Equilibrium Modeling

Optimization Techniques

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Special thanks to: Sonja Wogrin
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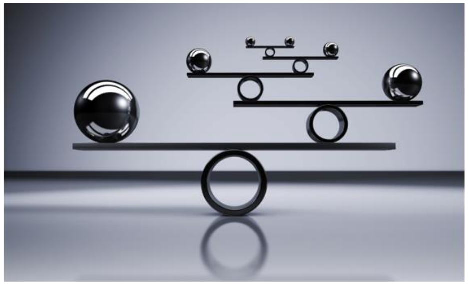
Outline

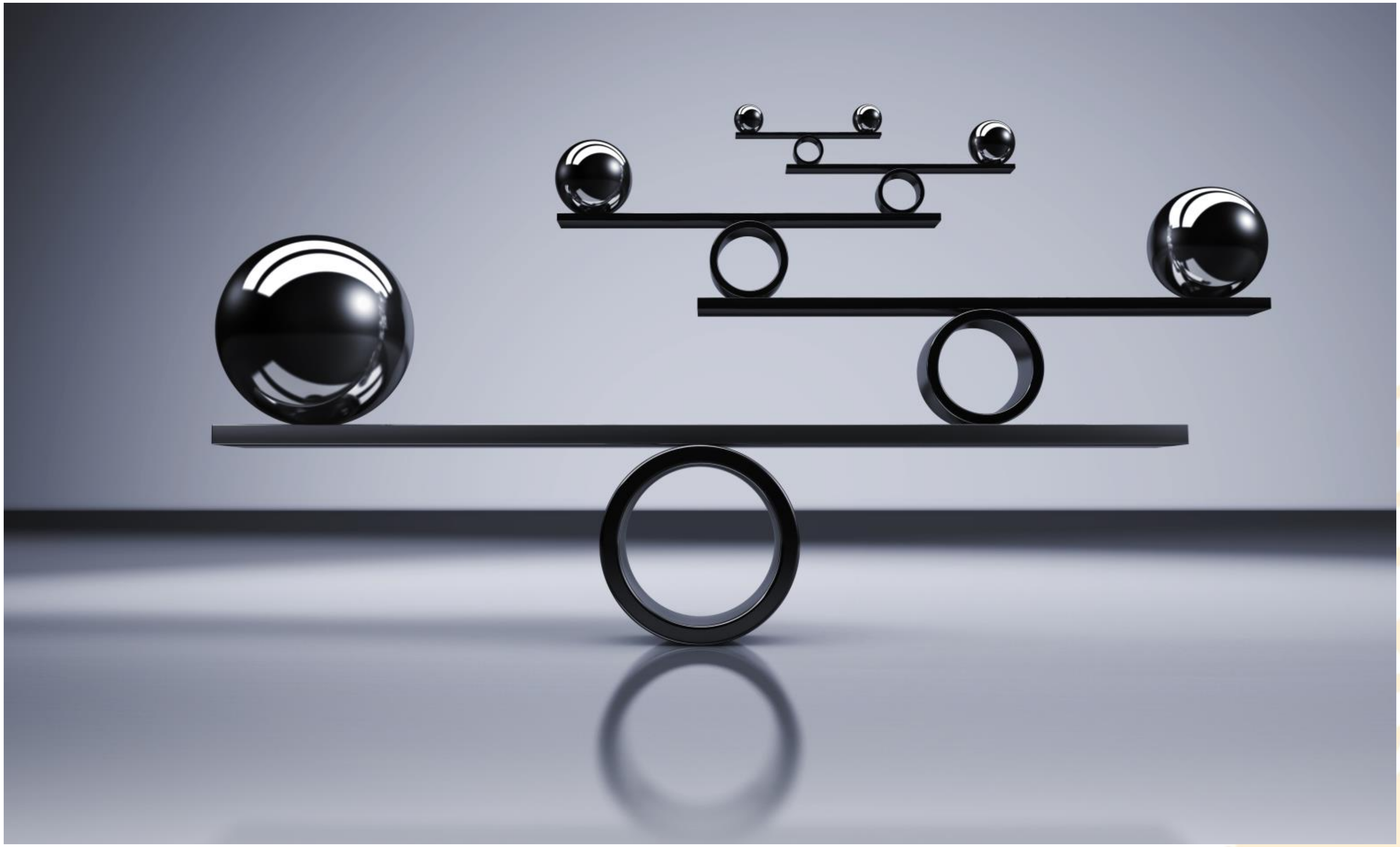
Introduction to Equilibrium Problems



Strategic Investment in Generation Expansion Planning

Annex

Introduction to Equilibrium Problems



So far, in the OT course…

In classical optimisation, we have one objective function subject to constraints, which could be interpreted as one player taking optimal decisions considering technical limitations.

Minimizing OF (=f(x1)) taking optimal decisions x1

s.t. Constraints (g(x1)<=0, h(x1)=0) involving x1 are

Equilibrium Problem

An equilibrium problem can be viewed as a situation where several players are considering an optimization problem at the same time, while the variables of other players can influence one outcome:

Player 1		**Player *i***		**Player I**
Min $f_1(x_1,x_{-1})$	…	Min $f_i(x_i,x_{-i})$	…	Min $f_I(x_1,x_{-I})$
s.t.		s.t.		s.t.
$g_1 (x_1,x_{-i})<=0$		$g_i (x_i,x_{-i})<=0$		$g_I (x_I,x_{-I})<=0$
$h_1 (x_1,x_{-i})=0$		$h_i (x_i,x_{-i})=0$		$h_I (x_I,x_{-I})=0$

How to define an equilibrium?

Game Theory – Basic Definitions

- When decisions of firms (or players) affect each others' outcome (e.g., profits) significantly, they are in a situation of **interdependence**.
- The study of behavior in (non cooperative) situations of interdependence is known as **game theory**.
- The reward received by a player in a game—such as the profit earned by an oligopolist—is that player's **payoff**.
- Economists use *game theory* to study firms' behavior when there is *interdependence* between their *payoffs*.
- A **payoff matrix** shows how the payoff to each of the participants in a two-player game depends on the actions of both. Such a matrix helps us analyze interdependence.

Game Theory – Basic Definitions

- A player's **strategy** is any of the options he/she can choose in a setting where the outcome depends *not only* on his/her own actions *but* on the action of others.
- The **strategy set** of a firm (or player) can be:
  - **Finite**: discrete number of different options. For example: produce either 30 or 40 million pounds.
  - **Infinite**: infinite number of options. For example, we can produce any amount between 30 and 40 million pounds.
- Given strategies x and y for one player, then x **dominates** y when x is better (in terms of payoff) than y, independent of the strategies of the opponents.
- Dominated strategies can be **eliminated** from payoff matrix.

Game Theory – Basic Definitions

- A **Nash equilibrium**, also known as a **noncooperative equilibrium**, is the result when each player in a game chooses the action that maximizes his or her payoff given the actions of other players, ignoring the effects of his or her action on the payoffs received by those other players.
- <u>Formal definition of Nash Equilibrium</u>:
  Let there be i=1,…,n players, and let x_i be a strategy of player i. Let $f_i(x)$ be the payoff function of player i. Then $x^*=(x_1^*,\dots, x_n^*)$ is a Nash equilibrium if:
  $$f_i(x_i^*, x_{-i}^*) \geq f_i(x_i, x_{-i}^*) \text{ for all } x_i$$

- In other words, a Nash equilibrium is a point where no player wants to move away from unilaterally.

Dominant Strategy Equilibrium

A dominant strategy is when one player strategy is always better off no matter what other players do. If all players have a strictly dominant strategy, we have a dominant strategy equilibrium.

		Prisoner 2	
		Testify	**Keep quiet**
Prisoner 1	**Testify**	(0,0)	(-1,3)
	Keep quiet	(3,-1)	(2,2)

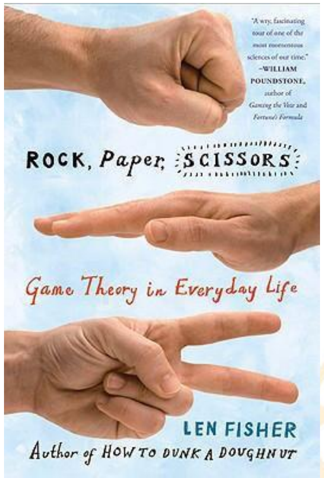
Dominant strategy equilibrium

Instead of Prisoners, think of two electricity companies bidding either high or low prices (strategies) on the market….

Multiple vs NO Equilibria

There exist games where there can exist multiple equilibria, or none at all (at least not in pure strategies).

