Robust Optimization

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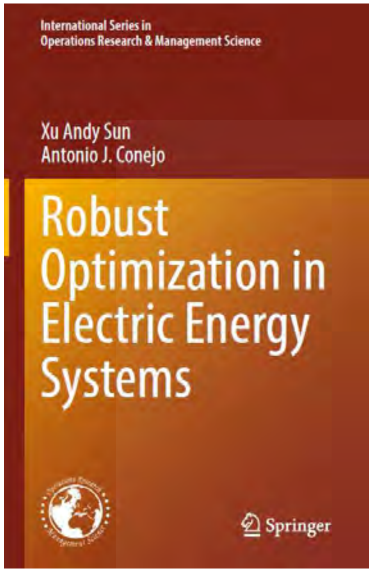
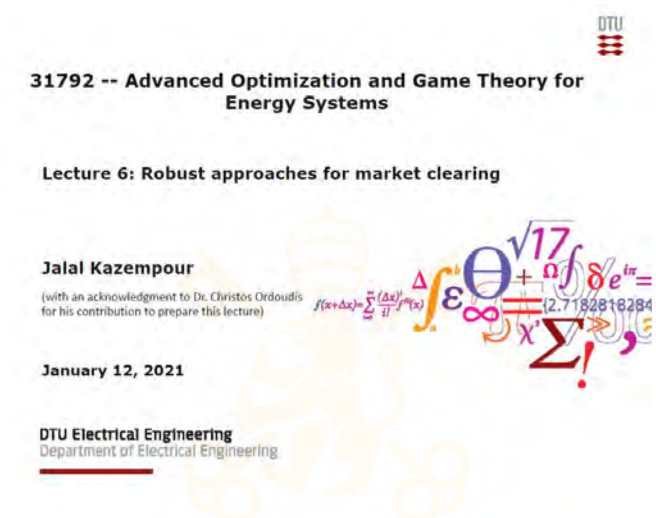
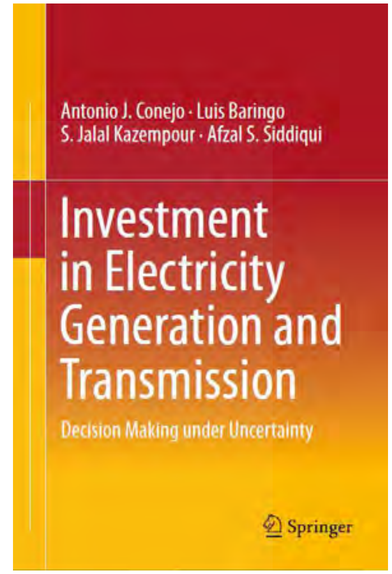
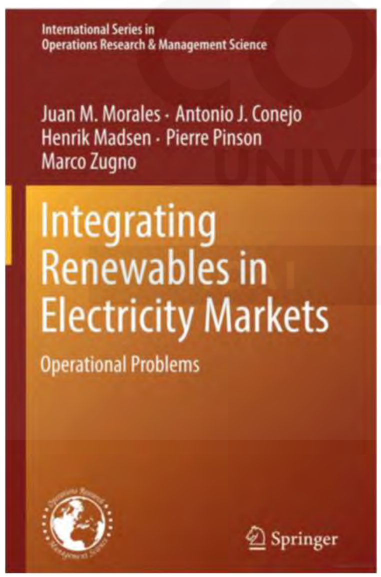
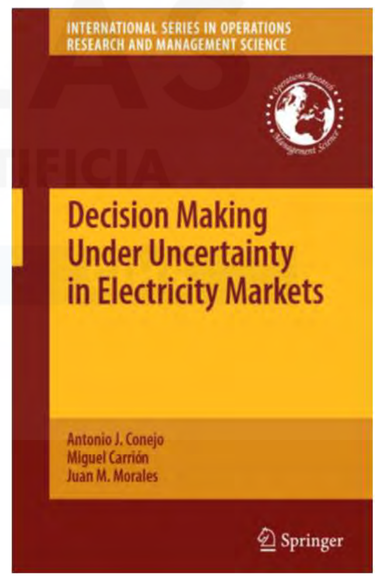
Main References











Contents

1

General Concepts

Uncertain Times in Power Systems

Uncertainty Representation

Two main methodologies allow representing uncertain parameters in an effective way:

- Stochastic Programming (SP) does so by considering a large enough, but finite number of realizations of the uncertain parameters, which are called scenarios
- Robust Optimization (RO) uses sets or distribution bounds to characterize the range of variability of the uncertain parameters

