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Discrete Event Simulation Modeling

Departamento de Organización Industrial

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Discrete Event Simulation

Discrete Event Simulation: Example

Time in queue/system
Occupancy of the counters
Money collection
Length of the queue

System

Time interarrivals

- Depending on the hour
- Batch appearance (bus)

Queueing

Service time

- Number of counters
- Different clients
  - Old/Young client
  - Number of products
- Different process
  - Serving the order and collecting money
  - Ask for the product to an external warehouse

Leaving the pharmacy

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Discrete Event Simulation: Basic Concepts - Example

- *System*: Model that considers the daily operation of a pharmacy.

- *Entities*: Customers that enter the bank branch.

- *Attributes (of each entity)*:
  - Number of orders each customer must carry out in the pharmacy
  - Cash each customer pays
  - Time of arrival/departure of each customer

- *Resources*: Tellers

- *System Variables*:
  - Number of busy tellers
  - Total number of customers in the pharmacy
  - Number of people in the queue

Discrete Event Simulation

- *Simulation*: imitates the operations of a real-world facility or process, usually via computer
  - What's being simulated is the *system*
  - A system could be *discrete* (State variables change instantaneously at separated points in time, e.g., when the customer arrives/leaves) or *continuous* (state variables change continuously as a function of time, e.g., airplane flight with state variables like position or speed).
  - The logical and mathematical assumptions about how the system works form a *model* of the system
  - If the model structure is simple enough, could use mathematical methods to get exact information on questions of interest — *analytical solution*
  - But most *complex systems require models that are also complex*. Must be studied via simulation — evaluate model numerically and collect data to estimate model characteristics

Discrete Event Simulation: Analytical Technique vs. Simulation

- Queueing Theory (QT) sets mathematical models to estimate the steady-state performance of waiting lines for different types of queueing systems

- Advantages QT vs. Simulation
  - More accurate when QT can be used
  - Much less computational effort
  - Initial system modeling may be studied with QT

- Disadvantages QT vs. Simulation
  - Need strong queueing modeling assumptions to use analytical formulas
  - Many real systems (that can be simulated) are not queueing systems

Discrete Event Simulation: Advantages, disadvantages, and pitfalls of simulation

- Advantages:
  - Simulation allows excellent flexibility in modeling complex systems so that simulation models can be highly valid
  - Easy to compare alternatives
  - Control experimental conditions
  - Can study systems with a very long timeframe
- Disadvantages:
  - Stochastic simulations produce only estimates – with noise
  - Simulation models can be expensive to develop
  - Simulations usually produce large volumes of output – need to summarize, statistical analysis done appropriately
- Implementation pitfalls:
  - Failure to identify objectives upfront
  - Inappropriate level of detail (both ways)
  - Inadequate design and analysis of simulation experiments
  - Inadequate education and training

Discrete Event Simulation: Application areas

- Some (not all) application areas
  - Designing and analyzing manufacturing systems
    - Example: A manufacturing company is considering extending its plant. Build it and see if it works out. Simulate current and expanded operations — could also investigate many other issues along the way quickly and cheaply
  - Evaluating military weapons systems or their logistics requirements
  - Determining hardware requirements for communication networks
  - Determining hardware and software requirements for a computer system
  - Designing transportation systems - airports, freeways, ports, and subways
  - Evaluating designs for service organizations such as call centers, fast-food restaurants, hospitals, and post offices
  - Determining ordering policies for an inventory system
  - Analyzing financial or economic systems

Discrete Event Simulation: Basic Concepts

- *System*: A collection of entities (people, parts, messages, machines, servers, etc.) that act and interact together toward some end (Schmidt and Taylor, 1970).

- *Entities*: Objects that compose a simulation model. Elements that go through the model.

- *Attributes:* Data values that characterize entities.

- *Resources*: Elements demanded by entities (machines, personnel,...)

- *State of a system*: Collection of *variables* and their values necessary to describe the system.

Example of Entities, Attributes and Variables

System: Warehouse

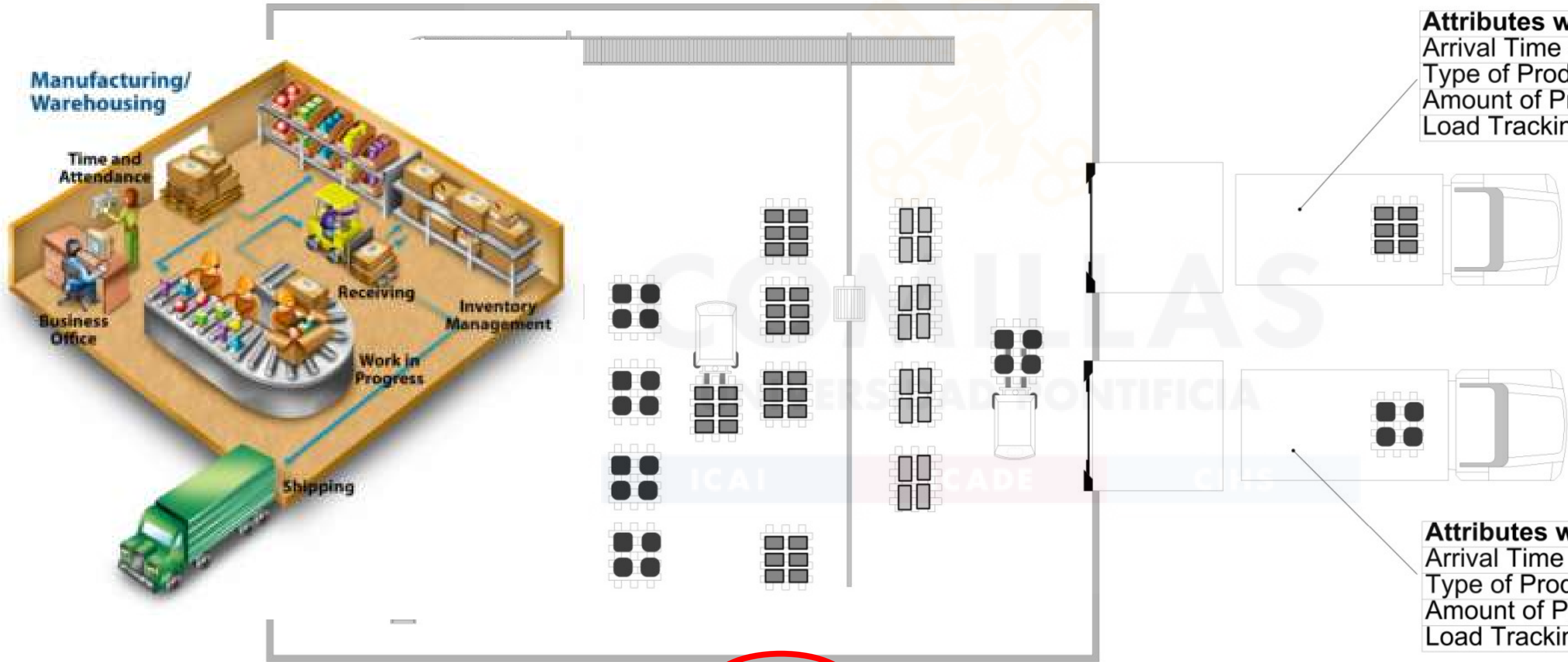
Entity: Part
Attributes: Units, Reference, Weight,…

