Investigación Operativa
Operations Research

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Contents

1

INTRODUCTION

Operations Research (OR) definition

- Application of advanced scientific analytical methods in improving the effectiveness of a company's operations, decisions, and management.
  - Design and improvements in operations and decisions
  - Problem-solving and support in management, planning, or forecasting functions
  - Provide knowledge and decision support
- Tasks:
  - Collect and analyze data
  - Develop and test mathematical models
  - Propose solutions and recommendations
  - Interpret the information
  - Help to implement improvement actions
- Results: computer applications, systems, services, or products.

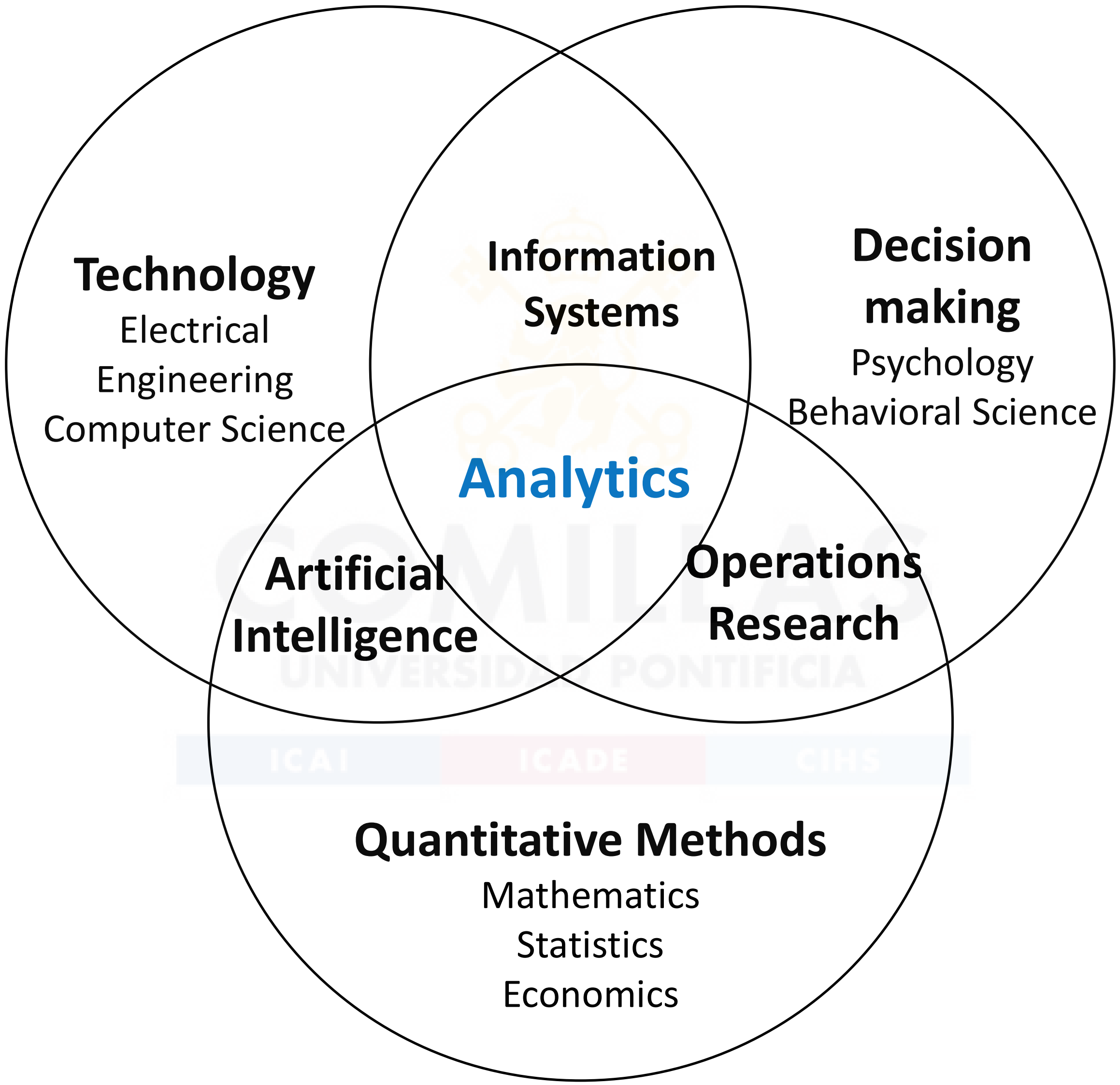
In short

- The science of better
- Decision support models
- Advanced analytical methods

Professionals

- Advanced Analytics & Optimization Leader
- Specialist in business modeling and analytics
- VP Global Analysis, Business Intelligence and Optimization
- Head of Operational Analysis
- Decision Analysis and Modeling Group
- Analytics Team Leader
- Senior Analytical Strategist
- Health & Government Analytics
- Finance & Risk, Infrastructure & Analytics
- Mathematical Optimization and Analytics

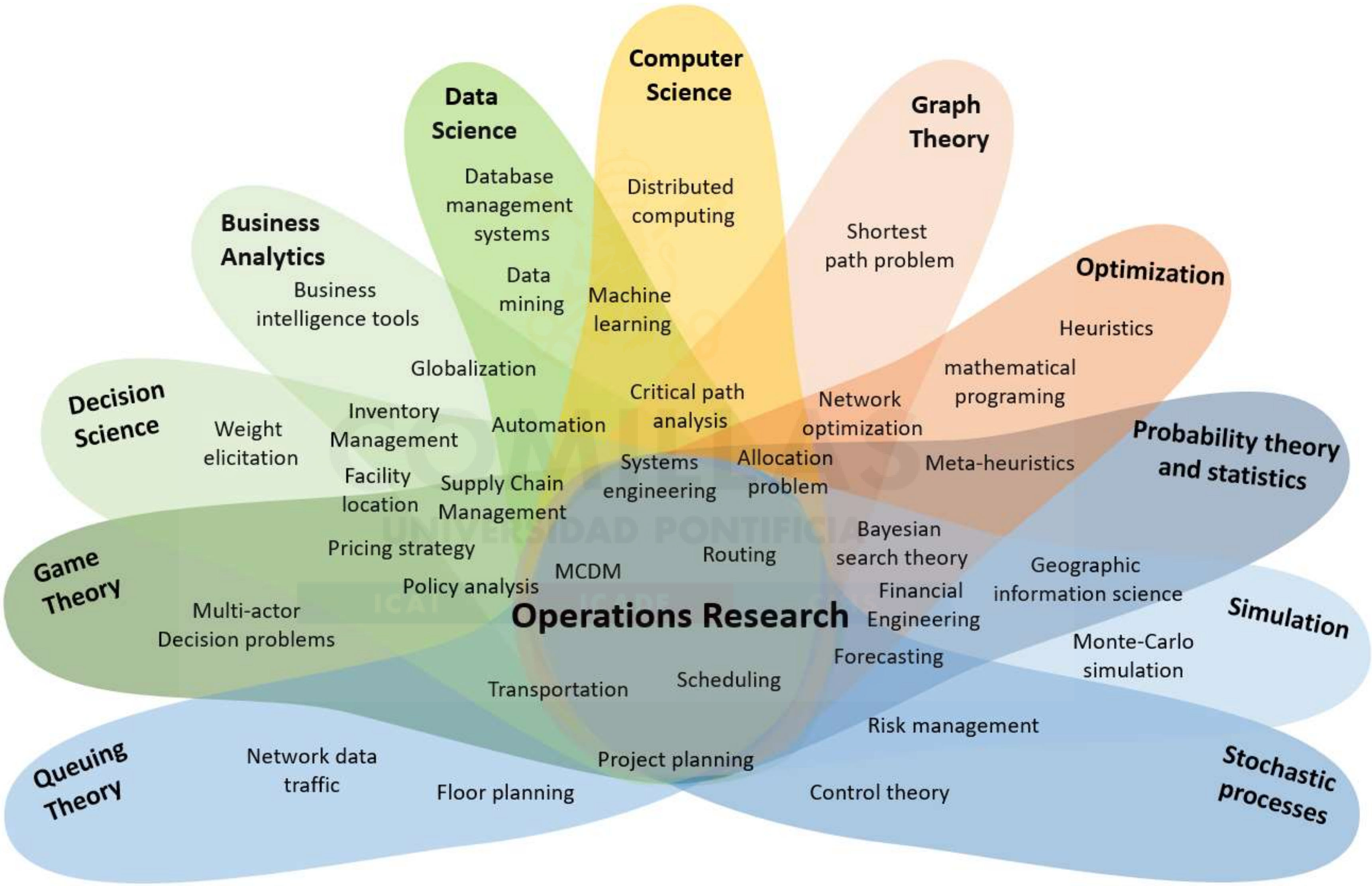
Analytics


Technology
Electrical
Engineering
Computer Science
Information
Systems
Decision
making
Psychology
Behavioral Science
Analytics
Artificial
Intelligence
Operations
Research
Quantitative Methods
Mathematics
Statistics
Economics

Holistic illustration of the disciplines and problems related to operations research.

https://towardsdatascience.com/why-operations-research-is-awesome-an-introduction-7a0b9e62b405

Photo by Alex Vasegaard



Skills for analytics

1. *Framing the problem and getting to the solution*
2. *Data preparation*
3. *Data presentation*
4. *Control of operations*
5. *Statistics, probability, forecasting*
6. *Optimization, simulation, queuing models, decision analysis*
7. *Enabling organizations to act intelligently*

Framing the problem and getting to the solution

- **Understanding the business issues and business/organizational concerns** using formal modeling processes that address purpose and environment and end-to-end capabilities and processes. This allows the analyst to understand the domain to frame problems and identify solutions.