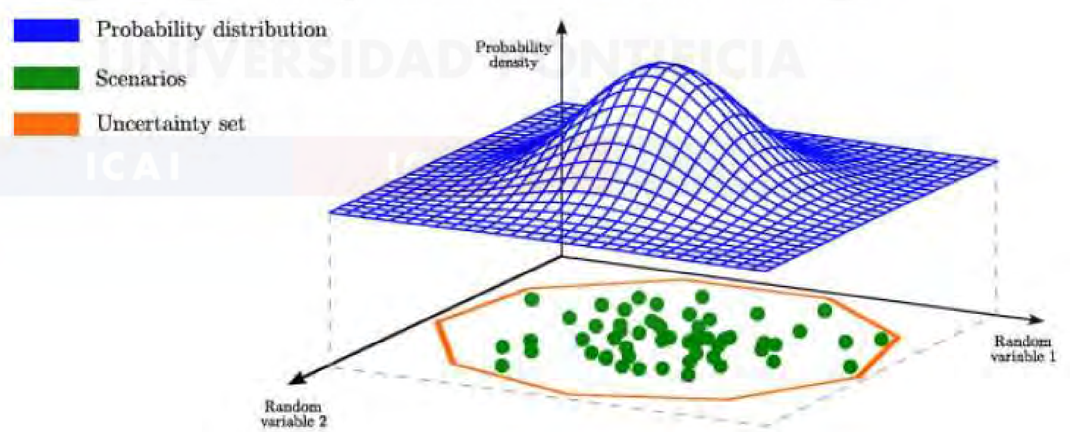


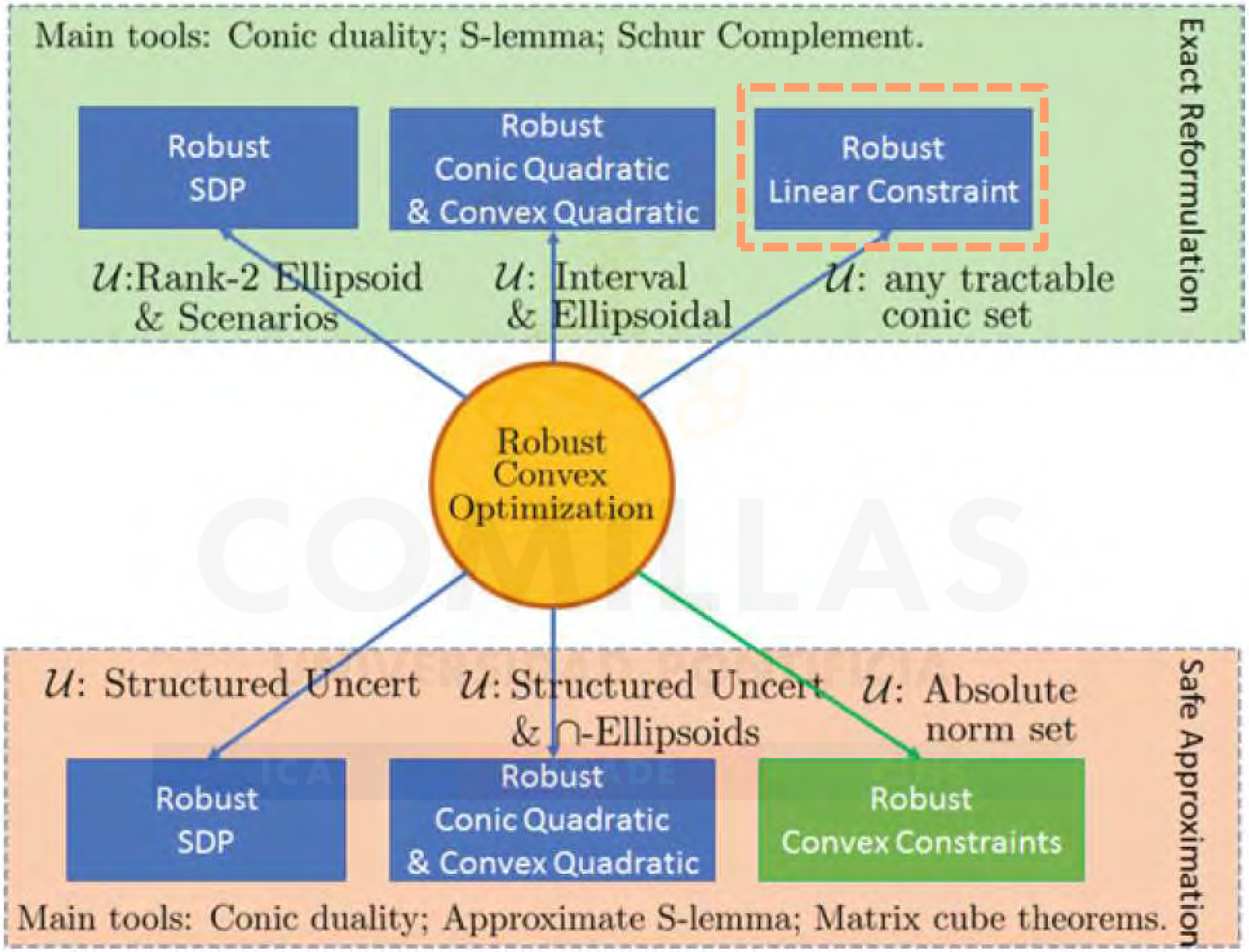
Figure from the Ph.D. thesis of Stefanos Delikaraoglou (DTU)

Robust Optimization - General Idea

- RO is a modelling paradigm that offers solutions when uncertainty in the input data can be bounded within a well described region.
- RO focuses on finding a solution that is feasible regardless of the realization of the uncertain values; hence the term robust
- When uncertainty is associated with the objective of a model, robust optimization returns a solution that is optimal with respect to the worst case of all realizations of the uncertain quantities.
- There is no probabilities associated to the scenarios or they are not used.