Entity: Truck

Examples of Delivery Truck Attributes
Arrival Time
Type of Product
Amount of Product
Load Tracking Number

Loading Area

Unloading Area



Attributes with values
Arrival Time = 10:45
Type of Product = A
Amount of Product = 6
Load Tracking Number = 0684432

Attributes with values
Arrival Time = 10:00
Type of Product = B
Amount of Product = 4
Load Tracking Number = 0687922

Examples of Global Variables
Number of Trucks Loading = 0
Number of Trucks Unloading = 2
Number of Busy Forklifts = 2
Number of Busy Operators = 2
Amount of Product A in Storage = 54
Amount of Product B in Storage = 20
Amount of Product C in Storage = 16

Attributes can be thought of
as variables attached to entities

Discrete Event Simulation: More Concepts

- *Discrete event simulation*: Modeling a system as it evolves, and state variables only change instantaneously at separated points in time
  - State changes at only a *countable* number of points in time
  - These points in time are when *events* occur

- *Event*: Instantaneous occurrence that <u>may</u> change the state of the system

- Simulation clock: Variable that keeps the current value of (simulated) time in the model
  - Must decide on and be consistent about time units
  - Usually, no relation between simulated time and (real) time needed to run a model on a computer

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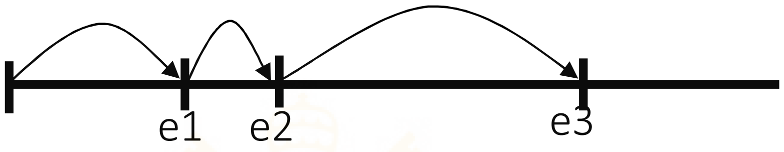
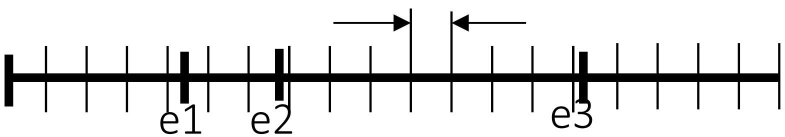
Discrete Event Simulation: Time-Advance Mechanisms

- Two approaches for time advance:
  - *Next-event time advance* (usually used) … described in detail below



    - Determine times of occurrence of future events - update the *event list*
    - Clock "jumps" from one event time to the next (until the stopping criterion is met) and doesn't "exist" for times between successive events; periods of inactivity are ignored
  - *Fixed-increment time advance* (seldom used: *virtual simulation*)
    - Generally, introduces some amount of modeling error in terms of when events *should* occur vs. *do* occur



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Example of a Simulation Model

Example of a Simulation Model: Problem Statement

Single-Server Queueing Model

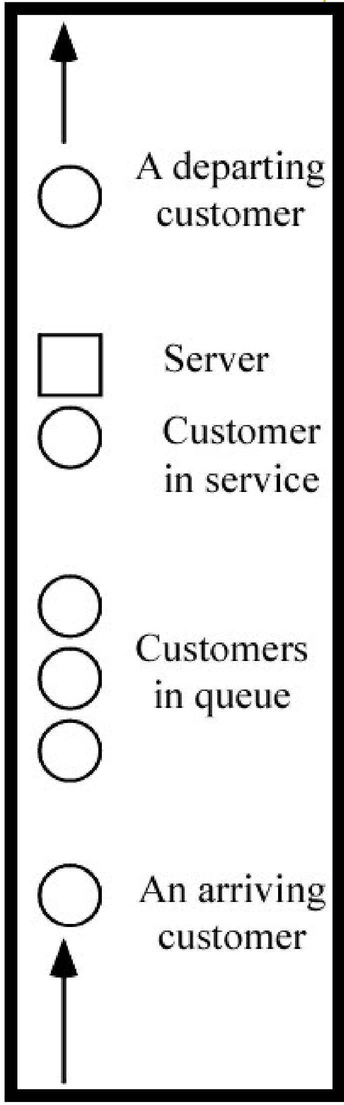
Parameters (initial state)

- Queue discipline is FIFO (First In, First Out)
- Starts empty and idle at time 0
- The first customer arrives after an interarrival time, not at time 0
- Stopping rule: When the n-th customer has completed the delay in the queue (i.e., *enters* the service) … n will be specified as input
- Assume interarrival times are independent and identically distributed (IID) random variables
- Assume service times are IID and are independent of interarrival times

Data

Variables

- Expected average delay in the queue (excluding service time)
- Expected average number of customers in queue (excluding any in service)
- Expected utilization (proportion of busy time) of the server



Example of a Simulation Model: Simulation by Hand

- Simulation by "hand":
  - Display system, state variables, clock, event list, and statistical counters … all *after* execution of each event
  - Use the lists below of interarrival and service times to "drive" the simulation
  - Stop when number of delays hits n = 6 (6^{th} customer starts being served), compute output performance measures

- Given (for now) interarrival times (all times are in minutes):

  0.4, 1.2, 0.5, 1.7, 0.2, 1.6, 0.2, 1.4, 1.9, …

- Given service times:

  2.0, 0.7, 0.2, 1.1, 3.7, 0.6, …

- $\boldsymbol{n}$ = 6 delays in queue desired

Example of a Simulation Model: Time Inputs

- Next-event time advance for the single-server queue is composed by:

  $\boldsymbol{t_i}$ = time of arrival of i-th customer ($t_0 = 0$)