- **Frame and structure problems in terms of objectives, constraints, risks and courses of action** to set the stage for meaningful results. Effective selection of a problem statement sets up the solution to align with the organizational structure, systems and culture; poor problem structures often lead to blind alleys during analysis or execution.

Framing the problem and getting to the solution

- **Project management** and exposure to the foundational principles of managing people, politics, products, and processes.

- **Communication skills, methods, and technologies that enable effective collaboration** throughout the project cycle, from effective information-gathering to collaborative decision support and change management

Enabling organizations to act intelligently

- **Design and governance of metrics portfolios to drive system-wide performance improvements**; this addresses the problem of figuring out which metrics are important and how to set targets and thresholds that address business needs.

- **Driving decisions from analysis**: helping decision-makers to use data-driven analyses effectively, often in conjunction with gut-feel or anecdotal methods.

- **Design and operation of analytics teams** and organizations in the context of business and IT functions.

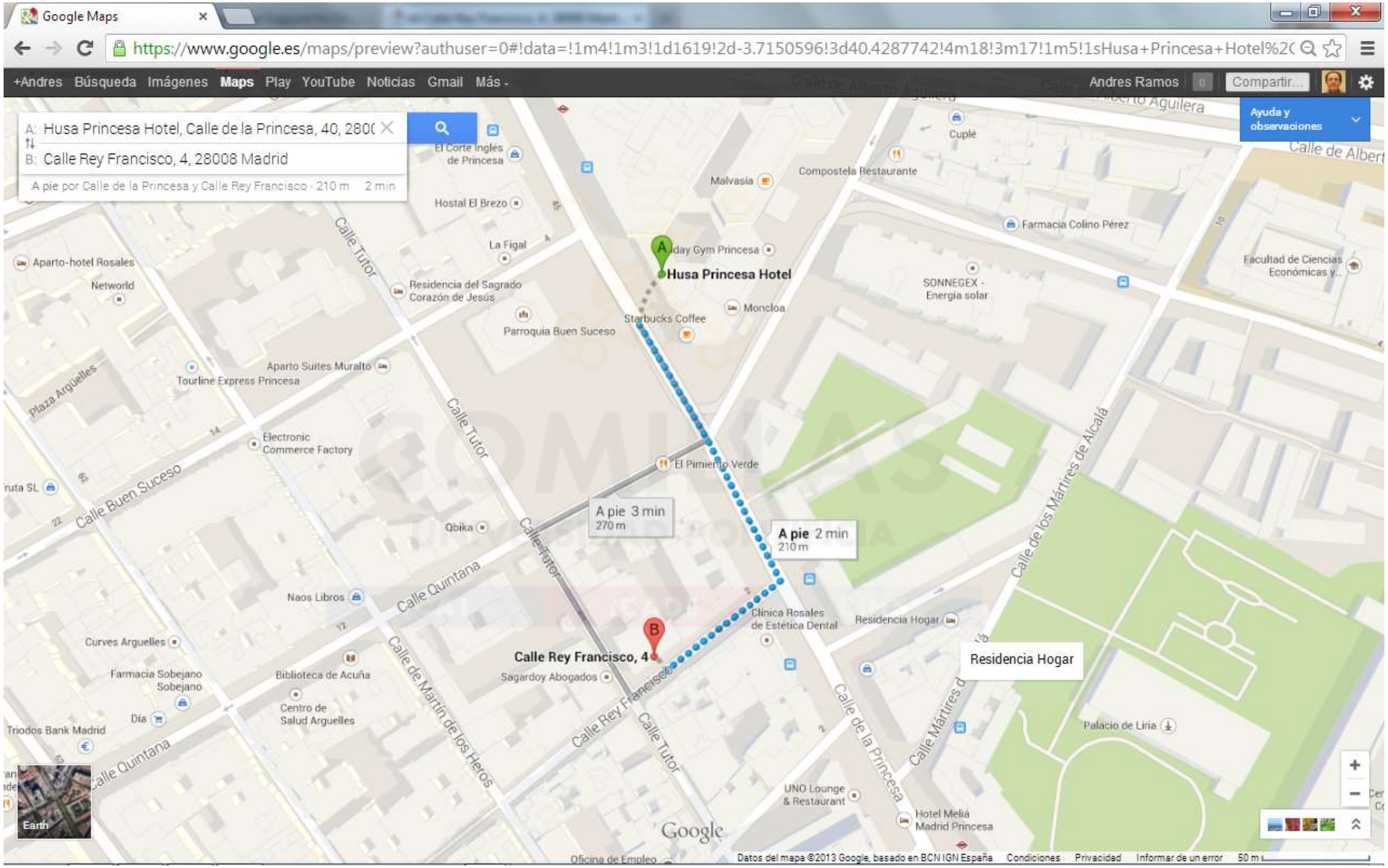
Book: “Conviértete en un profesional mejor, superior y diferente”. Javier Sánchez Álvarez. Ed. Debolsillo

- Planning to locate a florist shop. **Make numbers**
- Including a new machine to improve your production process. **Make numbers**
- Launching a new product. **Make numbers**
- Investing in a logistic process. **Make numbers**
- Setting prices or discounts. **Make numbers**
- Deciding necessary staff for a purpose. **Make numbers**
- Planning to expand your market. **Make numbers**
- Locating your company in another country. **Make numbers**
- And then, locating your company in yet another country. **Make numbers**
- Diversifying. **Make numbers**
- Investing in new technologies. **Make numbers**
- Changing your logo or your corporate communication plan. **Make numbers**
- Modifying your company scope in five years. **Make numbers**

Optimization is everywhere

- What is the first optimization problem you solved today before coming here?

Shortest path problem



Introduction to analytical techniques

- We will present mathematical tools that can be used for supporting decisions in different frameworks:
  - Operations management
  - Financial management
  - Marketing management
- How these techniques can be applied to business decisions
  - Inventory management and control
  - **Staff planning**
  - Fuel logistics
  - Maintenance scheduling
  - Forecasting electricity demand, prices, wind generation, etc.
  - **Detection of black losses**
  - **Short- and medium-term operations**
  - Market bidding
  - Market equilibrium

MISO Wins INFORMS Edelman Award

- **Chicago, April 11, 2011** – MISO, which manages one of the world's largest energy markets, won the 2011 Franz Edelman Award for Achievement in Operations Research and the Management Sciences at a banquet sponsored by the Institute for Operations Research and the Management Sciences (INFORMS) in Chicago tonight.
- The Franz Edelman competition attests to the contributions of operations research and analytics in the profit and non-profit sectors. Since its inception 40 years ago, cumulative dollar benefits from Edelman finalist projects has reached over $170 billion.
- "By leveraging operations research, MISO and its stakeholders added significant value to the region through improved reliability and increased efficiencies of the region's power plants and transmission assets," said John Bear, President and CEO of MISO. "We are honored and pleased that INFORMS recognized the value of this collaborative project."
- Driven by the goal of **minimizing delivered wholesale energy costs reliably**, MISO, with the support of Alstom Grid, The Glarus Group, Paragon Decision Technology, and Utilicast, **used operations research and analytics to design and launch its energy-only market** on April 1, 2005, **and introduced its energy and ancillary services markets** on Jan. 6, 2009.
- The MISO improved reliability and increased efficiencies of the region's power plants and transmission assets. Based on its annual Value Proposition study, the MISO region **realized between $2.1 and $3.0 billion in cumulative savings from 2007 through 2010**. The Midwest ISO estimated **an additional $6.1 to $8.1 billion of value will be achieved through 2020**.

http://www.youtube.com/watch?v=sz7C-aeO7Wc&list=PL094F34046FE684C7

Source: https://www.informs.org/About-INFORMS/News-Room/Press-Releases/Edelman-Winner-2011

Why is important to use analytical techniques for decision making?

- DIFFICULT DECISION FRAMEWORK
  - Making decisions in a complex world requires the use of analytical techniques

- ECONOMIC IMPACT
  - Huge economic relevance of the decisions
  - A small improvement in any decision-making will have a significant impact on the company's profit, operations, or processes

