

HYDRO RESOURCE MANAGEMENT IN A CONTEXT OF ELECTRICITY MARKETS INCOMPLETENESS: REGULATORY IMPLICATIONS

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Abstract.

Since the very beginning of the power system reform process, regulators' main concern has been whether the market, of its own accord, is able to lead agents' decisions in such a way that system welfare is maximized. As well known, particularly in the electric power context, different market failures may prevent markets from achieving this socially desirable result.

In this paper we contribute to this discussion. In particular, we analyze how market price risk aversion can affect the efficiency of the medium-term hydro resource planning and we discuss the key role hedging instruments should play to lead market outcomes to the desired objectives.

On the basis of a stylized mathematical model and a numerical case example, we illustrate the consequences derived from long-term market incompleteness (not having available hedging instruments) in a context of risk averse hydrothermal generators. We discuss the not so well-known welfare consequences of this market failure and the regulatory implications, which could justify the increasing trend towards the implementation of capacity mechanisms led to impose on electricity demand the obligation to enter into long-term contracts with generators.

Keywords: Medium-term planning, electricity markets, regulatory intervention

1 INTRODUCTION

The free market ability to provide efficient results

Since the very beginning of the power system reform process, one of the key questions posed has been whether the market, of its own accord, is able to lead agents' short-, medium- and long-term decisions in such a way that system welfare is maximized.

As well known, especially in the electric power system context, when considering that this might not be the case, and the market would significantly deviate from the social optimum, regulators may consider the suitability of introducing additional mechanisms to help the market reaching such ideal result. Real-time (in the US context) and balancing markets (in the EU one) as well as other kinds of reserves markets run by the System Operator are good examples of this regulatory intervention as an attempt to amend the potential inability of market agents to guarantee on their own the system security in the very short term. Capacity mechanisms (Batlle & Rodilla, 2012), implemented since the start of the market in the American continent (Batlle et al., 2014) and currently under deep discussion in the European context (EU Commission, 2012), aimed at guaranteeing the adequacy in the long term are also clear examples of this sort of regulatory interventions.

Before taking any regulatory measure, there is a certain consensus on the need to properly identify the problem, so as to effectively tackle the limiting market failure (the real illness)¹. There is no doubt that long has been debated on the associated market failures² and how to tackle them in the short term³

¹ It is worth noting that regulation is neither perfect. Therefore, before intervening by introducing any regulatory mechanism, a good regulator should make sure that the potential harm derived from "imperfect mechanism" does not outweigh the potential benefits being sought with it.

² Among others, lack of agents participation in the market, market power or externalities.

³ See for instance Batlle et al (2007) or Vandezande et al (2009).

(security) and in the very long term⁴ (adequacy). However, little attention has been devoted in the academic literature to the medium-term dimension of the problem, concerning the efficient medium-term resource management of already installed facilities.

The medium-term dimension of the problem

The medium-term management carried out by the generation companies, -i.e., the management of fuel stocks, of hydro reserves and of scheduled maintenances- conditions to a large extent the efficiency of economic dispatch (e.g. by conditioning the availability of resources when most needed). In the context of an electricity market, these medium-term decisions are exclusively driven by market signals.

The importance of carrying out a proper medium-term resource management has indirectly been acknowledged in practical regulation of electricity systems worldwide, since many regulatory mechanisms focused on enhancing adequacy, also include a valuable incentive for both new and existing generators to increase their availability in the medium term. This is the case for example with the explicit penalties for non-compliance embedded in the contracts signed in the Forward Capacity Market in New England, see Batlle & Rodilla (2010). This happens to be an increasingly relevant issue in face of the growing penetration of variable energy resources, e.g. wind or solar PV installations.

Hydro resource management and risk aversion

Among the different types of medium-term resource management affecting generation, we focus here on the problem of hydro-reservoir resource management. The flexibility provided by the “shiftable in time” hydro resources makes them highly valuable assets (when efficiently managed) to properly deal with potential high price situations. However, the uncertainty associated to inflows and the numerous reservoir constraints makes this resource management particularly complicated and inherently associated to the problem of risk management.

In this context, and associated to the need to manage risk, there is a relevant market failure that may hamper the efficiency of the medium-term hydro resource management, namely, the medium- and long-term electricity markets incompleteness. In this paper, we analyze how hydro (or hydro-thermal) generators’ risk aversion coupled with the generators’ inability to efficiently hedge medium- to long-term positions⁵ may compromise the efficient medium-term resource management. In other words, we study how the market incompleteness situation may alter the hydro resource management in such a way that the social welfare is not optimized.

From the point of view of generating companies a number of analyses on how risk aversion may affect hydro resource planning can be found in the literature, see for instance Unger (2002) or Fleten et al. (2002). Here we extend the scope so as to study the consequences of risk-averse behaviour from the social perspective (net social benefit) and the derived regulatory implications.

Market incompleteness

Long-term financial markets have been characterized in the literature as complete or incomplete (Duffie, 1996). Long-term markets are incomplete when perfect risk transfer between the agents is not possible. One of the most important reasons why this incompleteness could be the case is the so-called missing markets problem. This type of market incompleteness and its derived consequences in electricity markets have been analyzed in different contexts, including regional markets efficiency, see Smeers (2004), and long-term investments efficiency, see Willems and Morbee (2010).

As pointed out in Willems and Morbee (2010), the first main result of the literature on welfare effects and pricing of additional assets is that welfare in an incomplete market is lower than in a complete market because not all risk is perfectly allocated in the market⁶. This is precisely the effect that, in the context of the medium-term hydro planning, we aim at analyzing here.

⁴ See for instance Hogan (2005), Joskow (2007) and Pérez-Arriaga & Linares (2008).

⁵ This happens because of market incompleteness. If markets are incomplete, they will be in general Pareto inefficient, see for instance Magill and Quinzii (2002).

⁶ By completing the market, we do not make necessarily everyone better off. Complete markets are Pareto efficient, but not necessarily Pareto dominant with respect to all possible incomplete market allocations.

Objectives

In this paper we contribute to the theoretical analysis of the hydro resource management in a context of (i) perfect competition, (ii) risk averse agents and (iii) incomplete long-term markets. We analyze how long-term incomplete markets affect social welfare by driving generators to carry out a more socially-inefficient hydro resource management. To do so, we compare two scenarios: (i) a market where no financial instruments are available to generators, and (ii) a market where forward contracts for the next period are available (a more complete scenario).

Four multi-stage stylized models are built to drive and illustrate the discussion: first, a centralized risk-neutral welfare-maximizing problem (used as benchmark), second, a perfect competitive market with risk neutral generators, third a perfect competitive market with risk averse generators and no hedging instruments, and fourth, the same perfect competitive market of risk-averse agents with available forward contracting.

The paper is structured as follows:

- 1) Formulate and solve a benchmark optimization problem (section 2). The benchmark solution is the ideal central planner problem. This benchmark gives us a reference to compare the market results computed in following sections.
- 2) We analyze in detail the impact of generators' risk aversion. We study how in the absence of any kind of instrument providing generators with a hedge (i.e. a well-functioning long-term market), generators risk aversion can affect the efficient management of generation resources. Compare with the benchmark and draw conclusions (section 3).
- 3) Then we show how the existence of well-functioning markets for risk, represented in this paper by means of forward markets, helps the system to provide a more efficient hydro resource management. Formulate and solve the problem in this context and compare with the previous solutions (section 4).
- 4) To further illustrate the discussion, we resort to a case example with three stages and uncertainty regarding demand and available reservoirs (section 5).
- 4) Finally we conclude in section 6, where we also discuss whether in case a well-functioning long-term market does not arise of its own accord, some intervention of the regulator may be evaluated as a potential alternative to guarantee an adequate and efficient medium-term hydro resource management.

2 THE BENCHMARK PROBLEM

In this section we formulate and solve the optimization problem that we use as a benchmark for our regulatory analysis. We take the risk-neutral centralized problem as the reference for our work. We have resorted to a simplified version of the traditional formulation presented in Pérez-Arriaga & Meseguer (1997).

2.1 General modelling assumptions

Our model of the power system only includes the essential ingredients for the purposes of our study, which is the analysis of how risk aversion may affect the medium-term planning. We intentionally avoid unnecessary details that may obscure the regulatory analysis⁷.

We consider the following setting:

- The demand side is constituted by a large number of consumers with no bargaining power in the market context. At each time period t they consume an amount of electricity q_t which results in a

⁷ As previously pointed out, a much more detailed and elaborated benchmark problem should be developed to assist regulatory decisions. Here, for the sake of simplicity and clarity, we focus on the classic formulation where no risk aversion on the regulator side (assumed to act on behalf of the demand) is considered.

certain degree of satisfaction or utility $U_t^D(q_t)$. We assume demand utility functions to be strictly increasing ($dU_t^D / dq_t > 0$) and concave ($d^2U_t^D / dq_t^2 \leq 0$).

- The generation side is constituted by a large number of generation companies with no ability to affect the spot price of electricity (no market power) in the market context. At each time period t they produce an amount of electricity q_t , which is the sum of the output of their thermal units (T) and the output of their hydro units (H). Thus the total amount of electricity produced at each time period is given by the following expression:

$$q_t = q_t^T + q_t^H \quad (1)$$

- We assume thermal generation costs $C_t(q_t^T)$ to be strictly increasing and convex.

2.2 Problem formulation

We formulate the benchmark problem as the maximization of the expected social welfare, which is given by the difference between the expected utility of the demand and the expected generation costs subject to hydro reserve balance equations.

A usual approach to solve optimization problems under uncertainty is to adopt a discrete representation of the probability distribution, see for instance Birge & Louveaux (1997). Thus we assume a representation based on a multistage scenario tree as the one shown in Figure 1.

