Fuel Prices Scenario Generation based on a Multivariate GARCH Model for Risk Analysis in a Wholesale Electricity Market

Carlos Batlle and Julián Barquín

***Abstract*—This paper presents a fuel prices scenario generator in the frame of a simulation tool developed to support risk analysis in a competitive electricity environment. The tool feeds different exogenous risk factors to a wholesale electricity market model to perform a statistical analysis of the results. As the different fuel series that are studied, such as the oil or gas ones, present stochastic volatility and strong correlation among them, a multivariate Generalized Autoregressive Conditional Heteroskedastic (GARCH) model has been designed in order to allow the generation of future fuel prices paths. The model makes use of a decomposition method to simplify the consideration of the multidimensional conditional covariance. An example of its application with real data is also presented.**

***Index Terms*—Fuels, Monte Carlo methods, Power system modeling, Risk analysis, Stochastic processes.**

I. Introduction

UNTIL the early eighties, in the electricity supplying industry no market existed; instead the business was organized as regulated, vertically integrated utilities whose costs where fully asserted by the regulator. The development of the technology, led by the evolution of the gas turbines y combined cycles, together with new economic conditionings, launched the introduction of competition in the generation industry [1]-[3]. Beginning in Chile and more ambitiously followed by England & Wales and Argentina, new market-based ideas where put in place in the electricity sector. This scheme has been established since then in many other countries, such as Scandinavia, Spain, Australia...

From that point on, both economic and engineering science put their eyes in the study of the new electric environment, and both are trying to take advantage of their former expertise. The challenge is to adapt the models designed until now for other markets, in the case of economists, and for a regulated framework, in the case of engineers, in order to make them suitable for analyzing the singular electricity markets and for supporting decision-making.

From this point of view, the models applied up to now to the electricity markets may be classified in two areas, according to their origins, the ones that derive from the economic and more precisely financial science and the ones that come from the engineering science [4].

The latter ones face a more detailed representation of the production process that characterizes the electricity market. However, it is not easy to implement in these models ways to deal with the added uncertainties inherent to electrical markets, in addition to the classical uncertainty sources. This is especially true in case of being the agents' strategic behavior relevant, as opposed to the widely studied, but scarcely instantiated, perfect competition market.

One approach, used in the tool presented in [5], is to feed a strategic competition electricity market on a number of different scenarios of the relevant risk factors, which are considered to be exogenous (independent) of the electricity agents behavior. So it is the case of hidraulicity, demand in the short-term (2 years at most) study horizon, and fuel prices. Electricity prices are, on the other hand, endogenous variables resulting from the exogenous risk factors evolution and agents actions.

Fuel is the main raw material from which electricity is produced in most electricity markets, and thus it is one of the key risk factors that influence on electricity market prices. However, the way to model them to evaluate this influence it is not obvious. This paper focuses on the fuel prices scenario generator. First, a time series model is built, in this case over the weekly prices of a number of commodity indexes the modeler considers sufficiently relevant to model the fuel the plants in the system burn, named as *base fuels*. Then the final value of the costs of the fuel that every thermal plants uses to generate electricity is determined.

In this paper, we will just focus on the first step, the methodology required to sample possible matrices of the prices of the base fuels in the future. We are not going to delve on how the thermal plants variable costs can be obtained from these indexes, since only the plant operator is fit to decide the percentage of the variable cost of a plant that can be explicated by a change in the fuel indexes. Variable costs traditionally are composed by the cost of the fuel plus the O & M costs and a part of fungible cost. Even the indexation of the first component is not a simple question, since it depends on many factors (plant efficiency, transportation costs, storage

This work was supported in part by Endesa S.A. (Spain), in the framework of studies on competitive market behavior.

C. Batlle and J. Barquín are with Instituto de Investigación Tecnológica, Universidad Pontificia Comillas, Sta. Cruz de Marcenado 26, 28015 Madrid, Spain (e-mail: carlos.batlle@iit.upco.es, julian.barquin@iit.upco.es).

capabilities, agent's strategy...).

The time series model building is also a difficult task. Since the different fuel series that are studied, such as the oil or gas ones (see Fig. 1), present stochastic volatility and strong correlation among them [6], a multivariate Generalized Autoregressive Conditional Heteroskedastic (GARCH) model [7] has been designed in order to allow the generation of future fuel prices paths.