Robustverse

Focus in this lecture



Source: X. A. Sun, A. J. Conejo, Robust Optimization in Electric Energy Systems, International Series in Operations Research & Management Science 313, https://doi.org/10.1007/978-3-030-85128-6_2

Robust Linear Optimization Models

- ☑ Static Robust Optimization (Single-stage)
- ☑ Adaptive Robust Optimization
- ☒ Adaptive Robust Stochastic Optimization
- ☒ Adaptive Distributionally Robust Optimization

☒ Not included in this course

Robust Set

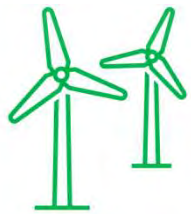
- Uncertainty is generally modeled using either robust sets or distribution bounds in RO.

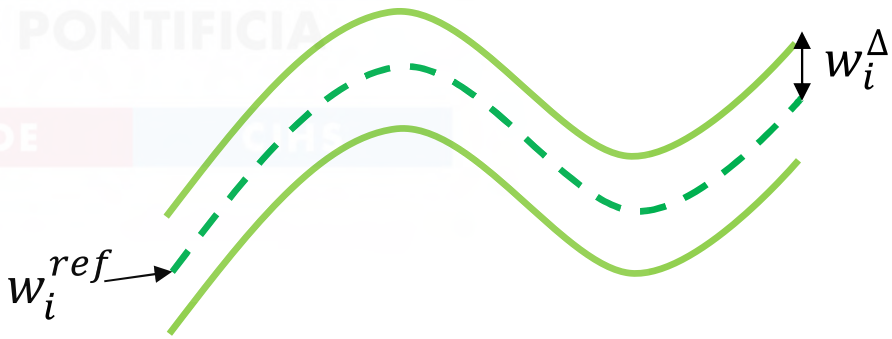
- Robust sets are generally based on boxes or polyhedral constraints

- For example, the wind speed in a particular location can vary within the interval:

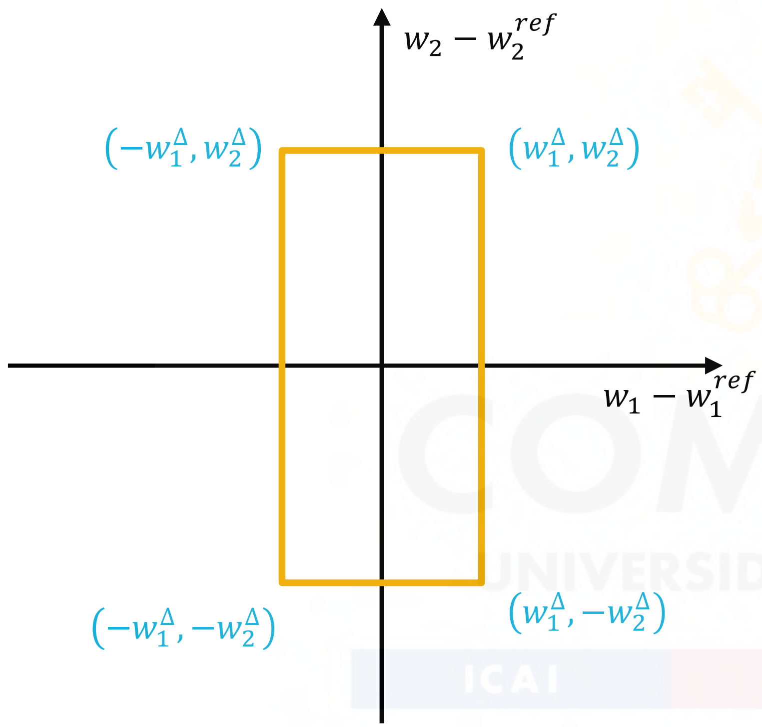
$$w_i \in \left[w_i^{ref} - w_i^{\Delta}, w_i^{ref} + w_i^{\Delta}\right] \quad \forall i$$

$$\left|w_i - w_i^{ref}\right| \leq w_i^{\Delta} \quad \forall i$$





Box Uncertainty Set with 2 Wind Farms



$$\left|w_1 - w_1^{ref}\right| \leq w_1^{\Delta}$$

$$\left|w_2 - w_2^{ref}\right| \leq w_2^{\Delta}$$

The optimal solution will be a vertex of the feasible region.

The number of vertices is a function of the number of uncertain parameters, in this case the number of wind farms (2^N)

Uncertainty Budget

- An uncertainty budget can be imposed by the constraint, where Γ varies between 0 and n:

$$\sum_{i}^{n} \frac{\left|w_i - w_i^{ref}\right|}{w_i^{\Delta}} \leq \Gamma$$

- $\Gamma = 0$ implies no uncertainty since each uncertain parameter equals its reference value.

- $\Gamma = n$ implies maximum uncertainty, as each uncertain parameter deviates the maximum from its reference value.

- Intermediate values of Γ represent different levels of uncertainty.

