

A strategic production costing model for electricity market price analysis

Carlos Batlle and Julián Barquín, *Member, IEEE*

Abstract—Production costing models (PCMs) have been extensively used to analyze traditional power systems for decades. These tools are based on the costs of production, but in oligopolistic electricity markets market prices can not be explained attending just to marginal costs but instead bid prices have to be considered, since market participants seize their dominant position in the market looking for higher profits. Thus, the merit order composition criteria applied in traditional PCMs has to be somehow reconsidered in order to be able to represent the agents' strategic bidding. The objective of the strategic production costing model (SPCM) presented in this paper is to evolve the PCM approach to adapt it to the actual wholesale electricity markets without losing its typical advantages. The generalization proposed allows to represent an oligopolistic hydrothermal electricity market and provides the system price-duration curve as well as the income and expected costs of every generating agent. Compared with other oligopolistic models, the main advantage of the SPCM is its potential computational speed, which makes it very suitable for risk analysis studies that require considering a large number of scenarios.

Index Terms— Marginal price, strategic behavior, oligopolistic markets, power system modeling, risk analysis.

I. INTRODUCTION

NO power market existed in the power industry until the Eighties; instead the business was organized as regulated, vertically integrated utilities whose costs were fully asserted by the regulator. The technology development, led by the evolution of the gas turbines and combined cycles, together with new economic conditionings, launched the introduction of competition in the power generation industry. Beginning in Chile and more ambitiously followed by England and Wales and Argentina and later many other countries, new market-based ideas have been put in place in the electricity sector.

From then on, economic and engineering sciences have put their eyes in the study of the new electric environment, and both are trying to take advantage of their previous expertise. One of the challenges is to adapt the models designed until now for other markets, in the case of economists, and for a regulated framework, in the case of engineers, to make them suitable for analyzing the singular electricity markets.

Production costing models (PCMs) can be classified among this latter category, since they seize the traditional knowledge of engineers and consider the electricity systems technical characteristics. Next, an evolution of this approach intended to

face the question of price analysis in a competitive electricity market is presented.

A. Production costing models

PCM have now been in existence for more than half a century and they have been subject of intense usage in the electricity field [1],[2]. Firstly, they were used for system planning and later on their scope was widened to analyze the effects of load management, fuel shortages and reliability.

PCMs take into account the generating units efficiency characteristics, including fuel costs per unit of energy supplied together with a representation of an economic scheduling and dispatching of the units in the system. Future energy costs are computed through expected load modeling and simulation of the operation of the generation. The results provided are the system price-duration curve as well as the expected costs of the units in the system. Besides, PCMs provide an excellent balance between actual electricity system representation, flexibility and calculation speed, what makes them a meaningful approach for price risk analysis, where a high number of scenarios have to be taken into consideration [4] (compared with the application of *equilibrium models* applied to long-term analysis [5], PCMs model demand in a much more extensive way, so peak values are not discarded).

But the underlying motivation of this paper is that the concept of *centralized power systems* has been recently replaced by *liberalized power markets* (we will consider a market organized around a power pool), and modeling a electricity market framework requires to handle additional complications. The model has to be able to represent the power pool, in which units are scheduled according to the bids they submit (commonly hourly) and remunerated at the system marginal price. In perfect competitive markets, if any, these PCMs models are still valuable without many refinements, as no agent is supposed to be able to influence the market-clearing price and thus it can be shown [6] that they are encouraged to bid revealing their true marginal costs. On the contrary, in oligopolistic markets as it is the case of most of the real electricity markets, one should expect that a big-sized firm withheld part of its generation to sustain prices at a higher level than the corresponding to marginal costs. Therefore, in this latter case, an adequate model cannot be just cost-based anymore.

Thus, if we want to use the PCMs approach for price analysis in this new electric business, a way of considering this oligopoly effect has to be found.

B. Structure of the paper

The strategic production cost model (SPCM) is presented as follows: First the modeling of the agents in the system is presented. Then the bidding algorithm is described, including the way agents owning a hydrothermal portfolio interact in the market, their strategies and the bids building procedure. Once the process to derive the market agents bids is settled, we describe the market clearing and liquidation algorithms. Finally, a brief case example of a real size market is added.

II. SYSTEM MODELING

A. Model scope

The model is designed to simulate the evolution of a power pool throughout a medium-term horizon consisting in T time blocks (e.g. $T = 8760$ hours) that are modeled with W time intervals or periods (e.g. groups of four weeks), containing each of them T_w time blocks, $w = 1, \dots, W$, (e.g. 672 hours).

B. System agents modeling

The electric system considered is configured by F generating firms, each owning U_f thermal plants, $f \in \{1, \dots, F\}$. Each thermal plant u is characterized by just two parameters (constant for every period w): its maximum capacity, \bar{p}_u and its variable cost of production, c_u , (it is assumed that the variable production costs of all the plants in the market are known or at least can be guessed adequately). As in traditional PCMs, no failure rates nor any operating constraint linking consecutive hours, such as start-up or shut-down costs are taken into account.