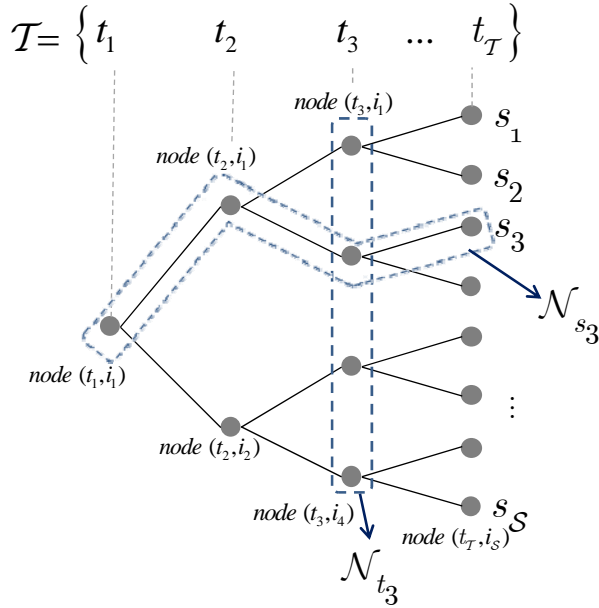


Figure 1. Example of scenario tree

In order to characterize the structure of the random parameters and the whole decision making process, it is convenient to introduce some notation:

- Each node of the multistage tree is identified and represented by (t, i) , i.e. the combination of two indexes: the time period (t), and the node identifier within each time period (i). For instance, the root node in the first period will be identified as (t_1, i_1) . The set representing all nodes considered in the problem is denoted by N . The set of nodes at each time period is denoted by N_t .

- The set of scenarios is denoted by \mathcal{S} . Each scenario s has a probability p_s , with $\sum_{s \in \mathcal{S}} p_s = 1$. The set of nodes belonging to scenario s is denoted by N_s .
- The set of time periods is denoted by \mathcal{T} .
- The probability of reaching node (t, i) is $p_{t,i}$. Notice that at given time period, the sum of the probabilities of all the nodes in that stage is one, i.e. $\sum_{i \in N_t} p_{t,i} = 1$
- Each node (t, i) has a set of descendant nodes $D_{(t,i)}$. The transition probability between (t, i) and a descendant—that is, the transition probability between node (t, i) , and a node $(t+1, j) \in D_{(t,i)}$ —will be referred to as $p_{(t,i),j}$ in order to simplify the notation.

Under this framework, the objective function of the central planer can be written as follows (note that the probability of the root node is 1):

$$\begin{aligned} \max_{q_{t,i}^T, q_{t,i}^H} & \left[U_{t_1,i_1}^D(q_{t_1,i_1}^T, q_{t_1,i_1}^H) - C_{t_1,i_1}(q_{t_1,i_1}^T) \right] + \sum_{(t,i) \in N_{t_2}} p_{t_2,i} \left[U_{t_2,i}^D(q_{t_2,i}^T, q_{t_2,i}^H) - C_{t_2,i}(q_{t_2,i}^T) \right] + \\ & + \dots + \sum_{(t,i) \in N_T} p_{t_T,i} \left[U_{t_T,i}^D(q_{t_T,i}^T, q_{t_T,i}^H) - C_{t_T,i}(q_{t_T,i}^T) \right] \end{aligned} \quad (2)$$

Thus, the maximization of the net benefits of power production and consumption, subject the limit of hydro power, can be formulated as follows:

$$\begin{aligned} \max_{q_{t,i}^T, q_{t,i}^H} & \sum_{t \in \mathcal{T}} \sum_{(t,i) \in N_t} p_{t,i} \left(U_{t,i}^D(q_{t,i}^T, q_{t,i}^H) - C_{t,i}(q_{t,i}^T) \right) \\ \text{s.t.:} & \sum_{(t,i) \in N_s} q_{t,i}^H = Q_s^H \quad : \mu_s, \quad \forall s \in \mathcal{S} \end{aligned} \quad (3)$$

To ease optimality conditions, we have avoided the usual complementarity conditions in the formulation. This way, no limits on thermal and hydro capacity are modeled and both the thermal and hydro outputs are assumed to be strictly positive. Also, network effects are disregarded in the analysis.

We have adopted a stylized formulation of the hydro reserves balance equation for each scenario s . Under this simplified modeling, the hydro generation is considered as a limited resource where the total available energy for the scenario s is Q_s^H . The decision maker has to decide the optimal way of allocating such uncertain energy among all the time periods in that scenario making a unique “here and now” decision at the first period. The Lagrange multiplier μ_s represents the additional expected welfare that could be obtained if Q_s^H were incremented one unit in that scenario.

2.2.1 Optimality Conditions

In order to obtain the first-order necessary conditions we formulate the Lagrangian function $\mathcal{L}(q_{t,i}^T, q_{t,i}^H)$, and compute its first partial derivatives with respect to the decision variables:

$$\mathcal{L} = \sum_{t \in T} \sum_{(t,i) \in N_t} p_{t,i} \left(U_{t,i}^D(q_{t,i}^T, q_{t,i}^H) - C_{t,i}(q_{t,i}^T) \right) + \sum_{s \in \mathcal{S}} \overbrace{p_s}^{\mu_s} \cdot \lambda_s \left(Q_s^H - \sum_{(t,i) \in N_s} q_{t,i}^H \right) \quad (4)$$

The probability of each scenario can be extracted from Lagrange multipliers μ_s in order to compute λ_s , which can be interpreted as the marginal water value in scenario s .

The optimality conditions stemming from the partial derivatives of $\mathcal{L}(q_{t,i}^T, q_{t,i}^H)$ with respect to the output of thermal units at each node, lead to a well established conclusion: in the optimum, the marginal thermal cost at each node (t, i) must be equal to the marginal utility of the demand:

$$\frac{\partial \mathcal{L}}{\partial q_{t,i}^T} = 0 \rightarrow \frac{dU_{t,i}^D}{dq_{t,i}^T} = \frac{dC_{t,i}}{dq_{t,i}^T}, \quad (t, i) \in N \quad (5)$$

where it has been considered that $\frac{dU_{t,i}^D}{dq_{t,i}^T} = \frac{dU_{t,i}^D}{dq_{t,i}} \cdot \frac{dq_{t,i}}{dq_{t,i}^T} = \frac{dU_{t,i}^D}{dq_{t,i}}$.

Setting the partial derivative of $\mathcal{L}(q_{t,i}^T, q_{t,i}^H)$ with respect to the hydro generation equal to zero yields to the next equation:

$$\frac{\partial \mathcal{L}}{\partial q_{t,i}^H} = 0 \rightarrow p_{t,i} \cdot \frac{dU_{t,i}^D}{dq_{t,i}} = \sum_{s/(t,i) \in N_s} \mu_s, \quad (t, i) \in N \quad (6)$$

where it has been considered that $\frac{dU_{t,i}^D}{dq_{t,i}^H} = \frac{dU_{t,i}^D}{dq_{t,i}} \cdot \frac{dq_{t,i}}{dq_{t,i}^H} = \frac{dU_{t,i}^D}{dq_{t,i}}$.

The interpretation of equation (6) is interesting as it allows to link the marginal utility in each node (t, i) and the expected marginal utilities in its descendant nodes $D_{(t,i)}$. For instance, terminal nodes at time period t_T are only “visited” by one possible scenario. Therefore, for the upper node at the last stage we could write:

$$p_{t_T, i_1} \cdot \frac{dU_{t_T, i_1}^D}{dq_{t_T, i_1}} = \mu_{s_1} \quad (7)$$

At its predecessor node (t_{T-1}, i_1) , apart from that scenario s_1 , it would be necessary to add the Lagrange multipliers that corresponds to the other descendants. In case it has three descendants, we could write:

$$p_{t_{T-1}, i_1} \cdot \frac{dU_{t_{T-1}, i_1}^D}{dq_{t_{T-1}, i_1}} = \mu_{s_1} + \mu_{s_2} + \mu_{s_3} \quad (8)$$

By substituting (7) in (8), it yields to:

$$p_{t_{T-1}, i_1} \cdot \frac{dU_{t_{T-1}, i_1}^D}{dq_{t_{T-1}, i_1}} = p_{t_T, i_1} \cdot \frac{dU_{t_T, i_1}^D}{dq_{t_T, i_1}} + p_{t_T, i_2} \cdot \frac{dU_{t_T, i_2}^D}{dq_{t_T, i_2}} + p_{t_T, i_3} \cdot \frac{dU_{t_T, i_3}^D}{dq_{t_T, i_3}} \quad (9)$$

By isolating the marginal utility at the predecessor node, we get:

$$\frac{dU_{t_{\mathcal{T}-1},i_1}^D}{dq_{t_{\mathcal{T}-1},i_1}} = \left(\frac{p_{t_{\mathcal{T}},i_1}}{p_{t_{\mathcal{T}-1},i_1}} \right) \cdot \frac{dU_{t_{\mathcal{T}},i_1}^D}{dq_{t_{\mathcal{T}},i_1}} + \left(\frac{p_{t_{\mathcal{T}},i_2}}{p_{t_{\mathcal{T}-1},i_1}} \right) \cdot \frac{dU_{t_{\mathcal{T}},i_2}^D}{dq_{t_{\mathcal{T}},i_2}} + \left(\frac{p_{t_{\mathcal{T}},i_3}}{p_{t_{\mathcal{T}-1},i_1}} \right) \cdot \frac{dU_{t_{\mathcal{T}},i_3}^D}{dq_{t_{\mathcal{T}},i_3}} \quad (10)$$

The normalization of the probabilities of the descendant nodes by the predecessor node probability is equivalent to use the transition probabilities between the father node and its children. Therefore, the general expression (11) can be obtained straightforwardly by applying the same idea to all the scenarios and time periods in a recursive way:

$$\frac{dU_{t,i}^D}{dq_{t,i}} = \sum_{(t+1,j) \in \mathcal{D}_{(t,i)}} p_{(t,i),j} \frac{dU_{t+1,j}^D}{dq_{t+1,j}}, \quad (t,i) \in N, t < \mathcal{T}, \quad (11)$$

Notice that according to (5), this condition can also be expressed in terms of marginal costs:

$$\frac{dC_{t,i}^T}{dq_{t,i}^T} = \sum_{(t+1,j) \in \mathcal{D}_{(t,i)}} p_{(t,i),j} \frac{dC_{t+1,j}^T}{dq_{t+1,j}^T}, \quad (t,i) \in N, t < \mathcal{T}, \quad (12)$$

This well-known result means that in the risk-neutral centralized context, hydro reserves are managed in order to balance thermal marginal costs between periods. In each node, (t,i) , the marginal cost is equal to the expected (observed from (t,i)) marginal cost in the descendant nodes.