Uncertainty Budget with 2 Wind Farms

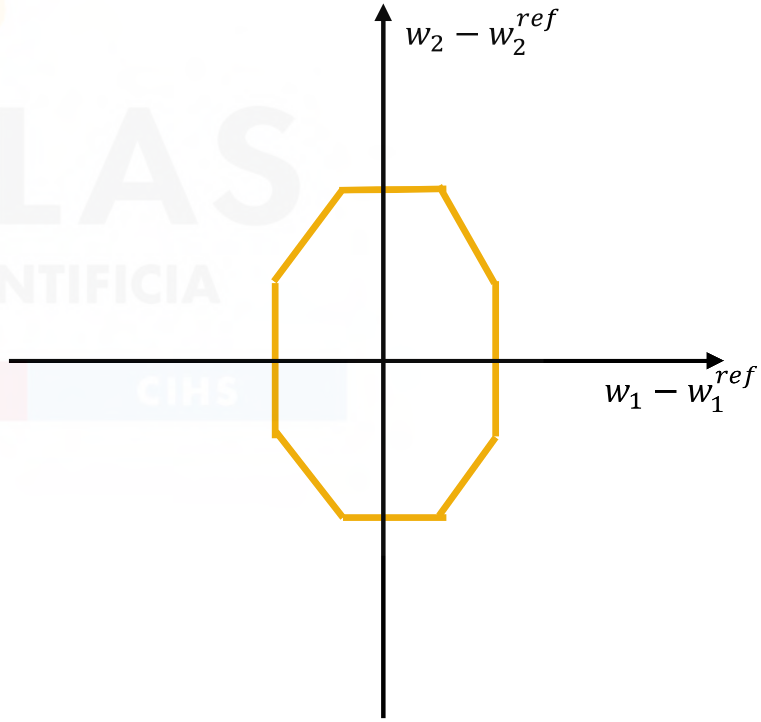
- The budget constraint reflects the correlation of the two farms:

$$\frac{|w_1 - w_1^{ref}|}{w_1^{\Delta}} + \frac{|w_2 - w_2^{ref}|}{w_2^{\Delta}} \leq \Gamma$$

0: both wind farms fixed to their reference values
1: only one can take its extreme value at the time
2: both can take one of its extreme values

- Example:

$$\frac{|w_1 - w_1^{ref}|}{10} + \frac{|w_2 - w_2^{ref}|}{20} \leq 1.4$$



Why Robust Optimization?

- Robust optimization is appropriate to identify optimal decisions under uncertainty if these decisions entail significant consequences, and thus, protection against bad decisions is a must (e.g., investment decisions).

- Robust optimization problems are generally tractable since their sizes do not generally depend on the accuracy of the uncertainty description. This is not the case with stochastic programming problems since the size of a stochastic problem depends on the number of considered scenarios.

2

Static Robust Optimization

Static RO with a Deterministic Uncertainty Set

- Minimize the total operational cost of the system under the worst-case scenario.

$$\min_{x \in \mathcal{X}} \max_{u \in \mathcal{U}} f(x,u)$$

- Where the decision variables are represented by vector $x \in \mathcal{X}$, and the uncertain parameters are represented by vector $u \in \mathcal{U}$
- Static because the solution does not depend on any specific realization of the uncertain parameter. Deterministic uncertainty set because the uncertain parameter is modeled as an unknown deterministic quantity taking value in a bounded set.
- RO provides ex-ante protection against the worst uncertainty realization, within the uncertainty set

Example – Generation Expansion Planning (RO)

We want to make a robust decision of the generation investment achieving the least total cost outcome under the worst uncertainty realization of the future demand.

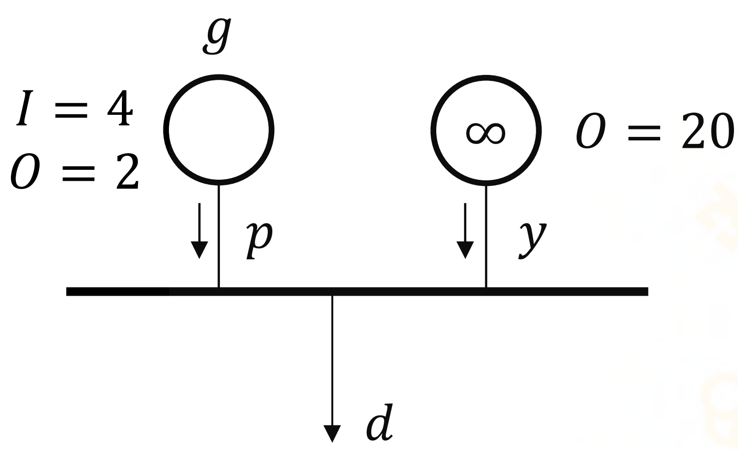
$$\min_{g_i \in \Omega_i} \max_{\substack{d_{j,t} \in \Delta_j \\ p_{i,t} \geq 0}} z = \sum_{i=1}^{n_G} I_i g_i + \sum_{t=1}^{n_T} \sum_{i=1}^{n_G} O_i p_{it}$$

$$s.t. \quad \sum_{i=1}^{n_G} p_{it} = \sum_{j=1}^{n_D} d_{jt}, \qquad \forall t$$

$$p_{it} \leq g_i, \qquad \forall i \forall t$$

The first-level decides on which generation capacity to build (minimize investment + operation cost), anticipating the worst demand realization (that results in maximum operation cost)