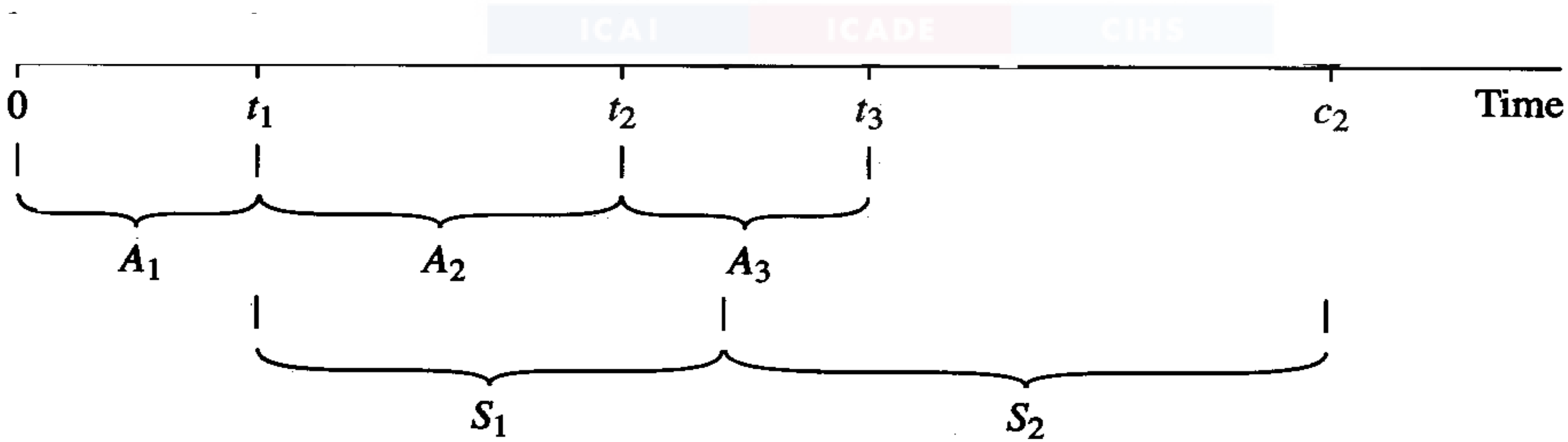
  $\boldsymbol{A_i} = \boldsymbol{t_i} - \boldsymbol{t_{i-1}}$ = interarrival time between $(i-1)$-th and i-th customers (usually assumed to be a random variable from some probability distribution)

  $\boldsymbol{S_i}$ = service-time requirement of i-th customer (another random variable)

  $\boldsymbol{D_i}$ = delay in queue of i-th customer

  $\boldsymbol{C_i} = \boldsymbol{t_i} + \boldsymbol{D_i} + \boldsymbol{S_i}$ = time i-th customer completes service and departs

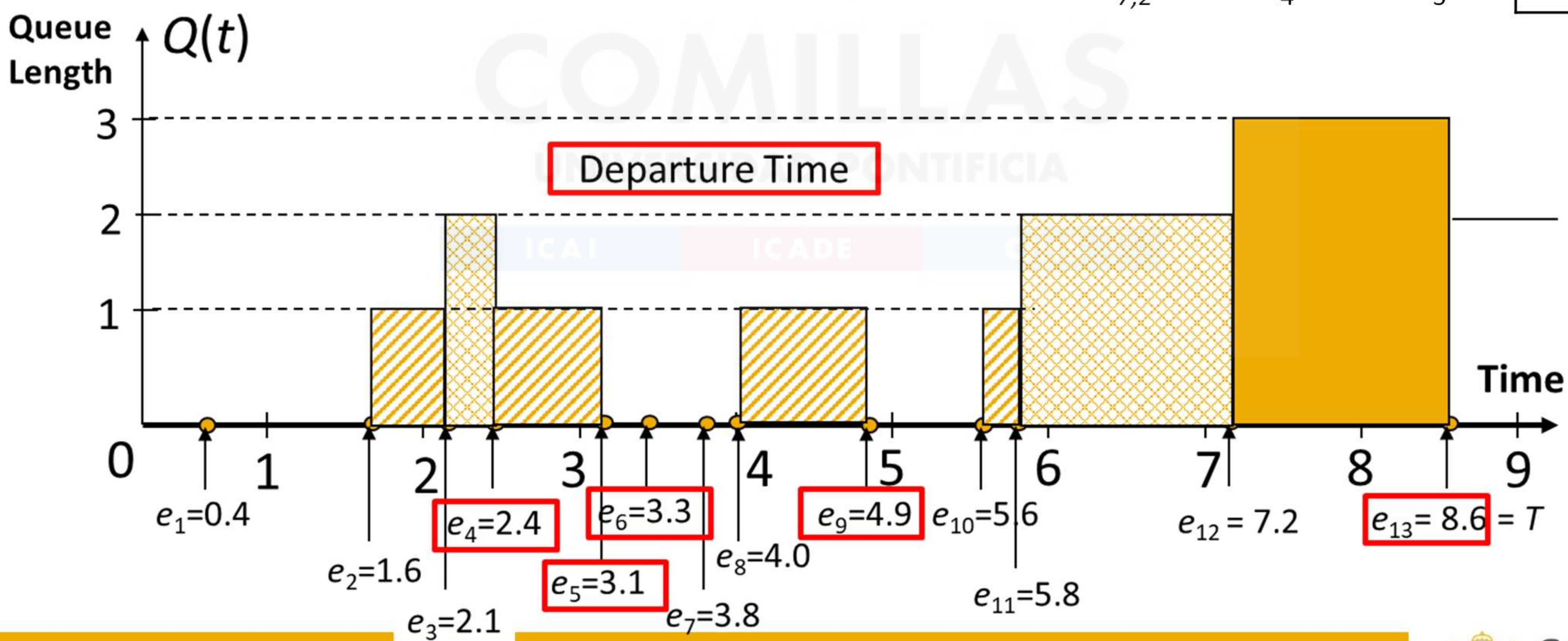
  $\boldsymbol{e_j}$ = time of occurrence of the j-th event (of any type), j = 1, 2, 3, …