3 MARKET EQUILIBRIUM: RISK NEUTRAL & RISK AVERSION IN THE GENERATION SIDE

3.1 The demand side

As it has just been mentioned, the demand side has no influence on the spot market price ($d\pi_{t,i} / dq_{t,i} = 0$, where $\pi_{t,i}$ stands for the spot price in node (t,i)). We also assume that consumers are risk neutral with respect to payments that they have to make to purchase electricity⁸. As a result, the joint decisions of all consumers can be modeled through the following optimization problem:

$$\max_{q_{t,i}} \sum_{t \in \mathcal{T}} \sum_{(t,i) \in N_t} p_{t,i} \left(U_{t,i}^D(q_{t,i}) - \pi_{t,i} q_{t,i} \right) \quad (13)$$

Let $\mathcal{L}^D(q_{t,i})$ be the Lagrangian function for the demand side. The optimality conditions are given by:

$$\frac{d\mathcal{L}^D}{dq_{t,i}} = \frac{dU_{t,i}^D}{dq_{t,i}} - \pi_{t,i} = 0, \quad (t,i) \in N \quad (14)$$

which is the well-known result that in the demand side consumes electricity until the marginal utility obtained is equal to its price.

3.2 The generation side

We assume that generators behave in a competitive way, and therefore, market prices will reflect the true marginal costs (no market power). This is achieved by introducing the market price as a constant exogenous variable in the generator's problem.

⁸ For example, demand preference about a certain payment of 10 Eur is the same as a lottery of paying 5 or 15 Eur with 1/2 probability.

We model generator's (G) risk aversion through a (concave) utility function U^G that serves to evaluate the utility of its total profit in a scenario. This way, the utility is expressed as a function of total profit over the whole time horizon in that scenario. The use of these utility functions is equivalent to assign more weight to the very low profit scenarios and less weight to the scenarios with very high profits.

3.2.1 Modeling generators' risk aversion by means of their utility function

From a generator's perspective, the medium-term planning and management of its generation plants in a market environment is subject to many sources of uncertainty. In case the generators are risk-neutral, their planning objectives would be to maximize the expected value of their bare market profits (defined as the difference between market incomes and the production costs). However, the distribution of profits over all the scenarios could lead to unacceptable low profits in some of them. In order to reduce their impact (left tail of the profit probability distribution), it is possible to apply several techniques to obtain a risk-based planning of the generation resources. For instance, Fleten *et. al* (2002) propose to minimise the negative deviations of each scenario's profit respect to a pre-fixed target. Other examples can be found in García-González *et. al* (2007), and Cabero *et. al* (2010), where Conditional Value at Risk (CVaR) is introduced in the optimization models given its tractability within the context of a linear programming. In this paper we have rather preferred to use the utility function approach since it embodies a compact (and differentiable) representation of the generators' preferences⁹, thus easing the conceptual model formulation and also easing the interpretation of the results.

For each generator G , this utility function, denoted as U^G , is defined over the market profit margin in scenario s , (P_s^G). As illustrated in Figure 2, a risk averse utility function is assumed to be strictly monotone ($dU^G / dP > 0$) and concave ($d^2U^G / dP^2 < 0$).

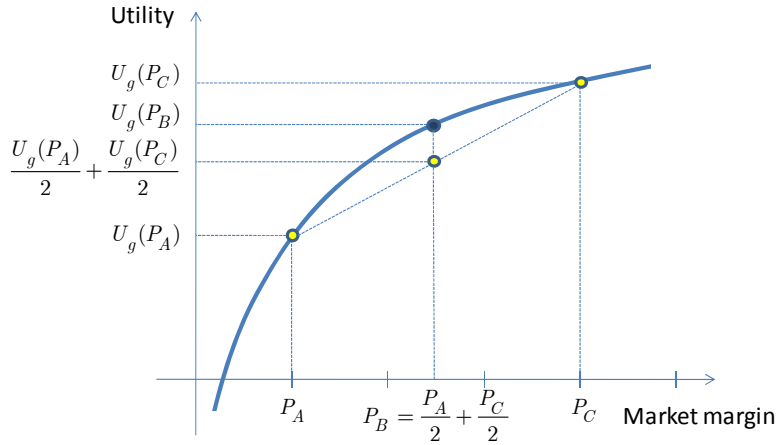


Figure 2. Generator utility function

As it can be observed in Figure 2, a concave utility function penalizes low profit scenarios. Note that this function also penalizes indirectly uncertainty. For example, let us suppose two different scenarios, A and C , each of them presenting the same probability of occurrence and characterized by the resulting profits P_A and P_C respectively. The expected utility under such an uncertain situation would be $U^G(P_A)/2 + U^G(P_C)/2$. This results in a lower utility than the one that would have been obtained if the average profit, $P_B = P_A/2 + P_C/2$, had been received with probability 1.

⁹ Similar approaches can be found in the literature, see for instance Fan *et al.* (2009), where the generator's utility function is used to make a conceptual analysis of the generation investment problem (and particularly under regulatory uncertainty regarding CO2 policies).

Without loss of generality, let assume that there is one generation company that owns both the thermal and hydro generation. By means of these utility functions, the problem of such generation company can be formulated as follows:

$$\begin{aligned} & \max_{q_{t,i}^T, q_{t,i}^H, s \in \mathcal{S}} \sum p_s U^G(P_s^G), \\ & \text{s.t.: } \sum_{(t,i) \in N_s} q_{t,i}^H = Q_s^H, \forall s \in \mathcal{S} \end{aligned} \quad (15)$$

where P_s^G is the profit of the generation company in scenario s given by (16):

$$P_s^G = \sum_{(t,i) \in N_s} \left(\pi_{t,i} (q_{t,i}^T + q_{t,i}^H) - C_{t,i}(q_{t,i}^T) \right) \quad (16)$$

Note that for the sake of simplicity, no discounting term has been included. Also notice that in this case we have adopted a scenario-wise representation because of the way we have defined the utility function for the generation side.

The Lagrangian function for the generation problem is given by:

$$\mathcal{L}^G = \sum_{s \in \mathcal{S}} p_s \left(U^G(P_s^G) + \lambda_s \left(Q_s^H - \sum_{(t,i) \in N_s} q_{t,i}^H \right) \right) \quad (17)$$

where now λ_s can be interpreted as the water value in scenario s for the generator in a market environment, and again we have intentionally extracted the probability from this Lagrange multiplier.

The first set of optimality conditions is:

$$\frac{\partial \mathcal{L}^G}{\partial q_{t,i}^T} = \left(\sum_{s|(t,i) \in N_s} p_s \frac{dU^G(P_s^G)}{dP^G} \cdot \frac{\partial P^G}{\partial q_{t,i}^T} \right) = \left(\sum_{s|(t,i) \in N_s} p_s \frac{dU^G(P_s^G)}{dP^G} \right) \cdot \left(\pi_{t,i} - \frac{dC_{t,i}}{dq_{t,i}^T} \right) = 0, \quad (18)$$

with $t \in \mathcal{T}, (t,i) \in N_t$. Since U^G has been defined as an increasing function, this condition is only fulfilled if $\pi_{t,i} = \frac{dC_{t,i}}{dq_{t,i}^T} \quad \forall t \in \mathcal{T}, (t,i) \in N_t$. In other words, the generator increases the output of its thermal units until its marginal cost is equal to the price of electricity.

The second set of optimality conditions is:

$$\begin{aligned} \frac{\partial \mathcal{L}^G}{\partial q_{t,i}^H} &= \left(\sum_{s|(t,i) \in N_s} p_s \frac{dU^G(P_s^G)}{dP^G} \frac{dP^G}{dq_{t,i}^H} \right) - \sum_{s|(t,i) \in N_s} p_s \lambda_s = \\ & \pi_{t,i} \left(\sum_{s|(t,i) \in N_s} p_s \frac{dU^G(P_s^G)}{dP^G} \right) - \sum_{s|(t,i) \in N_s} p_s \lambda_s = 0, \end{aligned} \quad (19)$$

with $t \in \mathcal{T}, (t,i) \in N_t$.

We will interpret the optimality condition (19) both under the assumption of risk neutrality and under the assumption of risk aversion.

3.2.2 The generation company is risk neutral

If the generation company is risk neutral, (i.e. $U^G(P_s^G) = P_s^G$) then, at any of the terminal nodes of the tree we have $\pi_{t_T, i} = \lambda_s$, $s \mid (t_T, i) \in N_s$, which means that the value of water for scenario s is equal to the price of electricity at the final node of that scenario. If we move backwards one stage in the tree, at each node we have:

$$\pi_{t_T-1, i} \left(\sum_{s \mid (t_T-1, i) \in N_s} p_s \right) - \sum_{s \mid (t_T-1, i) \in N_s} p_s \lambda_s = \pi_{t_T-1, i} \left(\sum_{s \mid (t_T-1) \in N_s} p_s \right) - \sum_{s \mid (t_T-1) \in N_s} p_s \pi_{t_T, i} = 0 \quad (20)$$

which yields to:

$$\pi_{t_T-1, i} = \sum_{s \mid (t_T-1, i) \in N_s} \left(\frac{p_s}{\sum_{s \mid (t_T-1, i) \in N_s} p_s} \right) \pi_{t_T, i} \Rightarrow \pi_{t_T-1, i} = \sum_{(t_T, j) \in D_{(t_T-1, i)}} p_{(t_T-1, i), j} \cdot \pi_{t_T, i} \quad (21)$$

This can be easily generalized to any node:

$$\pi_{t, i} = \sum_{(t+1, j) \in \mathcal{D}_{(t, i)}} p_{(t, i), j} \cdot \pi_{t+1, j}, \quad t \in \mathcal{T} - \{t_T\}, (i, t) \in N, \quad (22)$$

The meaning of equation (22) is that water resources are managed in such a way that the resulting spot price at each node is equal to the expected future spot price “seen” from that node. Taking into account the relationship between the prices and the marginal demand utility, this is an equivalent condition to (11). Then if generators are risk neutral, the medium term planning decisions are exactly the same as those observed in the centralized benchmark problem.