Additionally, each firm owns H_f hydro plants and every plant h is characterized by three parameters for each period w : the maximum capacity constraint, \bar{p}_h^w , the minimum production (run-of-the-river power), \underline{p}_h^w and the available energy, e_h^w of a hydro plant. The energy constraint represented through the parameter e_h^w is the key aspect that distinguishes hydro plants as *limited energy plants*. As discussed later, its direct consequence is that it links the firms' supply curves for the different time blocks t in an interval w .

No long-term management (no reservoir management, no *inter-period*, just *intra-period* management) is modeled for hydro plants. Thus, no long-term value to the water is assigned, i.e. the model assumes hydro energy variable cost is zero. The hydro entries are derived either from historical values or from a higher range model and it is supposed that both in one way or another take into account the water value to allocate the energy to be produced in each period.

C. Load modeling

The system load is assumed to be *homogeneous* and *inelastic*, although some elasticity could be considered, see below. Load has to be adapted to the time-dependent hydro modeling, thus load turns into W monotone vectors L^w .

III. BIDDING ALGORITHM

Traditional price models that analyzed centralized systems were based on the assumption that the supply curve was strictly cost-based. This hypothesis can hardly be held when analyzing actual liberalized electricity markets. Most of the by now known electricity markets are *very un-perfect*. They are often formed by a rather small number of agents with some capacity to control prices. Thus, it is to be expected that some firms may reduce its output to make prices raise from the marginal cost-based level. Next, a strategic postulate is formulated to allow a PCM to parameterize this effect.

A. The SPCM postulate

For each time block t , the last unit accepted in the market clearing, the marginal unit, sets the system price. Then, let us consider the problem that a firm f , which owns a portfolio of plants, has to solve to build its own supply function $B_f(p_f)$ expressing the bid price for each production level p_f .

The firm's strategy will be to maximize its profits $R_f = I_f - C_f$, i.e. the difference between its income, I_f , and its production costs, C_f . Therefore, we can expect that its marginal cost, i.e. the cost of incrementing the quantity produced in one unit will equal its marginal income. This is the first order condition where profits are maximized:

$$\frac{\partial R_f}{\partial p_f} = 0 \Rightarrow \frac{\partial C_f}{\partial p_f} = \frac{\partial I_f}{\partial p_f} \quad (1)$$

If this were not the case, the firm would be interested in producing a different quantity. Let us denote as M_f the firm's marginal cost function (i.e. the derivative of its cost function C_f) and let us consider that in a pure market (in particular without contracts) the income of the firm is the multiplication of the price of electricity times the produced quantity. Thus, if s denotes the system marginal price, the previous equation can be rewritten as:

$$M_f(p_f) = \frac{\partial(s \cdot p_f)}{\partial p_f} = s + \left(\frac{\partial s}{\partial p_f} \right) \cdot p_f \quad (2)$$

Then the system marginal price can be expressed as

$$s = M_f + \left\| \frac{\partial s}{\partial p_f} \right\| \cdot p_f \quad (3)$$

since $\frac{\partial s}{\partial p_f} < 0$, i.e. if the quantity produced by firm f increases (assuming constant demand and competitors bids), the system marginal price falls.

In general, it can be assumed that this conjectural variation of the price can be inferred from the past market evolution. In fact, this is nothing but the first derivative (slope) of the residual demand curve, which we will name as *strategic parameter* and we denote as:

$$\left\| \frac{\partial s}{\partial p_f} \right\| = a_f(p_f) \quad (4)$$

Then we get to the conclusive postulate statement. Under normal circumstances, every unit's bid is built under the assumption of being marginal, what in other words means that every bid is built under the assumption that it may settle the market price. Thus, the strategic supply function of a firm f can be derived from its marginal cost function as:

$$B_f(p_f) = M_f(p_f) + a_f(p_f) \cdot p_f \quad (5)$$

Fig. 1 illustrates how the consideration of the *strategic parameter* $a_f(p_f)$ implies that the supply function of a firm in an oligopoly does not just reflect the marginal cost function (plotted in a dotted line), but also strategic criteria.

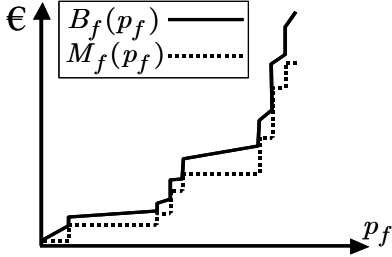


Fig. 1. A firm's strategic bidding function

The situation of perfect competition is achieved just by doing $a_f(p_f) = 0$. If firm f can not control the market price, the slope of its residual demand function will be zero. Also, it can be observed that the market power effect is equivalent to modify the marginal cost of the unit in $a_f(p_f) \cdot p_f$.

