered at every price within the aforementioned range; while reducing the distance between the clusters and the elastic demand curves they are meant to represent.

#### Algorithm

Let us consider a set of  $\Gamma$  historical elastic demand curves  $\pi = D_{\gamma}(q)$  Let H be the number of feature curves  $\pi = F_{\eta}(q)$  that will represent the whole set of historical data. The method to obtain the feature curves proceeds as follows:

Step 0. Initialize the iteration counter t = 0 and randomly choose a set of H feature curves from the  $\Gamma$  historical elastic demand curves.

In order to do the latter, generate a set of H random numbers  $\{x_1,...,x_H\}$  according to a discrete uniform distribution (DUI). As it may have been noticed, the DUI has been chosen as the elements of the original set of data represent equally likely scenarios (hourly elastic demand). Obviously, the possible values of the random numbers range from 1 to the number of vectors  $\Gamma$  within the original set of data. Thus, the set of feature curves will be  $\{F_1,...,F_H\} = \{D_{x_1},...,D_{x_H}\}$ 

Step 1. Another historical elastic demand curve  $\xi$  is randomly chosen

A random number x is generated according to a DUI as explained before. Consequently, the randomly chosen curve will be  $\xi = D_x$ 

Step 2. The distance of the *H* feature curves to the given historical elastic demand curve  $\xi$  is measured and the distance order is determined. Additionally, the feature curves are labelled accordingly.

 $k_{\eta}(\xi,F_{\eta}) = 0 \rightarrow F_{\eta} \,$  is the closest feature curve to  $\,\xi\,$ 

 $k_n(\xi, F_n) = 1 \rightarrow F_n$  is the second closest feature curve to  $\xi$ .



 $k_{\eta}(\xi, F_{\eta}) = H - 1 \rightarrow F_{\eta}$  is the most distant feature curve to  $\xi$ .

Step 3. Each feature curve is adapted according to the following expression:

$$\Delta F_{\eta}^{-1}(\pi) = \varepsilon(t) \cdot e^{\left(\frac{-k_{\eta}(\xi, F_{\eta})}{\lambda(t)}\right)} \cdot (\xi^{-1}(\pi) - F_{\eta}^{-1}(\pi) \qquad \forall \pi \in [\underline{\pi}, \overline{\pi}]$$

Where:

$$\mathcal{E}(t) = \mathcal{E}_i \left(\frac{\mathcal{E}_f}{\mathcal{E}_i}\right)^{\frac{t}{t_{\text{max}}}} \equiv \text{Adaptation step-size}$$

$$\lambda(t) = \lambda_i \left(\frac{\lambda_f}{\lambda_i}\right)^{\frac{t}{t_{\text{max}}}} \equiv \text{Neighbourhood range}$$

Step 4. The iteration counter is increased in one unit. As long as  $t \le t_{max}$  the algorithm goes back to Step 1. Otherwise, the procedure finishes.

After sufficiently many adaptation steps the feature curves cover the original data space (set of historical demand curves) with minimum representation error.

#### **Representation error**

The total representation error, denoted by TRE, is defined as the sum of the distances (defined as the area between two curves) between each original elastic demand curve  $D_{\gamma}$  and its closest feature curve  $F_n$ . Formally:

$$TRE = \sum_{\gamma=1}^{\Gamma} \min_{\eta} \delta(D_{\gamma}, F_{\eta})$$

This representation error can be used as a performance measure to evaluate the clustering algorithm, determine the optimal number of feature curves or initialize such parameters as the neighbourhood range or the adaptation-step-size.

#### 3.4 Analysis of the Time-Dependent Demand Elasticity

As shown in Chapter 2 the incorporation of demand elasticity in the PPC framework requires that the elasticity be the same for every hour in the simulation period. Nevertheless, a major aspect that can not be neglected is that demand is strongly time-dependent; see, for instance, Wood & Wollemberg (1996).

As a consequence, in order to address the "uniform behavior" assumptions required in the time abstracted PPC framework, the analysis needs to be split into multiple smaller-scale simulations. In particular, one simulation is needed for each time period in which a "uniform behavior" could be considered as a reasonable hypothesis for the elastic demand. Eventually, the results obtained from each one of simulations could be weighted according

to the number of hours they represented (i.e. the number of hours of the period in which a uniform behavior of elasticity is assumed).

In this Section we show how the clustering algorithm described in Section 3.3 is used to assess the number of time periods where the "uniform behavior" hypothesis can be considered as a reasonable assumption, how the optimal number of periods is determined and how the time scope of analysis is divided.

# 3.4.1 Determination of the Optimal Number of Clusters

As pointed out in Sugar & James (2003), the correct choice of the number of clusters is often ambiguous, with interpretations depending on the shape and scale of curves in the original data set and the desired clustering resolution.

However, increasing the number of clusters will tend to reduce the amount of representation error in the resulting clustering, to the extreme case of zero error if each original curve is considered its own cluster.

Intuitively then, the optimal choice of the number of clusters will strike a balance between maximum compression of the data using a single cluster, and maximum accuracy by assigning each data point to its own cluster.

# Balancing Accuracy and Simplicity

By means of an example, we will show how to choose qualitatively a set of clusters that is comprehensive enough but not unnecessarily exhaustive. Figure 3.4.1-I shows a set of hourly elastic demand curves (in yellow), and a single cluster (in green) that intends to represent the whole set. As it can be recognized, although the cluster is close to the vast majority of the demand curves, there are some outliers in the left side that greatly differ from the cluster that is supposed to be proxy for them. Hence, using a single cluster in this case is obviously not accurate.