3.2.3 The generation company is risk averse

By repeating the approach followed in steps (20), (21) and (22) to the general expression including the utility of the generator (eq. (19) we obtain the following expression:

$$\pi_{t, i} = \sum_{(t+1, j) \in \mathcal{D}_{(t, i)}} \tilde{p}_{(t, i), j} \cdot \pi_{t+1, j}, \quad (t, i) \in N / t \in \mathcal{T} - \{t_T\}, \quad (23)$$

where transition probabilities $\tilde{p}_{(t, i), j}$ take the form of a risk-modified probability:

$$\tilde{p}_{(t, i), j} = \sum_{s \mid (t+1, j) \in N_s} \left(\frac{p_s \cdot \frac{dU_s^G(P_s^G)}{dP^G}}{\sum_{s \mid (t, i) \in N_s} p_s \cdot \frac{dU_s^G(P_s^G)}{dP^G}} \right) \quad (24)$$

Using these new probabilities an analogous interpretation can be drawn: the water resources are used so as to make the price in each node (t, i) , equal to the expected price (now using the risk modified probabilities), in the descendant nodes $(t+1, j) / (t+1, j) \in \mathcal{D}_{(t, i)}$.

Since we defined the utility function as concave, the lower the income of a certain scenario the higher the value of the corresponding derivative dU_s^G / dP_s^G , and consequently the higher the value of the associated risk modified probability. This way, the prices in any of descendant nodes leading to low profits scenarios are given more weight in the objective function.

This condition obviously implies a different resource management than the one analyzed in the benchmark problem. Thus, when generation companies with hydro capacity are risk averse they may use their hydro reserves to hedge their risk exposure. This clearly leads to a market outcome different from the benchmark solution.

3.3 Computing the perfect competitive market equilibrium

The market equilibrium can then be computed by simultaneously solving the optimality conditions of the demand side (14) and the generation side (18) and (19).

In order to gain intuition, we assume that the generation side is constituted by companies with similar portfolios and similar utility functions. Under this assumption we can treat them as a single generation company in order to compute the market equilibrium.

Equation (14) together with equation (18) are equivalent to equation (5) and have the usual interpretation for a perfectly competitive market: At equilibrium, the marginal cost of electricity is equal to the marginal utility of demand and this determines the price that consumers should pay and generators should receive.

As pointed out previously, if we assume that generation companies are risk neutral, the optimality condition (22) is equivalent to (11). In other words, if generation companies are risk neutral the market equilibrium yields the same outcome as the benchmark (risk-neutral social maximizing) problem.

In contrast, if generation companies are risk averse, a different market equilibrium is reached. As we have discussed, we expect a shift of hydro production with respect to the benchmark solution, in order to hedge against low-profit scenarios and thus maximize their expected value of their utility function. This result is further explored in the case example.

4 MARKET EQUILIBRIUM WITH RISK AVERSION IN THE GENERATION SIDE AND A FORWARD MARKET

In this section we analyze the impact that the existence of a well-functioning forward market has on the medium-term management of hydro reserves under risk aversion from the generation side.

4.1 Forward market

In real systems, there is a variety of different instruments that agents can trade in order to hedge their risk. In this paper we assume a simple extension of the two-stage problem stated in Allaz & Villa (1993) which is generalized to the multistage formulation. However, no strategic behavior is considered, and market agents are assumed to behave competitively offering their marginal costs. Therefore, the issue of explaining how the dynamic interaction among market participants influence the strategic agents decisions both in the forward and in the spot market, and the discussion whether it is more convenient to follow a closed-loop or an open-loop approach to model these interactions is out of the scope of this paper. For the sake of simplicity, we follow a similar approach as the one described in Cabero et al. (2010) and formulate a single-level equilibrium problem in which the agents take their decisions for the forward and the spot market simultaneously (open-loop).

The extension of the two-stage problem to the multistage one is explained hereafter. We will assume a forward market where agents can sign contracts at a certain period to sell or buy energy with delivery at the next period. For instance, in case of having quarterly periods (which is a common time scope of the products traded in real markets), we assume that the generator can sign a contract in period t to sell a given quantity q^F at the price π^F in period $t + 1$. Typically, the seller will be a generation agent, and the buyer will be a demand agent. However, sometimes it can happen in the opposite way when agents need to correct their market positions. Thus, no negative constraints should be imposed to the traded quantities in the forward market.

From the point of view of the stochastic-tree shown in section 2.2, the existence of the described forward market implies the possibility of establishing a contract at each node of the tree with maturity at all its descendant nodes. The contracts will be signed by the generator (party) and the demand (counterparty).

From the demand side, we will assume that there is only one agent that aggregates all consumers. From the generation side, in case of existing several generation companies, all of them could have the possibility of signing independent contracts with the aggregated demand. Therefore, for each node of the tree there would be as many possible contracts as generation agents. However, for the sake of simplicity in the following exposition, and similarly to what was presented in section 3, we will assume that there is only one generation agent. Despite this fact, section 5 includes a brief example of the effect of allowing several generation agents endowed with different risk profiles.

4.2 The joint forward-spot equilibrium

In order to reproduce the behavior of the market participants in case of existing both a spot and forward market, the first step is to obtain the optimality conditions that define the joint forward-spot equilibrium. To derive them, it is necessary first to define the next variables:

- $q_{(t,i)}^F$: Quantity of the forward contract sold by the generator to the demand in node (t, i) with delivery at its descendant nodes $(t+1, j) \in \mathcal{D}_{(t,i)}$.
- $\pi_{(t,i)}^F$: Price of the forward contract sold by the generator to the demand in node (t, i) with delivery at its descendant nodes $(t+1, j) \in \mathcal{D}_{(t,i)}$.

4.2.1 The demand side

The optimization problem that represents the behavior of the demand side is formulated as follows:

$$\begin{aligned} \max_{q_{t,i}} \sum_{t \in \mathcal{T}} \left[\sum_{(t,i) \in N_t} p_{t,i} \left(U_{t,i}^D(q_{t,i}) - \pi_{t,i} q_{t,i} \right) \right] + \\ + \sum_{t \in \mathcal{T} - \{t_T\}} \left[\sum_{(t,i) \in N_t} \left[p_{t,i} (-\pi_{(t,i)}^F q_{(t,i)}^F) + \sum_{(t+1,j) \in \mathcal{D}_{(t,i)}} p_{(t+1,j)} \pi_{(t+1,j)} q_{(t,i)}^F \right] \right] \end{aligned} \quad (25)$$

where the first sum is the same as in (13), and the second one (which does not include terminal nodes) accounts for the fact that any time the demand signs a contract at time stage t , it implies a payment equal to $\pi_t^F q_t^F$ in order to reduce the exposure to spot price π_{t+1} just to the net quantity $(q_{t+1} - q_t^F)$.

Let $\mathcal{L}^D(q_{t,i}, q^F)$ be the Lagrangian function for the demand side. The first optimality condition is analogous to equation (14). The second optimality condition is:

$$\frac{d\mathcal{L}^D}{dq_{(t,i)}^F} = p_{t,i} (-\pi_{(t,i)}^F) + \sum_{(t+1,j) \in \mathcal{D}_{(t,i)}} p_{(t+1,j)} \pi_{(t+1,j)} = 0 \quad (26)$$

which yields to:

$$\pi_{(t,i)}^F = \frac{\sum_{(t+1,j) \in \mathcal{D}_{(t,i)}} p_{(t+1,j)} \pi_{(t+1,j)}}{p_{t,i}}, \quad \forall (t,i) \in N_s / t \in \mathcal{T} - \{t_T\} \quad (27)$$

Equation (27) means that the demand side will interchange electricity in the forward market up to the point at which the forward price is equal to the expected spot price in the descendant nodes where the contract is signed. It is important to highlight that this condition applies always, independently of the risk profiles of the generators.

4.3 The generation side

The optimization problem that models the generation side is again given by equations (18) and (19) except for the inclusion of the new decision variables, $q_{(t,i)}^F$ and $\pi_{(t,i)}^F$. Thus, the profit of each generation company in each scenario s must include the effect of the forward market:

$$P_s^G = \sum_{(t,i) \in N_s} \left(\pi_{t,i} (q_{t,i}^T + q_{t,i}^H) - C_{t,i}(q_{t,i}^T) \right) + \sum_{(t,i) \in N_s / t \in \mathcal{T} - \{t_T\}} \left((\pi_{t,i}^F - \pi_{(t+1,j)}) \cdot q_{t,i}^F \right), \forall s \in S \quad (28)$$

where as it is a scenario-based expression, given any node (t, i) there is only one single descendant $(t+1, j)$ that belongs to the scenario s which has been indicated as $(t+1, j) \in \mathcal{D}_{(t,i)} \cap N_s$. The index of this single descendant will be referred to as $(t+1, j_s)$. Notice that the first sum is the same as in (16), and that the second sum includes all the nodes expect the terminal node of such scenario.

