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Decision Theory

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Introduction

Introduction

Decision:

Choose the best from the possible → Define the best and the possible.

- The best:
  - Unique Criterion (Classical Optimization and Decision)
  - Several Criteria or several decision-makers (Multicriteria Games and Decision)
- Uncertainty
  - Stochastic Optimization
  - Classical Decision Theory
  - Game Theory with incomplete information
- The possible:
  - Discrete set
  - Continuous set

Decision under risk or uncertainty

Decision-maker decides with several states governed by uncertainty

- $\boldsymbol{E} = \{E_1, \dots, E_m\}$ States of nature
- $\boldsymbol{A} = \{A_1, \dots, A_n\}$ Possible decisions or alternatives
- x_{ij}: Result after decision A_i and state E_j
- p_j: Probability of state E_j
  - p_j known: Decision under risk
  - p_j unknown: Decision under uncertainty

- $\boldsymbol{E}$ and $\boldsymbol{A}$ finite $\rightarrow$ Table:

		E_1	E_2	...	E_m	States, scenarios
		p_1	p_2	...	p_m	$\rightarrow$ Probabilities
Decisions or alternatives	A_1	x_{11}	x_{12}	...	x_{1m}	
	A_2	x_{21}	x_{22}	...	x_{2m}	Pay off matrix
	:	:	:	...	:	
	A_n	x_{n1}	x_{n2}	...	x_{nm}	

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Criteria to assess decisions

Criteria to assess decisions

A) Known probabilities (decision under risk):

- Using the expected value or Laplace criterion: Assess alternatives using the mean value (adequate for repetitive cases).
- Using the mode: Assess the scenario, which is the mode (adequate for a relevant mode scenario). Choose the alternative that gives the best value for the most likely scenario.
- Based on the mean scenario: After obtaining the mean scenario (the average values for the state of nature), assess it (get the weighted average of the two payoffs closest to the mean scenario).

Criteria to assess decisions

B) Unknown probabilities (decision under uncertainty):

- Pessimistic or minimax-maximin or Wald criterion: For each possible decision, consider the worst outcome (depending on the scenarios) and choose the decision that yields the best among those worst cases → Costs: minimax Profits: maximin
- Optimistic criterion: For each possible decision, consider the best outcome (depending on the scenarios) and choose the decision that yields the best among those best cases (barely used)
- Hurwicz criterion: Between the pessimistic and optimistic: α $(0 \leq \alpha \leq 1)$ → optimistic index. Assessing: α • The highest profit + $(1 - \alpha)$ • The lowest profit
- Savage criterion or opportunity costs or minimize maximum regret

The opportunity cost of misforecasting the state of nature.

First, derive the regret matrix: For each state of nature, consider the best outcome and then calculate (per state of nature): the best of the state minus the value of the matrix cell. This yields the regret matrix (the cost of not choosing correctly).

Then, use the pessimistic criterion to choose the optimal decision.

Example of assessing decisions

Demand States: 1 (0.1), 2 (0.3), 3 (0.4), 4 (0.2).
Price current month: 6500, next month: 4000;
Cost per unit: 5000

	D_1=1	D_2=2	D_3=3	D_4=4
	P_1=0.1	P_2=0.3	P_3=0.4	P_4=0.2
A_1=1	1500	1500	1500	1500
A_2=2	500	3000	3000	3000
A_3=3	-500	2000	4500	4500
A_4=4	-1500	1000	3500	6000

A) Expected profit: $A1$: 1500 $A2$: 2750 $A3$: 3250 $A4$: 2750

Mode: $A1$: 1500 $A2$: 3000 $A3$: 4500 $A4$: 3500

Average scenario: D=2.7 $A1$: 1500 $A2$: 3000 $A3$: 3750 $A4$: 2750

B) Wald: $A1$: 1500 $A2$: 500 $A3$: -500 $A4$: -1500

Optimistic: $A1$: 1500 $A2$: 3000 $A3$: 4500 $A4$: 6000

Hurwicz: $A1$: 1500 $A2$: $3000\alpha+500(1-\alpha)$ $A3$: $4500\alpha-500(1-\alpha)$

$A4$: $6000\alpha-1500(1-\alpha)$ $\alpha<0.4$ $A1$ $\alpha\geq0.4$ $A4$

SAVAGE:

0	1500	3000	4500	4500
1000	0	1500	3000	3000
2000	1000	0	1500	2000
3000	2000	1000	0	3000

Expected Value of Perfect Information (EVPI)

EVPI = Expected profit with perfect information – Expected profit with uncertainty

- Expected profit with perfect information (EWPI): For each state, make the best decision and then average them.
- Expected profit with uncertainty: Fixed the chosen decision, average the profit for all states.