		Player 2		
Player 1		**Paper**	**Rock**	**Scissors**
	Paper	0	1	-1
	Rock	-1	0	1
	Scissors	1	-1	0



		Woman	
Man		**Soccer**	**TV-Show**
	Soccer	(2,1)	(0,0)
	TV-Show	(-1,-1)	(1,2)

Connection of EPs and classical optimization

- Let us write the equilibrium problem:

Player 1	...	Player *i*	...	Player I
Min $f_1(x_1,x_{-1})$ s.t. $g_1(x_1,x_{-i})<=0$ $h_1(x_1,x_{-i})=0$	...	Min $f_i(x_i,x_{-i})$ s.t. $g_i(x_i,x_{-i})<=0$ $h_i(x_i,x_{-i})=0$	...	Min $f_I(x_1,x_{-I})$ s.t. $g_I(x_I,x_{-I})<=0$ $h_I(x_I,x_{-I})=0$

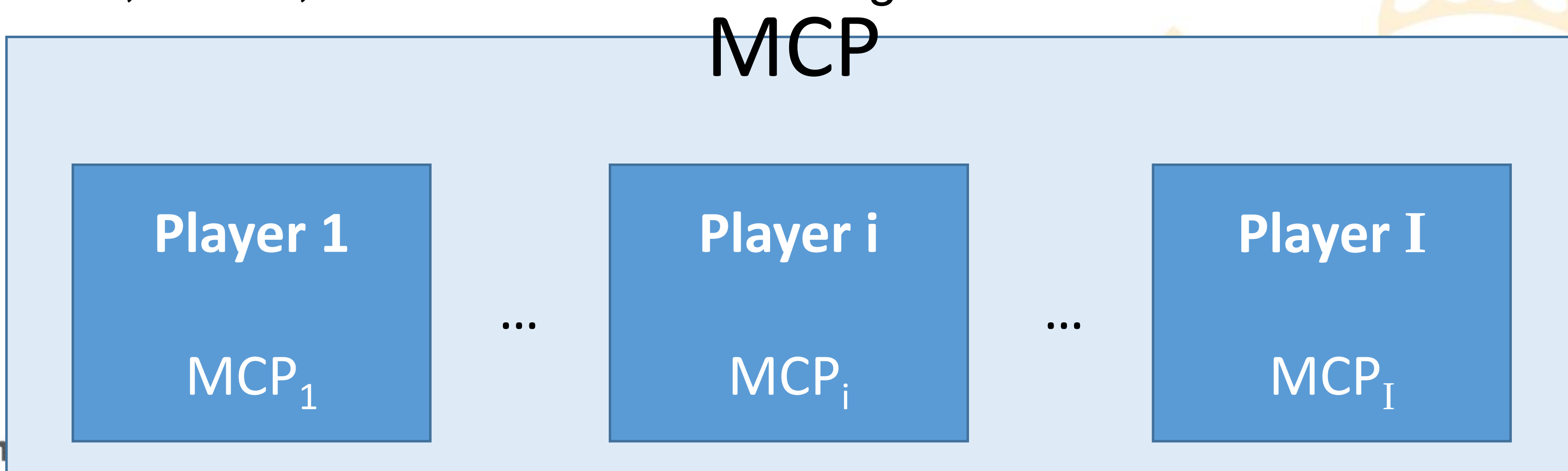
- As an NLP by using KKT conditions:

Player 1	...	Player i	...	Player I
KKT_1	...	KKT_i	...	KKT_I

Connection of Equilibrium Problems (EP) and Mixed Complementarity Problems (MCP)

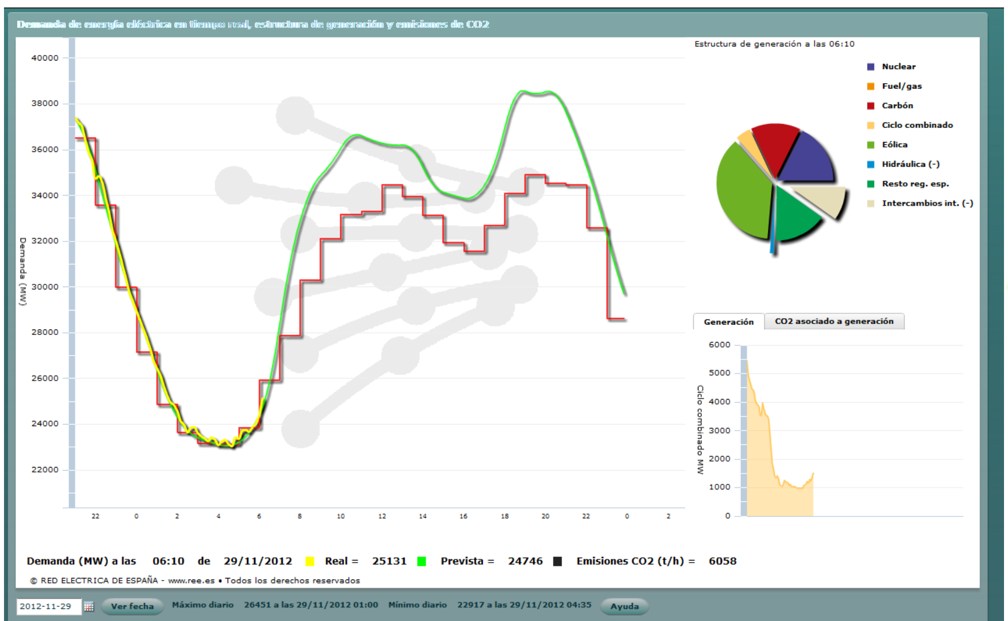
- KKT conditions can be written as MCPs:

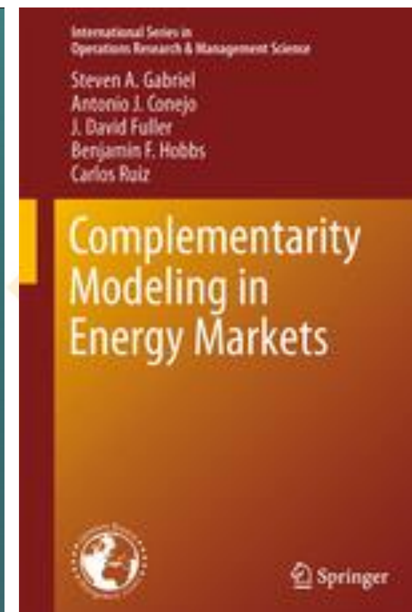
- Which, in turn, can be seen as one “large” MCP:



Energy markets and CPs

Due to the liberalization of electricity markets, many problems that arise are equilibrium problems and can be formulated as Complementarity problems.





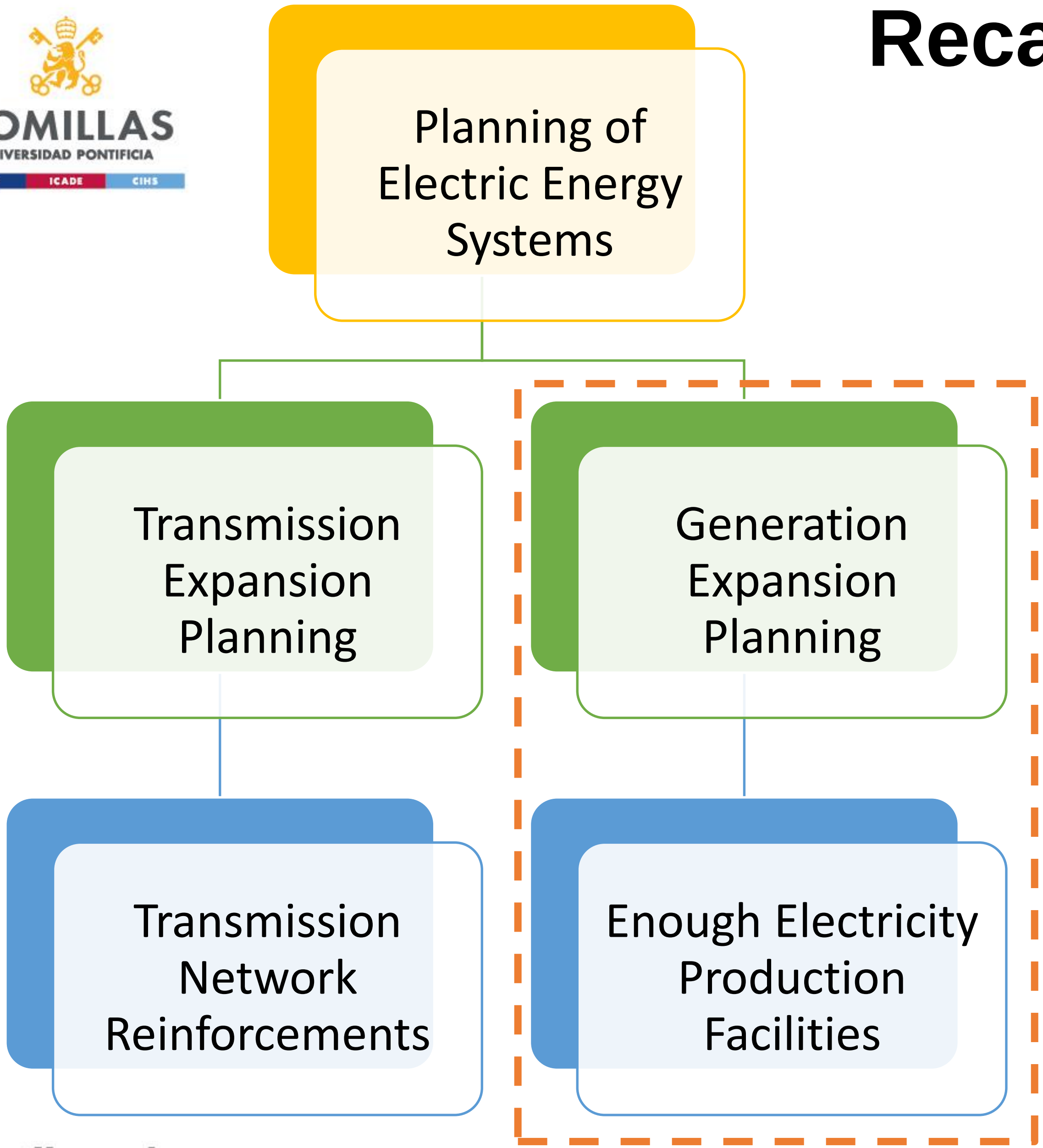
Perfectly Competitive Markets

<u>OLIGOPOLY:</u>

- COURNOT
  - ✓Each company decides their quantity at the same time.
- BERTRAND
  - ✓Each company decides their price at the same time.
- STACKELBERG (MPEC – see future classes)
  - ✓When taking production decisions, one of the companies acts as the leader and the other one as a follower who observes what the first company has done.
- CARTEL
  - ✓Both companies collude or come to an agreement in order to "divvy" up the market.