The model incorporates a_f as a deterministic function, as all the rest of the general parameters that describe a market scenario. The historical information about the competitors' behavior is reflected in the aggregated supply functions in the past. Granted that enough of these data are available, the modeler could obtain a good estimation of the function $a_f(p_f)$, (from the calculation of the residual demand curves).

In this paper we do not discuss which is the proper methodology to estimate the agents' residual demand functions, since it is still an open issue under deep research. See [7] and [8] for further insight on this issue.

As aforementioned, if some demand elasticity is to be represented, the SPCM can be easily refined: a *virtual* supplier, u_l , which competes against the actual suppliers, should just be added. For instance, a linear demand function could be assumed:

$$L(s) = L_{\max} - \lambda \cdot s \quad (6)$$

where L_{\max} represents the maximum load required (demand at price zero) and λ the parameter that reflects the demand's response to price.

Let denote the quantity of energy that demand withholds for each price level as $p_{u_l} = L_{\max} - L(s)$. The supply curve of this virtual unit u_l would be:

$$B_{u_l}(p_{u_l}) = \frac{1}{\lambda} \cdot p_{u_l} \quad (7)$$

B. The SPCM algorithm and the equilibrium

Since the introduction of competition in power markets, many *equilibrium models* have been formulated to face long term market price analysis [5]-[10]. These models, which have been applied to power markets with a lot of success, are based on *game theory* and simulate a market game with perfect information by calculating a *consistent conjectures equilibrium*.

The model proposed here is as well supported by *game theory*, but it simulates a market game with imperfect information. The model does not guarantee that the result is a *Nash equilibrium*, in which any agent could do nothing to improve market profits. In this sense, it is helpful to remind the definitions presented in [6]: 'A *conjectural variation* is a conjecture by one firm about how the other firm will adjust its decision variable with respect to potential adjustments in the first firm's action. A *consistent conjectural variation* is a conjectural variation that is correct: predicted (change) locally in the relevant decision variable (output or price) on the part of one's competitor is what actually occurs. A *consistent conjectures equilibrium* is a consistent conjectural variation equilibrium, in the sense that no individual change in a decision variable is profitable.'

The strategic parameter a_f sampled for each market firm is just a *conjectural variation*, but not at all *consistent*.

The SPCM model does not develop any iterative method (or a linear profit-optimization problem) to get to the set of strategies such that no player in the market can improve its position if the other competitors hold their strategies on (the *Nash equilibrium*). Eventually and casually, in certain scenarios the solution could be close to be the case¹, but this is not the objective of the model.

In the SPCM, suppliers build their bid curves basing on (5). Next we briefly show how the SPCM can be related to one of the main equilibrium models, the *Supply function equilibrium model* (a similar reasoning could be developed regarding *Cournot model*).

1) The SPCM and the supply function equilibrium model

Paul Klemperer and Margaret Meyer [10] proposed an oligopolistic model in which firms face uncertain demand choosing a supply function. They show how by considering uncertain demand, it can be justified why firms choose supply functions as strategic variables and how for a symmetric oligopoly under certain conditions, the existence of a *Nash equilibrium* can be proved.

In their model formulation, demand is assumed to depend on a scalar random variable ε , such that its expression is $L(s, \varepsilon)$. The model focuses on pure strategy Nash equilibria in supply functions: each firm supply function $P_f(s)$ maximizes the firm's profits for each of the values the aforementioned

¹ Besides, due to mathematical problems, the consistent conjectures equilibrium could just be precisely verified in a very theoretical model, in which, for example, continuous and differentiable cost and residual demand functions would be required.

variable ε may take subject that the firm's rivals choose $P_v(s), \forall v \in \Upsilon_f$, where Υ_f is the set of firms competing against f .

This way, the first-order condition for each value of ε , if $P_{\Upsilon_f} = \sum_{v \in \Upsilon_f} P_v$, is:

$$\begin{aligned} \frac{\partial R_f}{\partial s} &= \frac{\partial(s \cdot (L(s, \varepsilon) - P_{\Upsilon_f}) - C_f)}{\partial s} = \\ &= (L(s, \varepsilon) - P_{\Upsilon_f}) + (s - \frac{\partial C_f}{\partial s}) \cdot \frac{\partial(L(s, \varepsilon) - P_{\Upsilon_f})}{\partial s} = 0 \end{aligned} \quad (8)$$

Thus, a *consistent conjecture* is proposed. In this case, the *consistent conjectural variations* are the first derivatives (the slopes) of the inverse of the firms' *residual demand curves*:

$$\alpha_f = \frac{\partial N_f^{-1}}{\partial s} = \frac{\partial(L(s, \varepsilon) - P_{\Upsilon_f})}{\partial s} \quad (9)$$

This way, the profit-maximization problem formulation for each value of ε is:

$$\begin{aligned} \max_s \quad & R_f; \quad \forall f \in \Phi, \forall \varepsilon \\ \text{s.t.} \quad & P_f + (s - M_f) \cdot \alpha_f = 0 \\ & L(s, \varepsilon) = P_f + P_{\Upsilon_f} \end{aligned} \quad (10)$$

The simulation of the SFE with the SPCM is rather straightforward. In the SPCM, the firms' strategy is also to choose supply curves. The building of the latter is as well based on the conjectural variation of the market price with respect to the firms' output.