Example:

- Expected profit with perfect information EWPI:

$$D_1 : 1\,(0.1) \quad D_2 : 2\,(0.3) \quad D_3 : 3\,(0.4) \quad D_4 : 4\,(0.2)$$

$$A_1(1500) \quad A_2(3000) \quad A_3(4500) \quad A_4(6000) \qquad EWPI = 4050$$

- If the decision is A_3: Expected profit with uncertainty: 3250

EVPI = 4050 – 3250 = 800

(It is equivalent to the Savage criterion with expected regret.)

EVPI can be understood as the amount of money one is willing to pay to know precisely the state of nature that will happen (value of the information).

Introduction
Criteria to assess decisions
Decision Trees
Utility Function

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Decision Trees

Decision Trees

Multistage Decision Process → Decision Trees

Sequential Process of Decision - Uncertainty

Decision Tree Structure:

· Uncertainty vertex with as many arcs as possible states of nature at this vertex

· Decision vertex with as many arcs as possible decisions at this vertex

· Initial vertex or root with as many arcs as initial decisions

· Ending vertex or leaf with an associated profit or cost

The tree is built from the root to the leaves and is valued from the leaves to the root.

Decision Tree Assessment:

- Uncertainty vertices: Value them with one of the previous criteria (the expected value/Laplace is the most used)
- Decision vertices: Choose the best decision using the chosen criterion. Decisions that are not chosen are rejected (suppressed path)

Example of Decision Trees

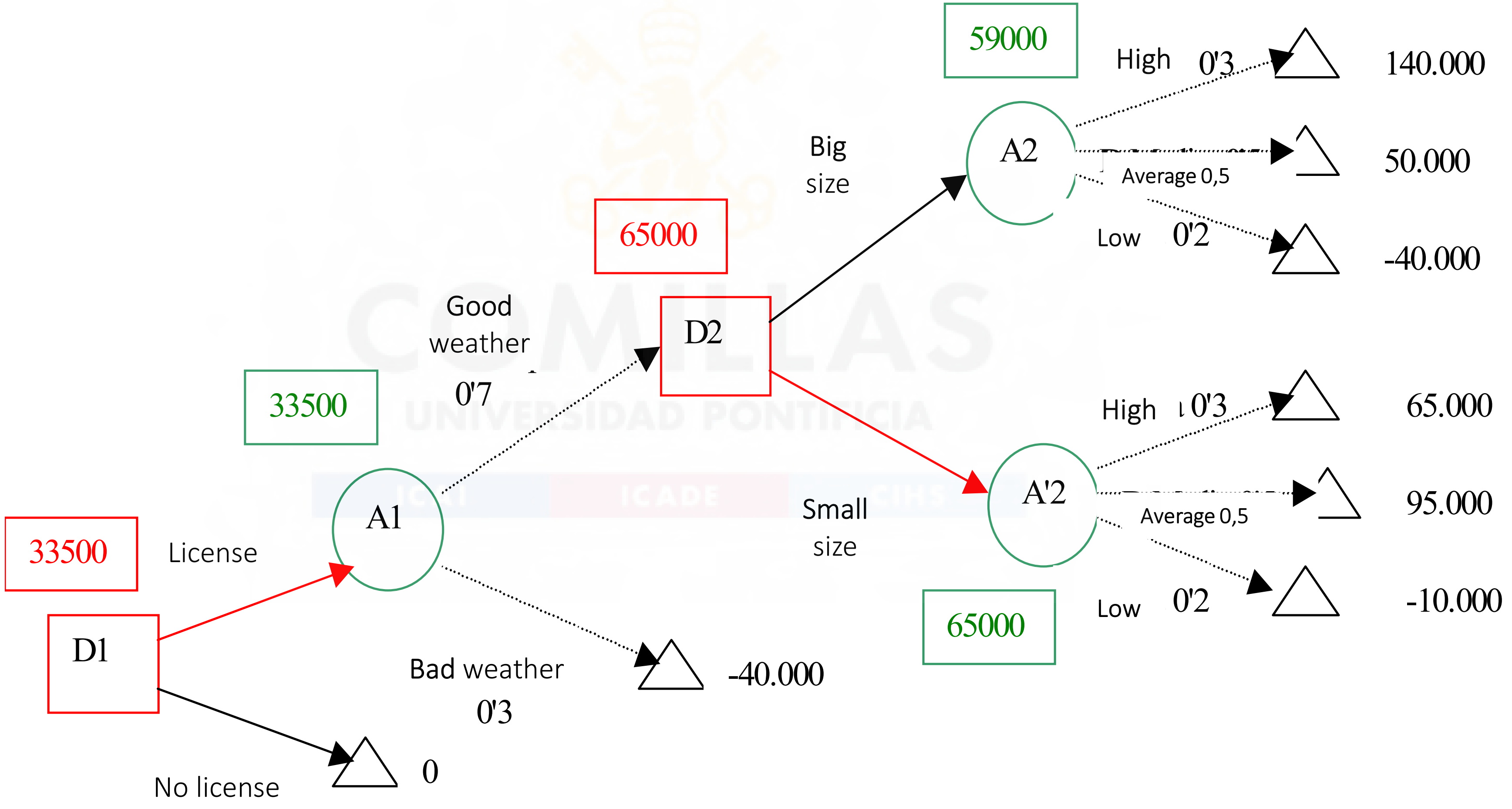
Salesman: January: pay 40000 Euros to get a license to attend a fair in September