Business Analytics Spectrum

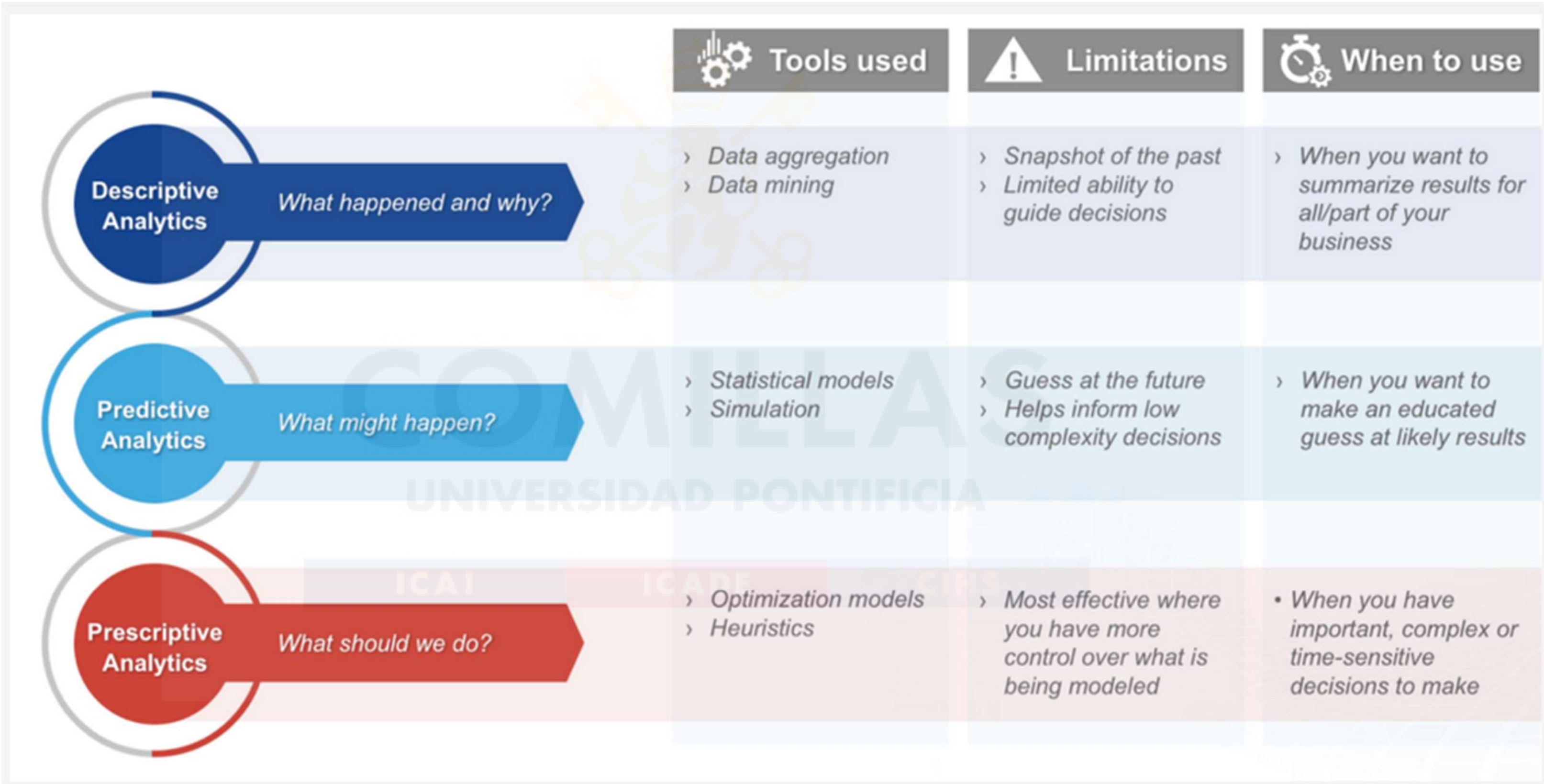
- **DESCRIPTIVE**: statistics (data analysis & visualization, analysis of variance, correlation, data mining)
- **PREDICTIVE**: simulation, regression, forecasting
- **PRESCRIPTIVE**: optimization, heuristics, decision analysis

Competitive advantage ↑

Stochastic Optimization	How can we achieve the best outcome including the effects of variability?	***PRESCRIPTIVE***
Optimization	How can we achieve the best outcome?	↑
Predictive modeling	What will happen next if?	***PREDICTIVE***
Forecasting	What if these trends continue?	↑
Simulation	What could happen…?	
Alerts	What actions are needed?	
Query/drill down	What exactly is the problem?	***DESCRIPTIVE***
Ad hoc reporting	How many, how often, where?	
Standard reporting	What happened?	

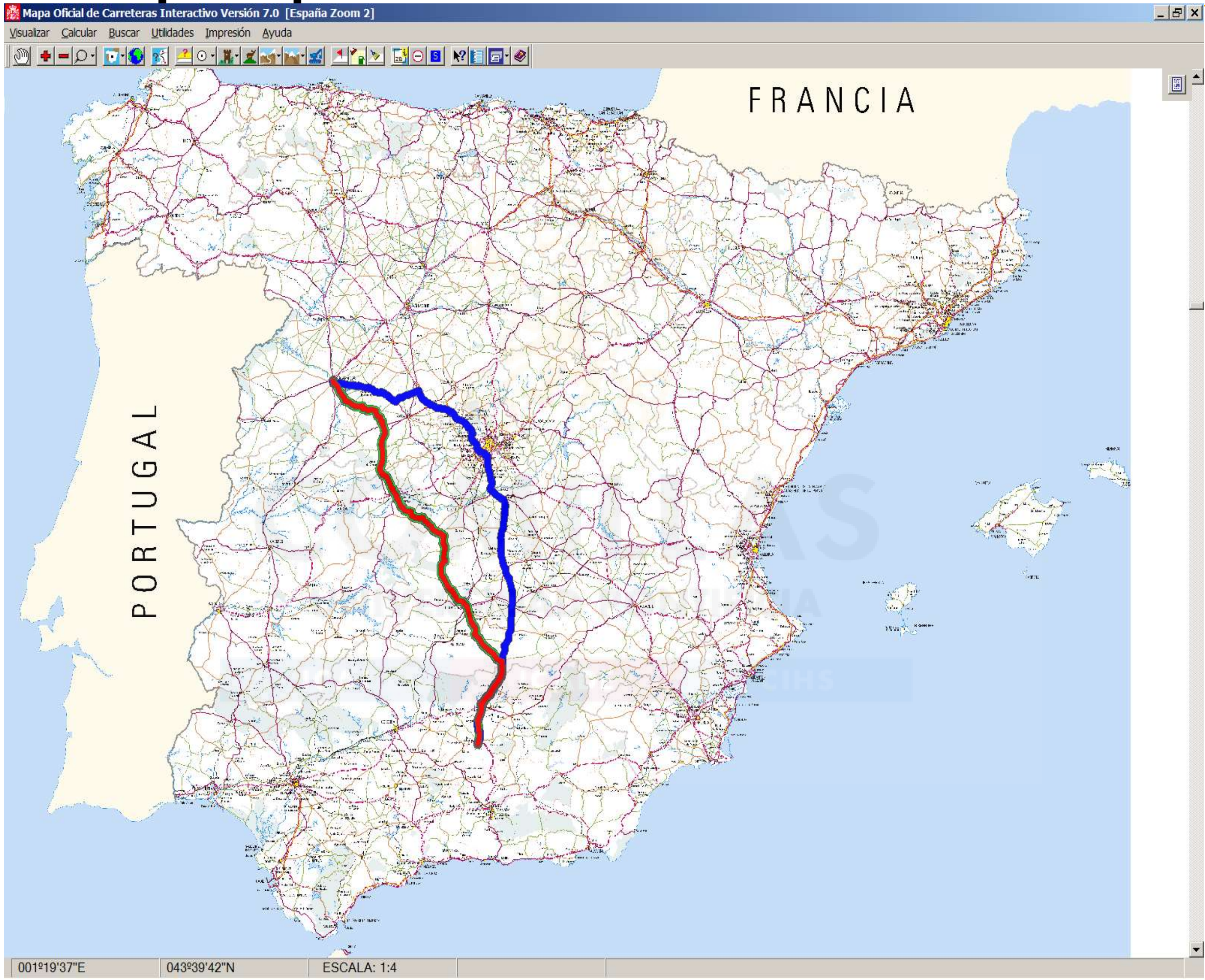
Source: A. Fleischer et al. *ILOG Optimization for Collateral Management*

Business Analytics



https://www.gurobi.com/faqs/prescriptive-analytics/

Shortest path problem

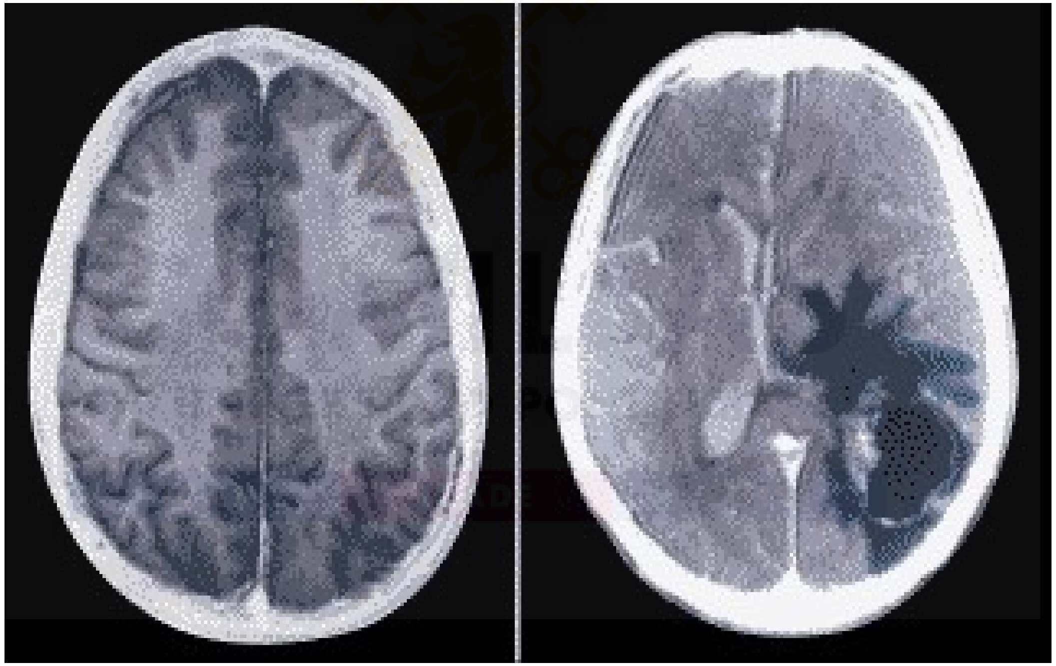


Car assembly line

- How to weld all the weld beads within a minimum time?