A new optimality condition arises when we derive the Lagrangian function defined in (17) with respect to the new variable $q_{t,i}^F$. Let $\mathcal{S}_{(t,i)}$ be the set of scenarios that include the node (t, i) in their pathway from the root node to the leaf node. Then, the new optimality condition can be expressed as follows:

$$\frac{\partial \mathcal{L}^G}{\partial q_{t,i}^F} = \sum_{s \in \mathcal{S}_{(t,i)}} p_s \frac{dU^G(P_s^G)}{dP^G} \left(\pi_{t,i}^F - \pi_{(t+1,j_s)} \right) = 0, \forall (t, i) \in N \quad (29)$$

Equation (29) establishes an interesting link among the forward price in node (t, i) and spot prices at its descendant nodes:

$$\pi_{t,i}^F = \frac{\sum_{s \in \mathcal{S}_{(t,i)}} p_s \frac{dU^G(P_s^G)}{dP^G} \left(\pi_{(t+1,j_s)} \right)}{\sum_{s \in \mathcal{S}_{(t,i)}} p_s \frac{dU^G(P_s^G)}{dP^G}}, \quad (t, i) \in N / t \in \mathcal{T} - \{t_T\} \quad (30)$$

and this expression can be re-arranged to be expressed in a similar way as (23), making use of the same risk-modified probabilities as the ones defined in (24):

$$\pi_{t,i}^F = \sum_{(t+1,j) \in \mathcal{D}_{(t,i)}} \tilde{p}_{(t,i),j} \cdot \pi_{t+1,j}, \quad (t, i) \in N / t \in \mathcal{T} - \{t_T\} \quad (31)$$

The immediate consequence of imposing (23) and (31) is that:

$$\pi_{t,i}^F = \pi_{t,i}, \quad (t, i) \in N / t \in \mathcal{T} - \{t_T\} \quad (32)$$

Thus, the price of the forward contract in the equilibrium must be equal to the spot price at the node where the forward contract is signed. As, the participation of the demand side as counterparty in the forward market imposed the equation (27), substituting (32) in such equation yields to:

$$\pi_{t,i} = \frac{\sum_{(t+1,j) \in \mathcal{D}_{(t,i)}} p_{(t+1,j)} \pi_{(t+1,j)}}{p_{t,i}}, \quad \forall (t, i) \in N / t \in \mathcal{T} - \{t_T\} \quad (33)$$

Thus, even in case of having risk-averse generators, the existence of forward market implies that at every node, the spot price is equal to the expected value of the spot prices at its descendant nodes with the original probabilities (not the risk-modified ones). This result shows that under the hypotheses presented here, a well-functioning long-term market brings back the optimal resource management solution.

It is important to bear in mind that our analysis of the optimality conditions does not take into account the impact of constraints. We have intentionally formulated the optimization problems without including constraints in order to obtain meaningful and reasonably simple optimality conditions. If the solution were limited by one or more of the constraints, then the optimality conditions would take a different form.

5 NUMERICAL EXAMPLE

In order to illustrate the ideas presented in this paper, this section presents a numerical example. The analysis will be based on the comparison of the results obtained for the next four settings that have been implemented in GAMS:

- Centralized planning (label **Cen**)
- Market with risk neutral agents (label **MrNe**)
- Market with risk averse agents and only spot market (label **MrAv**)
- Market with risk averse agents and forward markets (label **MrAF**)

The centralized planning problem has the structure of a multistage stochastic optimization problem that has been solved with the solver CONOPT3 as the resulting model is non-linear. Regarding the market, we will assume that the market consists of two agents: 1) the demand side that aggregates all the electricity consumption, and 2) a generation company with a hydrothermal portfolio. The simultaneous maximization of their utility functions, and the market clearing conditions of both the spot and the forward markets will result in a set of equations that have a mixed complementary problem (MCP) structure. The resulting MCP problems have been solved with the solver PATH.

5.1 Input data

We will consider a time horizon of one year divided into quarterly periods. The considered sources of uncertainty are the total hydro energy available in each scenario (Q_s^H , GWh), and the utility of the demand. In particular, we will assume that the utility of the demand at each node i is a quadratic function $U^D(q) = \alpha \cdot q - 0.5q^2$, where the linear coefficient α is unknown and will be node-dependent. As the generation will be measured in GWh, the utility function will be expressed in k€, so that the marginal utility of the demand will be expressed in €/MWh.

The stochastic tree used in this example is shown in Figure 3, where the values of the random parameters have been indicated. The probability of each scenario has been set to $1/24$. The nodes have been labelled indicating its time period, and the ordinal of the node among all the possible ones at that stage (1 in the 1st stage, 4 in the 2nd stage, 12 in the 3rd stage, and 24 in the 4th stage)

The considered thermal generation cost function is $C(q^T) = 10 \cdot (q^T)^2 + 0.1 \cdot (q^T)^3$ k€, which yields to a quadratic marginal cost function. Notice that this cost function could easily be considered as stochastic data or to be time-dependent if necessary. However, for making it easier to reproduce the results, it has been considered constant without any loss of generality.

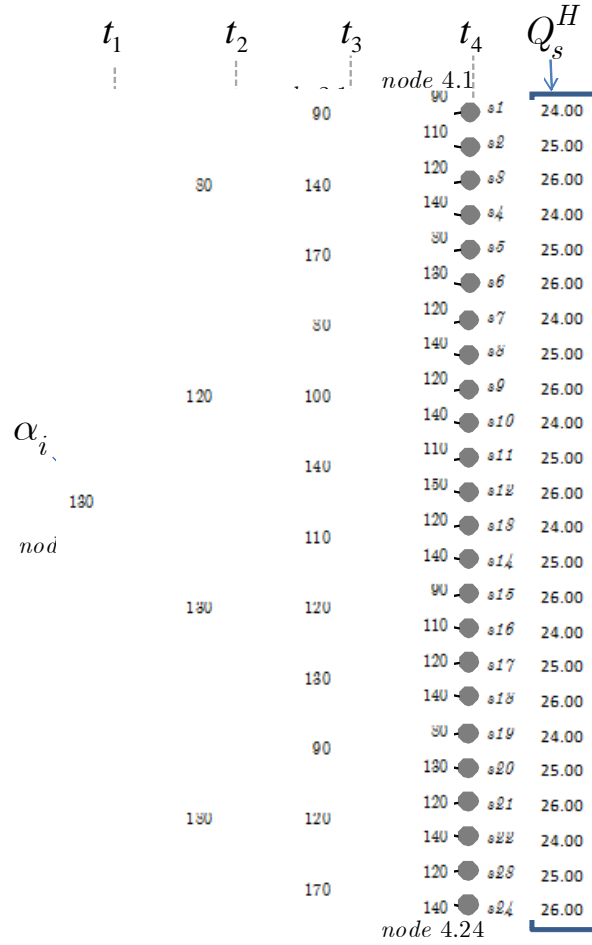


Figure 3. Stochastic tree data: linear term of the demand at each node, and total hydro energy per scenario

In order to model the risk aversion, according to Kalleberg & Ziemba (1983), there is variety of possible utility functions that could be used. Among them, in this paper an exponential utility function has been chosen as it presents some interesting properties such as having a constant Arrow-Pratt coefficient of absolute risk aversion (R_A). In particular, the following expression has been implemented:

$$U^G(P_s) = U_0 + K \cdot (1 - e^{-\beta \cdot P_s}) \quad (34)$$

where P_s is the total profit obtained by the generation agent in scenario s . Notice that Arrow-Pratt coefficient is $R_A = \beta$. The parameters U_0 and K correspond to an affine transformation of the exponential function in order to allow a better interpretation of the results. These parameters have been chosen in order to obtain a utility function as the one shown in Figure 4, where the straight line corresponds to the risk neutral utility function, and the concave one to the risk-averse utility function with parameters ($\beta = 0.002$, $U_0 = -4047.60$, $K = 6056.55$)

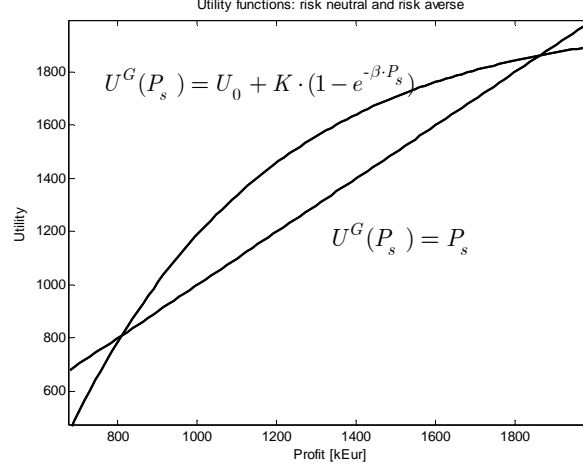


Figure 4. Utility functions.

5.2 Results

The obtained scheduling of the thermal and hydro generators is shown in Table i and Table ii respectively. For each problem setting, it can be observed the output power at each time stage (columns) for each one of the 24 scenarios (rows). The last row shows the average value. The cells corresponding to the scenarios that belong to the same node of the stochastic-tree have been filled with the same colour (grey or white). Thus, in stage t_1 all the scenarios share the same output power as it corresponds to the root node of the stochastic tree. In stage t_2 , something similar happens for scenarios $\{s_1, \dots, s_6\}$, $\{s_7, \dots, s_{12}\}$, \dots , $\{s_{19}, \dots, s_{24}\}$.

Table i. Thermal generation q^T (GWh)