One month before: bad weather forecast (0.3) → do not go to the fair

Types of order: Big 900 units Cost/u.:100 Price/u.:300

Small 600 units Cost/u.:125 Price/u.:350

Demand: 900 (0.3) 600 (0.5) 300 (0.2). If demand > order, Price/u. 50 euros off



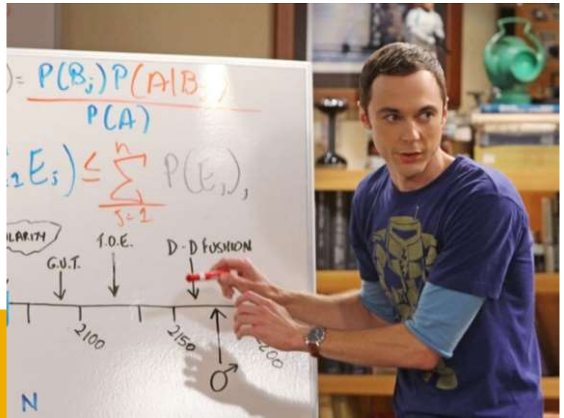
Optimal Decision Path: To get the license and, if the weather forecast is good, make a small order

Decision Trees
Including Partial information using Bayes

- A Priori probabilities:
  - Forecasted probabilities for states of nature
- A Posteriori probabilities:
  - Forecasted probabilities after knowing the result of an associated experiment
  - Example: In January, the salesman asks a weather expert about the weather in September. If this information is worth it, he will modify probabilities considering the expert's forecast
- Including additional information into the decision tree:
  - If a posteriori probabilities are known, directly
  - If these probabilities are unknown, the theorem of the total probability and Bayes' theorem