Again, the only difference is *consistency*. When simulating the supply curve building process, the SPCM assumes the firm just knows its costs and its conjecture about the derivative of its residual demand function. As no iterations are made, firms do not have the chance of refining its bids taking into account its rivals' reactions.

On the other hand, the SFE profit-maximization problem algorithm allows market participants to refine their beliefs about the other players' behavior through the information they get about productions and prices, what leads to the *consistent conjectures equilibrium*, the *Nash equilibrium* in supply functions.

This could be achieved with the SPCM by implementing an iterative procedure that, after each iteration allows market agents to recalculate the value of the strategic parameter attending to the scheduling results.

The strategic parameter is nothing but a way of characterizing how market agents expect their competitors may behave. Market agents' bidding strategy proposed is a representation of the behavior that would be the direct result of this assumption, but the model does not simulate that participants refine their bids subject to their rivals' strategies.

What at first sight might resemble as a poor simplification of this problem turns into an interesting feature. On the one hand, it reduces computation times, what is relevant when price risk analysis are going to be held. On the other, we think

it reflects real life. In real electricity markets, agents' daily bids are a kind of "bet" in which the suppositions about their most likely residual market reaction is internalized. Sometimes these bets lead them to good profits, sometimes do not. For example, one firm could opt for battling with cost-based bids in a hunt for increasing its market share, equivalent to a scenario in which the strategic parameter of the firm would be given a zero value. This has a lot to do with risk. Often, one firm in the market may decide temporarily to change its common strategy, what usually leads to results far from the theoretical equilibrium.

C. Building hydrothermal bid curves

It has been just stated that firms' supply functions are derived from the marginal cost M_f and the strategic functions a_f following expression (6). However, when hydro plants are considered, market firms' marginal cost functions M_f^t will not be the same for each time block t , since they will depend on the quantity of water that each firm decides to bid at this point in time, $\hat{p}_{H_f}^t$. In other words, we assume that the only aspect that makes the firms' cost function change from one time block to another is the quantity of water the firm decides to bid at each block, what, as illustrated in Fig. 2, it is equivalent to shift the thermal cost function from the origin according to this hydro output.

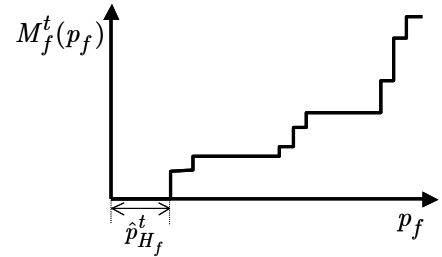


Fig. 2. A firm's marginal cost function shape for a time block t

The problem is that firms, when building their supply functions, have no way of knowing exactly which will be their resulting dispatched profile, P_f . Once a firm f knows this profile, it can dispatch its own load economically, allocating hydro energy in the peak hours to reduce costs. But again, unfortunately, this profile can not be known in advance.

While this is not a drawback when a centralized system is simulated, the way of modeling this decision-making process in a market environment is not obvious. As well as with their residual demand, in this latter case firms have to guess prior to bid how will they finally allocate their hydro energy when satisfying their scheduled load.

1) Non-strategic market approach

To ease the algorithm understanding, we refer to 'non-strategic market' since, in this section, the case of a centralized system is first reviewed. Afterwards, the algorithm proposed for a competitive market scheme is analyzed.

In the traditional production costing simulation algorithm,

when simulating a centralized system the system units are dispatched economically, as if they belong to a single firm. On the one hand, to minimize production costs, hydro plants are loaded first assuring that their available energy is fully used up and also attempting to seize as much as possible their maximum capacity, i.e. scheduling them in the peak hours (the so-called peak shaving). The thermal units' merit order is configured attending to their production costs, i.e. the system supply curve reflects the system marginal costs.

Since plants are dispatched sequentially, they have to be first arranged in a *merit order*. When a thermal dominated market is simulated and no failure rates are considered, the load-duration curve shape does not change as thermal plants are scheduled, so the criterion to configure the *hydro merit order* may be fully random. The *thermal load* obtained will be the same no matter how hydro units are ordered.

In practice, the solution to the problem of achieving an optimal hydro dispatch is often degenerated, i.e. there exist more than one optimal solution (often infinite), since usually the technical characteristics of the hydro plants in system are similar. If it is assumed that system operation is centralized and there is just one owner this does not pose any problem.