Brain cancer treatment

- Where to deploy radiotherapy to maximize impact on carcinogenic cells and minimize damage in other cells?



iMetro: Subway best route calculator (https://www.iit.comillas.edu/imetro/)

Parking place demand and offer assignment

- (https://www.iit.comillas.edu/aramos/papers/Parking%20place%20demand%20and%20offer%20assignment.pdf)

Train timetabling. EcoDriving

- A. Ramos, M.T. Peña, A. Fernández, P. Cucala *Mathematical programming approach to underground timetabling problem for maximizing time synchronization* Revista de Dirección, Organización y Administración de Empresas CEPADE 35: 88-95 Junio 2008 (http://www.revistadyo.com/index.php/dyo/article/view/60/60)

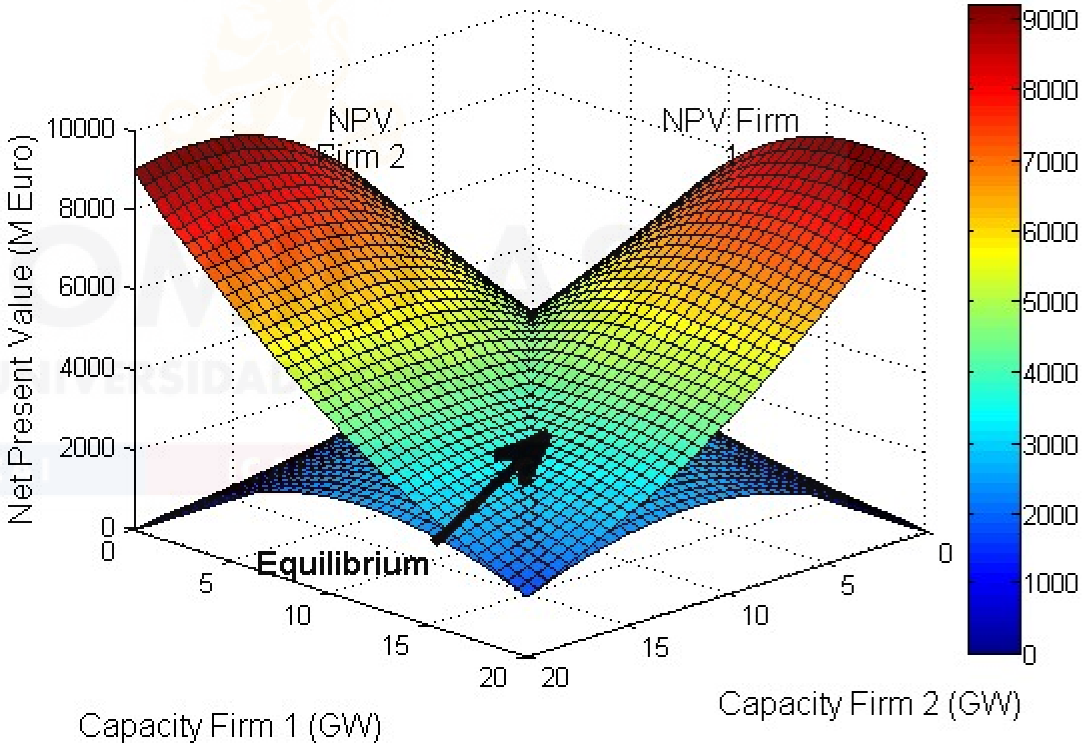
http://www.antena3.com/noticias/economia/madrid-presenta-metrolinera-estacion-carga-coches-electricos-que-aprovecha-frenada-metro_2014031400210.html

Unit commitment and economic dispatch

- S. Cerisola, A. Baillo, J.M. Fernandez-Lopez, A. Ramos, R. Gollmer *Stochastic Power Generation Unit Commitment in Electricity Markets: A Novel Formulation and A Comparison of Solution Methods* Operations Research 57 (1): 32-46 Jan-Feb 2009 (*http://or.journal.informs.org/cgi/content/abstract/57/1/32*)

Generation Capacity Expansion Problem

- S. Wogrin, E. Centeno, J. Barquín *Generation capacity expansion analysis: Open loop approximation of closed loop equilibria* IEEE Transactions on Power Systems vol. 28, no. 3, pp. 3362-3371, August 2013.

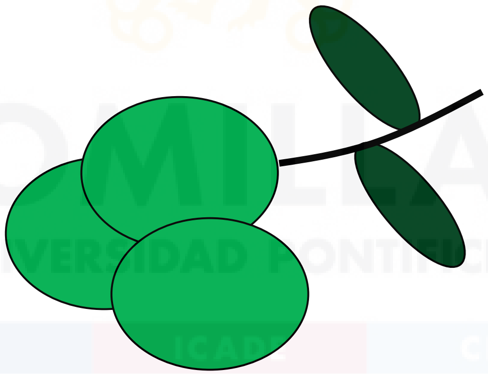


Smart Charging of Electric Vehicles

- P. Sánchez, G. Sánchez González *Direct Load Control Decision Model for Aggregated EV charging points* IEEE Transactions on Power Systems vol. 27 (3): 1577-1584 August 2012

Efficient Management of Olive Husk

- P. Sánchez-Martín, G. Rayón-Durán *An Improvement in the Efficiency of Olive Pomace Oil Extraction using an Optimal Pooling Decision Model* Biosystems Engineering (2013), http://dx.doi.org/10.1016/j.biosystemseng.2013.08.003



Off-shore wind farm electric design

- S. Lumbreras and A. Ramos *Optimal Design of the Electrical Layout of an Offshore Wind Farm: a Comprehensive and Efficient Approach Applying Decomposition Strategies* IEEE Transactions on Power Systems 10.1109/TPWRS.2012.2204906
- S. Lumbreras and A. Ramos *Offshore Wind Farm Electrical Design: A Perspective* Wind Energy 10.1002/we.1498
- M. Banzo and A. Ramos *Stochastic Optimization Model for Electric Power System Planning of Offshore Wind Farms* IEEE Transactions on Power Systems 26 (3): 1338-1348 Aug 2011 10.1109/TPWRS.2010.2075944

Optimization case studies

- Humanitarian aid
- Logistics
- Transportation industry
- Air transportation
- Space transportation
- Underground transportation
- Mail service
- Water supply service
- Call center
- e-business
- Production
- Banking industry
- Public Administration
- Energy industry
- Electricity industry
- Health system industry
- Livestock industry
- Project management
- Media industry
- Leisure and sports
- Teaching

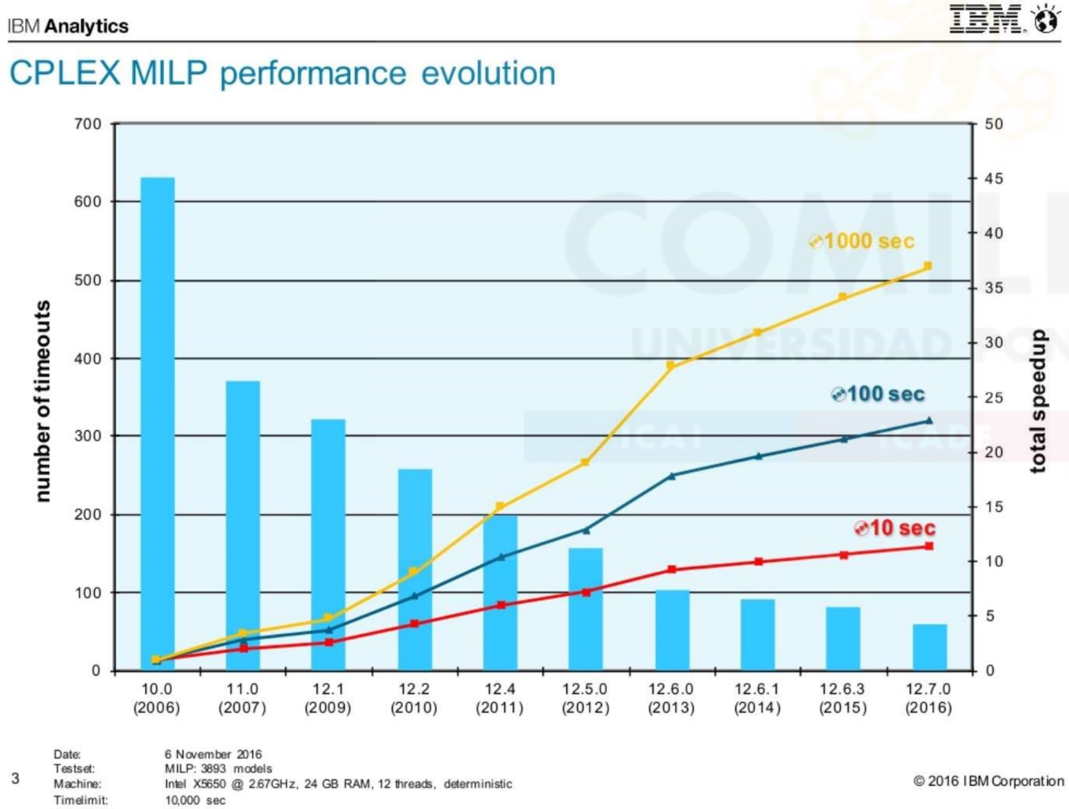
www.iit.comillas.edu/simio/apuntes/a_casos.pdf

OR History (www.iit.comillas.edu/aramos/timeline.pdf)

- Origin at the beginning of World War II (due to the urgent assignment of scarce resources in military operations, in tactical and strategic problems). The same techniques were then applied to companies.
- Rapid initial algorithmic progress (many techniques –LP, DP– were developed before 1960).
    - Game Theory: von Neumann y Morgenstern 1944
    - Simplex Method: Dantzig 1947
    - Optimality Principle: Bellman 1957
- Constant relation with computer hardware advance. Today, solving an LP problem with 1.000.000 equations and 1.00.000 variables in a PC is possible.