Figure 3.4.1-I Historical demand curves. #1 Cluster



In Figure 3.4.1-II the same set of hourly elastic demand curves is displayed. Nonetheless, two clusters (in green and red respectively) are calculated in this case. As expected, one of the clusters (the green one) is approximately equal to the one calculated in the single cluster case, while the other (the red one) is quite similar to the aforementioned outliers. Thus, adding a second cluster greatly enhances the accuracy of the analysis in this particular case.



Figure 3.4.1-II Historical demand curves. #2 Clusters

Finally, Figure 3.4-III shows the result of calculating an additional representative (in blue). Including another cluster improves the accuracy of the clustering. However, it is not clear whether it is worth it. The third cluster is very similar to the first one, so it does not provide a relevant understanding of the dynamics of the elastic demand whereas it significantly hinders further analysis.



Figure 3.4.1-III Historical demand curves. #3 Clusters

We can therefore conclude that the two-cluster approach is (qualitatively) the one that better combines both accuracy and simplicity.

#### The Elbow method

The analytical method used to deciding on the number of clusters is the so-called Elbow method, see Long et. al (2010). It looks at the total representation error as defined in Section 3.3 and is founded upon a rather intuitive idea: The number of clusters should be chosen so that adding another cluster does not give much better modelling of the data.

More precisely, if the total representation error is plotted against the number of clusters, the first clusters will add much information (reduce notably the error), but at some point the marginal gain will drop, giving an angle in the plot. The number of clusters is chosen at this point, hence the "elbow criterion"

The marginal reduction of the Total Representation Error is intuitively defined as follows:

$$\Delta TRE_{\#Clusters} = \frac{TRE_{\#Clusters-1} - TRE_{\#Clusters}}{TRE_{\#Clusters-1}}$$

Chart 3.4.1 shows the Total Representation Error (and its marginal reduction) committed when using different number of clusters to represent a given set of hourly curves (the hourly curves a whole month).

	#1 Cluster	#2 Cluters	#3 Clusters	#4 Clusters
TRE [€]	3,728E+07	2,989E+07	2,901E+07	2,867E+07
∆TRE [%]	-	19,82%	2,96%	1,17%

Chart 3.4.1 Total Representation Error

As it can be observed in the chart and even more clearly in Figure 3.4.1-IV, the marginal reduction of the error drops dramatically beyond two clusters. Thus, there is no sense in further complicating the analysis and two clusters should be chosen in this case.



Figure 3.4.1-IV Total Representation Error

#### 3.4.2 Division of the Time Scope of Analysis

In Section 3.4.1, we have outlined a procedure for determining the optimal number of clusters for a given set of hourly elastic demand curves. Now we undertake the problem of defining the periods in which the time scope of analysis should be divided. This division must be done in such a way that the "uniform behavior" of elastic demand could be considered as a credible hypothesis for each period.

Figure 3.4.2-I shows a set of hourly elastic demand curves (in yellow), and a two clusters (in green and red respectively) that are supposed to represent the whole set.



Figure 3.4.2-I Clustering Analysis. Clusters and original curves

After having calculated the clusters, the next step is to determine which cluster is closer to each one of the original curves, that is to say, which cluster represents each one of the hourly elastic demand curves better.

In Figure 3.4.2-II the results of the calculation of the closest cluster are displayed. For every single day in the time scope of analysis (28 days in this case), each hour is labelled in either red or green if it is closer to either of the clusters (the colour code is consistent).

	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Day 1	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Day 2	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2
Day 3	2	2	2	2	2	- 2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2
Day 4	1	2	2	2	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Day 5	1	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Day 6	2	2	2	- 2	- 2	- 2	2	- 2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Day 7	1	- 2	2	2	2	- 2	2	- 2	- 2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Day 8	2	1	2	- 2	- 2	- 2	2	- 2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Day 9	2	1	2	2	2	2	2	- 2	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1
Day 10	2	2	2	2	2	2	2	2	1	1	1	1	1	- 2	- 2	2	1	1	1	1	1	1	1	1
Day 11	2	2	2	2	2	- 2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Day 12	2	2	2	2	2	- 2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Day 13	2	2	2	2	2	- 2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Day 14	2	2	2	2	2	- 2	2	2	- 2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Day 15	2	2	2	2	2	- 2	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Day 16	2	2	-2	2	2	- 2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Day 17	2	1	1	2	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2
Day 18	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2
Day 19	2	2	2	2	2	- 2	2	2	- 2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Day 20	2	2	2	2	2	- 2	2	2	2	1	1	1	1	1	1	1	- 2	2	1	1	1	1	1	1
Day 21	- 2	2	2	2	2	- 2	2	2	1	1	1	1	1	1	1	1	1	- 2	- 2	1	1	1	1	1
Day 22	1	2	2	2	2	- 2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Day 23	2	2	2	2	2	- 2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Day 24	- 2	2	2	2	2	- 2	2	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Day 25	1	2	2	2	2	- 2	2	- 2	- 2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Day 26	1	2	2	2	2	2	2	1	1	1		1	1	1	1	2	1	1	1	1	1	1	1	1
Day 27	1	2	2	2	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Day 28	1	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2

Figure 3.4.2-II Clustering Analysis. Closest cluster

After carefully examining the results, a pattern can be recognized. In the first hours of the day, the so-called off-peak hours, elastic demand is quite similar. A certain affinity among the rest of the hours (peak hours) is also perceived. Therefore, it seems logical to divide the time scope of analysis into two periods, namely A and B, as illustrated in Figure 3.4.2-III.