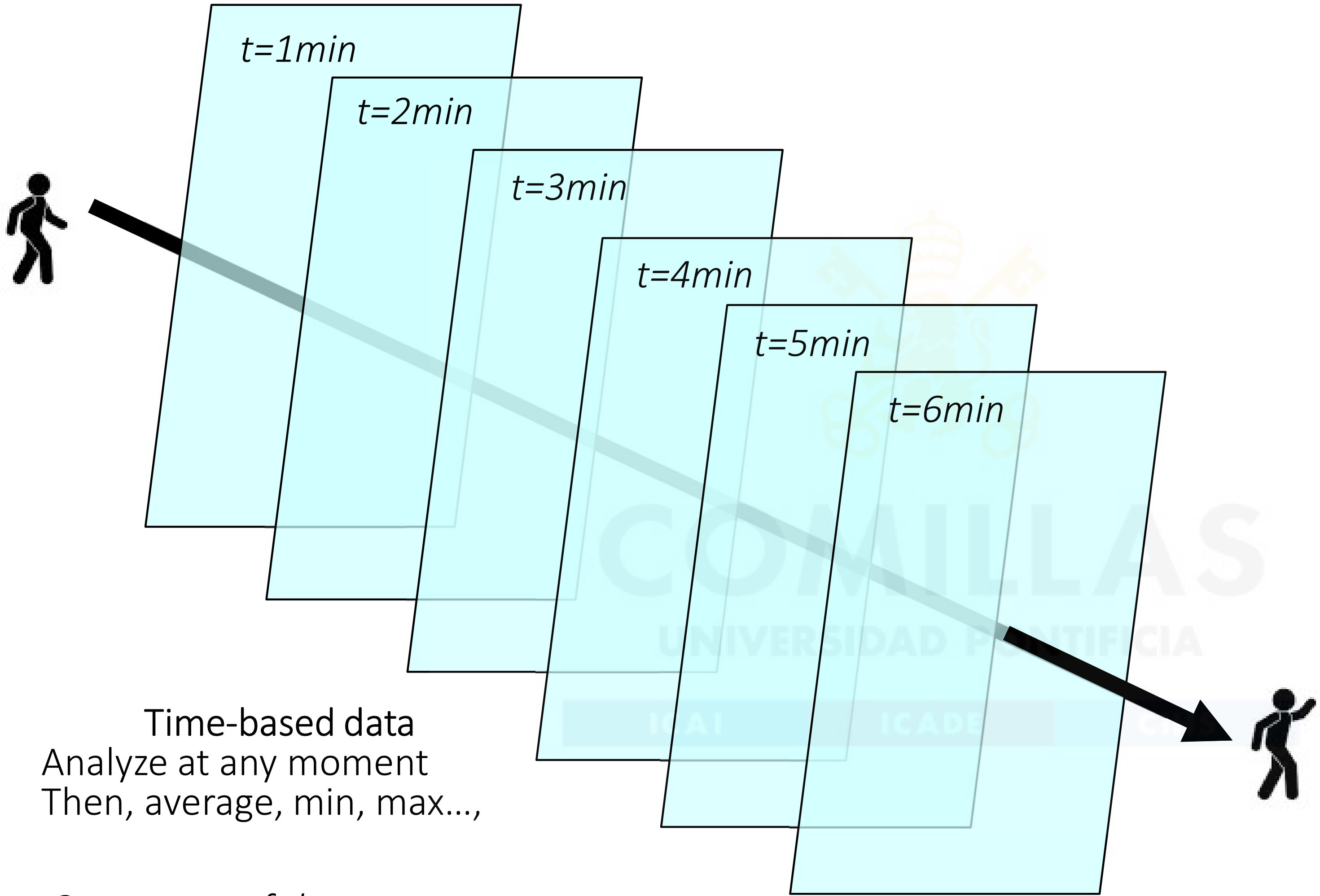
Example of a Simulation Model: Simulation by Hand (Cont'd)

	Interarrival time	Service time	Arrival time	Starts service	End service	Time queueing	Time at the system
Customer 1	0,4	2	0,4	0,4	2,4	0	2
Customer 2	1,2	0,7	1,6	2,4	3,1	0,8	1,5
Customer 3	0,5	0,2	2,1	3,1	3,3	1	1,2
Customer 4	1,7	1,1	3,8	3,8	4,9	0	1,1
Customer 5	0,2	3,7	4	4,9	8,6	0,9	4,6
Customer 6	1,6	0,6	5,6	**8,6**	9,2	3	3,6
Customer 7	0,2		5,8				
	1,4		7,2				

Events time	Client in system	Client in queue	Client in service
0	0	0	0
0,4	1	0	1
1,6	2	1	1
2,1	3	2	1
2,4	2	1	1
3,1	1	0	1
3,3	0	0	0
3,8	1	0	1
4	2	1	1
4,9	1	0	1
5,6	2	1	1
5,8	3	2	1
7,2	4	3	1



Type of variables



Time-based data

- Analyze at any moment
- Then, average, min, max…,

Occupancy of the counters
Length of the queue

Observation-based data

- Compute for each entity
- Then average, min, max,

Time in queue/system
Money collection

Example of a Simulation Model: Simulation by Hand (Cont'd)

	Interarrival time	Service time	Arrival time	Starts service	End service	Time queueing	Time at the system
						0,54	2,08
Customer 1	0,4	2	0,4	0,4	2,4	0	2
Customer 2	1,2	0,7	1,6	2,4	3,1	0,8	1,5
Customer 3	0,5	0,2	2,1	3,1	3,3	1	1,2
Customer 4	1,7	1,1	3,8	3,8	4,9	0	1,1
Customer 5	0,2	3,7	4	4,9	8,6	0,9	4,6
Customer 6	1,6	0,6	5,6	**8,6**	9,2	3	3,6
Customer 7	0,2		5,8				
	1,4		7,2				

…

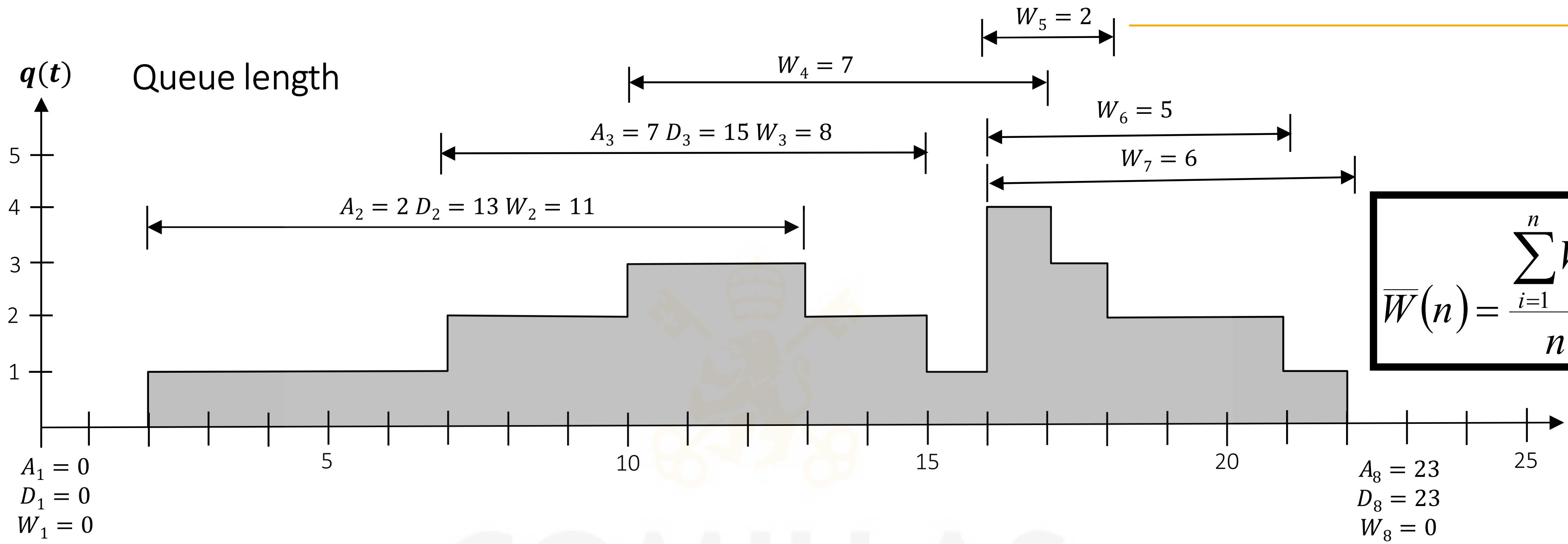
Events time	Client in system	Client in queue	Client in service	Length
		1,15	0,90	
0	0	0	0	0,4
0,4	1	0	1	1,2
1,6	2	1	1	0,5
2,1	3	2	1	0,3
2,4	2	1	1	0,7
3,1	1	0	1	0,2
3,3	0	0	0	0,5
3,8	1	0	1	0,2
4	2	1	1	0,9
4,9	1	0	1	0,7
5,6	2	1	1	0,2
5,8	3	2	1	1,4
7,2	4	3	1	1,4
8,6				