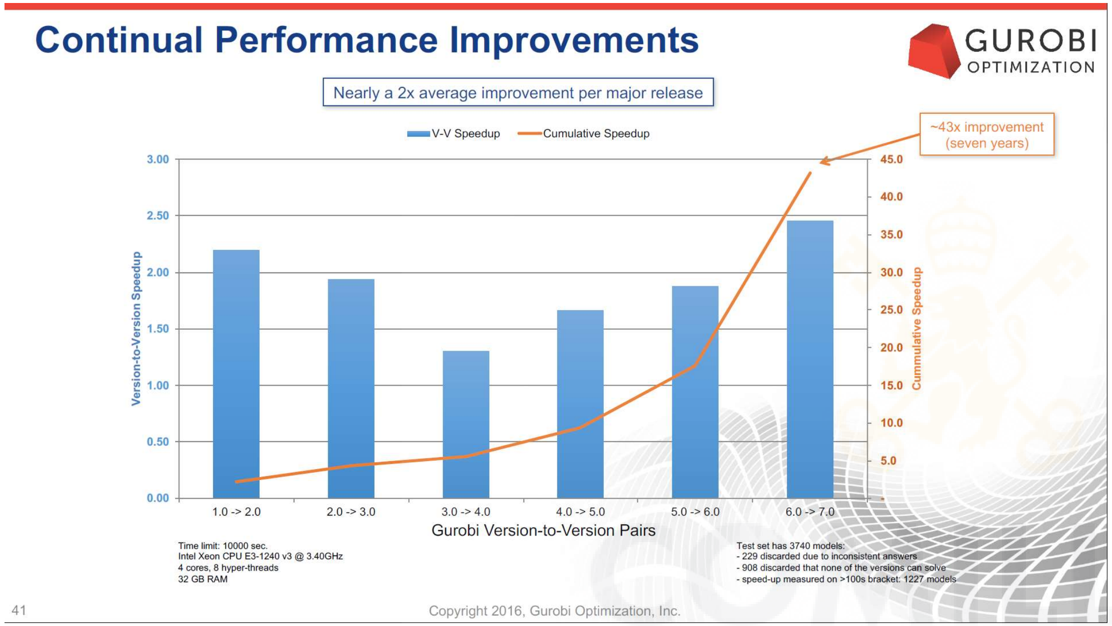
Logarithmic advances in solution time in CPLEX

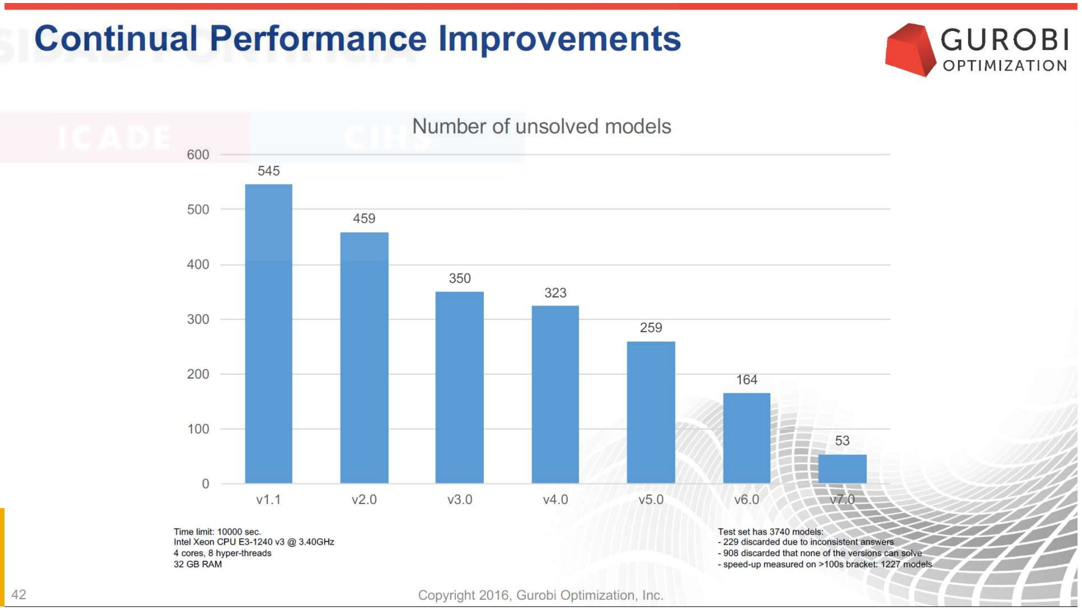
- Between CPLEX 1.0 from 1988 and CPLEX 9 del 2004, LP solver has observed a total improvement (software and algorithms) of 3300 times. As a reference, the performance improvement in hardware has been 1600 times in the same period
- Between CPLEX 1.2 from 1991 and CPLEX 11 del 2007 a total improvement (software and algorithms) of 29000 times has been observed in **MIP solver**



Today, it can be solved in seconds what a decade ago would have taken years.

Logarithmic advances in solution time in GUROBI





2

OPTIMIZATION

What is optimization?

- To find the value of the *variables* that make the *objective function* optimal while satisfying the set of *constraints*.

Components of an optimization problem

- *Objective function*
  - Quantitative performance function (fitness) of a system that we want to maximize or minimize
- *Variables*
  - Decisions that influence the objective function
- *Constraints*
  - Set of relations that variables are forced to satisfy

Particular cases

- Multiple objective functions: **Multicriteria Optimization**
  - Different optimum for each objective function
  - Multiobjective optimization
  - Satisfying methods
- **There is no objective function**
  - (Non) Linear system of equations
  - Find a feasible solution
- There are no constraints: **Unconstrained optimization**
  - Determine the minimum of a function
  - Least squares approximation

3

SOLUTION METHODS

Classic optimization vs. Metaheuristics (i)

Classical methods	•Linear Programming (LP) •Mixed Integer Linear Programming (MIP) •Quadratic Programming (QP) •Non Linear Programming (NLP) •Stochastic Programming (SP) •Dynamic Programming (DP) •Graph Theory or Network Optimization
Metaheuristic methods (Artificial Intelligence)	•Evolutionary algorithms (Genetic, Memetic, Scatter Search) •Heuristic (Simulated Annealing, Tabu Search) •Bio-inspired (Ant Colonies, Particle Swarm) •Other (Random, Greedy or Cross Entropy, Variable Neighborhood Search) •Gravitational Search Algorithm (GSA)

Other optimization methods

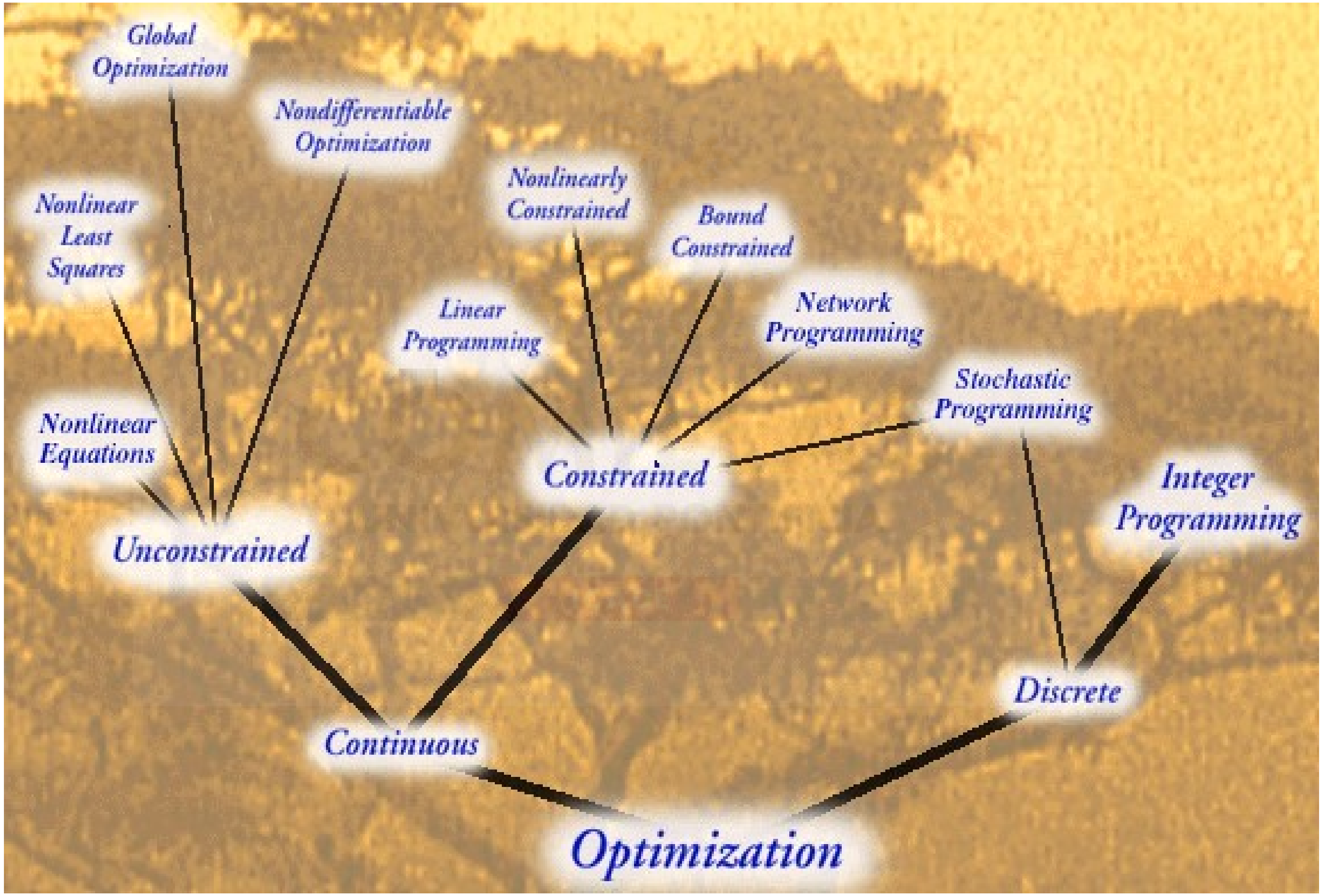
Hybrid methods	•Constraint Programming
Other classical decision methods	•Decision theory •Game theory
Other methods	•Fuzzy sets •Global optimization •Combinatorial optimization