			Pe	ric	d.	A			Period B															
	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Day 1	2	2	2	2	2	2	2	2	1	1		1	1	1	1	1	1	1	1	1	1	1	1	1
Day 2	2	2	2	2	2	2	2	2	1	i i	1	1		1	1	1	1			1		1	1	2
Day 3	2	2	2	2	2	2	2		1	1	1		- 1		1	1	1			1		1	1	2
Day 4	1	2	2	2	2	2	2		1		Ŧ	1	1	1	1	1	1	1	1		1	4	. 1	1
Day 5		2	2	2	2	2	2	2	1		1		4	1		1	1	1	1	1				
Day 6	2	2	2	2	2	2	2	2	1				- 1	1	1				1	1		1		
Day 7		2	2	2	2	2	2	2	2	2	1		1	1	1	1	1		1	1	4	1	. 1	14
Day 8	2		2	2	2	2	2	2	2		1			1	1	1		1	6	1		1		1
Day 9	2	- 1	2	2	2	2	2	2	1	- 1	1	4	1	1	1	2		1	1		1	1	. 1	1
Day 10	2	2	2	2	2	2	2	2	1	1	1	1	- 1	2	2	2	1		1		1		- 8	4
Day 11	2	2	2	2	2	2	2	2	1		1		1	1	1	1	1	1			- 1			
Day 12	2	2	2	2	2	2	2	2	1		1		1	1	1		1	1	1	1	1		1	11
Day 13	2	2	2	2	2	2	2	2	1		1		-1	1	1		1		1	1	1			1
Day 14	2	2	2	2	2	2	2	2	2	1	1			1	1	1	1		1	1	i i	1		1
Day 15	2	2	2	2	2	2	2	2	2	1	1		1	1	1	Ť	1	1	1	1		1	1	1
Day 16	2	2	2	2	2	2	2	2	1		1	1	1	1	1	1	1	1	1	1		1	T	1
Day 17	2	1	1	2	2	2	2		7		1		1	1	1	1	1	1		1		1	1	2
Day 18	2	2	2	2	2	2	2	2	1	1	1	1		1		1		1		1	1		2	2
Day 19	2	2	2	2	2	2	2	2	2	1	1			1	1	1	1	1	1	1		1	1	1
Day 20	2	2	2	2	2	2	2	2	2	14	1			1	1	i	2	2	1	1		1		1
Day 21	2	2	2	2	2	2	2	- 2	1		1	1		4		÷,	1	2	2			1	i ii	1
Day 22		2	2	2	2	2	2		1				4	1				1						
Day 23	2	2	2	2	2	2	2		1				1	1				1				1		
Day 24	2	2	2	2	2	2	2	2	2	2	1		1	1		1		1		1		1	- 1	
Day 25		2	2	2	2	2	2	12	- 2		1		1	1	1	1	1	1		1	1	1		1
Day 26		2	2	2	2	2	2		1	1	1	1	1	1	1	2	1	1		1	1	1	- 8	1
Day 27	1	2	2	2	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Day 28		2	2	2	2	2	2	2	2	11	1	1	1	1	1	1	1		1	1	1	1	1	2

Figure 3.4.2-III Clustering Analysis. Definition of periods

Eventually, in order to further improve the accuracy of the analysis while maintaining its comprehensiveness another step can be taken. Instead of using the clusters calculated previously we can divide the original set of hourly curves into two subsets (one corresponding to each period) and calculate a single cluster for each one of them, as shown in Figure 3.4.2-IV.



Figure 3.4.2-IV Clustering Analysis. Single cluster per period

# 3.4.3 Elastic Demand Clustering Tool

In order to carry out the analysis proposed in this section a robust and flexible clustering tool has been developed. This tool performs two rather distinct functions.

- On the one hand, it provides the user with the optimal number of clusters for a given time scope (e.g. four curves that represent the different types of hourly curves of a given month). Additionally, it helps to divide the time scope of analysis into different periods balancing accuracy and simplicity.
- On the other hand, it allows the user to obtain feature curves according to specified clustering criteria (i.e. clusters for peak and off-peak hours, working days and holidays, summer and winter seasons, etc.).

For the purposes of the model described in Chapter 2, the major outputs of this tool are both a definition of the periods  $\psi$  in which the time scope of analysis  $\Psi$  is divided, and the elastic demand curves  $\pi = D_{\psi}(q)$  that are supposed to apply for every hour within aeach one of those periods.



Figure 3.4.3 Time Structure



# 3.5 Conclusions

In this Chapter, the challenge of developing a tool that captures the time-dependent nature of elastic demand has been met. The tool proposed allows for finding patterns and obtaining clusters to realistically describe the elastic demand while avoiding the inconvenience of taking into account too many hourly demand curves.

This clustering tool will be used in Chapter 4 to characterize the elastic demand of the Spanish electricity market. That is to say, it will be used to determine how many different elastic demand scenarios are necessary to fairly represent the whole Spanish elastic consumption.

#### 3.6 References

Aldenderfer, M.S. & R.K. Blashfield (1984). "Cluster Analysis". Sage Publications, Newbury Park (CA), 1984.

Long, B., Z. Zhang & P.S. Yu (2010). "Relational Data Clustering. Models, Algorithms and Applications, 1<sup>st</sup> Edition". Chapman and Hall, CRC Data Mining and Knowledge Discovery Series, 2010.

Martinetz, T. & K. Schulten (1991). "A Neural Gas Network Learns Topologies". Artificial Neural Networks, Elsevier Science Publishers, pp. 397-402, 1991.

Romesburg, H.C. (2004). "Cluster Analysis for Researchers". Lulu Press, North Carolina, 2004.

Sugar, C. A. & G. M. James (2003). "Finding the number of clusters in a data set: An information theoretic approach". Journal of the American Statistical Association 98, pp. 750–763, January 2003

Wood, A.J. & B.F. Wollemberg (1996). "Power Generation, Operation and Control". , 1996

# 4. CASE EXAMPLE: APPLICATION TO THE SPANISH ELECTRICITY MARKET

# 4.1 Introduction

In Chapter 2 we formulated a Probabilistic Production Costing model to estimate the reliability of a power generation system. The model proposed is particularly suited for carrying out reliability assessments in electricity markets where elastic demand consumption is substantial. Notwithstanding, the model is also able to cope with a wide variety of peculiarities that arise in electricity wholesale markets.