Multivariate GARCH models present some difficulties related to the consideration of the multidimensional conditional covariance. In this paper, a decomposition method to simplify this problem is proposed as well.

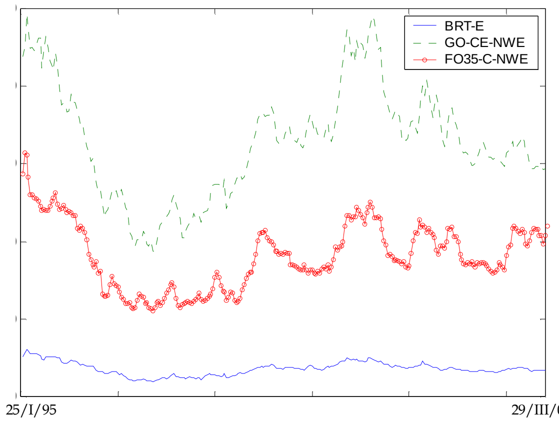

Fig. 1. Evolution of three fuel prices indexes from year 1995 to 2000.

The sequel of this paper is organized as follows. Section II includes a brief description of the general structure of the simulation model. Then, the paper core (the fuel prices model) is presented. Section IV is an example of its application with real data. Finally, our conclusions are stated.

II. The Risk Analysis Model Framework

In [5] a model for analyzing electricity market risk is proposed. The basic idea is to focus the modeling of the uncertainty on the price drivers' behavior rather than in the price itself, what in the literature is named as *fundamental* approach.

This way the future price distribution is obtained by feeding a market model with scenarios representing the possible realizations of the considered as *exogenous variables*. These variables are those that can be considered as independent of the electricity market itself, such as hydro inflows, demand or the fuel costs [5]. The way the model copes with the fuel prices modeling is the subject of this dissertation.

The model general framework, illustrated in Fig. 2, consists in a set of independent and interrelated modules with the aim of allowing the development of every one of them according to the market evolution.

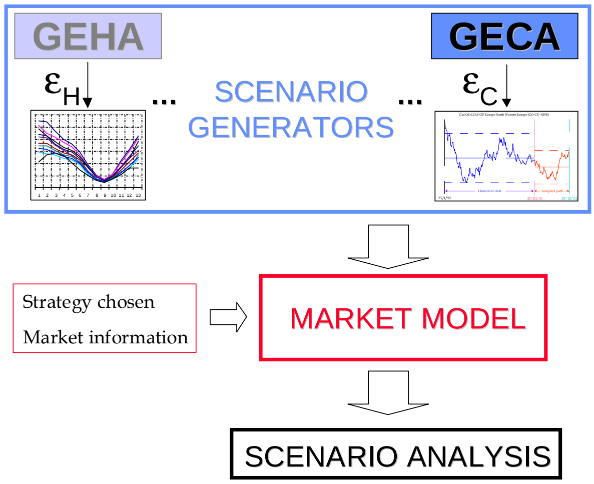

Fig. 2. The risk analysis model framework.

A. Uncertainty via scenario generation

The basic assumption behind the scenario generators modules is that the evolution of the variables that each one of them represents is correlated to no other. While this is rather straightforward in the case of hydraulicity or demand, it is true that it may be questioned in the fuel prices case.

Nevertheless, the model supposes the generating agent behaves as a mere observer in front of the evolution of the fuel prices in the commodity markets. In this sense, the *fuel prices scenario generator* (GECA) that is described next assumes the agent is a fuel price taker. Although it is clear that every electricity agent in the market may (and very often does) also exploit in any way his capability to manage his purchasing, in the model this is disregarded leaving the evaluation of the flexibility entailed open to a higher scope model.

B. The strategic market model

The main reason why the fundamental analysis is undertaken is that market agents' strategic behavior needs to be represented to properly analyze most of the actual electricity markets and particularly the Spanish wholesale electricity market.

While the demand or fuel prices modeling may be valid for many different markets, the representation of the way agents interact in different markets and even in the same market when the scheme changes requires a particular model. Also, depending on the time scope of the analysis different models may be considered suitable. For example, for long-term analysis a model that does not take into account start up costs may be suitable while for a short-term study a more detailed representation of the production cost function is recommended. The modular structure allows changing the market model depending on this casuistry. In fact, the model has already been applied to analyze a real electricity market, the Spanish one, using different market models, the ones described in [9] and [10].