Conversely, if hydro units belong to different owners, two optimal solutions could drive to different outcomes for some agents. Let us illustrate it with a very simple example.

Imagine two firms f_1 , f_2 , each owning one hydro unit, h_{f1} and h_{f2} respectively, with equal characteristic parameters: $\bar{p}_{h_{f1}} = \bar{p}_{h_{f2}}$, $\underline{p}_{h_{f1}} = \underline{p}_{h_{f2}} = 0$, $e_{h_{f1}} = e_{h_{f2}}$. Suppose that, in order to schedule the hydro plants, the merit order randomly arranged places f_1 before f_2 . We could think this would not have any impact on the final liquidation, however, we can check this is not always the case.

Fig. 3 shows a possible system load L , the *thermal load*, L^U , resulting from the whole hydro system dispatch and the schedule of hydro units h_{f1} and h_{f2} (where π denotes the power finally dispatched by each unit). As it can be seen, L^U is not completely *flat*, what leads to different prices in the peak. In this case, the *hydro merit order* is relevant as the outcome of the hydro plant scheduled in the first place gets higher profits. Prices in the peak sub-interval T_2^* of the time blocks in which hydro plants are scheduled, T_2^H , will be higher, so hydro plant h_{f1} will get higher profits than h_{f2} (the power it produces in these time blocks is higher). This does not pose any problem when analyzing a monopoly, since every unit belongs to the same owner, but unfortunately this is not the case when hydro plants belong to different firms.

There is nothing we can do to solve this problem. As the term itself means, the *degenerated* characteristic of the scheduling of hydro plants implies there is not a single solution. Indeed, it reflects real life, let us say *it is just the way it is*. It is not a problem derived from the algorithm used in the PCM. In fact, *unit commitment* and *equilibrium models* that solve the hydro scheduling using an optimization-based

method, e.g. the simplex, get to one of the many optimal solutions as well, i.e. the degenerated characteristics is inherent to the problem nature.

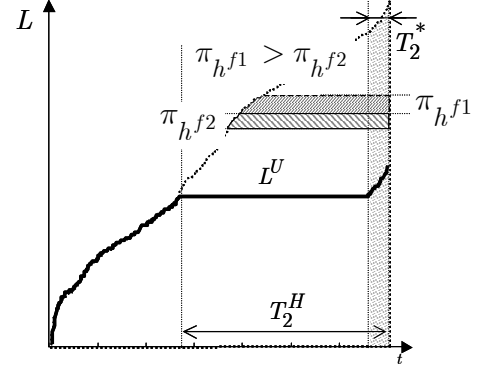


Fig. 3. Uncompleted peak shaving

The key of a market simulation model, as opposed to a centralized system simulation, is the modeling of the bidding procedure. A methodology to build the firms' supply curves from their marginal cost functions has been proposed; above it was stated that the main implication of considering hydro plants is that, at the time they have to build their bids, firms ignore their optimal hydro scheduling since they do not know the price for each time block that will result from the clearing.

So, as with their residual market, some assumption has to be made in advance. After evaluating many, looking for balancing simplicity, calculation speed and reasonability, we propose agents follow one particular strategy of the many we could think of, the one we guess it is "the less unreasonable". We could devise a more complex method, such as an iterative procedure, but it would not be worth, as we should introduce additional assumptions resulting in a significant loss of speed without a substantial results improvement.

2) Strategic market approach

We denote $P_{H_f}(t) = p_{H_f}^t$ as the vector containing the hydro production of a firm f in every time block t and denote by \hat{P}_{H_f} the firm's expected dispatch at the time the bidding curve building is done.

The firm's supply curve for every time block t , B_f^t , will then equal the expected marginal cost function \hat{M}_f^t , calculated from (6) with \hat{P}_{H_f} .

From the point of view of a certain firm f , attending to the phenomenon just discussed in the previous section, among all the set $\Pi_H = \{\Pi_H^j\}$, $\Pi_H^j = \{P_{H_1}^j, \dots, P_{H_f}^j, \dots, P_{H_F}^j\}$ of the J possible solutions to the hydro schedule problem, there is at least one that is the most favorable for its interests: $\Pi_{H_f}^o$, the one that allocates as much hydro energy of the firm as possible in the peak hours, $P_{H_f}^o$ (see Fig. 4). This solution can be calculated by prioritizing the hydro units of the firm in the *hydro merit order*.