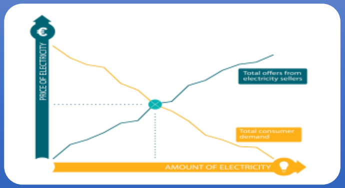
Strategic Investment in Generation Expansion Planning

Recap!



Generation expansion planning (a.k.a. Capacity expansion planning) involves the decision of what generation assets to build (or to close, or to acquire, or to sell) and when to do it.

Strategic Investment in GEP Motivation



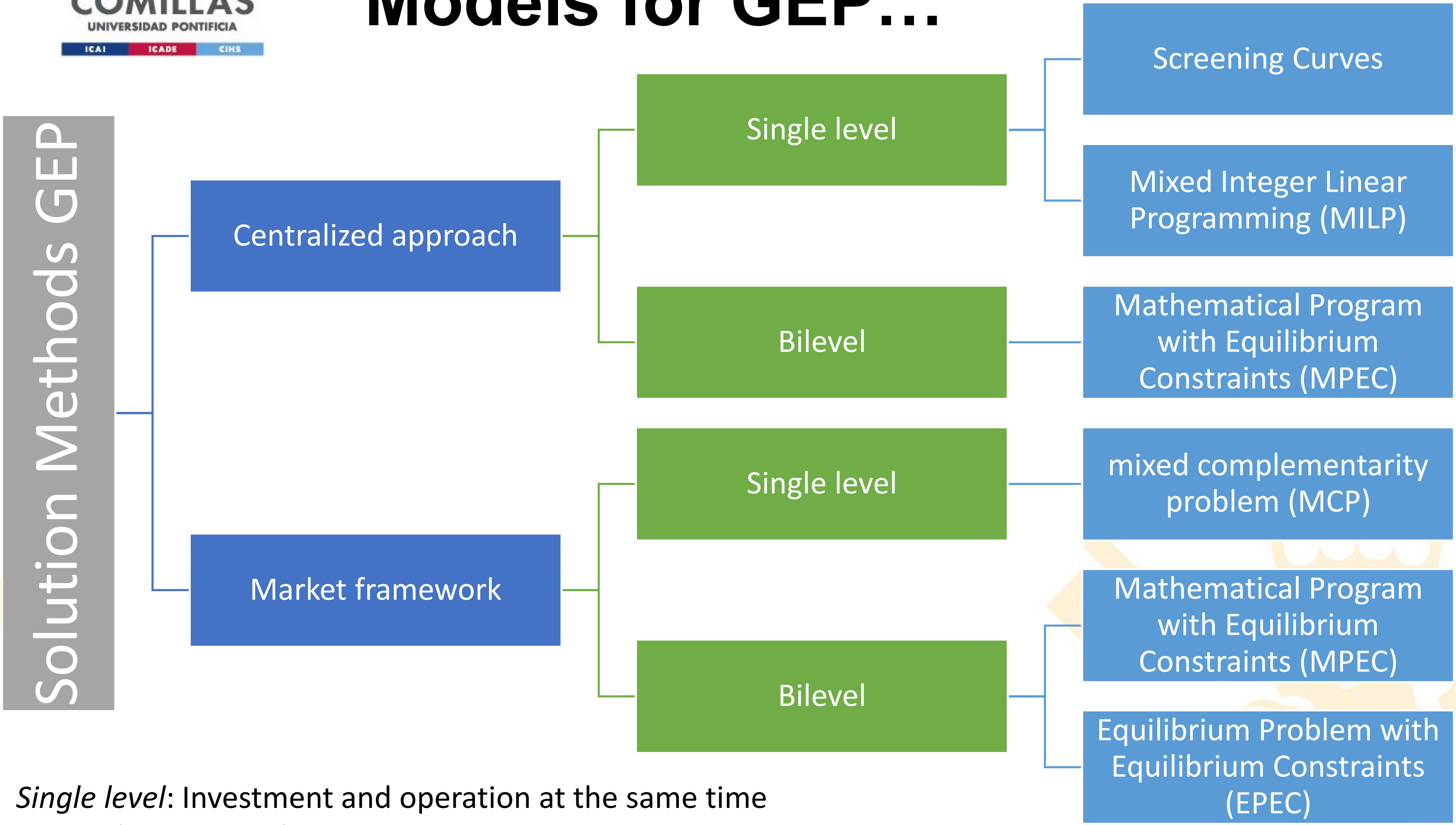
The liberalization of the electricity sector and the introduction of electricity markets have greatly complicated the organization of the electricity sector, especially for generation companies.

Under a centralized framework a central planner took decisions maximizing social welfare, whereas in electricity markets the responsibility of taking many decisions lies with public and private entities that interact.

From a game-theoretic point of view many decision-making problems in a liberalized power sector can be regarded and analyzed as sequential Stackelberg-type games among different players.

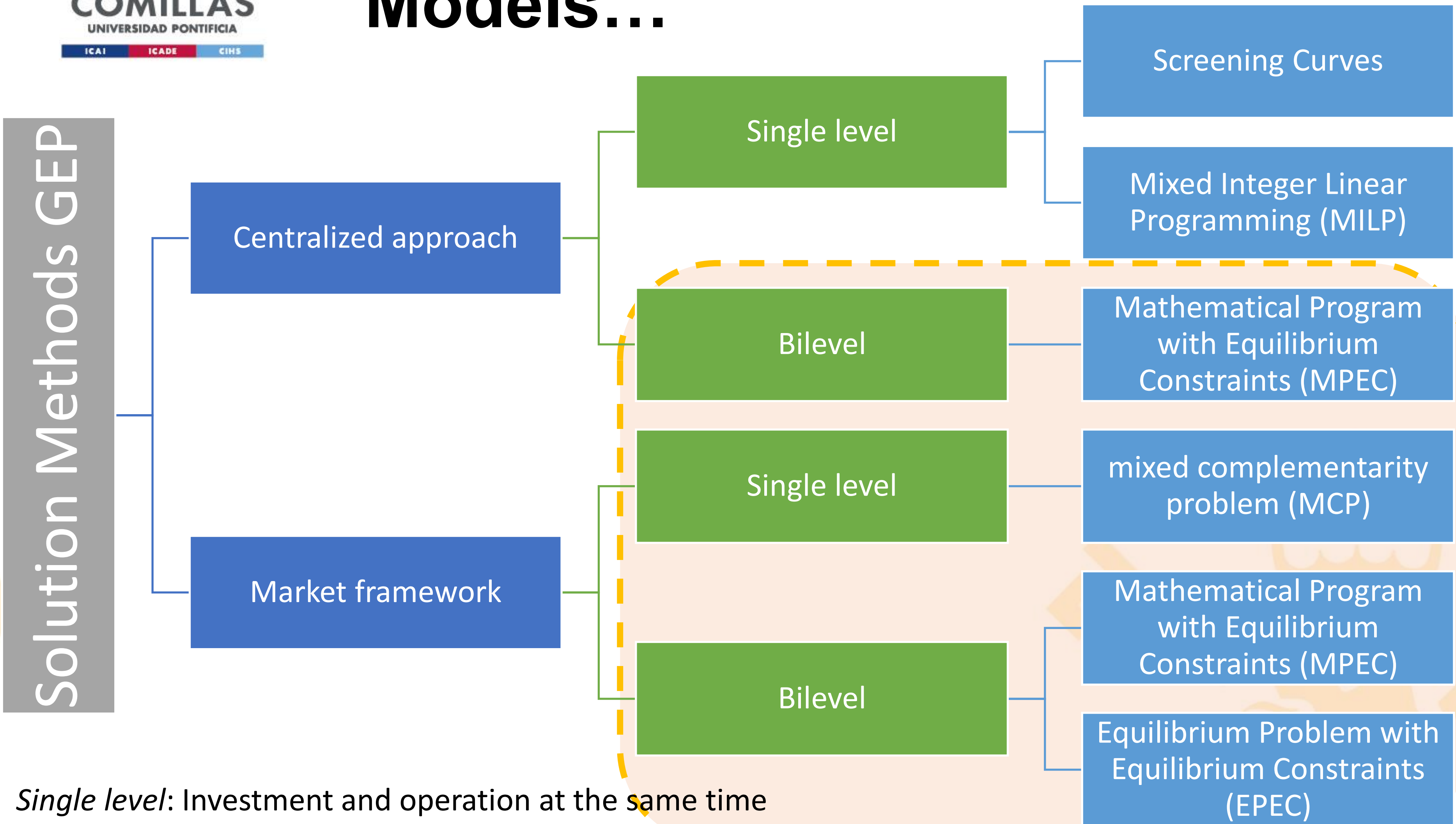
The sequence in which decisions are taken, can convert simple equilibrium games into complicated hierarchical/bilevel optimization or equilibrium problems whose outcomes can diverge significantly depending on the type of game.