Model classification

- According to the variables
  - Continuous, discrete, stochastic
- According to the objective function
  - Linear, quadratic, nonlinear, non smooth
- According to the constraints
  - Unconstrained, linear, nonlinear, non smooth

NEOS Guide Optimization Tree

http://neos-guide.org/optimization-tree



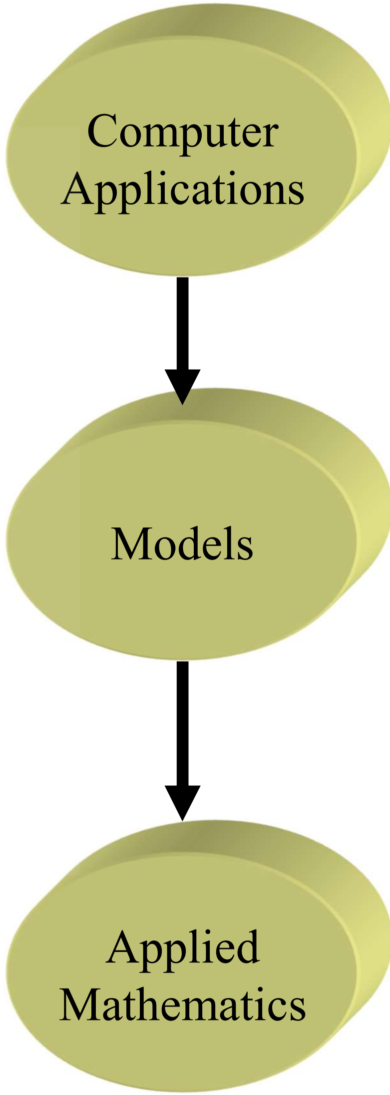
Classical optimization vs. Metaheuristics (ii)

- Classical methods
  - Look for the optimum "locally"
  - Guarantee the numerical optimum
  - Allow a high number of constraints

- Metaheuristic methods
  - Imitate simple phenomena found in nature
  - "Global", they have specific mechanisms to avoid local optima
  - Explore a huge number of solutions in very short time
  - Mainly applied to combinatorial or non differentiable problems
  - Do NOT guarantee the optimum. Allow getting multiple suboptima
  - Do NOT allow a high number of constraints
  - Require a tuning process of the parameter choice

Optimization yes, but which aspect

- Modeling
  - Solver is almost superfluous, it is standard
- Select the optimization method
  - Application of a known method to another context
- Implement and improve a method
  - Improve a decomposition method for subproblems with discrete variables
- Develop a new method
  - Interior point method for NLP



Classical methods

- *Linear Programming* (LP)
- *Mixed Integer Programming* (MIP)
- *Nonlinear Programming* (NLP)
- *Quadratic Programming* (QP)
- *Dynamic Programming* (DP)
- *Network Flow* (NF)

Linear Programming (LP) (i)

$$\min_x c^T x$$

$$Ax = b$$

$$x \geq 0$$

$$x \in R^n, c \in R^m, A \in R^{m\times n}, b \in R^m$$

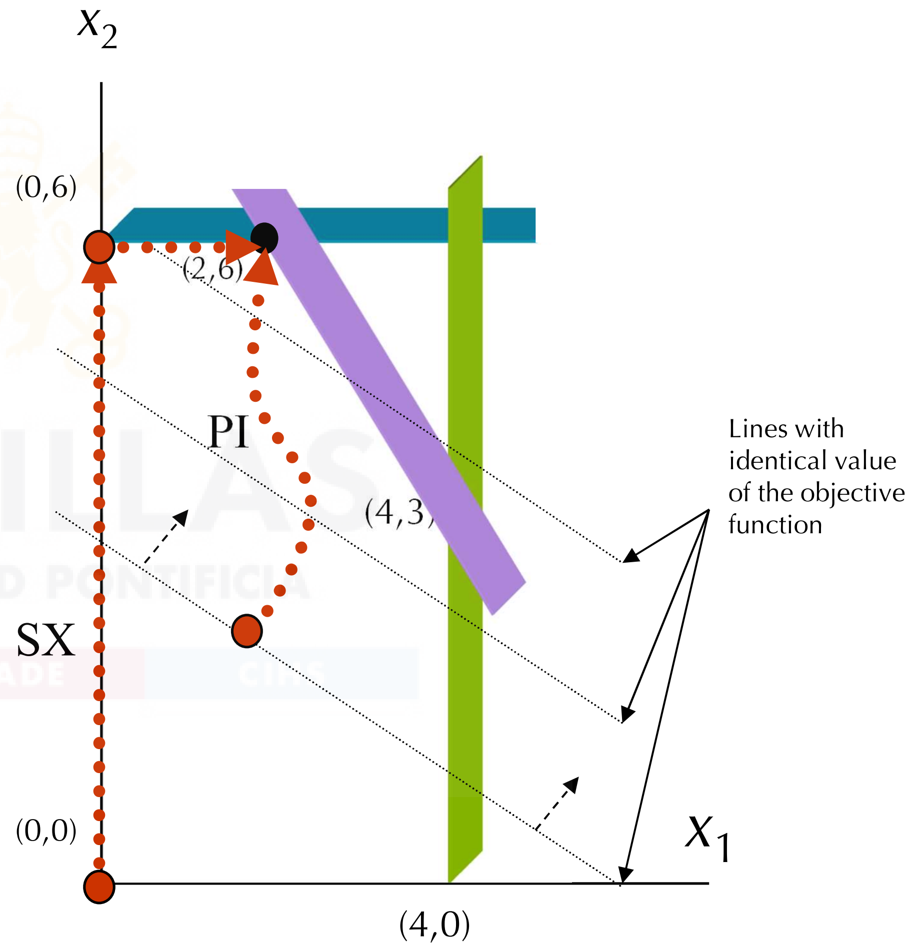
- Simplex primal and dual methods
- Interior point method (primal-dual, projective, affine scaling)

Why is 'x' the unknown? by Terry Moore

https://www.ted.com/talks/terry_moore_why_is_x_the_unknown

Linear Programming (LP) (ii)

$$\max \; z = 3x_1 + 5x_2$$
$$x_1 \leq 4$$
$$2x_2 \leq 12$$
$$3x_1 + 2x_2 \leq 18$$
$$x_1, x_2 \geq 0$$



Linear Programming. Which method to use?

- Simplex primal or dual method
  - Medium size problems (up to 100000x100000)
  - Sensitivity analysis or branch and bound method
  - Solution time depends on the cube of number of constraints
- Interior point method
  - Large size problems
  - Solution time depends almost linearly on the number of nonzero elements of the constraint matrix

Mixed Integer Linear Programming (MIP) (i)

$$\min_{x} c^T x + d^T y$$

$$A x + B y = b$$

$$x, y \geq 0$$

$$x \in Z^n, y \in R^l, c \in R^n, d \in R^l$$

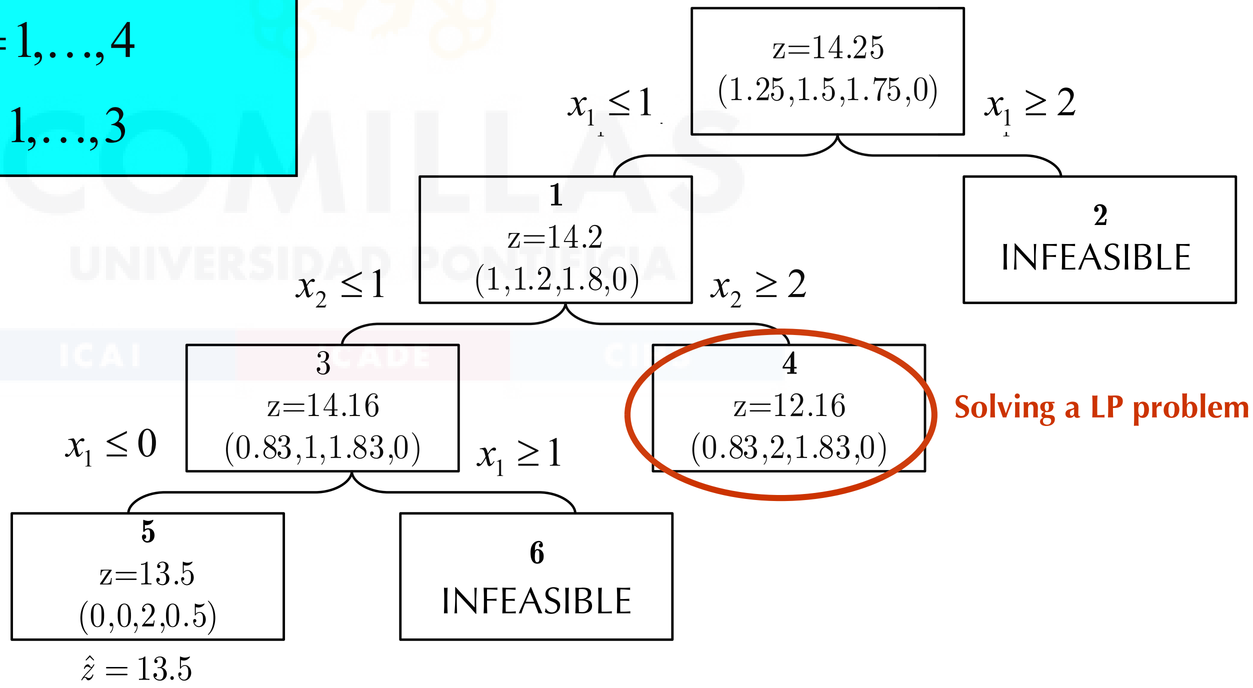
$$A \in R^{m \times n}, B \in R^{m \times l}, b \in R^m$$