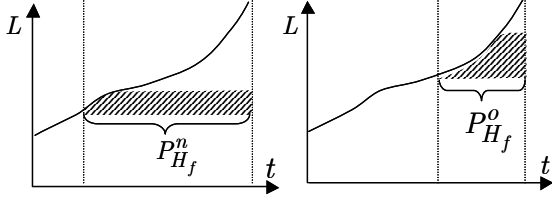


Fig. 4. Two possible solutions of the hydro plants scheduling of a firm

However, for risk aversion reasons, we propose establishing a rather more pessimistic agents' strategy. In our model, although it is clear that other strategy could be undertaken (e.g. the just reviewed), every agent builds its supply curves for each time block over the less favorable hydro schedule: the one derived from the assumption that its own hydro plants are placed at the end of the *hydro merit order*, $\hat{P}_{H_f} = P_{H_f}^n$, i.e.

$\forall t, p_{H_f}^n(t) \leq p_{H_f}^o(t)$. This strategy guarantees every agent that its profits will not be smaller than expected, as the firm's hydro schedule resulting for the market scheduling will not be in any case less favorable. The reason why we propose the *risk averse hydro predispatch* instead of other more optimistic is especially emphasized when dealing with the firms' bidding curves building under the assumption of a strategic market. Remember that in this scheme, the bid price depends upon the firms' inframarginal production (6). Thereby, if a firm overestimates its hydro regulation capacity and thus its inframarginal capacity and builds its bids expecting a very favorable hydro schedule (producing mostly in the peak blocks), it risks to bid too high and to be *underscheduled*, with the corresponding loss of income precisely in the time blocks in which market price is higher.

Thus, among the various algorithms we could think of to simulate the agents' strategies, we chose the risk averse one, considering it is always better to fall short when bidding and then get better outcomes than the opposite.

To sum up, the algorithm proposed to simulate the bid curve building of a firm f in the market for each time period w and time block t_w is as follows:

1. Considering the technical parameters of the hydro plants (capacity limits and available energy), the firm's *hydro merit order* is configured placing firm f 's hydro plants in the last place, preceded by the rest of the hydro units.
2. The firm's hydro schedule for each time block, the vector $\hat{P}_{H_f} = P_{H_f}^n$, resulting from dispatching the system hydro plants following this *hydro merit order* is then calculated.
3. The supply function for each time block t_w , $B_f^{t_w}$, equivalent to the expected marginal cost function, $\hat{M}_f^{t_w}$, is finally obtained from (6).

IV. CLEARING PROCESS

A. Scheduling algorithm

The scheduling procedure under the assumption that any

agent may behave strategically is analogous to the traditional production costing algorithm. As it has been explained, in the first scheme the only change is how market firms' bidding curves are built.

The system supply function B is obtained as the aggregation of the corresponding curves B_f , $f = 1, \dots, F$ of the F firms in the market:

$$\forall p = \sum_{v=1}^F p_v \left/ B_v(p_v) < B_f(p_f) < B_v(p_v + 1) \right. \quad (11)$$

$$\forall v \neq f / v \in \Phi \Rightarrow B(p) = B_f(p_f)$$

For each time block t , from $t = 1$ to $t = T$, the *system marginal price* s_t is settled by just crossing the aggregated system supply curve B_t and the system demand curve L_t (the latter, as it is considered inelastic will always be vertical).

$$S(t) = s_t = B^t(L_t) \quad (12)$$

B. Liquidation procedure

The vector P_f denoting firm f 's scheduled quantity for the scope of the study is obtained as:

$$P_f(t) = p_f^t = B_f^{t-1}(s_t) \quad (13)$$

Every unit offering at time block t a price below s_t is thus scheduled and receives this system marginal price. The total income of the firm is:

$$I_f = \sum_{t=1}^T (s_t \cdot p_f^t) \quad (14)$$

Once this latter vector is obtained for each firm f , the model recalculates the portfolio economic dispatch, of every firm in the market once they know the schedules resulting from the market clearing. To do so, the hydro energy of the firm is first *re-dispatched*, obtaining the final value of vector P_{H_f} , which not necessarily has to be equal to the one that was considered when building the firm's supply curve B_f^t .

The marginal cost function M_f^t for each time block resulting from the market schedule, P_{H_f} , is then calculated from (7). Thus, the total production costs turns to be:

$$C_f = \sum_{t=1}^T C_f^t = \sum_{t=1}^T \int_0^{p_f^t} M_f^t(p_f^t) \cdot dp_f \quad (15)$$

V. CASE EXAMPLE

To briefly illustrate the model capabilities, we have analyzed a look-like Spanish electricity market throughout year 2002 (8760 time blocks, i.e. hours) represented by $W = 13$ time intervals or periods, gathering four weeks each. Correspondingly, demand is modeled using thirteen load-duration curves (monotone T -dimensional vectors, $T_{w=1} = 696$ and $T_{w=2, \dots, 13} = 672$).

A. Market structure

The electric system analyzed is configured by four generating firms, $f = \{f1, f2, f3, f4\}$, each owning U_f thermal plants and H_f hydro units. Table I shows the number of thermal plants of different technologies and aggregated capacity of the market firms. The variable costs values have been selected arbitrarily trying to keep reasonable ranges and so we have expressed them in an invented currency € .