Example – Generation Expansion Planning (RO)



$$\min_{g\in\{5,7\}} \max_{\substack{d\in\{4,6\}\\ p\geq 0, y\geq 0}} z = 4g + 2p + 20y$$

$$s.t. \quad p + y = d$$

$$p \leq g$$

Investment (g)	Demand (d)	Total Cost
5	4	4*5+20*4=100
5	**6**	4*5+20*6=**140**
7	4	4*7+20*4=108
7	**6**	4*7+20*6=**148**

Static RO Reformulation

- Any static robust linear program can be modeled by a deterministic objective and robust constraints.

$$\min_{x} \max_{c \in \mathcal{C}} c^{\top} x \quad s.t.\ Ax \le b, \quad \forall (A,b) \in \mathcal{U}$$

$$\Rightarrow$$

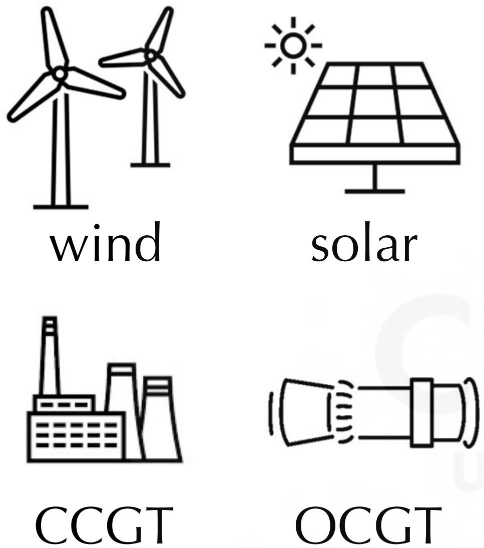
$$\min_{x,t}\ t \quad s.t.\ c^{\top} x \le t, \quad \forall c \in \mathcal{C}; \quad Ax \le b, \quad \forall (A,b) \in \mathcal{U}$$

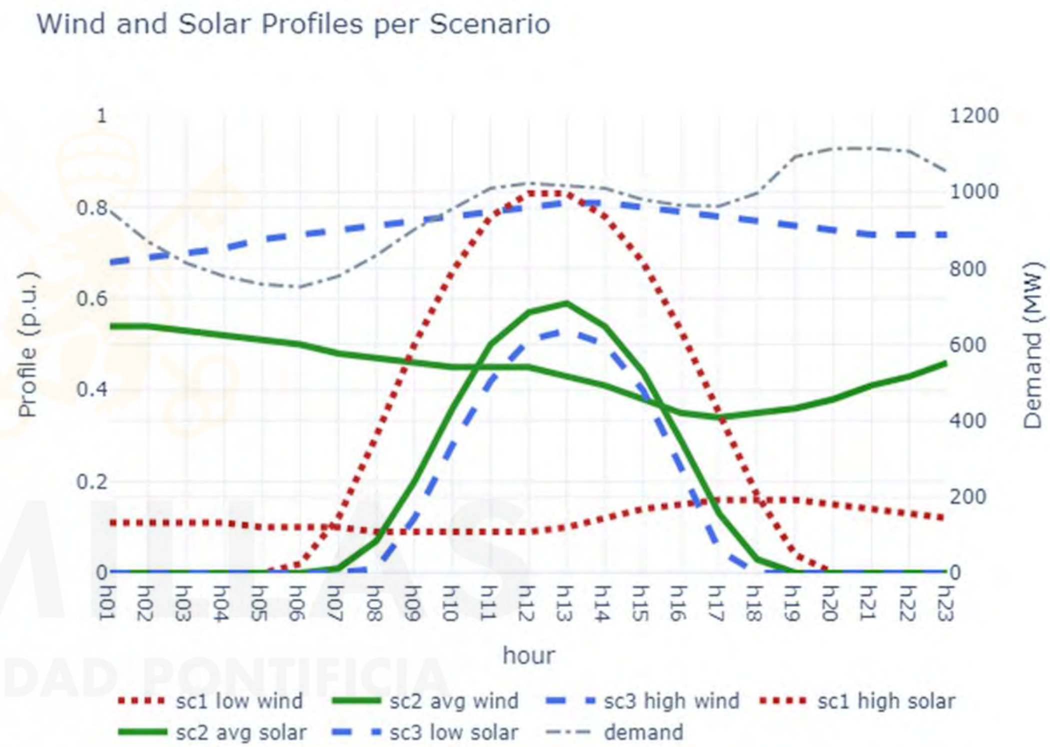
scenario uncertainty set

- Other names: single-stage robust optimization, scenario-based robust optimization.