Expected average delay in the queue (excluding service time)	Observation-based data	Average value for all the customers
Expected average number of customers in queue (excluding any in service)	Time-based data	Weighted average of the situation at any moment or
Expected utilization (proportion of busy time) of the server	Time-based data	Area under the $Q(t)$ (queueing length on time) and $B(t)$ (busy server on time)

Types of Statistical Variables: Observation- vs Time-based

- Within replication statistics, there are two primary types of statistical data:
  - Observation-based data:
    - Equally weighted data values associated with the entities that do not persist over time
      - *Example*: Average waiting time for a queue
  - Time-based data:
    - This type of data is associated with the duration or interval of time that the system is in a particular state
    - It is observed by marking the time that the system is in a particular state
    - They represent a sequence of values that persist over some specified amount of time, with that value being weighted by the amount of time over which the value persists
      - *Example*: Average number of people in the queue

Types of Statistical Variables: Example (I)



$$\overline{W}(n) = \frac{\sum_{i=1}^{n} W_i}{n}$$

- Let $\boldsymbol{A_i}$ the time that the i-th customer enters the queue
- Let $\boldsymbol{D_i}$ the time that the i-th customer exits the queue
- Let $\boldsymbol{W_i = D_i - A_i}$ the time that the i-th customer spends in the queue

Waiting Time

The $\boldsymbol{W_i}$ is an observation-based data and, once observed, the value never changes again with respect to time

Average waiting time

$$\overline{W}(8) = \frac{\sum_{i=1}^{n} W_i}{n} = \frac{0+11+8+7+2+5+6+0}{8} = \frac{39}{8} = 4.875$$

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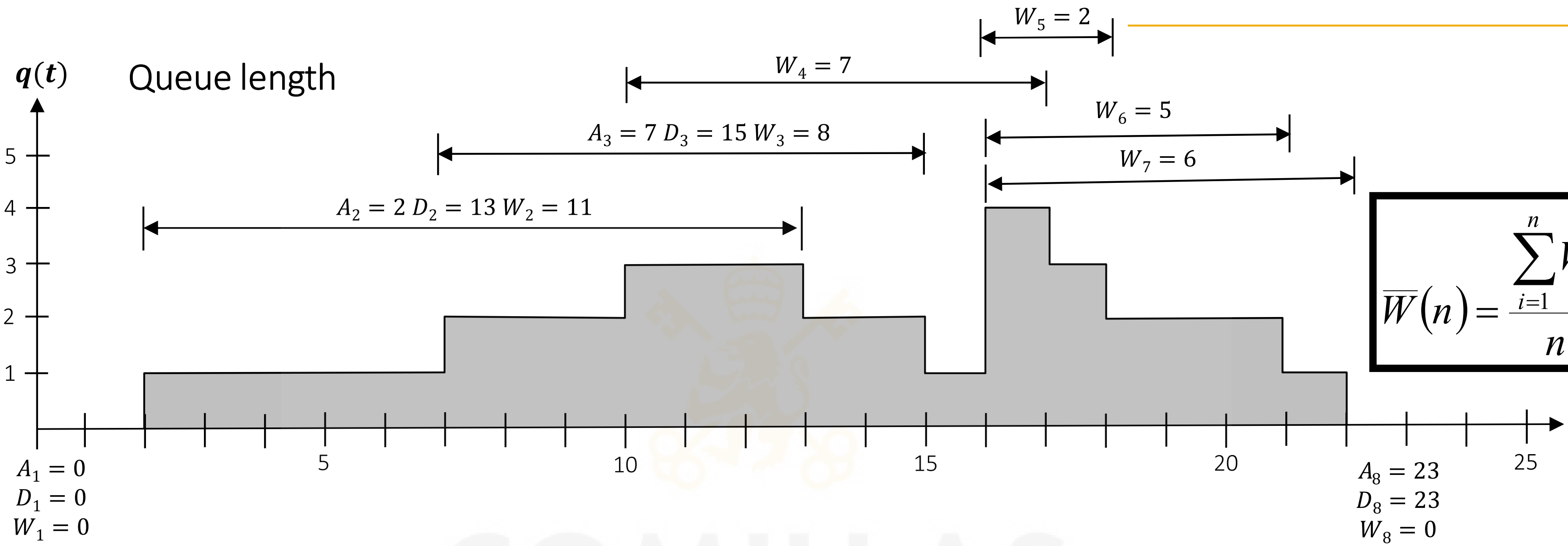
Types of Statistical Variables: Example (I)



$$\overline{W}(n) = \frac{\sum_{i=1}^{n} W_i}{n}$$

Number of Customers Waiting

$\boldsymbol{q(t)}$ (number of customers in the queue at time $\boldsymbol{t}$) is a time-based function whose value changes with respect to time