Additionally, in Chapter 3, we developed a tool that captures the time-dependent nature of demand elasticity. The tool produced allows for finding patterns and obtaining clusters to realistically describe the elastic demand of a given electricity market.

In this Chapter, we proceed to face the analysis of a real-size electricity market. We will test the capabilities of the models described in this dissertation by carrying out a simulation of the Spanish electricity market for 2016. In this context, we will put a strong emphasis on taking into consideration demand elasticity. Furthermore, we will thoroughly evaluate how traditional reliability measures (as, for instance, the Loss of Load Probability or Expected Non-Served Energy) are affected when such non-negligible demand elasticity is considered.

# 4.1.1 Structure of the Chapter

The Chapter proceeds as follows. In Section 4.2 we describe the characteristics of the Spanish electricity market that is going to be analysed, the assumptions made and the data that is going to be used as an input for the PPC model. Section 4.3 presents the structure of the simulation. Section 4.4 reviews and discusses the results obtained. Section 4.5 concludes and comments some applications and extensions.

# 4.2 The Spanish Electricity System

This section is devoted to describe the way the Spanish electricity market has been modelled. The structure of the section mirrors that of Chapter 2. In this case however, instead of modelling each one of the elements of the power system within the PPC framework we present the data that is going to be used in the simulation, along with the assumptions and projections made.

# 4.2.1 Load Scenario

In order to obtain the hourly values of load that are going to be used as an input to the PPC model as described in Section 2.3.1 we take, as a starting point, the chronological load profile from 2009. This hourly load profile is scaled up according to an estimation of demand growth as depicted in Figure 4.2.1.





Figure 4.2.1Load Estimated Profile for 2016

In order to estimate the demand growth, a central scenario is constructed from the historical series on demand growth at power plant bars and GDP from 1993 to date. Then an ARX-type model is used to fit the demand/GDP curve. The results of are shown in Chart 4.2.1.

	2009	2010	2011	2012	2013	2014	2015	2016
Annual Load [PWh]	254,1	249,7	250,2	253,2	257,7	263,5	270,8	279,5
∆L oad [%]	-3,73	-1,75	0,21	1,22	1,77	2,26	2,75	3,32

Chart 4.2.1 Evolution of the Annual Load

# 4.2.2 Thermal Generation

As outlined in Section 2.3.1 the parameters that are used to characterize the behaviour of a thermal generator t are its capacity  $\overline{q}_t$ , its forced outage rate  $FOR_t$  and its marginal  $\cot MC_t$ .

Both the capacities and the forced outage rates of the thermal plants that are currently operating in the Spanish system are available at REE (2010). With regards to the estimation of the installed capacity in Spain by 2016, the expected decommissioning of fuel-oil and coal plants is taken into account.

The fuel variable cost has been used as proxy for the marginal cost. Additionally, the carbon dioxide emissions price has been estimated. The parameters that characterize the set of thermal generators that have been used in the analysis are gathered in Chart 4.2.2.

4	$\mathbf{q}_{\mathrm{t}}$	MCt	FORt	4	qt	MCt	FORt		$\mathbf{q}_{t}$	MCt	FORt	4	$\mathbf{q}_{t}$	MCt	FORt
Ľ	[MW]	[€/MW]	[-]	Ľ	[MW]	[€/MW]	[-]	Ľ	[MW]	[€/MW]	[-]	Ľ	[MW]	[€/MW]	[-]
#1	1008	18,0	16,20%	<b>#</b> 28	361	43,4	4,80%	<b>#</b> 55	375	43,8	0,80%	#82	398	43,8	6,80%
#2	1017	18,0	15,70%	#29	400	43,8	12,70%	#56	400	43,8	6,30%	#83	160	44,7	3,10%
#3	1028	18,0	34,30%	#30	374	43,8	14,50%	#57	400	43,8	6,30%	#84	366	45,1	2,00%
#4	1027	18,0	7,70%	#31	387	43,8	0,00%	#58	400	43,8	29,50%	#85	366	45,1	3,70%
#5	1055	18,0	12,90%	#32	375	43,8	2,50%	<b>#</b> 59	400	43,8	29,50%	#86	367	45,1	6,30%
#6	466	18,0	10,60%	#33	396	43,8	2,30%	#60	393	43,8	0,50%	#87	368	45,6	22,40%
#7	1066	18,0	16,00%	#34	379	43,8	1,80%	<b>#</b> 61	393	43,8	0,50%	#88	368	45,8	21,20%
#8	1087	18,0	40,00%	#35	420	43,8	5,60%	<b>#</b> 62	390	43,8	5,20%	#89	366	45,8	0,50%
#9	364	36,5	8,10%	#36	420	43,8	5,60%	<b>#</b> 63	394	43,8	5,40%	#90	155	46,0	6,00%
#10	254	37,2	4,40%	#37	393	43,8	2,90%	#64	394	43,8	31,90%	#91	563	47,0	40,70%
<i>#</i> 11	556	37,3	2,40%	#38	393	43,8	2,60%	<b>#</b> 65	393	43,8	0,00%	<b>#</b> 92	221	47,5	16,80%
#12	350	37,6	0,90%	<b>#</b> 39	375	43,8	5,80%	#66	401	43,8	8,20%	<b>#</b> 93	65	50,3	0,00%
<b>#</b> 13	360	38,0	16,20%	#40	375	43,8	5,80%	<b>#</b> 67	396	43,8	3,10%	<b>#</b> 94	320	76,0	15,20%
#14	358	38,8	45,90%	#41	382	43,8	7,00%	#68	398	43,8	2,70%	<b>#</b> 95	120	97,6	0,00%
#15	568	39,0	7,70%	<b>#</b> 42	401	43,8	4,10%	<b>#</b> 69	380	43,8	25,10%	<b>#</b> 96	350	97,6	41,90%
#16	582	39,1	5,00%	#43	200	43,8	10,30%	#70	380	43,8	25,10%	#97	70	102,3	20,00%
#17	577	39,6	37,10%	#44	393	43,8	8,40%	#71	404	43,8	4,20%	<b>#</b> 98	140	102,3	10,50%
#18	371	39,8	7,70%	#45	425	43,8	12,70%	<b>#</b> 72	400	43,8	5,80%	<b>#</b> 99	289	102,3	11,40%
#19	166	39,9	0,20%	#46	425	43,8	13,20%	<b>#</b> 73	393	43,8	3,50%	#100	289	105,2	11,30%
#20	155	40,1	0,60%	#47	419	43,8	12,70%	<b>#</b> 74	397	43,8	6,80%	#101	520	110,4	0,10%
#21	284	41,1	7,00%	#48	398	43,8	3,40%	<b>#</b> 75	389	43,8	3,40%	<i>#</i> 102	350	112,5	3,60%
#22	350	41,7	0,30%	#49	393	43,8	24,40%	<b>#</b> 76	405	43,8	0,30%	<b>#</b> 103	350	113,2	10,90%
#23	369	41,9	35,20%	#50	378	43,8	2,10%	<b>#</b> 77	393	43,8	3,80%	#104	542	115,5	9,00%
#24	350	42,0	1,30%	#51	378	43,8	3,80%	#78	373	43,8	4,20%	<b>#</b> 105	377	115,6	10,30%
#25	365	42,9	0,00%	#52	375	43,8	0,40%	<b>#</b> 79	384	43,8	5,50%	<b>#</b> 106	314	116,7	3,10%
#26	217	43,1	0,10%	#53	375	43,8	0,40%	#80	393	43,8	6,80%	<b>#</b> 107	314	116,9	92,60%
#27	324	43,2	66,30%	#54	375	43,8	0,80%	#81	393	43,8	3,10%				