Both are *market equilibrium models* capable of taking into account the strategic behavior that characterizes most part of the actual electricity markets. Besides, both provide not only market prices but also production costs and benefits of the suppliers acting in the market.

C. Scenario analysis module

As mentioned, market prices, production costs and profits are the outcomes of the market model. Once the model is fed with all the set of possible realizations of the risk factors considered, density functions of the outcomes may be analyze in order to derive risk measures such as Value-at-Risk or the results sensitivity to each risk factor.

It is important to note that the model is just aimed to risk analysis and this way is disregarding the value of flexibility inherent to the managing capacity of the agents. The model provides the exposure of the agent supposing that nothing can be done in future steps in time. Thus, it provides the worst case scenario, the most pessimistic analysis possible.

III. THE FUEL PRICES SCENARIO GENERATOR (GECA)

The objective of the GECA is to generate fuel prices paths that feed the market model allowing an analysis of the impact of the fuel markets evolution over the electricity wholesale market price or even the analysis of the fuel market themselves.

GECA is conceived under the fundamental idea of facing MonteCarlo-based analysis, in which it could be possible to cluster and control in any way the scenario generation process, following the common idea presented in [5].

The market equilibrium models used are rather time-consuming due to the linear optimization problem that they solve. They represent a real electricity market with many plants (more than a hundred in the Spanish case) throughout a no less than a medium-term scope. For this reason and taking into account that at least up to three variables may be considered at a time (hydraulicity, demand and fuel prices), controlling the sampling process to achieve the objective of enhancing the computation turns a key issue.

Besides, leaving nuclear plants aside, *thermal plants* burn several types of fuel, mainly oil, gas and coal derivatives. In principle, this would lead to an unacceptable high number of variables. Thus the first assumption taken is that all the variable costs of thermal plants can be indexed to any available spot price series[1], e. g. fuel fired plant costs related to series for the crude oil such as *Brent*. The number of spot prices series required may be rather high (no smaller than three) what may complicate considerably the multivariate GARCH model. As it is going to be seen afterwards, this is overcome decomposing the fuel prices GARCH model through *Principal Components Analysis* (PCA).

A. The multivariate GARCH fuel prices model

As it has been formerly mentioned, the model assumes the generating utility acts as a mere fuel prices taker, what makes suitable to apply a quantitative model.

1) The quantitative modeling approach

Since the early thirties, the research aimed to obtain a model that allowed to parameterize the behavior of the financial markets has turned into an science community obsession.

After many frustrating attempts that proposed complicated models that in many cases depended on non-observable entries, Fisher Black y Myron Scholes [11] hit the target.

The method assumes that prices follow a random walk, i. e. proportional prices changes are normally distributed. This implies that prices at any point in time in the future follow a lognormal distribution.

The *Black & Scholes model* states the behavior of the underlying price may be described through a stochastic Gauss-Wiener process defined by two constant parameters, the expected average return and the volatility.

Let s_t be the underlying asset price at time t and $r_t = \ln(s_t)$ the expected return, then:

$$ds_t = \mu \cdot dt + \sigma \cdot d\varepsilon_t \quad (1)$$

where μ represents the expected return or *cost-of-carry*, σ the volatility and $d\varepsilon$ the white noise.

The process fulfils the *Markov property*, which means that the estimated value of the price at any time depends only on the foregoing value.

The model is based on a series of additional assumptions: no transaction costs (thus liquidity) nor taxes, fully divisible financial assets, no arbitrage opportunities... that, although arguable in any way, were good enough for short-term analysis of traditional stock markets.

However, these assumptions are no longer acceptable, particularly in the commodity markets. The analysis of the implied volatility calculated from the little fuel prices data available, such as oil and gas, shows that volatility can not be assumed constant [6]. Prices oscillate around a long-term average value (mean reversion). A strong correlation between volatility and price level can be easily detected.

Aiming to adapt the stochastic models to the casuistry of the commodity markets, many models that try to reflect these phenomena have been developed.

Reference shows some of these models, among which it is suitable to highlight the ones that assume a stochastic behavior of the volatility:

$$\sigma_t = F\left(\sigma_{t-1}, z_t, t\right) \quad (2)$$

where z_t is a white noise.