Multiple Approaches and Models for GEP…



Single level: Investment and operation at the same time
Bilevel (hierarchical): Investment first then operation

The ones using Equilibrium Models…



Single level: Investment and operation at the same time
Bilevel (hierarchical): Investment first then operation

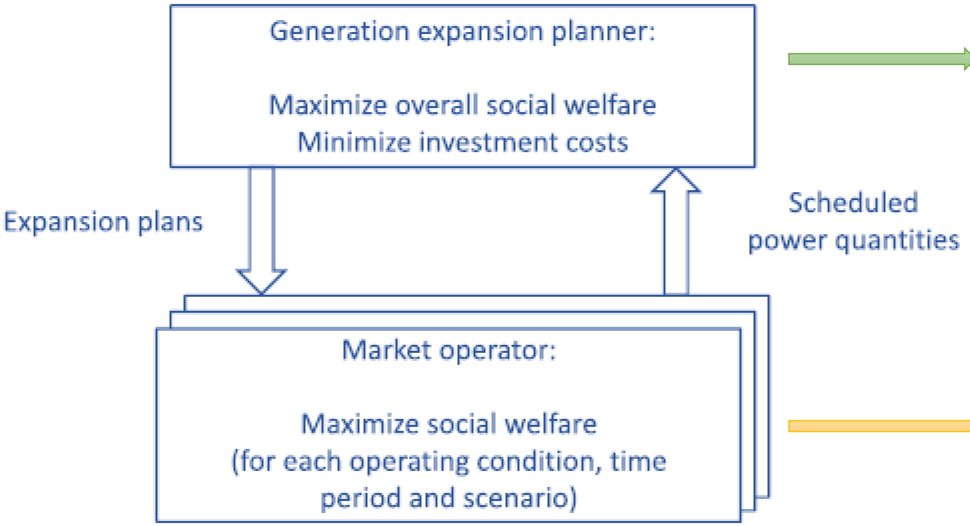
Solution Methods GEP

- Centralized approach
  - Single level
    - Screening Curves
    - Mixed Integer Linear Programming (MILP)
  - Bilevel
    - Mathematical Program with Equilibrium Constraints (MPEC)
- Market framework
  - Single level
    - mixed complementarity problem (MCP)
  - Bilevel
    - Mathematical Program with Equilibrium Constraints (MPEC)
    - Equilibrium Problem with Equilibrium Constraints (EPEC)

Single level: Investment and operation at the same time

Bilevel (hierarchical): Investment first then operation

Bilevel Centralized Approach
Hierarchical Approach



Production cost | Investment cost

$$\min_{p_c^{\mathrm{C}\max},\,\Delta_o^{\mathrm{DISM}}} \sum_o \rho_o \left[\sum_g C_g^{\mathrm{E}} p_{go}^{\mathrm{E}} + \sum_c C_c^{\mathrm{C}} p_{co}^{\mathrm{C}} \right] + \sum_c \tilde{I}_c^{\mathrm{C}} p_c^{\mathrm{C}\max}$$

subject to

$$0 \le p_c^{\mathrm{C}\max} \le \overline{P}_c^{\mathrm{C}\max} \quad \forall c$$

$$p_{go}^{\mathrm{E}}, p_{co}^{\mathrm{C}} \in \Omega_o\left(P_{do}^{\mathrm{D}}, P_g^{\mathrm{E}\max}, p_c^{\mathrm{C}\max}\right) \quad \forall g, c, o,$$

Production cost

$$\Omega_o\left(P_{do}^{\mathrm{D}}, P_g^{\mathrm{E}\max}, p_c^{\mathrm{C}\max}\right) = \Big\{ \min_{\Delta_o^{\mathrm{DISM}}} \sum_g C_g^{\mathrm{E}} p_{go}^{\mathrm{E}} + \sum_c C_c^{\mathrm{C}} p_{co}^{\mathrm{C}}$$

subject to

$$\sum_g p_{go}^{\mathrm{E}} + \sum_c p_{co}^{\mathrm{C}} = \sum_d P_{do}^{\mathrm{D}} \;:\; \lambda_o$$

$$0 \le p_{go}^{\mathrm{E}} \le P_g^{\mathrm{E}\max} \;:\; \mu_{go}^{\mathrm{E}} \quad \forall g$$

$$0 \le p_{co}^{\mathrm{C}} \le p_c^{\mathrm{C}\max} \;:\; \mu_{co}^{\mathrm{C}} \quad \forall c$$

$$\Big\}, \forall o,$$

The GEP problem becomes an optimization problem subject to other optimization problems. This is also known as Mathematical Program with Equilibrium Constraints (**MPEC**)

Conejo, A. J., Morales, L. B., Kazempour, S. J., & Siddiqui, A. S. (2016). Investment in Electricity Generation and Transmission: Decision Making under Uncertainty.

Bilevel Centralized Approach Example

Deterministic single-node static GEP problem:

1. There is one generating unit $\tilde{g}$ with capacity of 400 MW and production cost equal to \$35/MWh.
2. It is possible to build a new generating unit $\tilde{c}$ with capacity up to 500 MW and production cost equal to \$25/MWh. The annualized investment cost is \$70,000 per MW.
3. Demand conditions in the system are represented through two operating conditions. The first one, o_1, is defined by a demand of 290 MW and a weight of 6000 h, while the second one, o_2, is defined by a demand of 550 MW and a weight of 2760 h.

Conejo, A. J., Morales, L. B., Kazempour, S. J., & Siddiqui, A. S. (2016). Investment in Electricity Generation and Transmission: Decision Making under Uncertainty.

Bilevel Centralized Approach Example

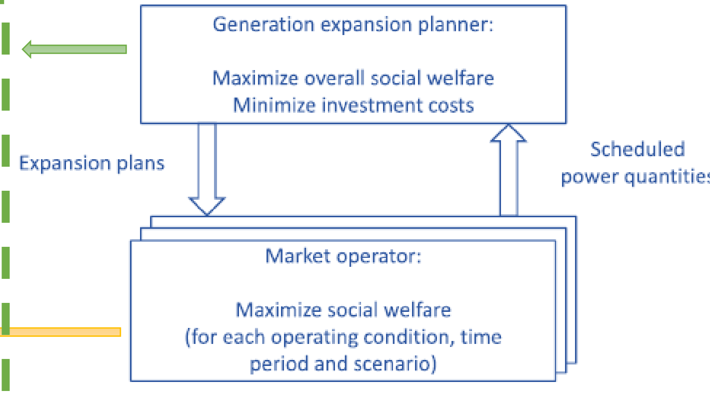
$$\min_{p^{E}_{\tilde{g}o_1},\, p^{C}_{\tilde{c}o_1},\, p^{E}_{\tilde{g}o_2},\, p^{C}_{\tilde{c}o_2},\, p^{C\max}_{\tilde{c}}} \quad 6000\left[35p^{E}_{\tilde{g}o_1} + 25p^{C}_{\tilde{c}o_1}\right] + 2760\left[35p^{E}_{\tilde{g}o_2} + 25p^{C}_{\tilde{c}o_2}\right] + 70000p^{C\max}_{\tilde{c}}$$

subject to

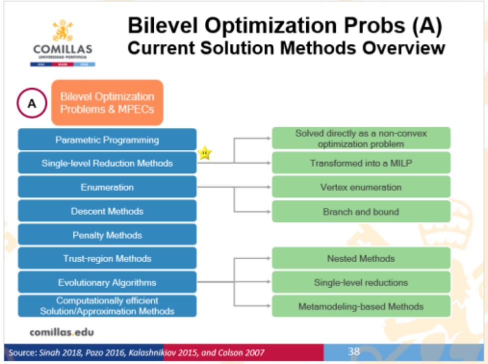
$$0 \le p^{C\max}_{\tilde{c}} \le 500$$

$$\begin{cases} \min_{p^{E}_{\tilde{g}o_1},\, p^{C}_{\tilde{c}o_1}} \quad 35p^{E}_{\tilde{g}o_1} + 25p^{C}_{\tilde{c}o_1} \\ \text{s.t.} \\ \quad p^{E}_{\tilde{g}o_1} + p^{C}_{\tilde{c}o_1} = 290 \\ \quad 0 \le p^{E}_{\tilde{g}o_1} \le 400 \\ \quad 0 \le p^{C}_{\tilde{c}o_1} \le p^{C\max}_{\tilde{c}} \end{cases}$$

$$\begin{cases} \min_{p^{E}_{\tilde{g}o_2},\, p^{C}_{\tilde{c}o_2}} \quad 35p^{E}_{\tilde{g}o_2} + 25p^{C}_{\tilde{c}o_2} \\ \text{s.t.} \\ \quad p^{E}_{\tilde{g}o_2} + p^{C}_{\tilde{c}o_2} = 550 \\ \quad 0 \le p^{E}_{\tilde{g}o_2} \le 400 \\ \quad 0 \le p^{C}_{\tilde{c}o_2} \le p^{C\max}_{\tilde{c}}. \end{cases}$$



How do we solve this type of problem?



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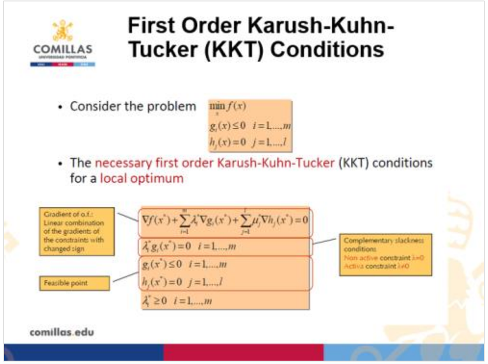
Bilevel Centralized Approach Single Level Reduction

Each market-clearing problem (one for each operating condition) is a linear programming (LP) problem. Thus, it is possible to replace each of these problems by its first-order optimality conditions. The first-order optimality conditions can be formulated using one of the two approaches below:

- **Primal–dual formulation**: In this case, each market-clearing problem is replaced by its primal constraints, its dual constraints, and its strong duality equality.
- **Karush–Kuhn–Tucker (KKT) formulation**: In this case, each market-clearing problem is replaced by its KKT conditions.