Chart 4.2.2 Thermal Generation Data

Figure 4.2.2 shows the implied supply function when every single generator is available.



Figure 4.2.2 Thermal generation supply curve



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# 4.2.3 Hydro Generation

As described in Section 2.4.1 the parameters that are used to characterize a hydro unit h in a period  $\psi$  are its maximum capacity  $\overline{q}_{h}^{\psi}$ , its minimum output  $\overline{q}_{h}^{\psi}$  and the available energy

 $\vec{e}_{h}^{\psi}$ . Those parameters are derived from historical values using the GEHA, developed in Batlle (2002).

# A Preliminary Division of the Time Scope of Analysis

As put across in Section 3.4, in order to address the "uniform elasticity" assumptions required in the PPC framework, the analysis needs to be split into multiple smaller-scale simulations. The characteristics of the data regarding hydro units, straightforwardly suggest a preliminary division of the time scope of analysis.

The available energy provided by the model is given monthly. Therefore, it seems reasonable to divide the time scope of analysis, at least, in months.

#### Hydro Scenarios

In order to capture the annual variability of hydro production, five different scenarios of hydro energy availability are considered. The scenarios are obtained from historical data as follows:

- Historical scenarios of hydro production are sorted based on the amount of energy produced.
- The hydro scenarios that represent the 5, 25, 50, 75 and 95-percentiles of the distribution are selected.

Figure 4.2.3 shows the five scenarios of hydro production that have been evaluated. Obviously, the results obtained from the analysis of each one of the scenarios must be weighted according to their likelihood.



Figure 4.2.3 Scenarios of Monthly Available Hydro Energy

#### 4.2.4 **RES** Generation

In order to obtain the hourly values of production of Renewable Energy Sources that are going to be used as an input to the PPC model as described in Section 2.5 we take, as a starting point, the chronological production profile from 2009. This hourly production profile is scaled up in consonance with the estimated RES production increase for 2016 based on governmental planning (that is to say, ~40%) as shown in Figure 4.2.4.



Figure 4.2.4 RES Generation Estimated Profile for 2016

# 4.2.5 Elastic Demand

In this case example we consider two different sources of demand elasticity: On the one hand, the elasticity reflected and extracted from the bids made by participants in the Spanish power pool in the past years and, on the other hand, the interruptible load bilaterally contracted by the Transmission System Operator to the largest industrial consumers in the system.

#### Demand Bids from Market Participants. Price-Sensitive Demand

We have taken into account the bids made hourly by market participants in the day-ahead Spanish electricity market along 2010.





Figure 4.2.5-I Aggregate supply and demand curves. Spanish electricity market (Source: OMEL)

Figure 4.2.5- shows, among other things, the aggregate demand curve (in light blue) for a given hour. As described in OMEL (2001), according to the market rules the bid price in the Spanish pool is capped at 180.3 €/MWh. Therefore, those bids whose price is equal to this cap can be considered to be inelastic. Consequently, the only bids that interest us are the ones made by agents who are price-sensitive (the agents who actually declare the maximum price they are willing to pay). We will now illustrate how the clusters of this elastic demand curves are obtained.

Step 1. As a first step then, we must obtain the historical aggregate demand curves and remove those bids whose price is equal to the maximum allowed price. In Figure 4.2.5- a set the elastic demand curves is shown.