Example – RO-GEP with Uncertainty in Renewables

4 technologies to choose





3 renewable scenarios:
- sc1: low wind, high solar (e.g., summer)
- sc2: avg wind, avg solar (e.g., spring or fall)
- sc3: high wind, low solar (e.g., winter)

Source: https://github.com/datejada/generation-expansion-planning-models

Example – RO-GEP with Uncertainty in Renewables

RO Reformulation of min-max

$$\min_{\substack{g_i,\beta \\ p_{sit},e_{st}}} z = \underbrace{\sum_{i=1}^{n_G} I_i g_i}_{investment\ cost} + \underbrace{\color{red}{\beta}}_{worst\ operating\ cost}$$

$$s.t. \underbrace{\sum_{i=1}^{n_G} p_{sit} + e_{st} = D_t}_{demand\ balance}, \qquad \forall s \forall t$$

$$\underbrace{p_{sit} \leq A_{sit} g_i}_{production\ limit}, \qquad \forall s \forall t \forall i$$

$$\underbrace{W \sum_{t=1}^{n_T} \left(C e_{st} + \sum_{i=1}^{n_G} O_i p_{sit} \right)}_{operating\ cost\ per\ scenario} \geq \color{red}{\beta}, \qquad \forall s$$

Sets

i: investment technologies
s: uncertainty scenarios
t: time periods (i.e., hours)

Variables

g_i: investment capacity
p_{sit}: generation production
e_{st}: energy not supplied
β: worst operating cost

Parameters

I_i: investment cost
O_i: variable cost
C: ENS cost
W: operating cost weight
D_t: demand
A_{sit}: availability profile

Example – RO-GEP with Uncertainty in Renewables

SO formulation (benchmark)

$$\min_{\substack{g_i,\beta \\ p_{sit},e_{st}}} z = \underbrace{\sum_{i=1}^{n_G} I_i g_i}_{\substack{investment \\ cost}} + \underbrace{W \sum_{t=1}^{n_T} \sum_{s=1}^{n_S} P_s \left(C e_{st} + \sum_{i=1}^{n_G} O_i p_{sit} \right)}_{expected\ operating\ cost}$$

$$s.t. \underbrace{\sum_{i=1}^{n_G} p_{sit} + e_{st} = D_t}_{demand\ balance}, \qquad \forall s \forall t$$

$$\underbrace{p_{sit} \leq A_{sit} g_i}_{production\ limit}, \qquad \forall s \forall t \forall i$$

Sets

i: investment technologies
s: uncertainty scenarios
t: time periods (i.e., hours)

Variables

g_i: investment capacity
p_{sit}: generation production
e_{st}: energy not supplied
β: worst operating cost

Parameters

I_i: investment cost
O_i: variable cost
C: ENS cost
W: operating cost weight
D_t: demand
A_{sit}: availability profile
P_s: scenario probability

Example – RO-GEP with Uncertainty in Renewables

Result	Scenario 1 Deterministic	Scenario 2 Deterministic	Scenario 3 Deterministic	Stochastic Optimization	Static Robust Optimization
Wind Invest (MW)	0	2550	1500	1700	450
Solar Invest (MW)	1440	0	0	490	1350
CCGT Invest (MW)	1200	0	0	800	1200
OCGT Invest (MW)	0	200	0	100	0
Total Cost (M€)	374.81	209.08	114.66	269.52	387.73

- The RO penalizes the variability of the wind across the scenarios and reduces its investment in favor of more solar and CCGT compared to the SO solution.
- The RO solution is closer to the deterministic solution of scenario 1 since it's the scenario with the highest cost. However, the worst-case depends on each time period, so not in all time periods the scenario 1 is the worst case.

3

Adaptative Robust Optimization

Why Adaptative?

- More often than not, once the uncertainty realizes, it is possible to react to mitigate the potentially pernicious effect of the uncertainty

$$\min_{x \in \mathcal{X}} \max_{u \in \mathcal{U}} \min_{y \in \mathcal{Y}(x,u)} f(x, y, u)$$

- ARO provides both a preventive and a corrective view: planning, then uncertainty realization, then mitigation.

- ARO provides both ex-ante protection (planning decisions) and ex-post correction (operation decisions). Remember that static RO only provides ex-ante protection.

Example – Generation Expansion Planning (ARO)

We want to make a robust decision of the generation investment achieving the least total cost outcome under the worst uncertainty realization of the future demand.

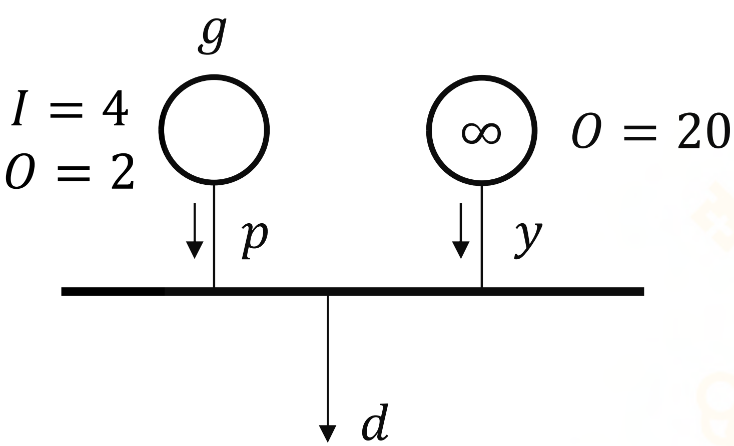
$$\min_{g_i \in \Omega_i} \max_{d_{j,t} \in \Delta_j} \min_{p_{i,t} \geq 0} z = \sum_{i=1}^{n_G} I_i g_i + \sum_{t=1}^{n_T} \sum_{i=1}^{n_G} O_i p_{it}$$

$$s.t. \quad \sum_{i=1}^{n_G} p_{it} = \sum_{j=1}^{n_D} d_{jt}, \qquad \forall t$$

$$p_{it} \leq g_i, \qquad \forall i \forall t$$

The first-level decides on which generation capacity to build (minimize investment + operation cost), anticipating the worst demand realization (that results in maximum operation cost), which in turn anticipates the best operating decision (that seeks minimum operation cost) at the third-level subproblem