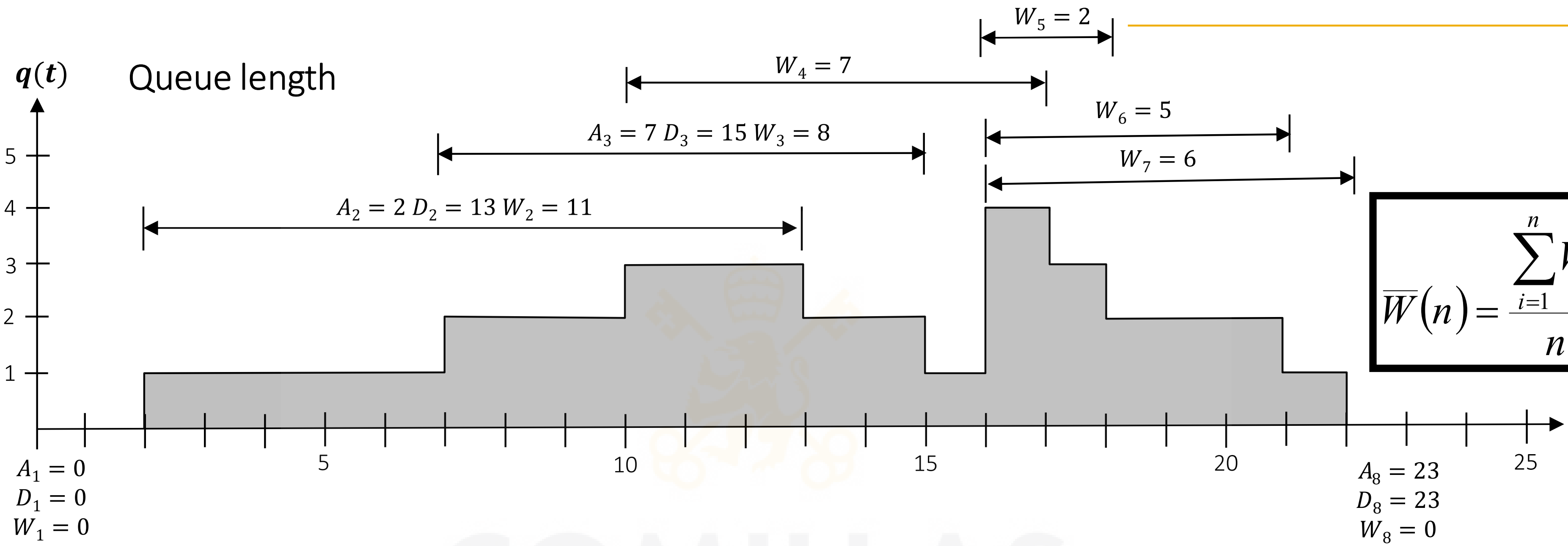
However, its value takes constant values during intervals of time corresponding to when the queue has a certain number of customers.

To estimate the percentage of time that the variable takes a particular value, $\boldsymbol{p_i}$, is computed as the proportion of time that the queue has $\boldsymbol{i}$ customers during the simulation time

$$\overline{L}_q(n) = \frac{\int_{t_0}^{t_n} q(t)\,dt}{t_n - t_0} = \sum_{k=1}^{n} \frac{q_k (t_k - t_{k-1})}{t_n - t_0}$$

$$\overline{L}_q(n) = \frac{0(2-0)+1(7-2)+2(10-7)+3(13-10)+2(15-13)+1(16-15)}{25} + \frac{4(17-16)+3(18-17)+2(21-18)+1(22-21)+0(25-22)}{25} = \frac{39}{25} = 1.56$$

Types of Statistical Variables: Example (I)



$$\overline{W}(n) = \frac{\sum_{i=1}^{n} W_i}{n}$$

Number of Customers Waiting

$\boldsymbol{q(t)}$ (number of customers in the queue at time $\boldsymbol{t}$) *is* a time-based function whose value changes with respect to time

However, its value takes constant values during intervals of time corresponding to when the queue has a certain number of customers.

To estimate the percentage of time that the variable takes a particular value, $\boldsymbol{p_i}$, is computed as the proportion of time that the queue has $\boldsymbol{i}$ customers during the simulation time

$$p_i = \frac{T_i}{t_n - t_0}$$

Where $\boldsymbol{T_i}$ is the time where the queue has $\boldsymbol{i}$ length

$$p_0 = \frac{T_0}{T} = \frac{(2-0)+(25-22)}{25} = 0.2$$

$$p_1 = \frac{T_1}{T} = \frac{(7-2)+(16-15)+(22-21)}{25} = 0.28$$

$$p_2 = 0.32;\ p_3 = 0.16;\ p_4 = 0.04$$

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Simulation Software

Simulation Software:
Simulation Packages vs. Programming languages

- Packages
  - Software in high-level programming code very specialized in simulation modeling
  - Examples: ARENA, SIMIO, SimEvents, WITNESS, AUTOMOD, GPSS, …
- Programming languages
  - General programming languages that can be used for simulation modeling
  - Examples: Python/Simpy, C++, Visual Basic, ...

Simulation Software:
Simulation Packages vs. Programming languages (Cont'd)

- Advantages of using packages
  - Natural framework for simulation modeling
  - Usually make it easier to modify models
  - Better error detection for simulation-specific errors

- Pitfalls of using packages
  - Require a specific package learning process
  - Higher execution time
  - Less modeling flexibility
  - More expensive software

- Advantages of using programming languages
  - More widely known, available, and more modeling flexibility.
  - Usually executes faster … if well written
  - Software cost is usually lower

Simulation Software: Desirable features of packages

STATISTICAL CAPABILITIES

- Adequate **random-number generator** (RNG) for basic U(0,1) variables
  - Statistical properties, cycle length, adequate streams, and substreams
  - RNG seeds should have good defaults, be fixed – not dependent on the clock
- Comprehensive list of input probability distributions
  - Continuous, discrete, empirical
- Ability to make independent replications
- Confidence-interval computation for output performance measures
- Warm-up
- Experimental design
- Optimum-seeking

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Simulation Software: Common elements

- Common elements of simulation packages:
  - Entities: Elements that go through the model
  - Attributes: Specific characteristics for each entity (type, size, ...)
  - Resources: Elements demanded by entities (machines, personnel, ...)
  - Queues: Waiting lines in front of resources

  - Examples:

	Entities	Attributes	Resources
• Manufacture	Products	Due date	Machines
• Communication	Calls	Destination	Phone operators
• Airports	Planes	Capacity	Runways
• Bank branch	Customer	Waiting time	Tellers

Simulation Software: Commercial packages

- ARENA
  - General purpose package
  - Includes basic and advanced modeling modules
  - Graphical model design
  - 2D and 3D Animation
  - Includes transport components (conveyors, trucks, and AGVs)
  - Visual Basic Applications

  https://www.rockwellautomation.com/es-es/products/software/arena-simulation.html

- EXTEND
  - General purpose package
  - http://www.extendsim.com/

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Analyzing Simulation Output

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Contents – Analyzing Simulation Output

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iv. Analysis of Infinite-Horizon Simulations

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Types of Statistical Variables

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Types of Statistical Variables: Definitions