This way, the intention is to capture the fact volatility is price dependent. The model GECA, to capture the heteroskedasticity[2] of the fuel prices series, is based on routines that allow to adjust a model of the GARCH-type (*Generalized Autoregressive Conditional Heteroskedasticity*)

[1] How to do this indexation is not a simple question, as it depends on many factors (plant efficiency, transportation costs, storage capabilities, agent's strategy...) and this is not going to be subject of this paper.

[2] A series presents an heteroskedastic behavior when volatility is not constant over time.

proposed in [7]. Besides, the model permits to consider satisfactorily multivariate series, representing the existent covariance among the different fuel data series (e. g. oil and gas).

B. The multivariate GARCH model

The GARCH model proposed by Bollerslev in [7] is a generalization of the *Autoregressive Conditional Heteroskedasticity* model (ARCH) enounced by Engle in [12].

The more general expression of it, a multivariate GARCH follows.

Let $S_t = (s_{1t}, \ldots, s_{Kt})'$ be a vector containing the values at time t of K variables considered, e. g. different commodity prices. Thus, the variables evolution can be modeled as a *vector autoregressive process VAR*(p), where p is the order of the process, i. e. the number of relevant precedent values:

$$S_t = M + A_1 \cdot S_{t-1} + \ldots + A_p \cdot S_{t-p} + U_t \quad (3)$$

M is a vector of dimension [$K \times 1$], A_i , $i = 1, \ldots, p$, are fixed coefficient matrices of dimension [$K \times K$] and U_t is a white noise with nonsingular covariance matrix Σ_U :

$$\Sigma_U = \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{1k} \\ \vdots & \ddots & \vdots \\ \sigma_{k1} & \cdots & \sigma_{KK} \end{bmatrix} \quad (4)$$

The *vector autoregressive processes VAR*(p) express the value of the variable in a more general sense in two ways, one associated to the *autoregressiveness* and the other to the *vectorization*.

The first relates the value of the asset price with more than the previous point in time. The *mean-reverting model* [17] is the first step that can be taken to move away from the *Markov property*, as the information in more than the previous point in time is relevant. In fact, it represents a particularization of the *autoregressive processes*, the one of first order, $p = 1$.

On the other hand, the *vectorization* allows to relate the asset price value to other assets that could present a correlated behavior. This is of a lot of use in the case of commodities, e. g. gas prices are deeply correlated with oil prices.

The vector GARCH(m , r) model states that, in each period, the non-singular covariance matrix Σ_U of U_t depends not only on the last r residuals, but on its own m previous values:

$$\begin{aligned} \Sigma_t &= V + \\ &+\Delta_1 \cdot U_{t-1} \cdot U'_{t-1} + \ldots + \Delta_r \cdot U_{t-r} \cdot U'_{t-r} + \quad (5) \\ &+\Theta_1 \cdot \Sigma_{t-1} + \ldots + \Theta_m \cdot \Sigma_{t-m} \end{aligned}$$

where V , Δ , and Θ are square matrices of dimension K .

C. Multivariate GARCH decomposition

As previously described, the multivariate model analyzes a number, K , of price series jointly. This analysis turns out into a non-singular square covariance matrix Σ_U of $K \times K$ dimension, whose out-of-the-diagonal values express the conditional crossed covariance among the series. Unfortunately, usually these parameters are not null, what deeply complicates the further stochastic variance analysis.

As the number of parameters that appear when analyzing the covariance matrix grows in such a way that it makes it unaffordable, most part of the literature proposes to add restrictions that reduced this number and that guaranteed that the matrix was positive definited to assure the process stability. These restrictions are quite straightforward in the univariate context (e. g. in a GARCH(1,1) every model parameter has to be positive) but less obvious in a multivariate model.

In the introductory section of [13], where Engle presents a proposal to solve the problem, a detailed description of the attempts found in the literature up to the moment can be found. They are either to decompose the problem in a set of univariate models, either to assume that the covariance values are constant over time what reduces the problem to the analysis of the values in the diagonal.

We opt for applying the *Principal Component Analysis* (PCA) technique. PCA has been intensively used in engineering (signal analysis, circuits theory...) but also in almost every scientific field, such as medical statistics, ecology, econometrics or even history[3]. Reference [15] applies PCA to develop a forward curve model for energy derivatives valuation.