	t1				t2				t3				t4			
	Cen	MrNe	MrAv	MrAvF	Cen	MrNe	MrAv	MrAvF	Cen	MrNe	MrAv	MrAvF	Cen	MrNe	MrAv	MrAvF
s1	1.774	1.774	1.611	1.774	1.394	1.394	1.286	1.394	0.696	0.696	0.845	0.696	0.547	0.547	0.711	0.547
s2	1.774	1.774	1.611	1.774	1.394	1.394	1.286	1.394	0.696	0.696	0.845	0.696	0.840	0.840	0.996	0.840
s3	1.774	1.774	1.611	1.774	1.394	1.394	1.286	1.394	1.707	1.707	1.801	1.707	1.184	1.184	1.366	1.184
s4	1.774	1.774	1.611	1.774	1.394	1.394	1.286	1.394	1.707	1.707	1.801	1.707	2.181	2.181	2.339	2.181
s5	1.774	1.774	1.611	1.774	1.394	1.394	1.286	1.394	1.707	1.707	1.785	1.707	1.184	1.184	1.383	1.184
s6	1.774	1.774	1.611	1.774	1.394	1.394	1.286	1.394	1.707	1.707	1.785	1.707	2.181	2.181	2.354	2.181
s7	1.774	1.774	1.611	1.774	1.661	1.661	1.653	1.661	1.380	1.380	1.463	1.380	1.248	1.248	1.346	1.248
s8	1.774	1.774	1.611	1.774	1.661	1.661	1.653	1.661	1.380	1.380	1.463	1.380	1.509	1.509	1.603	1.509
s9	1.774	1.774	1.611	1.774	1.661	1.661	1.653	1.661	1.577	1.577	1.619	1.577	1.041	1.041	1.183	1.041
s10	1.774	1.774	1.611	1.774	1.661	1.661	1.653	1.661	1.577	1.577	1.619	1.577	2.059	2.059	2.181	2.059
s11	1.774	1.774	1.611	1.774	1.661	1.661	1.653	1.661	2.004	2.004	2.046	2.004	1.635	1.635	1.762	1.635
s12	1.774	1.774	1.611	1.774	1.661	1.661	1.653	1.661	2.004	2.004	2.046	2.004	2.350	2.350	2.465	2.350
s13	1.774	1.774	1.611	1.774	1.770	1.770	1.792	1.770	1.829	1.829	1.892	1.829	1.706	1.706	1.783	1.706
s14	1.774	1.774	1.611	1.774	1.770	1.770	1.792	1.770	1.829	1.829	1.892	1.829	1.950	1.950	2.024	1.950
s15	1.774	1.774	1.611	1.774	1.770	1.770	1.792	1.770	1.521	1.521	1.567	1.521	0.980	0.980	1.086	0.980
s16	1.774	1.774	1.611	1.774	1.770	1.770	1.792	1.770	1.521	1.521	1.567	1.521	2.007	2.007	2.097	2.007
s17	1.774	1.774	1.611	1.774	1.770	1.770	1.792	1.770	1.950	1.950	2.011	1.950	1.829	1.829	1.905	1.829
s18	1.774	1.774	1.611	1.774	1.770	1.770	1.792	1.770	1.950	1.950	2.011	1.950	2.069	2.069	2.142	2.069
s19	1.774	1.774	1.611	1.774	2.236	2.236	2.148	2.236	1.652	1.652	1.726	1.652	1.124	1.124	1.322	1.124
s20	1.774	1.774	1.611	1.774	2.236	2.236	2.148	2.236	1.652	1.652	1.726	1.652	2.130	2.130	2.300	2.130
s21	1.774	1.774	1.611	1.774	2.236	2.236	2.148	2.236	2.251	2.251	2.352	2.251	1.771	1.771	1.915	1.771
s22	1.774	1.774	1.611	1.774	2.236	2.236	2.148	2.236	2.251	2.251	2.352	2.251	2.693	2.693	2.820	2.693
s23	1.774	1.774	1.611	1.774	2.236	2.236	2.148	2.236	2.749	2.749	2.853	2.749	2.640	2.640	2.759	2.640
s24	1.774	1.774	1.611	1.774	2.236	2.236	2.148	2.236	2.749	2.749	2.853	2.749	2.856	2.856	2.971	2.856
avg	1.774	1.774	1.611	1.774	1.765	1.765	1.720	1.765	1.752	1.752	1.830	1.752	1.738	1.738	1.867	1.738

Table ii. Hydro generation q^H (GWh)

	t1				t2				t3				t4			
	Cen	MrNe	MrAv	MrAvF	Cen	MrNe	MrAv	MrAvF	Cen	MrNe	MrAv	MrAvF	Cen	MrNe	MrAv	MrAvF
s1	6.733	6.733	7.389	6.733	3.234	3.234	3.645	3.234	6.765	6.765	6.251	6.765	7.268	7.268	6.715	7.268
s2	6.733	6.733	7.389	6.733	3.234	3.234	3.645	3.234	6.765	6.765	6.251	6.765	8.268	8.268	7.715	8.268
s3	6.733	6.733	7.389	6.733	3.234	3.234	3.645	3.234	8.004	8.004	7.623	8.004	8.029	8.029	7.343	8.029
s4	6.733	6.733	7.389	6.733	3.234	3.234	3.645	3.234	8.004	8.004	7.623	8.004	6.029	6.029	5.343	6.029
s5	6.733	6.733	7.389	6.733	3.234	3.234	3.645	3.234	11.004	11.004	10.688	11.004	4.029	4.029	3.277	4.029
s6	6.733	6.733	7.389	6.733	3.234	3.234	3.645	3.234	11.004	11.004	10.688	11.004	5.029	5.029	4.277	5.029
s7	6.733	6.733	7.389	6.733	6.191	6.191	6.222	6.191	3.287	3.287	2.970	3.287	7.789	7.789	7.419	7.789
s8	6.733	6.733	7.389	6.733	6.191	6.191	6.222	6.191	3.287	3.287	2.970	3.287	8.789	8.789	8.419	8.789
s9	6.733	6.733	7.389	6.733	6.191	6.191	6.222	6.191	4.525	4.525	4.357	4.525	8.551	8.551	8.032	8.551
s10	6.733	6.733	7.389	6.733	6.191	6.191	6.222	6.191	4.525	4.525	4.357	4.525	6.551	6.551	6.032	6.551
s11	6.733	6.733	7.389	6.733	6.191	6.191	6.222	6.191	6.782	6.782	6.606	6.782	5.294	5.294	4.783	5.294
s12	6.733	6.733	7.389	6.733	6.191	6.191	6.222	6.191	6.782	6.782	6.606	6.782	6.294	6.294	5.783	6.294
s13	6.733	6.733	7.389	6.733	6.750	6.750	6.661	6.750	4.508	4.508	4.252	4.508	6.009	6.009	5.698	6.009
s14	6.733	6.733	7.389	6.733	6.750	6.750	6.661	6.750	4.508	4.508	4.252	4.508	7.009	7.009	6.698	7.009
s15	6.733	6.733	7.389	6.733	6.750	6.750	6.661	6.750	6.745	6.745	6.560	6.745	5.772	5.772	5.389	5.772
s16	6.733	6.733	7.389	6.733	6.750	6.750	6.661	6.750	6.745	6.745	6.560	6.745	3.772	3.772	3.389	3.772
s17	6.733	6.733	7.389	6.733	6.750	6.750	6.661	6.750	6.008	6.008	5.752	6.008	5.509	5.509	5.198	5.509
s18	6.733	6.733	7.389	6.733	6.750	6.750	6.661	6.750	6.008	6.008	5.752	6.008	6.509	6.509	6.198	6.509
s19	6.733	6.733	7.389	6.733	9.794	9.794	10.172	9.794	3.224	3.224	2.927	3.224	4.249	4.249	3.511	4.249
s20	6.733	6.733	7.389	6.733	9.794	9.794	10.172	9.794	3.224	3.224	2.927	3.224	5.249	5.249	4.511	5.249
s21	6.733	6.733	7.389	6.733	9.794	9.794	10.172	9.794	3.727	3.727	3.283	3.727	5.746	5.746	5.156	5.746
s22	6.733	6.733	7.389	6.733	9.794	9.794	10.172	9.794	3.727	3.727	3.283	3.727	3.746	3.746	3.156	3.746
s23	6.733	6.733	7.389	6.733	9.794	9.794	10.172	9.794	6.486	6.486	5.999	6.486	1.987	1.987	1.440	1.987
s24	6.733	6.733	7.389	6.733	9.794	9.794	10.172	9.794	6.486	6.486	5.999	6.486	2.987	2.987	2.440	2.987
avg	6.733	6.733	7.389	6.733	6.492	6.492	6.675	6.492	5.922	5.922	5.606	5.922	5.853	5.853	5.330	5.853

The first conclusion that can be drawn from these results is that the scheduling obtained by the centralized operator, i.e., the one that maximizes the global social welfare, is equal to the one obtained in a competitive market context when agents are risk neutral. However, in case the generator is risk averse, the obtained scheduling deviates from such optimal solution. In particular, in this example case it can be seen that the hydro generator prefers to increase its hydro generation in stage t_1 (7.389 GWh instead of 6.733 GWh). This is due to the existing volatility of market prices in future stages. For a risk-neutral agent this volatility does not represent any problem as low-price scenarios can be “compensated” by the ones of highest prices. However, for the risk-averse agent, the lower utility assigned to low-profit scenarios makes the hydro generator to prefer to release more water in the first stage where the uncertainty is lower. This rational behavior could compromise the availability of hydro resources in future stages in a more realistic system representation.

This can be checked by analysing the obtained spot prices π for each case. As the hydro-thermal scheduling has proven to be the same for the cases Cen, and MrNe, their corresponding prices will also be identical, and they will be different from the ones obtained for the MrAv case (see Table iii).

Table iii. Spot prices π (€/MWh)

	Cen, MrNe, MrAvF				MrAv			
	t1	t2	t3	t4	t1	t2	t3	t4
s1	44.9264	33.719	15.384	11.847	39.9987	30.685	19.043	15.742
s2	44.9264	33.719	15.384	18.920	39.9987	30.685	19.043	22.894
s3	44.9264	33.719	42.887	27.875	39.9987	30.685	45.761	32.915
s4	44.9264	33.719	42.887	57.899	39.9987	30.685	45.761	63.185
s5	44.9264	33.719	42.887	27.875	39.9987	30.685	45.266	33.399
s6	44.9264	33.719	42.887	57.899	39.9987	30.685	45.266	63.692
s7	44.9264	41.484	33.326	29.632	39.9987	41.254	35.673	32.352
s8	44.9264	41.484	33.326	37.019	39.9987	41.254	35.673	39.778
s9	44.9264	41.484	38.988	24.077	39.9987	41.254	40.242	27.852
s10	44.9264	41.484	38.988	53.899	39.9987	41.254	40.242	57.874
s11	44.9264	41.484	52.138	40.713	39.9987	41.254	53.482	44.550
s12	44.9264	41.484	52.138	63.563	39.9987	41.254	53.482	67.521
s13	44.9264	44.799	46.629	42.850	39.9987	45.471	48.567	45.192
s14	44.9264	44.799	46.629	50.409	39.9987	45.471	48.567	52.777
s15	44.9264	44.799	37.347	22.481	39.9987	45.471	38.721	25.251
s16	44.9264	44.799	37.347	52.214	39.9987	45.471	38.721	55.136
s17	44.9264	44.799	50.420	46.619	39.9987	45.471	52.366	48.977
s18	44.9264	44.799	50.420	54.220	39.9987	45.471	52.366	56.604
s19	44.9264	59.704	41.239	26.269	39.9987	56.799	43.463	31.673
s20	44.9264	59.704	41.239	56.209	39.9987	56.799	43.463	61.885
s21	44.9264	59.704	60.221	44.831	39.9987	56.799	63.648	49.296
s22	44.9264	59.704	60.221	75.612	39.9987	56.799	63.648	80.247
s23	44.9264	59.704	77.651	73.727	39.9987	56.799	81.483	78.012
s24	44.9264	59.704	77.651	81.575	39.9987	56.799	81.483	85.893
avg	44.9264	44.9264	44.9264	44.9264	39.9987	43.5525	47.3095	48.8623