Figure 4.2.5-II Historical hourly elastic demand curves. March 2010

Step 2. Secondly, we use the clustering tool described in Chapter 3 to determine the number of clusters that better represent the historical data gathered. As argued in

Section 4.2.3, the time scope of analysis will, at least, be divided in months. Accordingly, the clustering process will aim to determine the optimal number of clusters for each month.

Figure 4.2.5-III illustrates the result of calculating different numbers of clusters for the data presented in Figure 4.2.5-II Historical hourly elastic demand curves. March 2010.



Figure 4.2.5-I Different number of clusters. March 2011

As explained in Section 3.4.1, in order to determine the optimal number of clusters the total representation error committed has to be measured. Chart 4.2.5 shows the marginal reduction of the total representation error committed when using different numbers of clusters to represent the hourly curves of each month.



	$\Delta TRE_{1-2}$	$\Delta \text{TRE}_{2-3}$	$\Delta TRE_{3-4}$
January	14,79%	2,31%	0,19%
February	15,82%	12,76%	5,34%
March	19,19%	-6,57%	3,10%
April	19,82%	2,96%	2,21%
May	14,70%	3,26%	1,92%
June	22,48%	1,89%	1,57%
July	22,65%	2,32%	9,36%
August	23,58%	2,62%	0,19%
September	15,22%	3,06%	0,19%
October	19,80%	-1,43%	-1,29%
November	24,55%	2,02%	0,31%
December	19,85%	8,04%	-1,04%
Average	19,37%	2,77%	1,84%

Chart 4.2.5 Marginal reduction of the representation error

As it can be noticed, the marginal reduction of the error does not decrease significantly when adding a third or a fourth cluster. Therefore, it does not make sense to further complicate the analysis and two clusters per month will be calculated. This decision is reinforced by merely having a look at Figure 4.2.5-III, in which the average Total Representation Error committed per month is depicted. The so-called "elbow" (see Section 3.4.1) is clearly in the neighbourhood of the point that represents the error committed when using two clusters.



Figure 4.2.5-III Average representation error per month

Step 3. Finally, the time scope of analysis must be divided appropriately. Each month will be split into two periods. In order to define those periods, we must calculate

which cluster is closer to the elastic demand of each hour. In Figure 4.2.5-III the hours of a given month are labelled according to which their closest cluster is.

	1	1	O	ff-l	Pe	ak	ġ.		Peak															
	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Day 1	1	1	2	2	2	2	2	2	T.	1	1	1	i.	1	1	1	1	1	1	1	1	2	1	1
Day 2		2	2	2	2	2	2	2	1			- 1		1		1				1		1	1	
Day 3	1	2	2	2	2	2	2			1		1	1	1		1		1		1			1	1
Day 4	1	2	2	2	2	2	2		1			1				1				1		1	1	11
Day 5	1	2	2	2	2	2	2	- 2	1		1			1					T	1	1	1		1
Day 6	2	ET.	2	2	2	2	2	2	2	1				1		1		- 1			1	*	1	1
Day 7	1	1	2	2	2	2	2	2	2	1		Ŧ		1		T.		1	1	1	1	1	Ť.	2
Day 8	2	2	2	2	2	2	2	2	1					4	1			1	2	2	2	1	1	2
Day 9	2	2	2	2	2	2	2		1	1		1	1	1		1	11	- 1	1	1	1	1	1	3
Day 10	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	2	1	1	2
Day 11	2	2	2	2	2	2	2	2	Ť	1			1	Ť	1	1	1		1	1	1	Ť	1	1
Day 12	2	2	2	2	2	2	2	2	Ŧ	1		1		t.	1	1	1	1	1	1		1	1	1
Day 13	2	2	2	2	2	2	2	1	1			1		1	2	2	2		1	1		1	2	
Day 14	2	2	2	2	2	2	2	2	Ť.	1		1	1	1	1	2	1	3		1		1	1	
Day 15	2	2	2	2	2	2	2	2	1	2		1		1	1	1				- 1			1	
Day 16	2	2	2	2	2	2	2	2	1	T	Ť			1		1		1	1	1		1		
Day 17	2	2	2	2	2	2	2	2	2	1	1	2		1		1					1	1		
Day 18	2	2	2	2	2	2	2	2	1			1		1		1		1			1	2		
Day 19	2	2	2	2	2	2	2	2	1							*				1	1	2	T	
Day 20	2	2	2	2	2	2	2		1	1	1	1	1	1		1	11	-3	1	1	2	2	2	13
Day 21	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	1
Day 22	1	1	2	2	2	2	2	1	Ť.	1			1	1	1	1	1		1	1		1	1	1
Day 23	2	2	2	2	2	2	2	2	T.			1		đ.		1		1	1	1		2	1	1
Day 24	2	2	2	2	2	2	2	-2	11			1	2	2	1					1		1	1	
Day 25		2	2	2	2	2	2	1	1	1		1	4	1	1	1		1		1		1	1	1
Day 26	2	2	2	2	2	2	2	2	T			1		1						1		1	1	
Day 27	2	2	2	2	T	2	2	2	1	1				1		1		1	1			T		
Day 28	1	2		2	1	2	2		T.					1		- 1		3			1	*	Ť	
Day 29	1	2	2	2	2	2	2	1	1	1		1		1		1					1	1	Ť	1
Day 30	1	2	2	2	2	2	2	2	2					1						4			1	2

Figure 4.2.5-III Division of a month in periods

The division of the month in this case in peak and off-peak hours seems to be more than reasonable. The division of the rest of the months is carried out analogously, and the results obtained are similar. Therefore, every month is divided into a peak and an off-peak period. It is important to note, however that the definition is not exactly the same for all of them (i.e. the off-peak period may be1h-8h, 1h-9h, 23h-8h or so).