$$P(A) = \sum_i P(A/B_i)P(B_i)$$

$$P(B/A) = P(A/B)\frac{P(B)}{P(A)}$$



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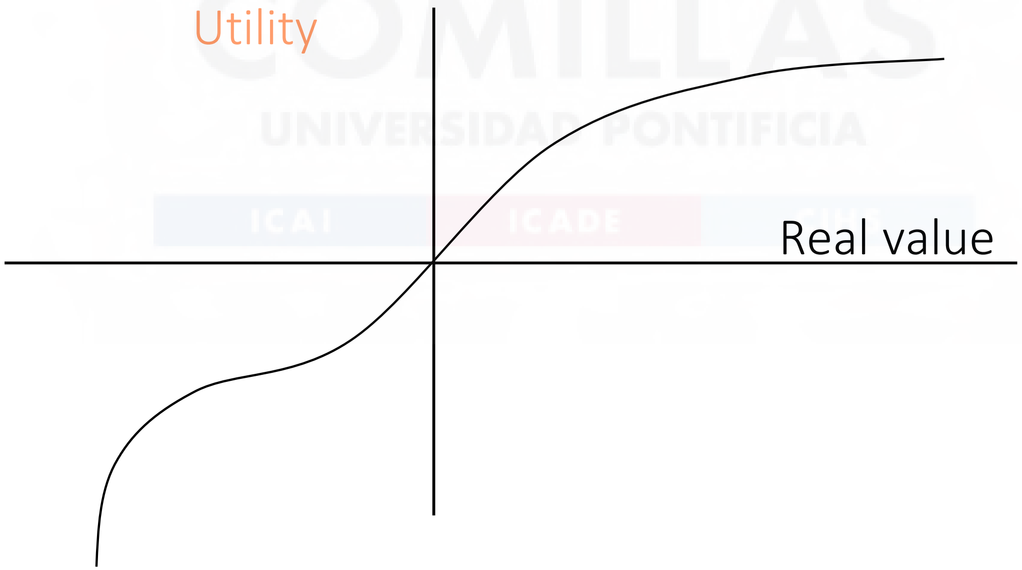
Utility Function

Utility: Concept and Utility Functions

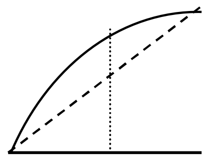
- Utility: Personal assessment of an amount

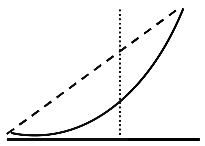
- Utility Function: resumes it bears in mind the personal relevance of different amounts
  - Individual scale or index, non-decreasing.
  - This function can be used with decision criteria.

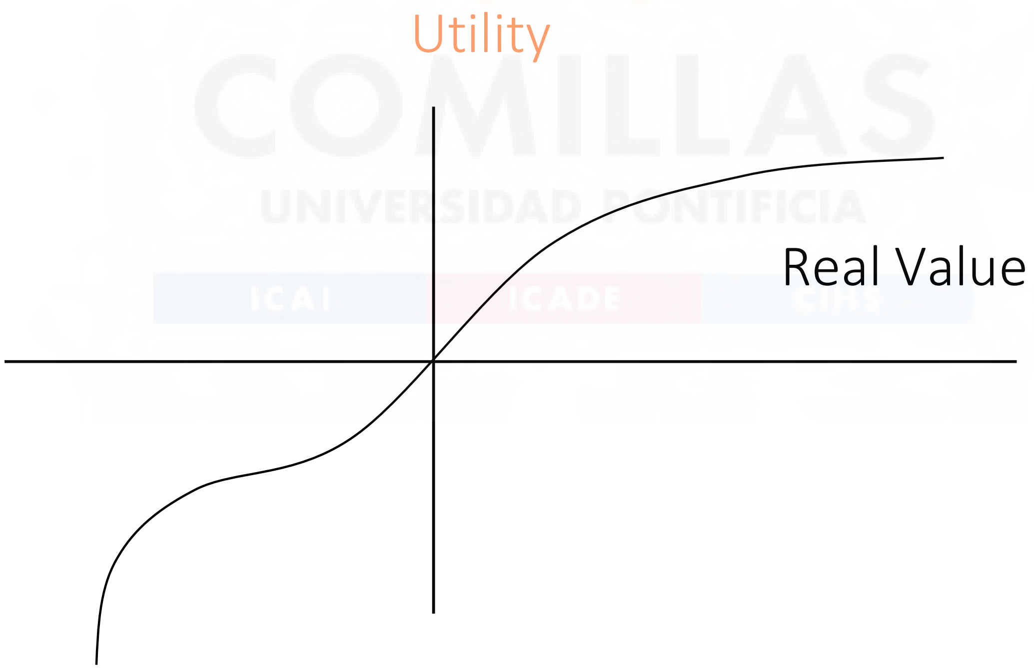


Utility

- Concave Zones: Risk aversion
- Convex Zones: Risk preference
- Linear Zones: Neutral







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