As explained in equations (14) and (18), in the market equilibrium the spot price equals both the marginal utility of the demand, and the marginal cos. For instance, in the node 2.2 (scenarios s₇-s₁₂ in state t₂) the spot price for the risk neutral case is 41.484 €/MWh, and the total generation is $q = 7.852$ GWh, ($q^T = 1.661$, and $q^H = 6.191$). The marginal cost function is:

$$\frac{dC(q^T)}{dq^T} = 20 \cdot (q^T) + 0.3 \cdot (q^T)^2.$$

The marginal demand function in the node 2.2 is:

$$\frac{dU^D(q)}{dq} = 120 - 10 \cdot q$$

It can easily be checked that: $20 \cdot (1.661) + 0.3 \cdot (1.661)^2 = 120 - 10 \cdot (7.852) = 41.484$.

Moreover, in the risk neutral case, the spot price at each node will be the expected value of the spot prices of its descendant nodes. For instance, the descendant nodes of node 2.2, are 3.4, 3.5 and 3.6. Therefore, as in this example all the scenarios share the same probability, it has to be satisfied the next condition:

$$41.484 = \frac{33.326 + 38.988 + 52.138}{3}$$

However, in the risk aversion case (without forward market) this does not apply, as the expected values should be computed according to the risk modified probabilities shown in equation (24). Figure 5 shows the obtained spot prices for the Cen and MrNe cases. Figure 6 shows the obtained spot prices for the MrAv case. The average prices have been added in dashed lines in both figures.

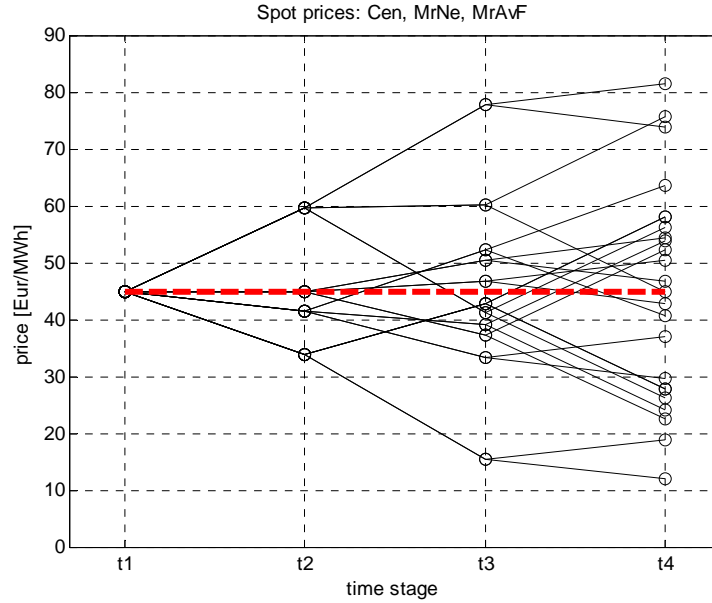


Figure 5. Spot prices π for cases Cen, MrNe, and MrAvF.

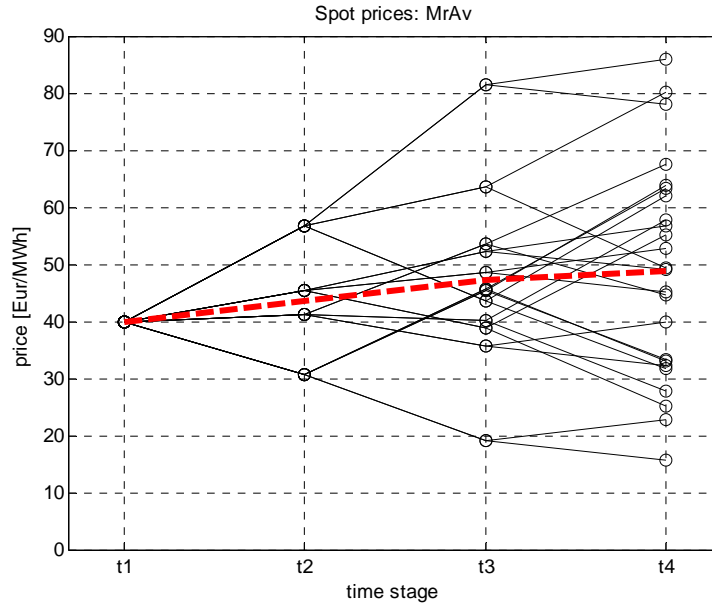


Figure 6. Spot prices π for case MrAv

In case of allowing that risk-averse market participants hedge their risk by buying-selling forward contracts, the obtained results are the same as in the centralized and risk-neutral cases. As shown in previous Table i, Table ii, and Table iii, the hydro-thermal scheduling, and the spot prices obtained in case MrAvF are the same as the ones obtained in MrNe or Cen.

The information of the contracts signed to achieve these results is summarised in Table iv. Given that the generator can sign a forward contract at each node of the tree (except in the terminal nodes), the information has been presented indicating the node of the tree where the contract is signed. As it was explained in section 4, we assume that the maturity date of a contract signed in time t will be $t + 1$. For instance, the generator signs at the stage t_1 a forward contract to sell a quantity of 23.961 GWh at a price of 44.926 Eur/MWh in all the nodes that belong to stage t_2 . In stage t_2 , the same agent sells a quantity of 18.052 GWh at a price of 33.719 Eur/MWh. This contract will be valid for the nodes 3.1,

3.2, and 3.3 (scenarios s_1 to s_6), which are the descendant nodes of node 2.1. In the same way, the contract to sell a quantity of 18.231 GWh at a price of 41.484 Eur/MWh signed in node 2.2 will be valid at its descendant nodes 3.3, 3.4, and 3.5 (scenarios s_7 to s_{12}).

Finally, as discussed in equation (32), notice that the resulting forward contract prices are the same as the spot market price at the node where the contract is signed.

Table iv. Contracts signed in the forward market

node	t	q^F (GWh)	π^F (€/MWh)
1.1	t1	23.961	44.926
2.1	t2	18.052	33.719
2.2	t2	18.231	41.484
2.3	t2	17.124	44.799
2.4	t2	14.263	59.704
3.1	t3	10.638	15.384
3.2	t3	5.871	42.887
3.3	t3	7.656	42.887
3.4	t3	14.180	33.326
3.5	t3	6.504	38.988
3.6	t3	10.076	52.138
3.7	t3	14.507	46.629
3.8	t3	3.771	37.347
3.9	t3	14.592	50.420
3.10	t3	7.770	41.239
3.11	t3	3.078	60.221
3.12	t3	15.129	77.651

5.3 Three agents case

In the previous example it has been assumed that both the thermal and the hydro generators belonged to the same company. The conclusions derived in section 4 are also valid in case of considering a greater number of generation companies that maximise simultaneously their utility functions. For instance, let assume that the thermal generator belongs to agent-1 and the hydro generator belongs to agent-2. The demand will act as agent-3 and will have the capability of entering into bilateral agreements to buy-sell forward contracts with both generator agents.

Moreover, it is possible to consider different risk profiles among the agents. Assume that $\beta = 0.001$ for the thermal generator, and $\beta = 0.005$ for the hydro generator. The resulting utility functions are the ones shown Figure 7 and Figure 8 where the circles represent the risk-neutral profits obtained for the 24 scenarios.

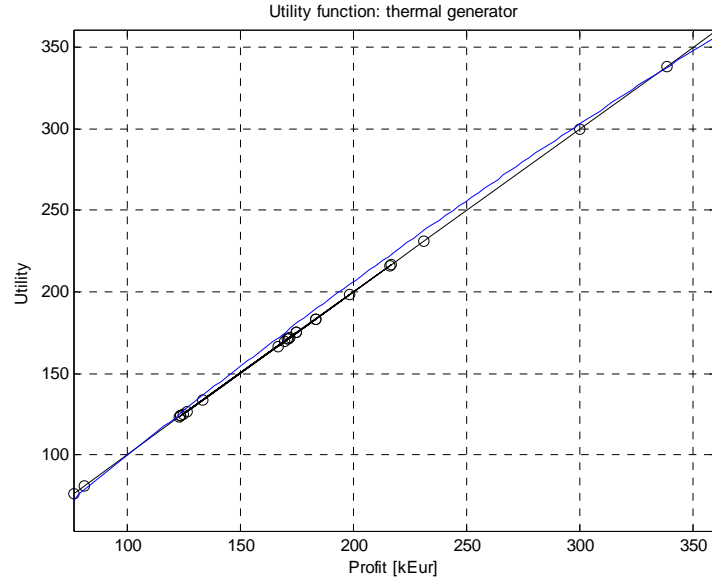


Figure 7. Utility function of the thermal generator

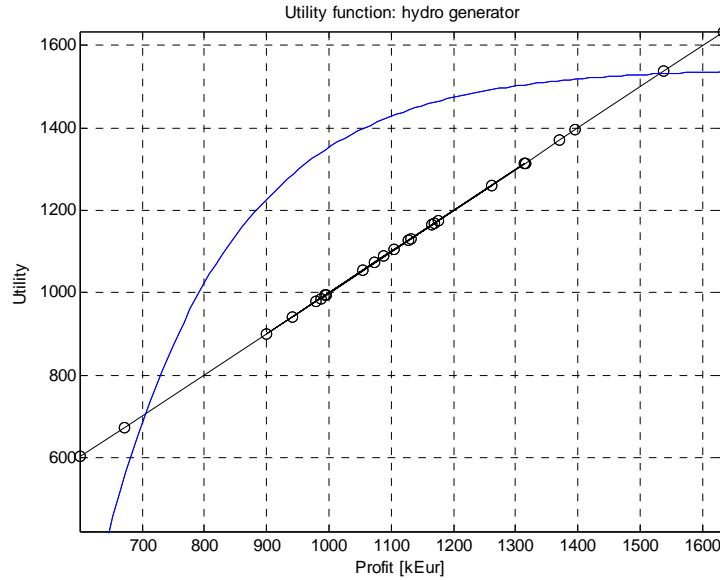


Figure 8. Utility function of the hydro generator

Table v shows the obtained scheduling of the thermal generator. It can be checked that the risk-neutral solution is again the same as the centralized one. However, for the risk-averse setting, the scheduling is different than the one obtained in the previous-subsection. This is due to the fact that now the two generation agents behave independently and with different risk profiles. However, what it is important to highlight is that when agents participate in the forward market, they can hedge their market risk resulting in the same operation of the generators as the centralized planning.