TABLE I

THERMAL UNITS' CHARACTERISTICS

# (MW)	Nuclear	Coal	Fuel	Gas
$U_{f1}(\bar{p})$	3 (2981)	20 (6044)	21 (5683)	5 (1785)
$U_{f2}(\bar{p})$	3 (2856)	5 (1137)	14 (4310)	5 (1877)
$U_{f3}(\bar{p})$	2 (1147)	7 (2000)	4 (1046)	1 (390)
$U_{f4}(\bar{p})$		5 (1477)		

Additionally, each one of the generating firms owns H_f hydro plants, characterized by three parameters (capacity, \bar{p}_h^w , minimum, \underline{p}_h^w and available energy e_h^w) for each interval w in which year 2002 is divided into. As an example, Table II shows the number of hydro units of each firm and their aggregated characteristics for period $w = 4$ of the third scenario of hydro production. Data is expressed aggregated by firm for simplicity reasons just to give a hint about the system size, but the model considers each unit independently.

TABLE II

HYDRO UNITS' CHARACTERISTICS

# (MW, MW, GWh)	Hydro
$H_{f1}(\bar{p}_h^w, \underline{p}_h^w, e_h^w)$	10 (1682, 522, 405)
$H_{f2}(\bar{p}_h^w, \underline{p}_h^w, e_h^w)$	5 (3881, 259, 832)
$H_{f3}(\bar{p}_h^w, \underline{p}_h^w, e_h^w)$	3 (775, 83, 1625)
$H_{f4}(\bar{p}_h^w, \underline{p}_h^w, e_h^w)$	1 (141, 19, 27)

The market game is solved for five scenarios of each one of the three risk factors that are more relevant in the Spanish market price formation: *load*, *fuel prices* and *hydro production*. (i.e. we have run the model for $5 \cdot 5 \cdot 5 = 125$ different scenarios). To illustrate the different results when a competitive market or an oligopolistic market are considered, we have analyzed the 125 scenarios for three different case studies assuming agents behave in three different ways.

Case 1: the first market case study simulates a competitive market. Firms just build their bids revealing their production costs (the strategic parameter value is assumed to be zero).

Case 2: the second market case simulates an oligopoly (firms aim to exploit their capacity to control prices).

As well as in the next case, for simplicity reasons, the strategic parameter a_f has been assumed constant for every output level of each firm f . The values considered, calculated as the average slope of each firm's residual marginal cost curve (i.e. the curve considering the aggregated marginal costs

of the rivals), are shown in the following table.

TABLE III
FIRMS' STRATEGIC PARAMETER VALUES

$\text{€}/\text{GW}$	$f1$	$f2$	$f3$	$f4$
a_f	0.1292	0.1758	0.0868	0.1102

Case 3: finally, the third case simulates an oligopoly in which market firms behave strategically but the price that 80% of the system load pays is already predetermined. This percentage may stem from former agreements between generation and demand (or imposed by the regulator) such that in each time block every firm is responsible for a fixed proportion J_f of system load.

In this case example, these proportions (in p.u.) are fixed at $J_f = [0.52, 0.27, 0.16, 0.05]$, $f = \{f1, f2, f3, f4\}$. Thus, the amount of inframarginal capacity that influences on the bidding price formation is reduced up to values around 20%.

For each time block t , bids are built following the formula:

$$B_f(p_f, t) = M_f(p_f, t) + a_f(p_f, t) \cdot (p_f - 0.8 \cdot J_f \cdot L(t)) \quad (16)$$

B. Case example results

In the two figures below, we can check how the average annual market price changes as the three risk factors and market agents' behavior assumption change. If we compare the resulting prices from *Case 1* and *Case 2*, we can observe that prices rise notably when firms behave as an oligopoly. *Case 3* shows that this effect is notably reduced when firms have already committed 80% of their output.

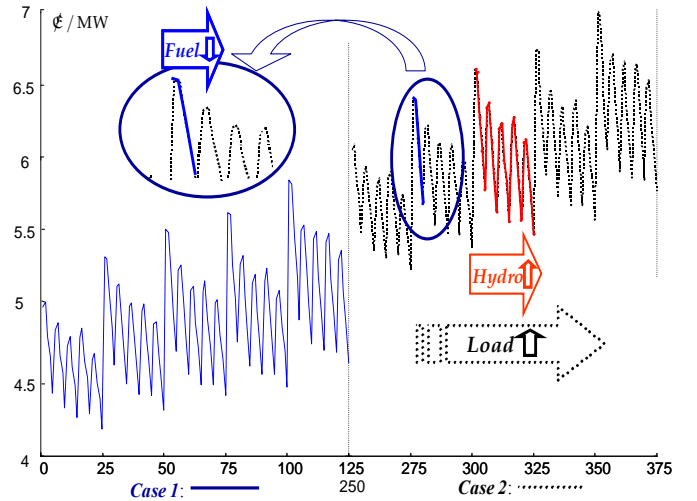


Fig. 5. Market price sensitivity to the three main risk factors

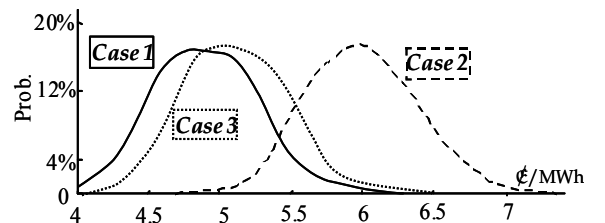


Fig. 6. Average annual prices

The model provides not only market prices but also firms'