The application to the GARCH model decomposition that is presented afterwards is analogous to the one presented in detail in [8].

The reason why we develop this approach is that it adapts very well to the particular application pursuit: the fuel prices scenario generation.

First, decomposing the covariance matrix into its principal components allows to estimate its parameters considering just K univariate GARCH models.

Besides, PCA offers a key additional piece of information of a lot of use for the scenario generation process. The eigenvalues obtained in the PCA represent the weights of each component in descending order. So, taking advantage of the nature of the series considered, which are very correlated (see Fig. 1), it is possible to resume most of the information in just one component, i. e. just one parameter or risk factor.

This is seized to cluster the scenarios in order to better cover the universe of possible realizations of the risk factor *fuel prices* ϕ_f . Being able to resume the set of series in just one, the first principal component, eases the scenario

[3] Reference [14] establishes a very curious relation between the primitive migrations and the principal components of the human proteins.

clustering task, as for example makes simpler to build an scenario tree. Reference [16] contains a review of the different approaches to create a scenario tree from a set of samples.

As the desired number of scenarios that wanted to be taken into consideration was rather small, a much more simple clustering process, outlined next in section III.E., was developed for the model.

Next, the PCA of the non-singular covariance matrix Σ is presented.

Decomposing the matrix Σ we have:

$$\Sigma = E\left[U_t \cdot U_t'\right] = \\ = \left[\Omega_1 \quad \dots \quad \Omega_K\right] \cdot \begin{bmatrix} \sigma_1^2 & & \\ & \ddots & \\ & & \sigma_K^2 \end{bmatrix} \cdot \begin{bmatrix} \Omega_1' \\ \vdots \\ \Omega_K' \end{bmatrix} \tag{6}$$

Let $G \cdot \Sigma^{-1} = G \cdot G'$, then

If we define $\mathrm{M}_t = G^{-1} \cdot U_t$, then

$$U_t = G \cdot \mathrm{M}_t = \\ = \left[\Omega_1 \quad \dots \quad \Omega_K\right] \cdot \begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_K \end{bmatrix} \cdot \begin{bmatrix} \Omega_1' \\ \vdots \\ \Omega_K' \end{bmatrix} \cdot \mathrm{M}_t = \tag{7} \\ = \sigma_1 \cdot \Omega_1 \cdot \left(\Omega_1' \cdot \mathrm{M}_t\right) + \dots + \sigma_K \cdot \Omega_K \cdot \left(\Omega_K' \cdot \mathrm{M}_t\right)$$

Let

$$W_t = \begin{bmatrix} \sigma_1 \cdot \Omega_1' \cdot \mathrm{M}_t \\ \vdots \\ \sigma_K \cdot \Omega_K' \cdot \mathrm{M}_t \end{bmatrix} \tag{8}$$

thus

$$U_t = \left[\Omega_1 \quad \dots \quad \Omega_K\right] \cdot W_t \tag{9}$$

If we calculate the typical elements of $E\left[W_t \cdot W_t'\right]$:

$$E\left[W_{it} \cdot W_{it}\right] = E\left[\sigma_i^2 \cdot \left(W_i' \cdot \mathrm{M}_t\right)^2\right] = \\ = \sigma_i^2 \cdot W_i' \cdot E\left[\mathrm{M}_t \cdot \mathrm{M}_t'\right] \cdot W_i = \\ = \sigma_i^2 \cdot W_i' \cdot I \cdot W_i = \sigma_i^2 \tag{10}$$

and

$$E\left[W_{it} \cdot W_{jt}\right] = \\ = E\left[\sigma_i \cdot \sigma_j \cdot W_i' \cdot \mathrm{M}_t \cdot \mathrm{M}_t' \cdot W_j\right] = \\ = \sigma_i \cdot \sigma_j \cdot W_i' \cdot E\left[\mathrm{M}_t \cdot \mathrm{M}_t'\right] \cdot W_j = \\ = \sigma_i \cdot \sigma_j \cdot W_i' \cdot I \cdot W_j = 0 \tag{11}$$

Thus

$$E\left[W_t \cdot W_t'\right] = \begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_K \end{bmatrix} \tag{12}$$