# Interruptibility Service. Reserve-Sensitive Demand

The so-called "Interruptibility Service", see MITyC (2007, is a demand management tool that allows the Transmission System Operator giving a rapid and efficient response to the needs of the electricity system in emergency situations. It consists of a reduction of the active power demanded to a residual power level in response to a power reduction order issued by Red Eléctrica de España, the TSO, to consumers subscribed to this service.



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In 2010, there were 142 existing contracts in the Spanish mainland system. According to those contracts, the total interruptible power manageable by the TSO in periods of maximum demand reaches approximately the amount of 2112 MW.

The criterion to draw upon this load shedding is not based on prices but on the TSO's evaluation of the reserve margin at any given moment. Actually, the TSO can issue an order when this margin is below the 10% of the system's load.

Therefore, in line with the methodology described in Section 2.6.2, the interruptible load (IL) will be modelled for each period  $\psi$  as a fictitious generator characterized by the following parameters:

- $\bar{q}_{IL}^{\psi} = 2112 \text{ [MW]}$
- $FOR_{IL} = 0$  [-]
- $MC_{IL} \equiv$  Marginal cost of fictitious generator IL in period  $\psi$ . It is set in such away that the position of the fictitious generator IL allows for the desired level of reserve margin.

The reserve margin for each period, namely  $RM^{\psi}$  (in MW), is calculated as a given percentage ( $\varepsilon_{RM}$ ) of the maximum load of that period, denoted by  $l_{Max}^{\psi}$ . Formally:

$$RM^{\psi} = \varepsilon_{RM} \cdot l_{Max}^{\psi} = \frac{10}{100} \cdot l_{Max}^{\psi}$$

# 4.3 Simulation Structure

The simulation proceeds as follows. For each one of the five hydro scenarios a whole year is simulated. Each one of those years is subsequently divided up into months (hydro parameters vary monthly), and in both a peak and an off-peak period (elastic demand is supposed to behave uniformly in each one of those periods). Therefore, a total of 120 simulations are carried out (five hydro scenarios times twelve months times two periods of elastic demand). Figure 4.3-I summarizes the structure of the simulation.



Figure 4.3-I Simulation Structure

For each one of the simulations the input data is comprised of:

1) The de-cumulative distribution function of the hydro thermal load for that period.



Figure 4.3-II DDF of the hydro-thermal load for a given period

2) The cluster of the historical elastic demand curves of that period.



Figure 4.3-III Cluster of the elastic demand curves for a given period

3) The data related to thermal plants, which is supposed to be equal for every period.



Figure 4.3-IV Thermal Supply Curve when every generator is available



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- 4) The data related to the interruptible load, which is supposed to be equal for every period as well.
- 5) And the data related to hydro units. The disaggregation presented (by peak and offpeak hours) allows us to better represent the dispatch of hydro plants. The manageable production (which was termed as available energy over minimum in Section 2.3.1) is only used in peak hours while the run-of-the-river output is kept both in peak and off-peak hours.

Figure 4.3-V shows the equivalent supply curve for a given period. This curve is built upon the modeling assumptions described in Chapter 2. As it can be noted, elastic demand bids are modeled as equivalent thermal generators; hydro generation is split into the run-of-the-river output (offered at zero price) and the manageable production and the interruptible load is modeled as an equivalent generator that allows for a certain reserve margin.





Figure 4.3-V Equivalent Supply Curve for a given period

# 4.4 Case Example Results

In order to illustrate the discussion regarding the need for new reliability measures presented in Chapter 1, we ran two simulations:

- "Active Role of Demand" scenario. In this simulation demand elasticity was taken into account as described in Section 4.2.
- "Business As Usual" scenario. In this case, demand elasticity is neglected and all the load is supposed to be inelastic (as in conventional PPC models)

Figure 4.4-I shows the result of the probabilistic loading for a given period when demand elasticity is taken into consideration.



Figure 4.4-I Successive Equivalent Load Curves. Active Role of Demand

Figure 4.4-II shows the result of the probabilistic loading for the same period when demand elasticity is left aside.



Figure 4.4-II Successive Equivalent Load Curves. Business As Usual



# 4.4.1 Classical Reliability Measures

We can compare the different values of the traditional reliability measures obtained in the two simulations.

	Active Role of Demand	Business As Usual
LOLP [-]	2,16·10 <sup>-16</sup>	1,76·10 <sup>-12</sup>
ENSE [MWh/hour]	5,164·10 <sup>-14</sup>	5,193·10 <sup>-10</sup>

#### Chart 4.4.1 Classical Reliability Measures. Results

Even in the case of the Spanish electricity system, characterized by a large overcapacity, the magnitude order of the difference is significant. As expected, there is a considerable difference in the values of the LOLP and the ENSE that depends on whether elastic demand is taken into account or not.

As anticipated in Rodilla & Batlle (2009) traditional reliability measures leave aside very meaningful information when used to assess the performance of a system where elastic demand plays an important role. In neither of the scenarios evaluated the information provided by those metrics is fully comprehensive, as they do not capture the total value of the bids that were not accepted.

In the case of the "Active Role of Demand" scenario, this value may be exorbitant but it would lie hidden behind a ridiculously low LOLP that only takes into account the fully inelastic demand. Alternatively, in the case of the "Business As Usual" scenario, the total value of the bids that were not accepted may be negligible but this fact would be concealed by a high LOLP that would lead to think that the system does not meet the minimum reliability requirements.

# 4.4.2 Value of Non-Purchased Energy

In order to attain a better understanding of the reliability of the system we evaluate the distribution function of the Value of Non-Purchased Energy (see Section 2.7). Figure 4.4.2 shows the distribution function calculated in the "Active Role of Demand" scenario (obviously, it does not make sense to evaluate it in the "Business As Usual" scenario as demand is assumed to be inelastic).