- *Branch and bound* method
- *Branch and cut* method

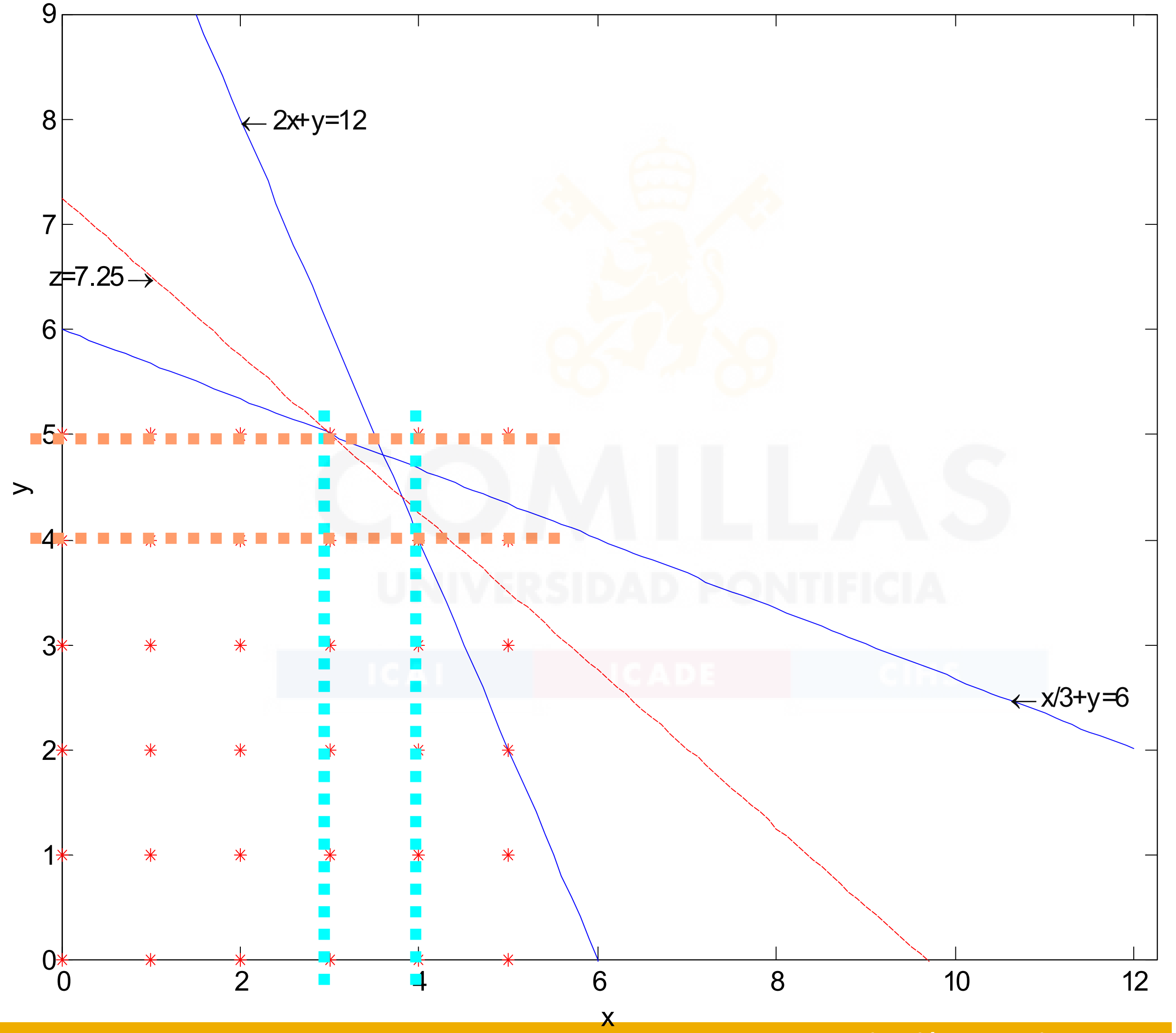
Mixed Integer Linear Programming (MIP) (ii)

$$
\begin{array}{llllll}
\max & Z = 4x_1 - 2x_2 + 7x_3 - x_4 \\
x_1 & & +5x_3 & & \leq 10 \\
x_1 & +x_2 & -x_3 & & \leq 1 \\
6x_1 & -5x_2 & & & \leq 0 \\
-x_1 & & +2x_3 & -2x_4 & \leq 3 \\
x_j \geq 0 & & j = 1, \ldots, 4 \\
x_j \text{ enteras} & & j = 1, \ldots, 3
\end{array}
$$

$$Z^* = 13.5$$
$$(x_1, x_2, x_3, x_4) = (0, 0, 2, 0.5)$$



Mixed Integer Linear Programming (MIP) (iii)



$$\max_{x,y} z = \frac{3}{4}x + y$$

$$\frac{x}{3} + y \le 6 \quad : \pi_1$$

$$2x + y \le 12 \quad : \pi_2$$

$$x, y \text{ enteras}$$

$$(x, y) = (3, 5)$$

$$(\pi_1, \pi_2) = (2.25, 0)$$

Nonlinear Programming (NLP)

- Unconstrained
  - Do not require derivatives
    - Cyclic coordinates, Hooke & Jeeves and Nelder & Mead methods
  - Require first derivatives
    - Steepest descent, conjugate gradient methods
  - Require second derivatives
    - Newton and quasi-Newton methods
- Constrained
  - Feasible methods
    - Gradient, Newton and reduced quasi-Newton methods
    - Sequential quadratic programming
  - Penalty methods
    - Penalty and barrier methods
    - Augmented lagrangian method

$$\min_x f(x)$$
$$g(x) = 0$$
$$h(x) \leq 0$$
$$l \leq x \leq u$$
$$f : R^n \rightarrow R$$
$$g, h : R^n \rightarrow R^m$$

Techniques used in Nonlinear Programming (NLP)

- Feasible methods
  - Approximate the functions by Taylor expansion series (linear or quadratic, first or second derivatives)
  - Stop when the first derivatives are 0

- Penalty methods
  - Solve sequences of unconstrained optimization problems
  - Penalty method
    - Penalize the constraint violation
  - Barrier method
    - Avoid reaching the neighborhood of a constraint

Newton’s method for unidimensional function (i)

$$f(x) = (x-1)^3 + 2(x-1)^2 + 3$$

$$f'(x) = 3(x-1)^2 + 4(x-1)$$

$$f''(x) = 6(x-1) + 4$$

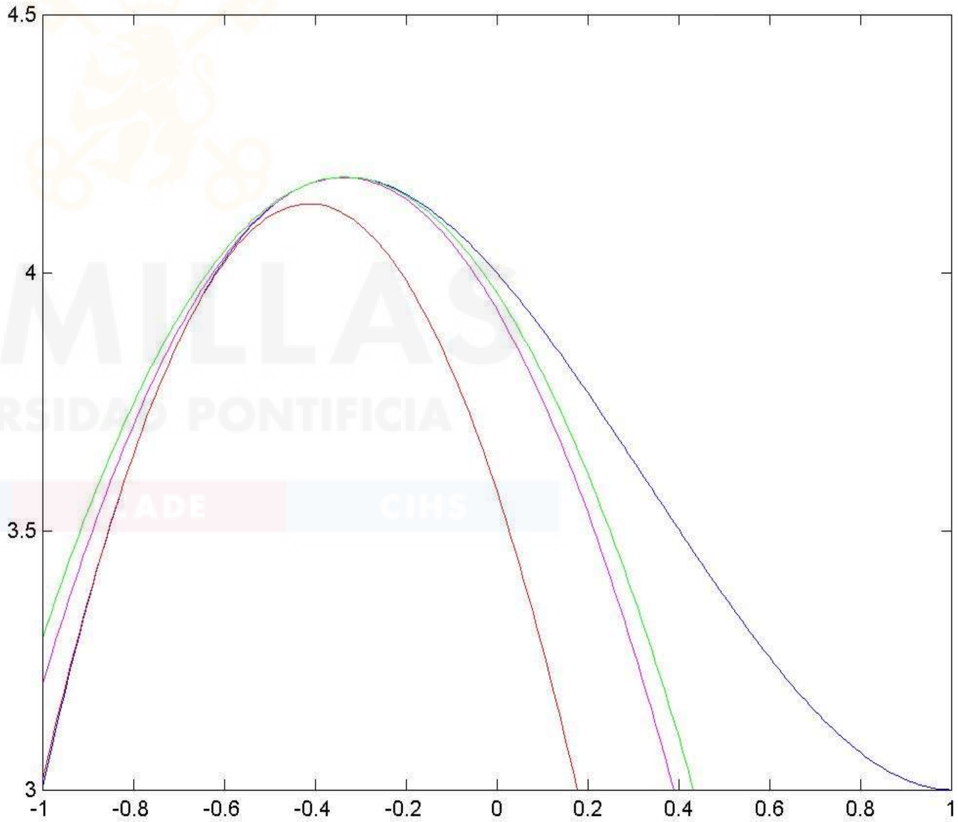
Sequence of points

$$x_0 = -0.75$$

$$x_1 = x_0 - \frac{f'(x_0)}{f''(x_0)} = -0.4135$$

$$x_1 = -0.3376$$

$$x_1 = -0.3333$$



Quadratic Programming (QP)