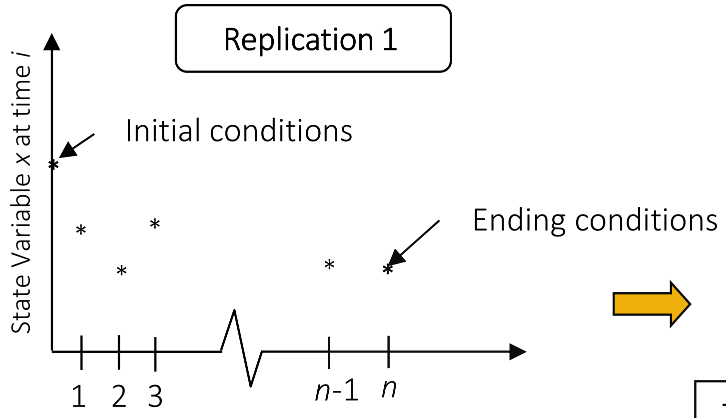
- A simulation experiment occurs when the modeler sets the input parameters (including initial conditions) to the model and runs the simulation

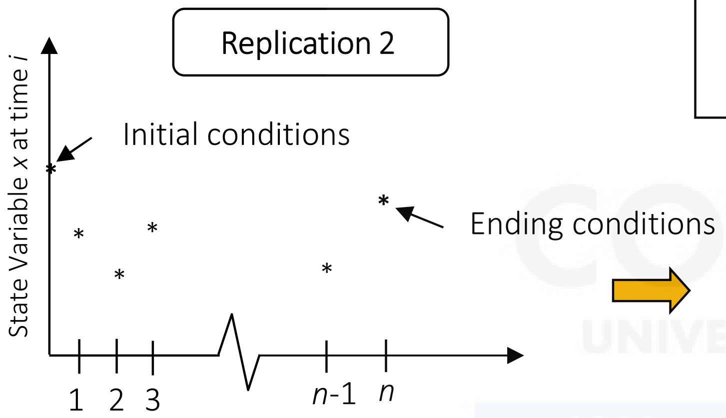
- Events occur, and the simulation model evolves over time
- Statistical quantities are computed, and at the end of the simulation, those quantities are summarized in output reports

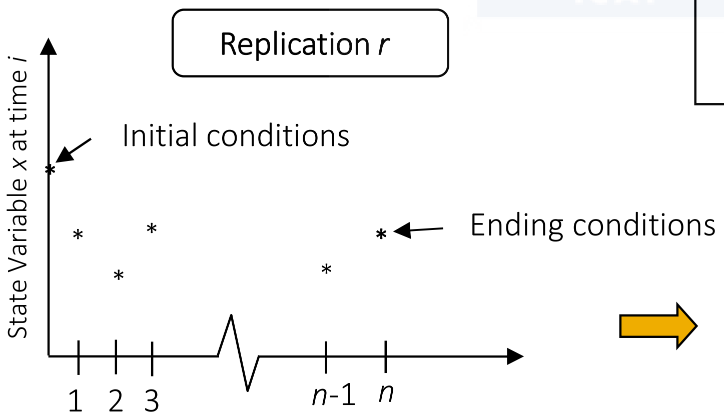
- A replication is the generation of one sample path, representing the evolution of the system from its initial conditions to its ending conditions

- If the simulation experiment has multiple replications within an experiment, each replication represents a different sample path (with the same initial conditions and input parameter settings)
- The underlying random numbers within each replication can be made to be independent

Types of Statistical Variables: Within/Across Replication Stats







Within Replication Statistics

$$\overline{X}_1 = \frac{1}{n}\sum_{i=1}^{n} X_{1,i}$$

The statistical quantities collected within a replication

$$\overline{X}_2 = \frac{1}{n}\sum_{i=1}^{n} X_{2,i}$$

$$\overline{X}_r = \frac{1}{n}\sum_{i=1}^{n} X_{r,i}$$

Across Replication Statistics

The statistical quantities collected across the replications

$$\overline{Y} = \frac{1}{2}\sum_{j=1}^{2} \overline{X}_j$$

$$\overline{Y} = \frac{1}{r}\sum_{j=1}^{r} \overline{X}_j$$

The statistical properties of both types are different and require different methods of analysis.

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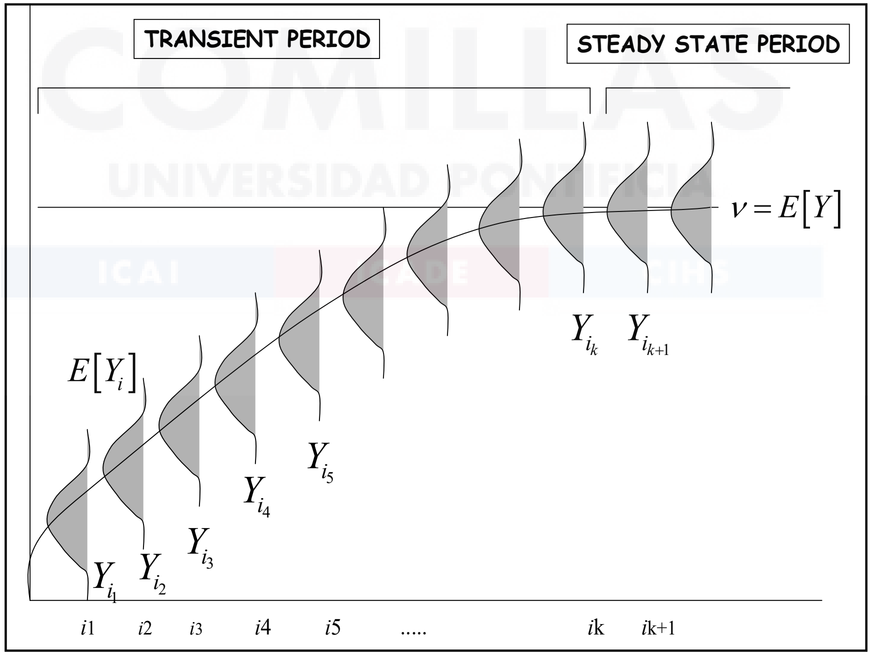
Types of Simulation concerning Output Analysis

Types of Simulation with respect to Output Analysis

- When planning the experimental analysis, it is helpful to think of simulations as consisting of two main categories related to the period over which a decision needs to be made:
  - Finite horizon: A well-defined ending time or condition can be specified (*terminating* simulations). They are focused on the *transient* period.

  - Infinite horizon: There is no well-defined ending time or condition. The planning period is over the life of the system. They are also called *steady-state* simulations.