Bilevel Centralized Approach Example as NLP Formulation

MPEC formulation

$$\min_{p^{\mathrm{E}}_{\tilde{g}o_1}, p^{\mathrm{C}}_{\tilde{c}o_1}, p^{\mathrm{E}}_{\tilde{g}o_2}, p^{\mathrm{C}}_{\tilde{c}o_2}, p^{\mathrm{C^{max}}}_{\tilde{c}}} \quad 6000\left[35p^{\mathrm{E}}_{\tilde{g}o_1} + 25p^{\mathrm{C}}_{\tilde{c}o_1}\right] + 2760\left[35p^{\mathrm{E}}_{\tilde{g}o_2} + 25p^{\mathrm{C}}_{\tilde{c}o_2}\right] + 70000p^{\mathrm{C^{max}}}_{\tilde{c}}$$

subject to

$$0 \le p^{\mathrm{C^{max}}}_{\tilde{c}} \le 500$$

$$\left\{\begin{array}{l} \min_{p^{\mathrm{E}}_{\tilde{g}o_1}, p^{\mathrm{C}}_{\tilde{c}o_1}} \quad 35p^{\mathrm{E}}_{\tilde{g}o_1} + 25p^{\mathrm{C}}_{\tilde{c}o_1} \\ \text{s.t.} \\ \quad p^{\mathrm{E}}_{\tilde{g}o_1} + p^{\mathrm{C}}_{\tilde{c}o_1} = 290 \\ \quad 0 \le p^{\mathrm{E}}_{\tilde{g}o_1} \le 400 \\ \quad 0 \le p^{\mathrm{C}}_{\tilde{c}o_1} \le p^{\mathrm{C^{max}}}_{\tilde{c}} \end{array}\right.$$

$$\left\{\begin{array}{l} \min_{p^{\mathrm{E}}_{\tilde{g}o_2}, p^{\mathrm{C}}_{\tilde{c}o_2}} \quad 35p^{\mathrm{E}}_{\tilde{g}o_2} + 25p^{\mathrm{C}}_{\tilde{c}o_2} \\ \text{s.t.} \\ \quad p^{\mathrm{E}}_{\tilde{g}o_2} + p^{\mathrm{C}}_{\tilde{c}o_2} = 550 \\ \quad 0 \le p^{\mathrm{E}}_{\tilde{g}o_2} \le 400 \\ \quad 0 \le p^{\mathrm{C}}_{\tilde{c}o_2} \le p^{\mathrm{C^{max}}}_{\tilde{c}}. \end{array}\right.$$

Primal-dual Formulation

Equivalent NLP formulation

$$\min_{\Delta} \quad 6000\left[35p^{\mathrm{E}}_{\tilde{g}o_1} + 25p^{\mathrm{C}}_{\tilde{c}o_1}\right] + 2760\left[35p^{\mathrm{E}}_{\tilde{g}o_2} + 25p^{\mathrm{C}}_{\tilde{c}o_2}\right] + 70000p^{\mathrm{C^{max}}}_{\tilde{c}}$$

subject to

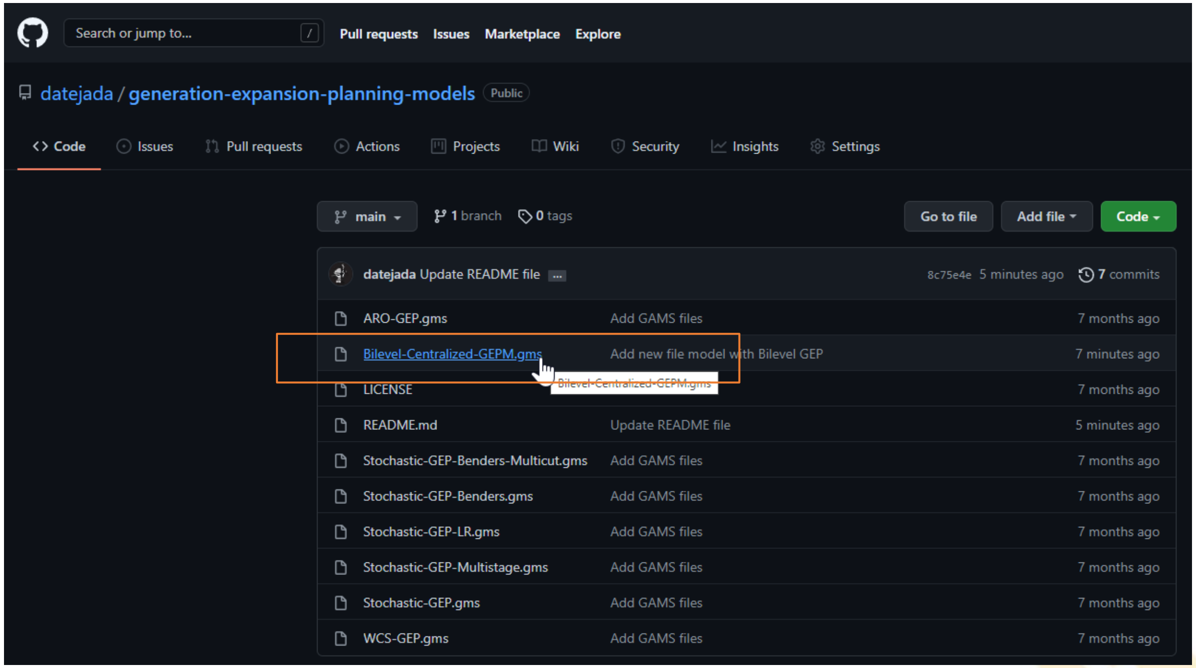
$$0 \le p^{\mathrm{C^{max}}}_{\tilde{c}} \le 500$$

$$\left\{\begin{array}{l} p^{\mathrm{E}}_{\tilde{g}o_1} + p^{\mathrm{C}}_{\tilde{c}o_1} = 290 \\ 0 \le p^{\mathrm{E}}_{\tilde{g}o_1} \le 400 \\ 0 \le p^{\mathrm{C}}_{\tilde{c}o_1} \le p^{\mathrm{C^{max}}}_{\tilde{c}} \\ 35 - \lambda_{o_1} + \mu^{\mathrm{E^{max}}}_{\tilde{g}o_1} \ge 0 \\ 25 - \lambda_{o_1} + \mu^{\mathrm{C^{max}}}_{\tilde{c}o_1} \ge 0 \\ \mu^{\mathrm{E^{max}}}_{\tilde{g}o_1} \ge 0 \\ \mu^{\mathrm{C^{max}}}_{\tilde{c}o_1} \ge 0 \\ 35p^{\mathrm{E}}_{\tilde{g}o_1} + 25p^{\mathrm{C}}_{\tilde{c}o_1} = 290\lambda_{o_1} - 400\mu^{\mathrm{E^{max}}}_{\tilde{g}o_1} - p^{\mathrm{C^{max}}}_{\tilde{c}}\mu^{\mathrm{C^{max}}}_{\tilde{c}o_1} \end{array}\right.$$

$$\left\{\begin{array}{l} p^{\mathrm{E}}_{\tilde{g}o_2} + p^{\mathrm{C}}_{\tilde{c}o_2} = 550 \\ 0 \le p^{\mathrm{E}}_{\tilde{g}o_2} \le 400 \\ 0 \le p^{\mathrm{C}}_{\tilde{c}o_2} \le p^{\mathrm{C^{max}}}_{\tilde{c}} \\ 35 - \lambda_{o_2} + \mu^{\mathrm{E^{max}}}_{\tilde{g}o_2} \ge 0 \\ 25 - \lambda_{o_2} + \mu^{\mathrm{C^{max}}}_{\tilde{c}o_2} \ge 0 \\ \mu^{\mathrm{E^{max}}}_{\tilde{g}o_2} \ge 0 \\ \mu^{\mathrm{C^{max}}}_{\tilde{c}o_2} \ge 0 \\ 35p^{\mathrm{E}}_{\tilde{g}o_2} + 25p^{\mathrm{C}}_{\tilde{c}o_2} = 550\lambda_{o_2} - 400\mu^{\mathrm{E^{max}}}_{\tilde{g}o_2} - p^{\mathrm{C^{max}}}_{\tilde{c}}\mu^{\mathrm{C^{max}}}_{\tilde{c}o_2}, \end{array}\right.$$

Conejo, A. J., Morales, L. B., Kazempour, S. J., & Siddiqui, A. S. (2016). Investn
Electricity Generation and Transmission: Decision Making under Uncertainty.

Hands on! GAMS



https://github.com/datejada/generation-expansion-planning-models

1) Run the bilevel centralized approach
2) Run the centralized approach
3) Compare the results

Bilevel Centralized Approach Example Results

Bilevel centralized approach: of = 109.35 M$

Result	O1	O2
Existing generating unit production [MW]	0	250
Candidate generating unit production [MW]	290	300
Investment capacity [MW]	300	
Prices [$/MWh]	25	35