Table v. Thermal generation q^T (GWh)

	t1	t1	t1	t1	t2	t2	t2	t2	t3	t3	t3	t3	t4	t4	t4	t4
	Cen	MrNe	MrAv	MrAvF	Cen	MrNe	MrAv	MrAvF	Cen	MrNe	MrAv	MrAvF	Cen	MrNe	MrAv	MrAvF
s1	1.774	1.774	1.475	1.774	1.394	1.394	1.199	1.394	0.696	0.696	0.960	0.696	0.547	0.547	0.843	0.547
s2	1.774	1.774	1.475	1.774	1.394	1.394	1.199	1.394	0.696	0.696	0.960	0.696	0.840	0.840	1.121	0.840
s3	1.774	1.774	1.475	1.774	1.394	1.394	1.199	1.394	1.707	1.707	1.897	1.707	1.184	1.184	1.487	1.184
s4	1.774	1.774	1.475	1.774	1.394	1.394	1.199	1.394	1.707	1.707	1.897	1.707	2.181	2.181	2.444	2.181
s5	1.774	1.774	1.475	1.774	1.394	1.394	1.199	1.394	1.707	1.707	1.855	1.707	1.184	1.184	1.531	1.184
s6	1.774	1.774	1.475	1.774	1.394	1.394	1.199	1.394	1.707	1.707	1.855	1.707	2.181	2.181	2.483	2.181
s7	1.774	1.774	1.475	1.774	1.661	1.661	1.657	1.661	1.380	1.380	1.521	1.380	1.248	1.248	1.422	1.248
s8	1.774	1.774	1.475	1.774	1.661	1.661	1.657	1.661	1.380	1.380	1.521	1.380	1.509	1.509	1.676	1.509
s9	1.774	1.774	1.475	1.774	1.661	1.661	1.657	1.661	1.577	1.577	1.660	1.577	1.041	1.041	1.278	1.041
s10	1.774	1.774	1.475	1.774	1.661	1.661	1.657	1.661	1.577	1.577	1.660	1.577	2.059	2.059	2.262	2.059
s11	1.774	1.774	1.475	1.774	1.661	1.661	1.657	1.661	2.004	2.004	2.075	2.004	1.635	1.635	1.859	1.635
s12	1.774	1.774	1.475	1.774	1.661	1.661	1.657	1.661	2.004	2.004	2.075	2.004	2.350	2.350	2.553	2.350
s13	1.774	1.774	1.475	1.774	1.770	1.770	1.812	1.770	1.829	1.829	1.938	1.829	1.706	1.706	1.846	1.706
s14	1.774	1.774	1.475	1.774	1.770	1.770	1.812	1.770	1.829	1.829	1.938	1.829	1.950	1.950	2.085	1.950
s15	1.774	1.774	1.475	1.774	1.770	1.770	1.812	1.770	1.521	1.521	1.624	1.521	0.980	0.980	1.148	0.980
s16	1.774	1.774	1.475	1.774	1.770	1.770	1.812	1.770	1.521	1.521	1.624	1.521	2.007	2.007	2.151	2.007
s17	1.774	1.774	1.475	1.774	1.770	1.770	1.812	1.770	1.950	1.950	2.057	1.950	1.829	1.829	1.966	1.829
s18	1.774	1.774	1.475	1.774	1.770	1.770	1.812	1.770	1.950	1.950	2.057	1.950	2.069	2.069	2.202	2.069
s19	1.774	1.774	1.475	1.774	2.236	2.236	2.121	2.236	1.652	1.652	1.768	1.652	1.124	1.124	1.447	1.124
s20	1.774	1.774	1.475	1.774	2.236	2.236	2.121	2.236	1.652	1.652	1.768	1.652	2.130	2.130	2.410	2.130
s21	1.774	1.774	1.475	1.774	2.236	2.236	2.121	2.236	2.251	2.251	2.448	2.251	1.771	1.771	1.969	1.771
s22	1.774	1.774	1.475	1.774	2.236	2.236	2.121	2.236	2.251	2.251	2.448	2.251	2.693	2.693	2.868	2.693
s23	1.774	1.774	1.475	1.774	2.236	2.236	2.121	2.236	2.749	2.749	2.916	2.749	2.640	2.640	2.835	2.640
s24	1.774	1.774	1.475	1.774	2.236	2.236	2.121	2.236	2.749	2.749	2.916	2.749	2.856	2.856	3.045	2.856
avg	1.774	1.774	1.475	1.774	1.765	1.765	1.697	1.765	1.752	1.752	1.893	1.752	1.738	1.738	1.955	1.738

To achieve these results, the forward contracts signed between both generation agents and the aggregated demand are the ones shown in Table vi (notice that forward price is the same for all the agents at the same node).

Table vi. Contracts signed in the forward market

node	t	qT^F (GWh)	qH^F (GWh)	π^F (€/MWh)
1.1	t1	5.616	18.352	44.926
2.1	t2	2.568	15.458	33.719
2.2	t2	3.486	14.748	41.484
2.3	t2	3.159	13.931	44.799
2.4	t2	4.334	9.928	59.704
3.1	t3	0.696	9.943	15.384
3.2	t3	0.696	4.172	15.384
3.3	t3	1.699	5.957	42.887
3.4	t3	1.380	12.800	33.326
3.5	t3	1.568	4.936	38.988
3.6	t3	2.000	8.076	52.138
3.7	t3	1.829	12.678	46.629
3.8	t3	1.512	2.260	37.347
3.9	t3	1.950	12.642	50.420
3.10	t3	1.644	6.126	41.239
3.11	t3	2.245	0.833	60.221
3.12	t3	2.749	12.380	77.651

This example case has shown that a well-functioning forward market can replicate the efficient benchmark solution. This would imply the generation side hedging against risk by purchasing or selling forwards electricity and then selling or buying it at the spot price. A perfect competitive demand side would have clear incentives to be the counterpart of such contracts, even if it were risk neutral.

The problem is that up to now, due to a number of different possible reasons demand is not yet actively playing this important role. This lack of participation supports the regulators intervention to avoid inefficient market outcomes.

6 CONCLUSIONS

With the advent of electricity markets, regulation is required to supervise that the market is capable to ensure an adequate and efficient supply of electricity. This is particularly relevant at the generation activity, where the liberalization process has been more intense.

In this paper we have focused on the medium-term efficiency associated to the hydro resource management. In particular, we have isolated the welfare implications involved by the market-driven medium-term decisions when risk management plays a key role.

We have shown that when the generation side is risk averse and there is not a well-functioning long-term market, the market equilibrium can lead to a loss of welfare, when compared to the scenario with financial instruments. The reason is that generation companies use their hydro resources to hedge against low-profit scenarios. And this use is inefficient from the social perspective.

A well-functioning forward market would improve the outcome. This would imply the generation side hedging against risk by purchasing or selling forwards electricity and then selling or buying it at the spot price. A perfectly competitive demand side would have clear incentives to be the counterpart of such contracts, even if it were risk neutral (the case analyzed here).

The problem is that up to now, due to a number of different possible reasons demand is not yet actively playing this important role. This lack of participation poses the question of the potential necessity of introducing regulatory measures to avoid inefficient market outcomes.

Regulatory implications: ensuring also an efficient medium-term planning

It has been studied how the net social benefit increases when both generators and demand enter into long-term contracts. The problem is that up to now, and due to a number of different reasons, in the vast majority of electricity markets, demand is not yet actively playing this important role, see e.g. (Rodilla and Batlle, 2012) or (Neuhoff and De Vries, 2004). No matter the reason, the final result is a malfunctioning of long-term markets that hampers the efficiency of the electricity supply.

In this context, regulators worldwide are evaluating the necessity of intervening or they are actually intervening by administratively introducing any kind of reinforcement of the long-term signal perceived by generators. This can be achieved either by compelling demand to enter into long-term contracts or by directly acting on its behalf, see Batlle & Rodilla (2010).

Although this kind of regulatory mechanism has commonly been aimed and justified as a way to enhance system adequacy, a rigorous analysis of the mechanisms put in place shows that in many cases, the objective is also to affect the way generators operate and plan their generating units in the medium-term planning, particularly the one corresponding to the so-called LEPs (Limited Energy Plants, hydro in most cases, but also gas-fired plants, for instance). This is for example the view in the Colombian electricity market (a hydro-dominated system), where the objective of long-term mechanisms has always been that of ensuring an efficient hydro resource management not only of new investment, but also of already installed facilities. Indeed, after the scarcity conditions occurred in 2009/2010 with the prevailing long-term mechanism, the Colombian Wholesale Electricity Market Monitoring Committee (CSMEM, 2010) stated that hydro generators preferred to assume a future non-compliance of their physical commitment than to face a certain economic loss in the present. This was pointed out as something that needs to be revisited.

Our analysis, focused on the implications of market incompleteness on the optimal medium-term planning suggests that when designing these reliability mechanisms, the focus should also be put on assessing the suitability of adding additional incentives for generators to properly manage their energy resources in this term.

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