Figure 4.4.2 Distribution function of the VNPE & Risk Measures

In addition to the distribution function and the probability mass function of the VNPE, conventional risk measures such as the expected value (mean of the distribution), the Valueat-Risk and the Conditional Value-at-Risk have been included; see Philippe (2006) and Rockafellar & Uryasev (2002) respectively. Chart 4.4.2 summarizes the results obtained.

Expected Value	[k€/hour]	59,31
Expected Value*	[M€/year]	519,56
VaR <sub>95</sub> (VNPE)	[k€/hour]	91,60
CVaR <sub>95</sub> (VNPE)	[k€/hour]	96,00

Chart 4.4.2 VNPE.	Risk	Analysis
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# 4.5 Conclusions

In this Chapter we have faced the analysis of a real-size electricity market. By carrying out a simulation of the Spanish electricity market for 2016, some of the capabilities of the models developed in this dissertation have been tested.

- The model described in Chapter 3 has proven itself as a useful and powerful clustering tool. It has allowed us to efficiently characterize the elastic demand of a real-size electricity market.
- The Probabilistic Production Costing model developed in Chapter 2 has allowed for evaluating the reliability of a real-size power system both in terms of conventional reliability measures such as the LOLP and the ENSE, and in terms of the Value of Non Purchase Energy. The distribution function of the latter could be compared with

that resulting from a benchmark, as shown in Figure 4.5-1. This comparison would be useful to provide a more precise idea on how well the market is performing its function.



Figure 4.5-I VNPE benchmarking

Additionally, the simulations carried out have reinforced the points of view made explicit all along Chapter1. There we presumed that classical reliability measures were not meaningful when used to assess the performance of a system where elastic demand was important. In this Chapter we have verified that statement. Eventually, we have made evident the potential of the Value of Non-Purchased Energy as an alternative or complementary reliability metric in a context where elastic demand is substantial.

# 4.6 References

Batlle, C. (2002). "A Model for Electricity Generation Risk Analysis". PhD Thesis. IIT, Universidad Pontificia Comillas, 2002.

MITyC, (2007). Ministerio de Industria y Comercio, Orden ITC/2370/2007, July 2007.

OMEL, (2011). "Reglas de Funcionamiento del Mercado de Producción Eléctrica". Boletín Oficial del Estado , 5 April, 2001.

REE, (2010). "El Sistema Eléctrico Español". Red Eléctrica de España, pp. 42-48, July, 2010.

Philippe, J. (2006). "Value at Risk: The New Benchmark for Managing Financial Risk,3rd Edition". McGraw-Hill, 2006.

Rockafellar, R.T. & S. Uryasev (2002). "Conditional Value-at-Risk for general loss distributions". Journal of Banking and Finance, Elsevier, 2002.

Rodilla, P. and C. Batlle (2010). "Redesigning Probabilistic Production Costing Models and Reliability Measures in the Presence of Market Demand Elasticity". IIT Working Paper IIT-10-028A, June 2010. Submitted to IEEE Transactions on Power Systems. Available at www.iit.upcomillas.es/batlle.



#### 5. CONCLUSIONS

This last chapter is dedicated to evaluating the conclusions that result from the research conducted in this dissertation. It includes a brief summary of the analysis, developments and findings that constitute the core of this work.

#### 5.1 Summary

This dissertation has addressed the problem of developing an adequacy analysis model for electricity markets in which elastic demand plays a significant role. This issue is currently of the most relevance due to the reforms that have been introduced in the power industry worldwide and the upcoming technical and regulatory developments of smart grids and demand response tools that will allow for an increasingly active role of demand.

The following points summarize the analysis carried out and the results obtained in this dissertation.

# Development of a PPC model

We have developed a PPC model that is able to cope with a wide variety of peculiarities of electricity wholesale markets. The proposed formulation meets the following modelling challenges:

- Firstly, the model allows for analysis in a conventional thermal generation system • with fully inelastic load.
- Secondly, the conventional PPC model has been adapted for loading energy limited • units.
- Thirdly, the conventional PPC model has been extended with regard to taking into account intermittent renewable generation.
- Fourthly, an equivalent load and generation model has been constructed to integrate price sensitive demand into the conventional PPC model.
- Fifthly, the PPC model has been adjusted to properly reflect the effect of the reservesensitive demand.
- Sixthly, an algorithm to compute the distribution function of the Value of the Non-• Purchased Energy has been implemented within the PPC frame-work.

#### Addressing the Time-Dependent Nature of Demand Elasticity

We have developed as well a methodology that captures the time-dependent nature of demand elasticity. The tool produced allows for finding patterns and obtaining clusters to realistically describe the elastic demand. Additionally, the clustering process is aimed at being comprehensive enough but not unnecessarily exhaustive. The tool performs two rather distinct functions.
- On the one hand, it provides with the optimal number of clusters for a time scope (e.g. four curves that represent the different types of hourly curves of a given month). Additionally, it helps to divide the time scope of analysis into different periods balancing accuracy and simplicity.
- On the other hand, it allows obtaining feature curves according to specified clustering criteria (i.e. clusters for peak and off-peak hours, working days and holidays, summer and winter seasons, etc.).

## Power System Reliability Assessments

Two results regarding reliability assessments of power systems could be highlighted.

- We have verified, in a real-size case example, that traditional reliability measures are not fully comprehensive in a context where elastic demand consumption is substantial.
- We have made evident the potential of the Value of Non-Purchased Energy as a substitute or complementary reliability metric in a context where elastic demand plays an active role. The VNPE could help to reveal a scarcity situation that would otherwise lie hidden behind ridiculously low values of LOLP and ENSE that only take into account the fully inelastic demand.



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