income, production costs and thus spot market returns. Below, we compare the scenarios run for the *competitive Case 1*, and the ones run for a pure oligopoly, *Case 2*. Fig. 7 shows how firm f_1 , when behaves as an oligopolist, reduces significantly its production costs, as a result of withholding part of its output. However, in Fig. 8 it can be seen that this output reduction leads the firm to a returns growth. On the other hand, f_2 increases its production costs, since it produces more to cover part of f_1 's withholding, and, as the rest of firms, it also improves its returns in terms of profits. Notice that f_1 is the biggest company in the system and the one with a better ability to influence price.

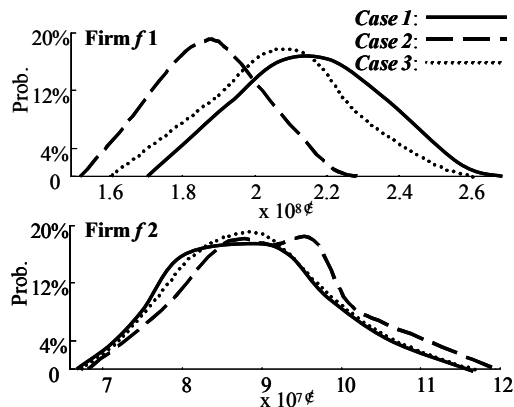


Fig. 7. Production costs of firms f_1 and f_2

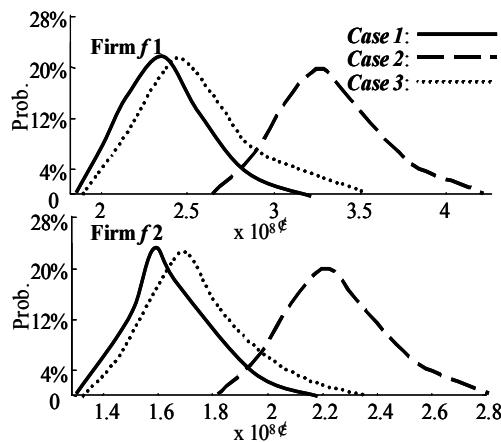


Fig. 8. Returns of firms f_1 and f_2

VI. CONCLUSION

Besides the factors that traditionally have been considered, such as hydro inflows, demand values, etc., any model aiming to face a proper analysis of the wholesale electricity market price risk has to cope with an additional feature: the uncertain behavior of generation competitors and consumers.

In these markets the best way of defining price or profits data and series, is to use a well defined price behavior model where the agents strategies are modeled under different market structures, competitors behavior or other issues.

The model proposed generates supply-side bid curves. Each agent manages a hydrothermal portfolio and tries to maximize

its profits taking into account its cost structures and the expected behavior of its competitors, modeled through a strategic parameter, which represents the slope of the residual demand function for each production level of the firm.

The model simulates how firms manage their hydro energy in the short term to build strategically their bidding curves. Based on these supply curves, the SPCM computes wholesale prices, the resulting dispatch of the units, and the revenues and costs of each market participant.

The model proposed can be used either to test the effect of different bidding strategies and market circumstances (ranging from price wars to oligopolistic equilibria) in real size markets or to perform risk studies through a fundamental analysis.

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Carlos Batlle was born in Madrid in 1972. He obtained his Electrical Engineering degree from the Universidad Pontificia de Comillas (I.C.A.I.), Madrid, Spain after having spent his final year at the Ecole Polytechnique Fédérale de Lausanne (E.P.F.L.), Switzerland where he submitted his diploma dissertation in artificial intelligence. He received the doctoral degree in 2002 for his dissertation "A model for electricity generation risk analysis". Currently he is a Researcher in the Instituto de Investigación Tecnológica and his areas of interest are risk analysis and regulation of electric power systems.

Julián Barquín (M'94) obtained a Degree in Industrial Engineering (1988) and a Ph.D. in Industrial Engineering (1993) at the Universidad Pontificia Comillas, Madrid, and the Licenciado en Ciencias Físicas Degree in 1994. He

belongs to the research staff at the Instituto de Investigación Tecnológica. His present interests include control, operation and planning of power systems.