Example – Generation Expansion Planning (ARO)



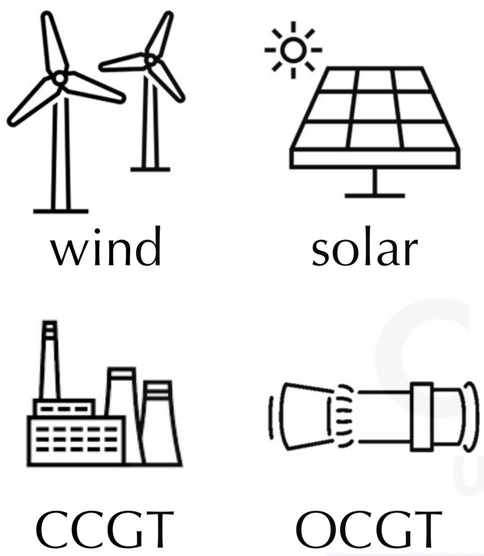
$$\min_{g \in \{5,7\}} \max_{d \in \{4,6\}} \min_{p \geq 0, y \geq 0} z = 4g + 2p + 20y$$
$$s.t. \quad p + y = d$$
$$p \leq g$$

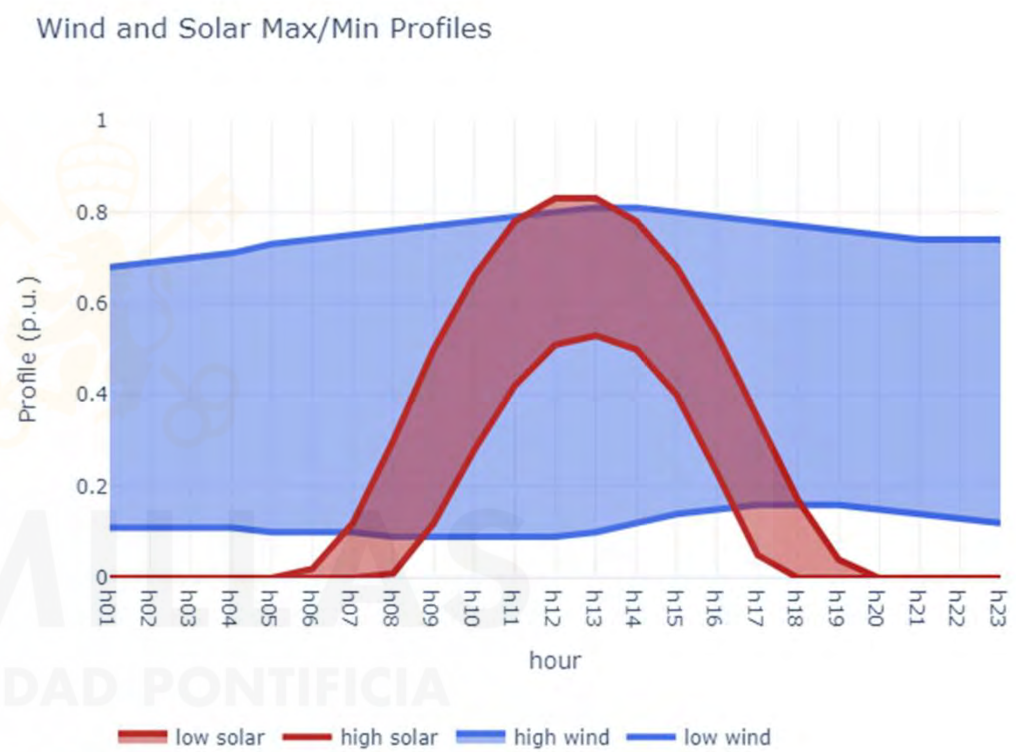
Investment (g)	Demand (d)	Total Cost
5	4	4*5+2*4=28
5	**6**	4*5+2*5+20*1=**50**
7	4	4*7+2*4=36
7	**6**	4*7+2*6=**40**

The ARO solution is different from the RO solution, since first decisions anticipate second-level ones, which in turn anticipates third-level decisions.

Example ARO-GEP with Uncertainty in Renewables

4 technologies to choose





⚠ Not a "scenario-based" approach; instead of scenarios, we use the low and high values to determine the min/max values of renewables' uncertainty sets

Source: https://github.com/datejada/generation-expansion-planning-models

Example ARO-GEP with Uncertainty in Renewables

ARO formulation

$$\min_{g_i} \quad \max_{A_{it}} \quad \min_{p_{it},e_t} \underbrace{\sum_{i=1}^{n_G} I_i g_i}_{\substack{investment\\ cost}} + \underbrace{W \sum_{t=1}^{n_T} \left(C e_t + \sum_{i=1}^{n_G} O_i p_{it} \right)}_{operating\ cost}$$