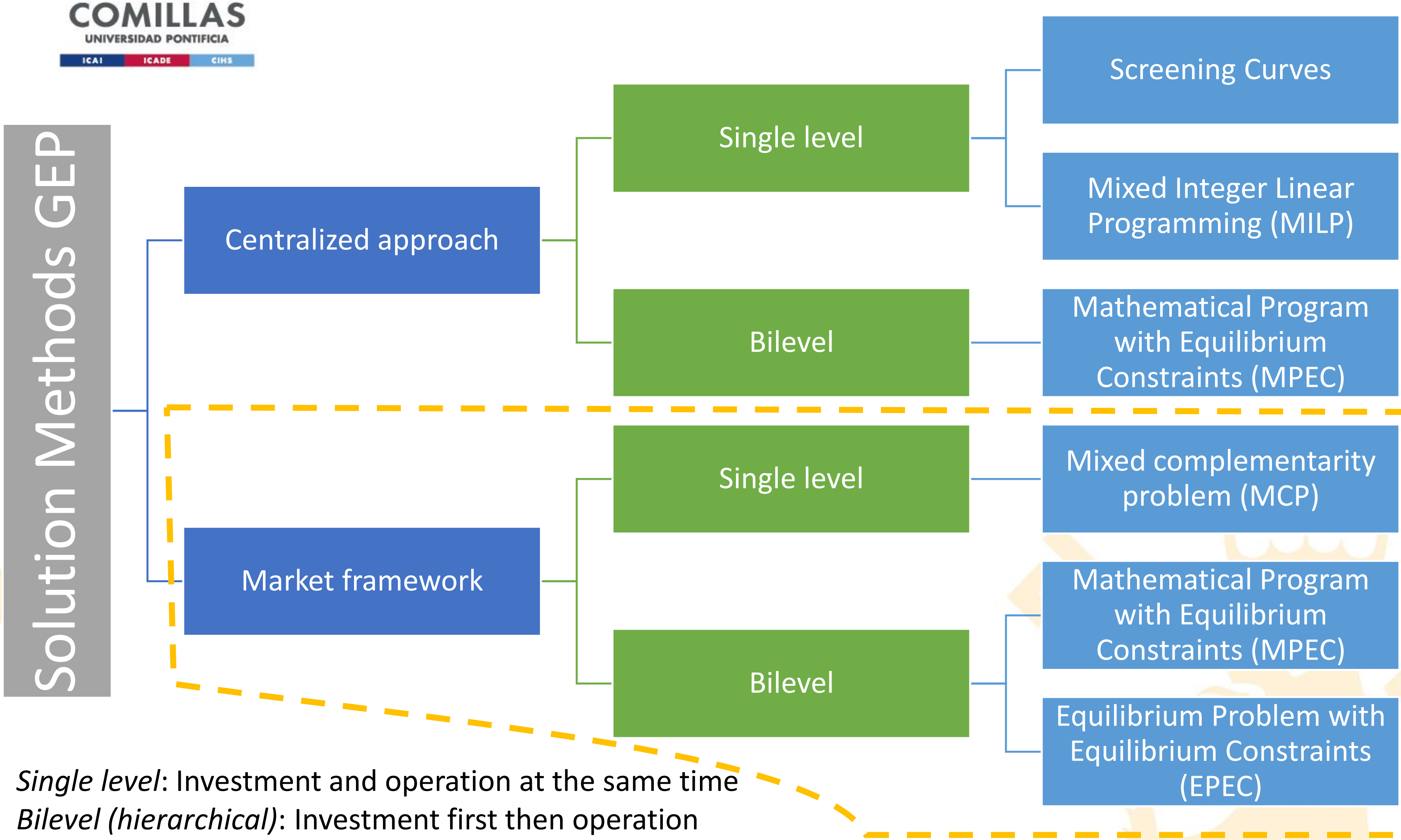
Single level centralized approach: of =108.93 M$

Result	O1	O2
Existing generating unit production [MW]	0	260
Candidate generating unit production [MW]	290	290
Investment capacity [MW]	290	
Prices [$/MWh]	35	35

Bilevel Centralized Approach
Hierarchical Approach

- Investment is defined first, and then the operational decisions (representing the market-clearing) are taken.
- Integer nature of investment decisions could be considered in the central planner problem.

- It is a non-linear programming (NLP) problem. Therefore, it is hard to solve for large-scale systems.
- The market-clearing problem is commonly replaced by its first-order optimality conditions (e.g., KKTs), but it means that it must be a linear problem. Therefore, unit commitment constraints are hard to represent.



Single level: Investment and operation at the same time
Bilevel (hierarchical): Investment first then operation

Strategic Investment in GEP Considerations

We have n-identical firms with perfectly substitutable products, facing either a one-stage or a two-stage competitive situation.

One-stage situation (open loop/ single-level model)

Two-stage situation (closed loop/ bilevel model)

Investment and operation decisions are made simultaneously.

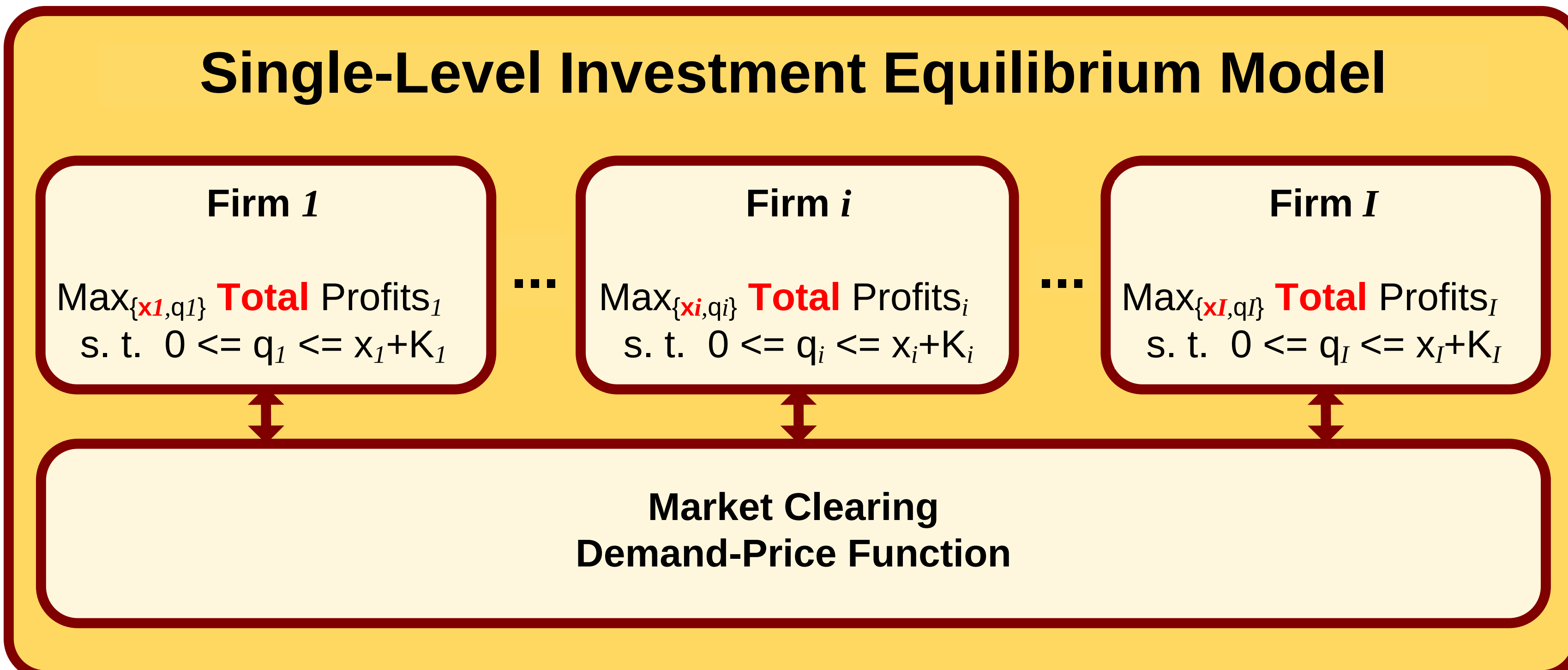
First, firms choose capacities that maximize their profit anticipating the second stage, where…

…quantities and prices are determined by a conjectured price response market equilibrium.

One-stage situation or Single-Level GEP Investment Equilibrium

All Generation Companies (**GENCOs**) simultaneously maximize their total profits (market revenues minus investment costs minus production costs) subject to lower and upper bounds on production and a demand balance.

This equilibrium problem can be formulated as a mixed complementarity problem (**MCP**)



Two-stage situation or Bilevel Problem - Basic Concepts

Bilevel Problem

- A *bilevel programming* problem is a hierarchical optimization problem which is constrained by another optimization problem.

MPEC

- Mathematical Program with Equilibrium Constraints (**MPEC**) – this is a bilevel optimization problem

EPEC

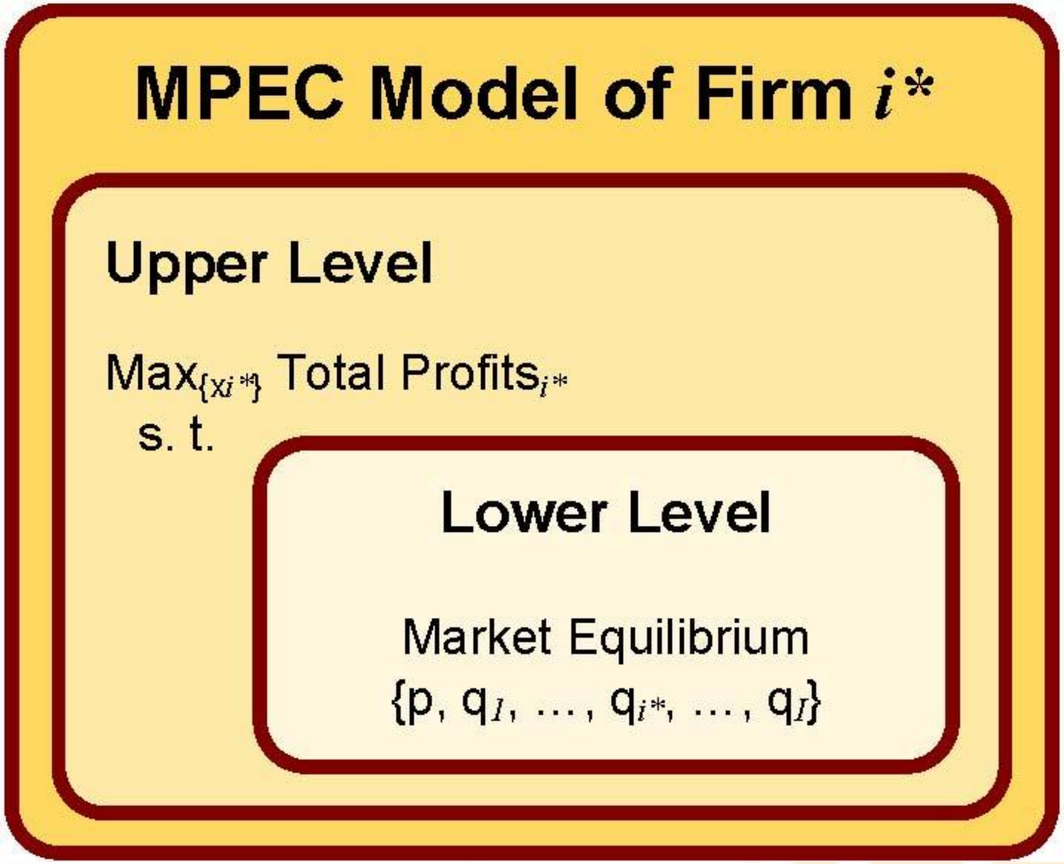
- Equilibrium Problem with Equilibrium Constraints (**EPEC**) – this is a bilevel equilibrium problem

GEP Investment Equilibrium MPEC - Bilevel Investment Optimization

This model assists one **GENCO** in taking capacity decisions while considering the competitors’ investments as fixed.