As it can be easily seen, the covariance matrix obtained from the decomposition in singular vectors is diagonal, what lets to decompose the multivariate GARCH process in a set of K univariate models ($k = 1, \dots, K$) of the form:

$$\sigma_{kt}^2 = \nu_k + \delta_{k1} \cdot \varepsilon_{kt-1}^2 + \dots + \delta_{kr} \cdot \varepsilon_{kt-r}^2 + \\ + \theta_{k1} \cdot \sigma_{kt-1}^2 + \dots + \theta_{km} \cdot \sigma_{kt-m}^2 \tag{13}$$

As previously mentioned, as the singular components are multiplied by its corresponding eigenvalues and these are always of decreasing value, often some of them may be disregarded. In fact, usually when analyzing oil and gas price series, the first eigenvalue is considerably bigger than the rest, what implies that the first component gathers most of the series information.

D. Estimation of the multivariate GARCH parameters

The multivariate model decomposition into univariate models eases significantly the parameters' calculation. However, the estimation of a GARCH model is not an easy task to implement. Getting into the subject it is a worthy experience. The reason is double as it allows us, on the one hand, to better understand the model and on the other to avoid problems that often appear in some of the commercial software as not always converge when trying to solve the problem.

The estimation process of the parameters of the GARCH(m, r) model is faced via *maximum likelihood*, as detailed in [7]. The procedure is equally of use for the adjustment of the ARCH(r) models, as these ones are a particular case of the generalized models.

As suggested in [5], to obtain the estimators of maximum likelihood, the algorithm applied is the Berndt, Hall, Hall and Hausman (a non-linear optimization with no restrictions based in the gradient).

E. Scenario clustering process

Once the model parameters are estimated, a high number of possible fuel prices paths are generated for the scope of the analysis. Then the whole set of scenarios are clustered following the hierarchical criteria described in [5] that share all the scenario generators involved in the general model framework.

Assume that a high number of paths, e. g. a thousand, are pseudo-randomly sampled for a two-year period. The first clustering criterion, ϕ_{f1}, is the average value of the first principal component calculated in the decomposition process just presented. As mentioned, this component is the one that gathers most of the series information. As the model is focused on long-term analysis and at the same time the market model represents the electric market operation with detail, the number of samples considered E has to be limited.

The density function of $f(\phi_{f1})$ can be easily calculated and scenarios whose values of ϕ_{f1}^{e} were closer to the "e-*percentile*", $f'(100 \cdot e/(E+1))$, $e = 1,\ldots,E$, can be chosen as representatives. This process is illustrated in Fig. 3.

Fig. 3 shows the principal component resultant of the analysis of the series shown in Fig. 1 together with the two representatives, $e=1$ and $e=19$ for $E=19$. The sampled paths shown in the figure correspond to 95% and 5% percentile representatives ($f(\phi_{f1}^{19}) = 0.95$).

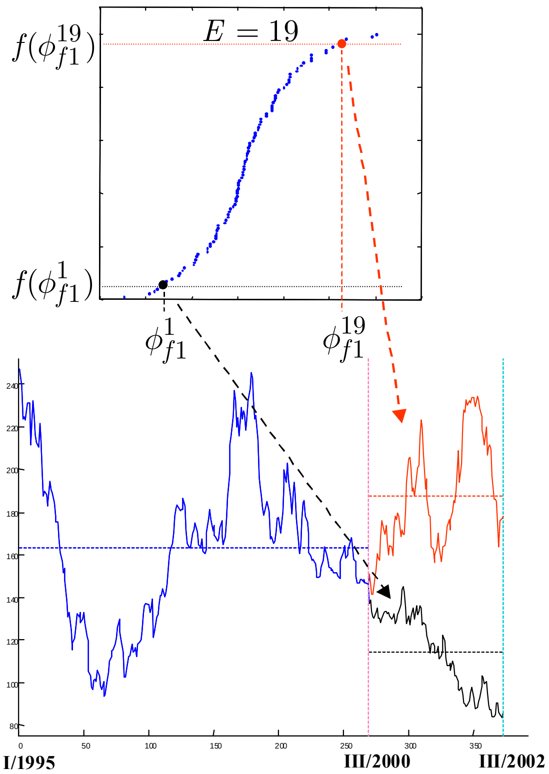

Fig. 3. Scenario paths clustering.