$s.t.$

$$\underbrace{A_{it} \in \left[A_{it}^{min}, A_{it}^{max}\right]}_{uncertainty\ set}, \quad \forall i \forall t$$

$$\underbrace{\frac{A_{it} - A_{it}^{min}}{A_{it}^{max} - A_{it}^{mi}} \leq \Gamma_i}_{uncertainty\ budget}, \quad \forall i \forall t$$

$s.t.$

$$\underbrace{\sum_{i=1}^{n_G} p_{it} + e_t = D_t}_{demand\ balance}, \quad \forall t$$

$$\underbrace{p_{it} \leq A_{it} g_i}_{production\ limit}, \quad \forall i \forall t$$

New Parameters

A_{it}^{min}: minimum technology availability

A_{it}^{max}: maximum technology availability

Γ_i: uncertainty budget

Not a "scenario-based" approach, so the set scenario is not needed

Example ARO-GEP with Uncertainty in Renewables

ARO formulation

$$\min_{g_i} \quad \max_{A_{it}} \quad \min_{p_{it}, e_t} \underbrace{\sum_{i=1}^{n_G} I_i g_i}_{\substack{investment \\ cost}} + \underbrace{W \sum_{t=1}^{n_T} \left(Ce_t + \sum_{i=1}^{n_G} O_i p_{it} \right)}_{operating\ cost}$$

$$s.t. \qquad s.t.$$

$$\underbrace{A_{it} \in \left[A_{it}^{min}, A_{it}^{max}\right]}_{uncertainty\ set}, \quad \forall i \forall t \qquad \underbrace{\sum_{i=1}^{n_G} p_{it} + e_t = D_t}_{demand\ balance}, \quad \forall t$$

$$\underbrace{\frac{A_{it} - A_{it}^{min}}{A_{it}^{max} - A_{it}^{min}} \leq \Gamma_i}_{uncertainty\ budget}, \quad \forall i \forall t \qquad \underbrace{p_{it} \leq A_{it} g_i}_{production\ limit}, \quad \forall i \forall t$$

⚠ A_{it} is a variable for the outer maximization problem, but a parameter for the inner minimization problem!

Example ARO-GEP with Uncertainty in Renewables

ARO formulation

$$\min_{g_i} \quad \max_{A_{it}} \quad \min_{p_{it},e_t} \underbrace{\sum_{i=1}^{n_G} I_i g_i}_{\substack{investment\\ cost}} + \underbrace{W \sum_{t=1}^{n_T} \left(Ce_t + \sum_{i=1}^{n_G} O_i p_{it} \right)}_{operating\ cost}$$

$s.t.$

$$\underbrace{A_{it} \in \left[A_{it}^{min}, A_{it}^{max}\right]}_{uncertainty\ set}, \quad \forall i \forall t$$

$$\underbrace{\frac{A_{it} - A_{it}^{min}}{A_{it}^{max} - A_{it}^{min}}}_{uncertainty\ budget} \leq \Gamma_i, \quad \forall i \forall t$$

$s.t.$

$$\underbrace{\sum_{i=1}^{n_G} p_{it} + e_t = D_t}_{demand\ balance}, \quad \forall t$$

$$\underbrace{p_{it} \leq A_{it} g_i}_{production\ limit}, \quad \forall i \forall t$$

The first-level decides on which generation capacity to build (minimize investment + operation cost), anticipating the worst generation availability realization (that results in maximum operation cost), which in turn anticipates the best operating decision (that seeks minimum operation cost) at the third-level subproblem

Solution Techniques

Min-max-min

Benders' decomposition + KKT's of the inner min problem	A. J. Conejo, L. Baringo, S. J. Kazempour and A. S. Siddiqui, Investment in Electricity Generation and Transmission, Cham, Zug, Switzerland:Springer, 2016.
Benders' decomposition + Dual problem with bilinear term	Zugno, M., & Conejo, A.J.,(2015). A robust optimization approach to energy and reserve dispatch in electricity markets. European Journal of Operational Research, 247, 659-671
Enumeration of vertices	Morales, J. M., Conejo, A. J., Madsen, H., Pinson, P., & Zugno, M. (2013). Integrating renewables in electricity markets: operational problems (Vol. 205). Springer Science & Business Media.

This solution techniques is available in GitHub for the example: https://github.com/datejada/generation-expansion-planning-models

Example ARO-GEP with Uncertainty in Renewables

Result	Stochastic Optimization	Static Robust Optimization	Adaptive Robust Optimization
Wind Invest (MW)	1700	450	0
Solar Invest (MW)	490	1350	1920
CCGT Invest (MW)	800	1200	1200
OCGT Invest (MW)	100	0	0
Total Cost (M€)	269.52	387.73	310.62

- The ARO reduces the conservativeness of the SRO solution (lower total cost), but still is higher than the SO solution.
- Once more, the wind is penalized in the robust optimization framework since it has the wider range of variation.

4

Final Remarks

Final Remarks

- In RO, under which outcome of uncertainty do we get the optimal solution?

- Worst case

- What about the other outcomes of uncertainty? Do we know something about these cases?

- Feasibility

- How can we reduce conservativeness of the solution?

- Well structured uncertainty sets

Source: Jalal Kazempour, DTU 31792 -- Advanced Optimization and Game Theory for Energy Systems, January 12, 2021.

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