In the **upper level** investment decisions of firm $i*$ are taken.

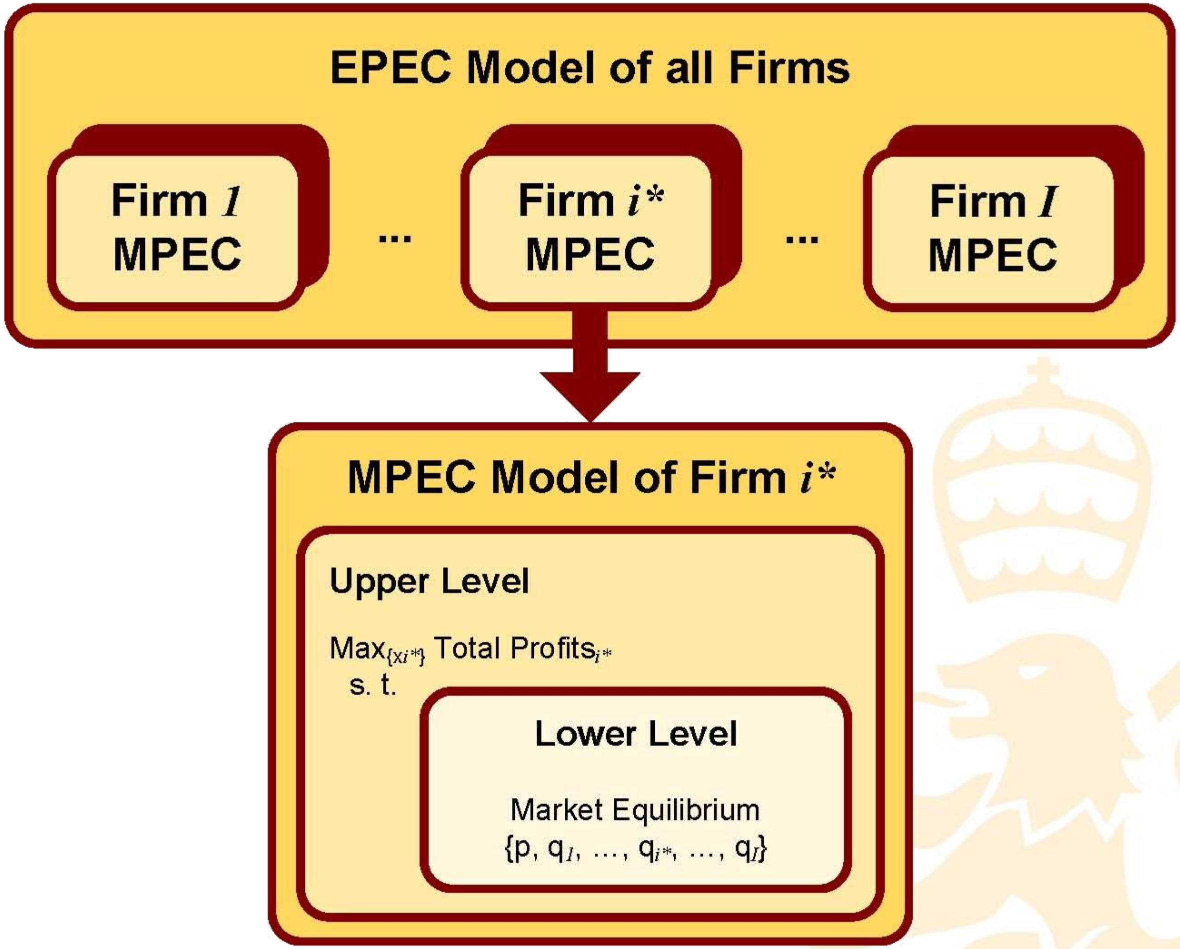
The **lower level** corresponds to the definition of **market equilibrium**.



GEP Investment Equilibrium EPEC - Bilevel Investment Equilibrium

This model assists ALL **GENCOs** in taking capacity decisions.

This problem is an **EPEC**: all GENCOs simultaneously face an **MPEC**.



Advantages/Disadvantages of Models

MODELS	Advantages	Disadvantages
Equilibrium 1 level (MCP)	Easy to solve.	Simplified representation (investment and production decisions taken at same time)
Model 2 levels (MPEC)	Good representation of investing agent.	Decides investments of one agent while competition is fixed.
Stochastic Model 2 levels (stochastic MPEC)	Stochastic model (risk evaluations). Evaluate various scenarios (decision analysis). Better representation of investing agent.	More difficult to solve (depends on the number of scenarios).
Equilibrium 2 levels (EPEC)	Equilibrium also in capacities (not only in productions).	Very complicated problem – hard to solve.

Overview

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Solution Methods

A. Bilevel Optimization Problems & MPECs

- Parametric Programming
- Single-level Reduction Methods
- Enumeration
- Descent Methods
- Penalty Methods
- Trust-region Methods
- Evolutionary Algorithms
- Computationally efficient Solution/Approximation Methods

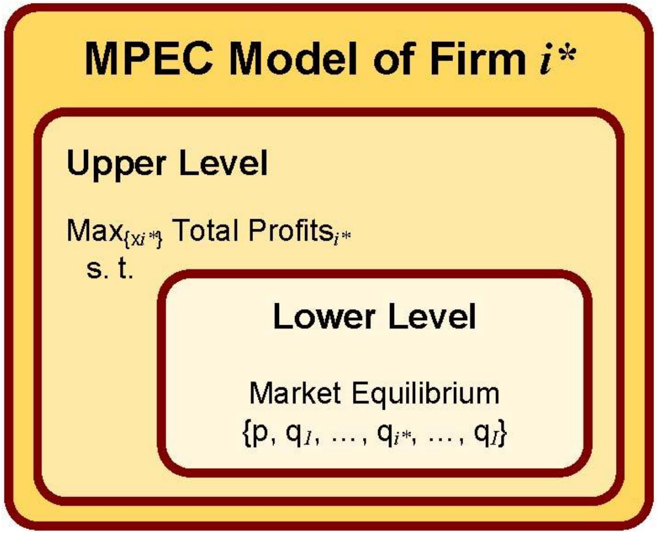
B. Bilevel Equilibrium Problems & EPECs

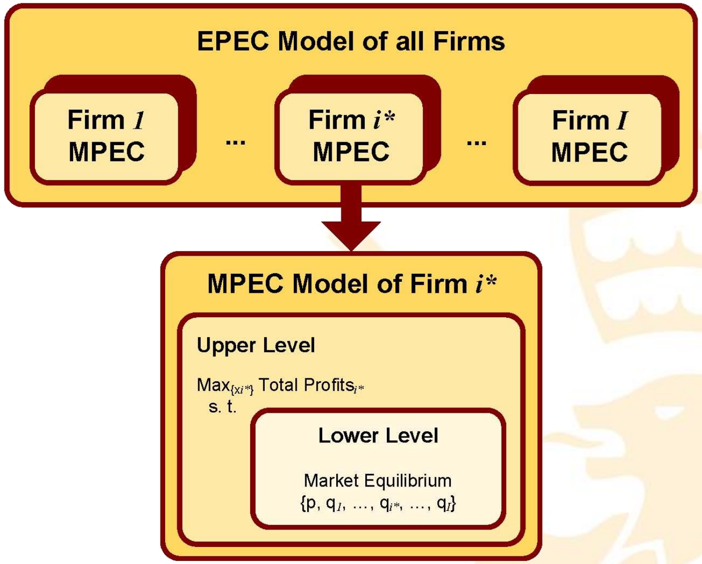
- Single-level reduction of Bilevel Equilibrium
- Diagonalization
- Computationally efficient Solution/Approximation Methods

Classification of Solution Methods

A Bilevel Optimisation Problems & MPECs

B Bilevel Equilibrium Problems & EPECs





Bilevel Optimization Probs (A)
Current Solution Methods Overview

A

Bilevel Optimization Problems & MPECs

- Parametric Programming
- Single-level Reduction Methods
  - Solved directly as a non-convex optimization problem
  - Transformed into a MILP
- Enumeration
  - Vertex enumeration
  - Branch and bound
- Descent Methods
- Penalty Methods
- Trust-region Methods
- Evolutionary Algorithms
  - Nested Methods
  - Single-level reductions
  - Metamodeling-based Methods
- Computationally efficient Solution/Approximation Methods