As it can be seen, both representatives' shapes are very different, what serves also to illustrate that using ϕ_{f1} as the only clustering criteria may not be good enough due to the inherent seasonality of electricity demand. This may have some impact when analyzing electricity price risk, due to the inherent seasonality of electricity demand. Two samples for one-year period with the same average value, but one with high prices in the first and the last three months and another with inverse shape (low prices in winter periods and high prices in the summer ones) result in different electricity market prices. A higher level of detail, e. g. building a four-stage scenario tree would be then required if we wanted to analyze how does this affect electricity prices.

F. Determination of the variable costs of the system thermal plants

We are not going to delve on how the thermal plants variable costs can be obtained from the matrix containing the base fuel prices for each scenario.

Once the base fuel indexes matrix are sampled, the variable cost of thermal plant u, for a scenario ζ will be obtained as:

$$c_u^{\zeta} = \sum_K \chi_k \cdot S_k^{\zeta} + c_u^{*} \qquad (14)$$

where χ_k expresses the relation between the fuel cost component of the plant with each of the spot prices of the K *base fuels* and c_u^{*} represents the other components of the final variable cost (O & M and fungibles).

It is up to the modeler to decide the percentage of the variable cost of a plant that can be explicated by a change in the fuel index price, i. e. to determine the values of χ_k. This indexation depends on many factors (plant efficiency, transportation costs, storage capabilities, agent's strategy...).

IV. Case Example

Finally, Fig. 4 illustrates the decreasing trend of the average annual spot price of the electricity in a synthetic real-sized hydrothermal market for $E = 10$ scenarios of fuel prices generated with GECA, clustered attending to its annual average price and solved using the market equilibrium model presented in [10].

Just a look at the graph may provide interesting conclusions. For example, one could observe the sensitivity of the electricity market price to fuel prices. In this synthetic system, it can be seen that a decrease in the fuel cost results in a larger percent reduction in the electricity spot price.

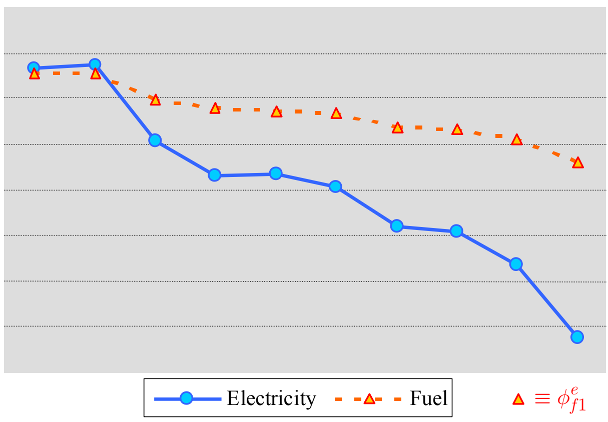

Fig. 4. Electricity spot prices vs. fuel prices

Obviously, there is a direct correlation between both prices, an increase in the fuel prices leads to an increase in electricity prices and *vice versa*. However, as the values in the graph are averaged across the year periods and the impact of the fuel prices' changes on the electricity prices depends on load pattern, then cases with inverse correlation among the annual average values may appear, such as the two first scenarios shown in the figure. For instance, assume we consider two annual fuel prices paths, e_1, e_2, with the same average value. e_1 is characterized by high prices in the beginning of the year and low in the end while e_2 starts with low prices that steadily increase along. As it can be assumed that electricity load presents an increasing trend through time, e_1 is expected to result in a lower average electricity price for the whole year.

V. Conclusion

A fuel prices scenario generator in the frame of a simulation tool developed to support risk analysis in a competitive electricity environment has been presented. The model proposed considers fuel prices as exogenous regarding the wholesale electricity prices and faces the generation of future paths through a multivariate Generalized Autoregressive Conditional Heteroskedastic (GARCH) model, in order to allow to take into consideration the stochastic volatility detected in actual commodity markets.

The decomposition method proposed makes possible the consideration of the multidimensional conditional covariance and shows the possibility of representing the various fuel series by just its first principal component. This way, it is possible to generate and cluster scenarios to feed a market model in the frame of a fundamental electricity price risk model.

In markets such as the electricity ones, where price is affected by various factors (hydrology, demand, fuel prices,...), to be able to quantify the actual impact of each one of them is a key task of generating firms. The model is a useful tool when it comes to analyze the effect on electricity market price of the evolution of the fuel prices in commodity markets.