$$\min_x c^T x + \frac{1}{2} x^T Q x$$

$$Ax = b$$

$$x \geq 0$$

$$x \in R^n, c \in R^n, A \in R^{m \times n}$$

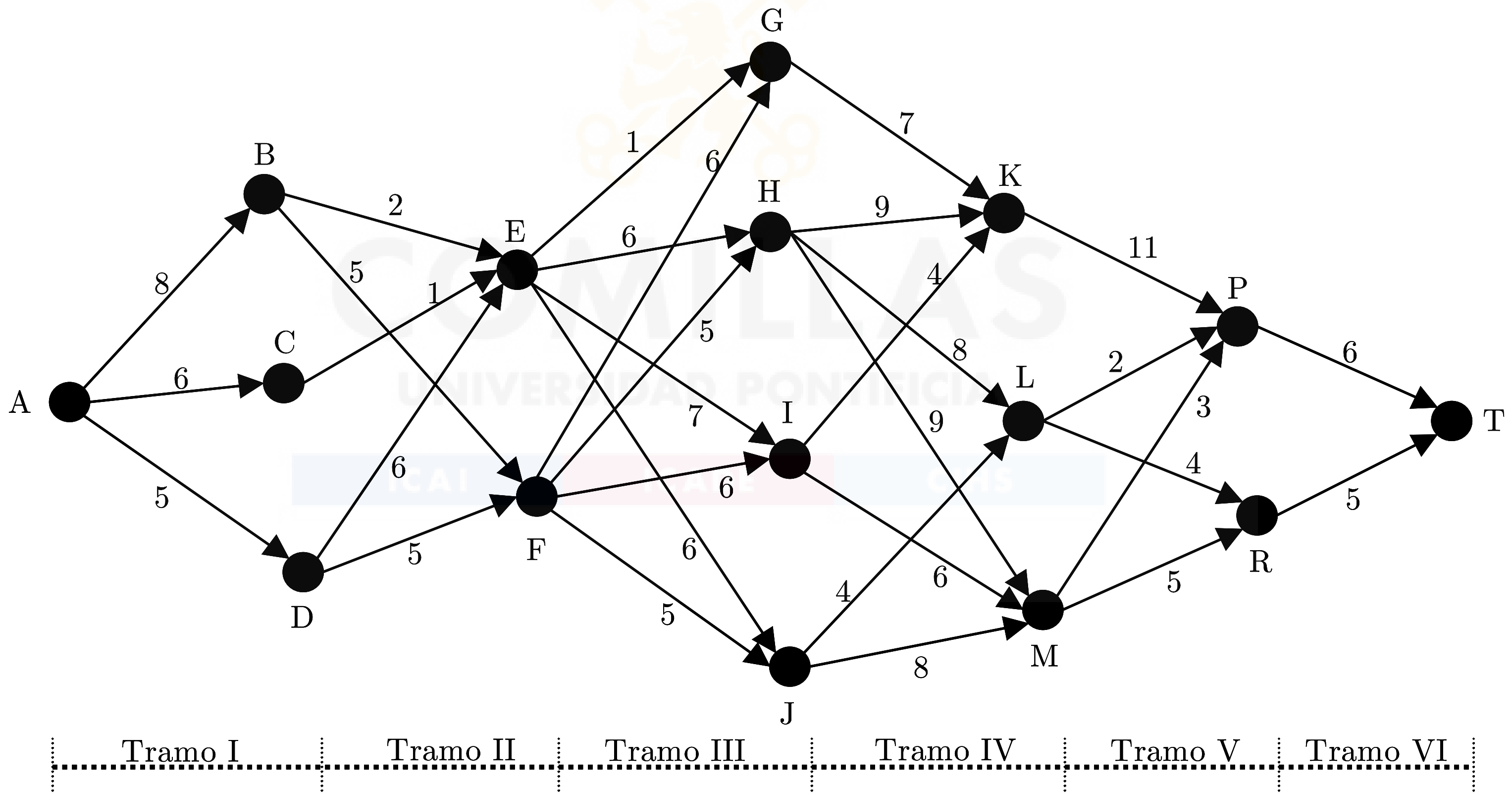
$$Q \in R^{n \times n}, b \in R^m$$

Advantage

Hessian matrix is constant, therefore second derivative approximation is constant

Dynamic Programming (DP) (i)

- Mathematical technique oriented to solving problems with decisions in successive stages (spatial or temporal) where the total cost of the decisions will be minimized



Dynamic Programming (DP) (ii)

- In each stage the optimal decision to go from a state of a certain stage to the next stage taking into account current and future costs
- Each state keeps the necessary information for taking future decisions without knowing how this state is reached
- It is a recursive procedure that solves in an iterative way, adding one stage each time, bigger sizes of the original problem. It can be solved forward or backward

$$\min_{u_k} J = \sum_{k=0}^{N} f_k(x_k, u_k)$$

$$x_{k+1} = \phi_k(x_k, u_k)$$

$$g_{k,i_k}\left(x_k, u_k\right) = 0$$

$$x_k \in X_k,\ u_k \in U_k$$

Graph Theory or Network Optimization

- **Shortest path**: find the minimum distance between one origin and one destination through a non-directed connected network knowing the distance between each pair of nodes
- **Minimum spanning tree**: find the chain of minimum distance that connects through all the nodes without cycles
- **Maximum flow**: given a directed connected network with arcs of bounded capacity find the maximum flow that can be sent from one origin to one destination
- **Transportation problem or minimum cost flow**: minimize the transportation cost of a product from several origins to different destinations given the per unit transportation cost between each origin and each destination, the maximum capacity of each origin and the demand of each destination
- **Task assignment**: minimize the total assignment cost knowing that each person does one task and each task is done by one person and the assignment cost of each task to each person

Metaheuristic methods (http://www.metaheuristics.net/)

- Evolutionary
  - *Genetic Algorithm* (GA)
  - *Memetic Algorithm*
  - *Scatter Search (SS)*
- Heuristic
  - *Tabu Search* (TS)
  - *Simulated Annealing* (SA)
- Bio-inspired
  - *Ant Colony Optimization* (ACO)
  - *Particle Swarm Optimization* (PSO)
- Other
  - *Cross Entropy* (CE)
  - *Variable Neighborhood Search* (VNS)
  - *Gravitational Search Algorithm* (GSA)

Genetic Algorithm (GA) (i)

- Idea: in nature only survive the best ones
- Selection process as a function of a quality measure (*fitness function*) of the solution and must include constraint violation. Recombining process to search for new solutions
- Combine guided and stochastic search. Two apparently opposite goals: search of the best solutions and exploration of the state space
- Difficulty: code the solutions so in the recombining process feasible solutions are obtained easily

Genetic Algorithm (GA) (ii)

1. Iteration i=0
2. Let be a **solution population** (**chromosomes**) for iteration i, $P(i)$
3. **Evaluate** the solution population $P(i)$
4. For a while, do the following steps:
   - Iteration i+1
   - Select the solution population of the iteration i+1 as the **best** of the previous iteration
   - **Recombine**, i.e., crossover and mute solutions randomly chosen to determine new solutions
     - **Crossover**: exchange between components (genes) of the solutions
     - **Mutation**: random modification of one component (gen)
   - Evaluate the solution population $P(i+1)$

Memetic Algorithm

- Add the cultural inheritance besides the physical inheritance to the genetic algorithm

Scatter Search (SS)

- Operates on a set of solutions, the reference set, by combining these solutions to create new ones
- **Steps**:
  - A **Diversification Generation Method** to generate a collection of diverse trial solutions, using an arbitrary trial solution (or seed solution) as an input.
  - An **Improvement Method** to transform a trial solution into one or more enhanced trial solutions.
  - A **Reference Set Update Method** to build and maintain a reference set consisting of the "best" solutions found. Solutions gain membership to the reference set according to their **quality** or their **diversity**.
  - A **Subset Generation Method** to operate on the reference set, to produce a subset of its solutions as a basis for creating **combined** solutions.
  - A **Solution Combination Method** to transform a given subset of solutions produced by the Subset Generation Method into one or more combined solution vectors.

Tabu Search (TS)

- **Idea**: try to extract information of what has happened (save the memory) and to act accordingly.
- **Short-term memory**:
  - Move iteratively from one solution to another in its reduced neighborhood.

    Reduced neighborhood = neighborhood – tabu solutions
  - Tabu solutions are those solutions just visited. Tabu solution **set is adjusted dynamically**. It is defined by **attributes** to avoid saving the solutions as them.
- **Long-term memory**:
  - Store frequencies and events of attributes in visited solutions to identify or differentiate regions. Two long-term strategies:
    - **Intensification**: search in explored regions
    - **Diversification**: visit non explored regions

Simulated Annealing (SA)

- Idea: based on local search where each improvement move is accepted and non improvement moves are allowed with a certain probability (which is a direct function of the system temperature)
- It begins with high temperature
- Important parameters:
  - High initial temperature: fraction of accepted moves T_0
  - Length: size of the reduced neighborhood
  - Exponential cooling sequence $$T_k = T_0 e^{-\alpha k}$$
  - Acceptance probability of a new solution

    Z^k and Z^{k+1} objective functions of one iteration and the next one

$$p = \min\left[1, e^{\frac{Z^k - Z^{k+1}}{T_k}}\right]$$

Ant Colony Optimization (ACO)

- Ants walk randomly to find food and once they find food return to the colony. Ants trace trails with pheromone.
- If other ants find this trail instead of walking randomly they follow this path and **reinforce** (positive feedback) it.
- Over time pheromone starts to **evaporate** (negative feedback) reducing its attractive. Shortest paths are more frequently used and pheromone remains.
- Pheromone evaporation avoids locally optimal solutions

Particle Swarm (PS)

- **Idea**: based on the concept of collective intelligence (*swarm intelligence*) with two mechanisms
  - Social influence
  - Social learning
- It needs additionally a communication structure (social network)
- Algorithm:
  - Each agent obtains randomly a set of initial candidate solutions
  - Each agent evaluates the candidate solutions and keep record of the best for him and where it has appeared
  - Each agent communicates to their neighbors and observe where they have been successful
  - All the agents move in the success direction

Variable Neighborhood Search (VNS)

- Facts:
  1. A local minimum with respect to one neighborhood structure is not necessary so for another;
  2. A global minimum is a local minimum with respect to all possible neighborhood structures.
  3. For many problems local minima with respect to one or several neighborhoods are relatively close to each other.
- Idea: escape from local optima trap by changing the neighborhood structure