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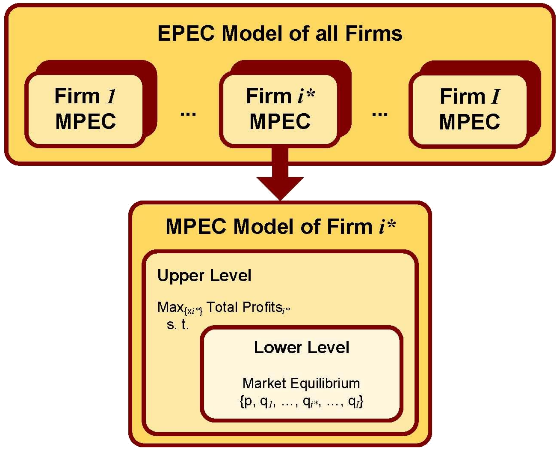
Source: *Sinah 2018, Pozo 2016, Kalashnikiov 2015, and Colson 2007*

Bilevel Equilibrium Probs (B)
Current Solution Methods Overview



B

Bilevel Equilibrium Problems & EPECs

- Single-level reduction of Bilevel Equilibrium
  - Transformed into MIP
  - Formulated as Complementarity Problem
- Diagonalization
- Computationally efficient Solution/Approximation Methods
  - Open-loop approx. of closed-loop equilibria
  - Closed-form solution

Centralized Approach vs Strategic Bilevel Approach

peak demand and load shedding costs are equal to 1000 MW and 300 €/MWh.

Table 1 Demand and solar capacity factor profiles for the representative day

	t01	*t02*	*t03*	*t04*	*t05*	*t06*	*t07*	*t08*	*t09*	*t10*	*t11*	*t12*
Demand (p.u.)	0.65	0.60	0.50	0.28	0.31	0.46	0.65	0.74	0.79	0.86	0.88	0.82
Solar (p.u.)	0.00	0.00	0.00	0.00	0.03	0.35	0.51	0.59	0.58	0.51	0.23	0.54

	t13	*t14*	*t15*	*t16*	*t17*	*t18*	*t19*	*t20*	*t21*	*t22*	*t23*	*t24*
Demand (p.u.)	0.69	0.59	0.56	0.66	0.79	0.94	1.00	0.98	0.88	0.75	0.69	0.65
Solar (p.u.)	0.28	0.34	0.45	0.69	0.70	0.61	0.32	0.02	0.00	0.00	0.00	0.00

For the sake of illustration, we assume a greenfield approach, i.e., no initial generating capacity is considered. We consider three different technologies:

- *Thermal power generation* consisting of combined cycle gas turbine (CCGT) units with a capacity of 100 MW, a linear production cost of 60 €/MWh, and an annualized investment cost of 42000 €/MW. Failures of thermal units are not considered and therefore $\rho_{gt} = 1$.
- *Solar power generating* consisting of solar farms with a capacity of 100 MW, a variable production cost equal to 0 €/MWh, and an annualized investment cost of 85000 €/MW. The capacity factor of these units is provided in Table 1.
- *Energy storage* consisting of Lithium-Ion batteries with a power capacity of 100 MW, the discharge time of 4 hours, and an annualized investment cost of 4000 €/MW, which is associated with a low-cost projection of this technology in the following years.

Sonja Wogrin, Salvador Pineda, Diego A. Tejada-Arango
“Applications of Bilevel Optimization in Energy and Electricity Markets”

https://github.com/datejada/SIGASUS

Centralized Approach vs Strategic Bilevel Approach

	Strategic	Centralized
Thermal capacity (MW)	600	700
Solar capacity (MW)	300	1000
Storage capacity (MW)	400	300
Load shedding (%)	1.8	0
Investment costs (M€)	52	116
Operating costs (M€)	348	218
Total costs (M€)	400	334
Average price (€/MWh)	300	60
Power producer profit (M€)	1431	32

- The Strategic Investor (SI) withholds thermal and solar capacity to create scarcity in the system, which causes the total investment costs to be higher in the centralized approach.
- The lack of capacity investment of the SI leads to some demand shedding, which, in turn, increases the operating costs if compared with the centralized approach.
- The total cost obtained is significantly higher due to the exercise of market power.
- In the strategic approach, the electricity price is always equal to energy not supplied cost due to the load shedding actions caused by the limited investments in generation.
- The power producer profit is much higher for the strategic approach because of the price increase caused by withholding generating capacity.
- A centralized planner would have never captured the fact that a SI strategically withholds capacity to drive up market prices (and even cause load shedding) in order to increase profits.
- Bilevel models provide invaluable insight when exploring the strategic behavior of agents in electricity markets.

Final Comments on Strategic Investment using Equilibrium Models

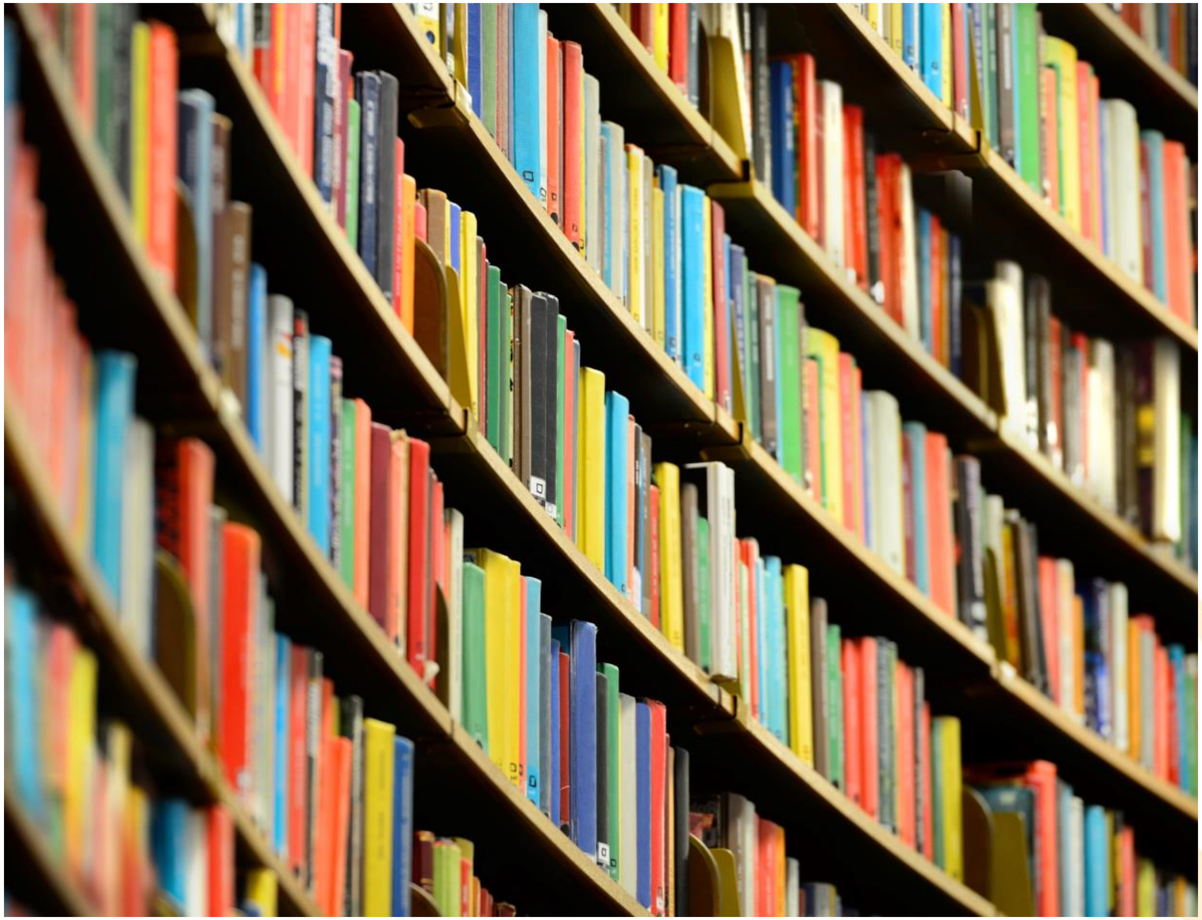
Hierarchical equilibrium models are important when analyzing liberalized electricity markets.

They provide dynamic insight that single-level models cannot capture.

There are many applications of bilevel problems in power systems, e.g., storage investment, and TEP/GEP

Challenges: Require efficient numerical techniques to handle integrality (UC), non-convexity (AC-OPF), stochasticity.

Annex



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First Order Karush-Kuhn-Tucker (KKT) Conditions

- Consider the problem

$$\min_x f(x)$$
$$g_i(x) \le 0 \quad i = 1,\dots,m$$
$$h_j(x) = 0 \quad j = 1,\dots,l$$

- The necessary first order Karush-Kuhn-Tucker (KKT) conditions for a local optimum

Gradient of o.f.: Linear combination of the gradients of the constraints with changed sign

$$\nabla f(x^*) + \sum_{i=1}^{m} \lambda_i^* \nabla g_i(x^*) + \sum_{j=1}^{l} \mu_j^* \nabla h_j(x^*) = 0$$

Complementary slackness conditions
Non active constraint $\lambda=0$
Activa constraint $\lambda\neq0$

$$\lambda_i^* g_i(x^*) = 0 \quad i = 1,\dots,m$$

Feasible point

$$g_i(x^*) \le 0 \quad i = 1,\dots,m$$
$$h_j(x^*) = 0 \quad j = 1,\dots,l$$

$$\lambda_i^* \ge 0 \quad i = 1,\dots,m$$