VI. Acknowledgements

The authors wish to thank Angel Garro and Carlos Vázquez for the contributions and valuable comments provided.

VII. References

[1] P. L. Joskow, R. Schmalensee, *Markets for power: an analysis of electrical utility deregulation*, MIT Press, 1983.
[2] F. C. Schweppe, M. C. Caramanis, R. D. Tabors, R. E. Bohn, *Spot pricing of electricity*, Kluwer Academic Publishers, 1988.
[3] I. J. Pérez-Arriaga, "Principios económicos marginalistas en los sistemas de energía eléctrica," Working paper IIT-93-044, 1994.
[4] C. Batlle, J. Barquín, "Modeling electricity market risks," Working paper IIT-01-115A, Instituto de Investigación Tecnológica, Universidad Pontificia Comillas, Dec. 2001.
[5] C. Batlle, I. Otero-Novas, J. J. Alba, C. Meseguer, J. Barquín, "A model based in numerical simulation techniques as a tool for decision-making and risk management in a wholesale electricity market. Part I: General structure and scenario generators," in *Proc. 6th International conference on Probabilistic Methods Applied to Power Systems (PMAPS)*, Madeira, Portugal, Sept. 2000.
[6] D. Duffie, "Volatility in energy prices," in *Managing energy price risk*, Risk books, 1999.
[7] T. Bollerslev, "Generalized Autoregressive Conditional Heteroskedasticity," *Journal of econometrics*, vol. 31, pp. 307-327, 1986.
[8] C. Alexander, "Orthogonal GARCH," in *Mastering Risk Volume 2*, Financial Times – Prentice Hall, pp 21-38, 2001.
[9] I. Otero-Novas, C. Batlle, J. J. Alba, C. Meseguer, J. Barquín, "A model based in numerical simulation techniques as a tool for decision-making and risk management in a wholesale electricity market. Part II: The market model," in *Proc. 6th International conference on Probabilistic Methods Applied to Power Systems (PMAPS)*, Madeira, Portugal, Sept. 2000.
[10] J. Barquín, E. Centeno, A. López-Nicolás, R. Quinteros, "Forecasting the Chilean Short-Term Electricity Market Behavior under a New Proposed Regulation," Working paper IIT-02-007A, Instituto de Investigación Tecnológica, Universidad Pontificia Comillas, 2002.
[11] F. Black, M. Scholes, "The pricing of options and corporate liabilities," *Journal of Political Economy*, vol. 81, May-June 1973.
[12] R. F. Engle, "Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation," *Econometrica*, vol. 50, no. 4, pp. 987-1007, July 1982.
[13] R. F. Engle, K. Sheppard, "Theoretical and Empirical properties of Dynamic Conditional Correlation Multivariate GARCH," Working Paper No. 8554, *National Bureau of Economic Research*, October 2001. Available at http://www.nber.org/papers/w8554
[14] L. L. Cavalli-Sforza, M. Seielstad, *Genes, Peoples, and Languages*, North Point Press, 2000.
[15] L. Clewlow, C. Strickland, *Energy derivatives: Pricing and risk management*, Lacima publications, London, 2000.
[16] N. Gulpinar, B. Rustem, R. Settergren, "Simulation and Optimization Approaches to Scenario Tree Generation, " Imperial College of Science, Technology and Medicine, London, UK, September 2001.
[17] O. Vasicek, "An equilibrium characterization of the term structure," Journal of Financial Economics, vol. 5, pp. 177-188, 1977.

VIII. Biographies

Carlos Batlle (Madrid, 1972) obtained his Electrical Engineering degree from the Universidad Pontificia Comillas (ICAI), Madrid, after having spent his final year at the Ecole Polytecnique Fédéral de Lausanne (EPFL), Switzerland where he submitted his diploma dissertation in artificial intelligence. Currently he is a researcher at the Instituto de Investigación Tecnológica (IIT) where he develops his Ph.D. work on generation company risks in a liberalized electricity market. His areas of interest also cover regulation of electric power systems.

Julián Barquín Gil (M'94) obtained a Degree in Industrial Engineering (1988) and a Ph.D. in Industrial Engineering (1993) at the Universidad Pontificia Comillas, Madrid, and the Licenciado en Ciencias Físicas Degree in 1994. He belongs to the research staff at the Instituto de Investigación Tecnológica. His present interests include control, operation and planning of power systems.