Types of Simulation with respect to Output Analysis

- Examples of Finite horizon: Its ending time of each replication is specified or a random variable (for example, when no entities in the model)
  - **Bank**: Bank doors open at 9 a.m. and close at 5 p.m.
  - **Military battle**: Simulate till force strength reaches a critical value
  - **Filling a customer order**: Simulate the production of the first 100 products
- Examples of Infinite horizon: There is no natural ending point. However, a finite replication length must be specified
  - A **factory** in which you are interested in measuring the steady-state throughput
  - A **hospital emergency room** that is open 24 hours a day, 7 days a week
  - A **telecommunications system** that is always operational
- Infinite-horizon simulations often model situations that involve modeling of non-stationary processes. It is often possible to find a period where the non-stationary behavior repeats (steady state cyclical estimation, Law (2007))
- Of the two types of simulations, Finite-Horizon simulations are easier to analyze

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Analysis of Finite-Horizon Simulations

Analysis of Finite-Horizon Simulations

- The analysis of the system will require some replications to obtain an output result for decision-making with a confidence interval half-width

Arena computes a 95% confidence interval
and reports the half-width value for that interval

- To get that interval size, it is necessary to estimate in advance the sample size, i.e., the number of replications of the experiment

- **For example**: Estimating $E[W_r]$ it is possible to be 95% confident that the average of the random variable is the true value within ± 2 minutes

- In practice, this half-width ratio method is used to estimate the adequate number of replications n to achieve the desired half-width h:
  - Set an initial pilot run with n_0 replications and compute the half-width h_0 (corresponding to this initial run). Then, n is approximated by

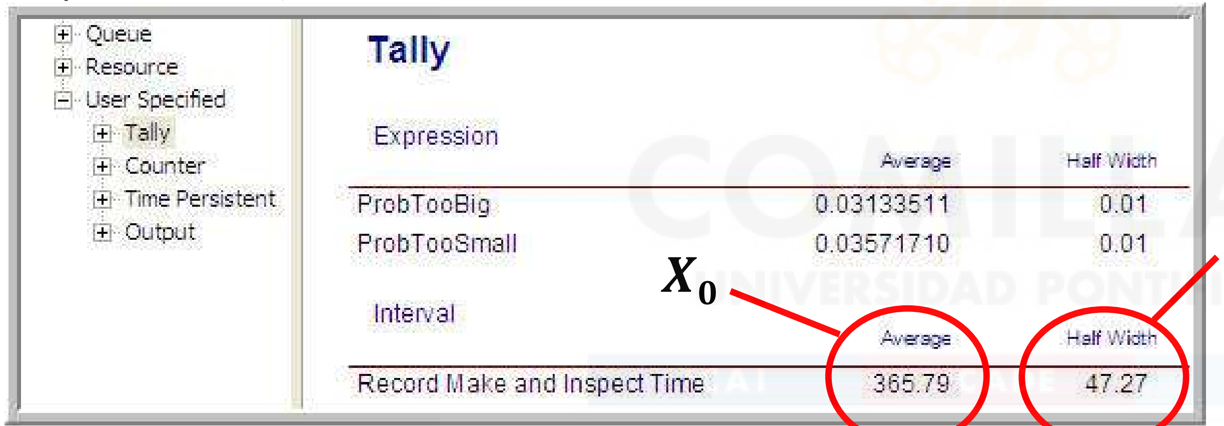
Quadratic behavior

$$\boxed{n \approx n_0 \left(\frac{h_0}{h} \right)^2}$$

Example in Arena
How to estimate the number of replications n

• How to estimate the number of replications n to ensure a 95% confidence interval with an error bound of ± 20 min (i.e., half width of 20 minutes) for variable B.

• Solve the model with the initial number of replications n_0 (we choose this number, e.g., n_0 =10) to obtain the corresponding initial half-width h_0 (Run>Setup>Number of Replications)

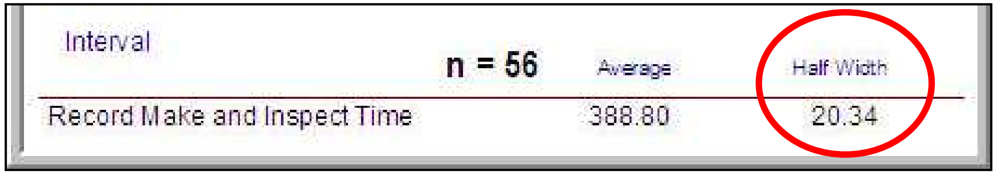


X_0 h_0

Number of replications to get 20 min accuracy using half-width ratio method

$$n \approx n_0\left(\frac{h_0}{h}\right)^2 = 10\left(\frac{47.27}{20}\right)^2 = 56$$

After sampling the required number of replications, the new average time obtained as follows:



i. Types of Statistical Variables
ii. Types of Simulation concerning Output Analysis
iii. Analysis of Finite-Horizon Simulations
iv. Analysis of Infinite-Horizon Simulations

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Analysis of Infinite-Horizon Simulations

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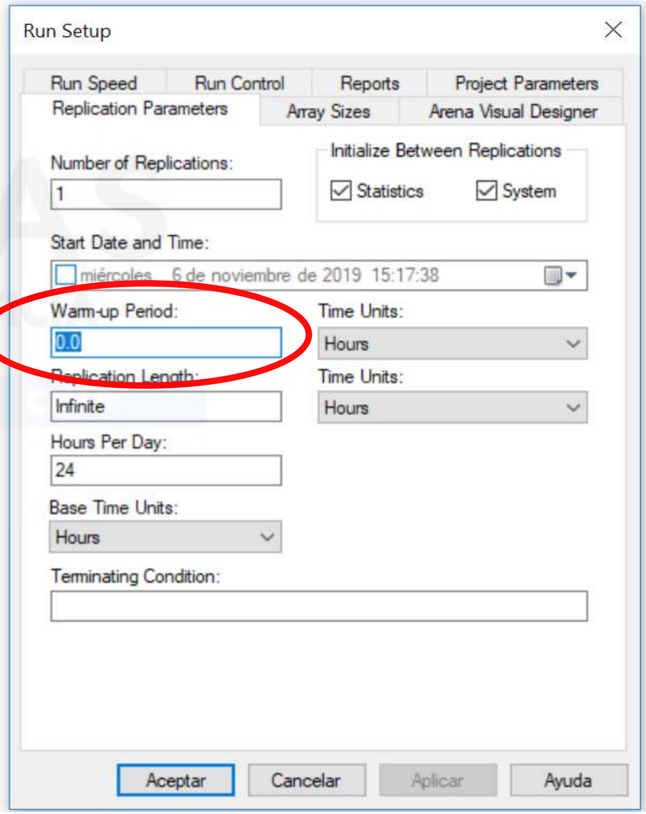
Analysis of Infinite-Horizon Simulations

- The initial conditions of the system impact model results and statistics and can distort the actual value of those statistics.

- Analyzing the effect of initial conditions mitigates the nonstationary aspect of within-replication data for infinite-horizon experiments.

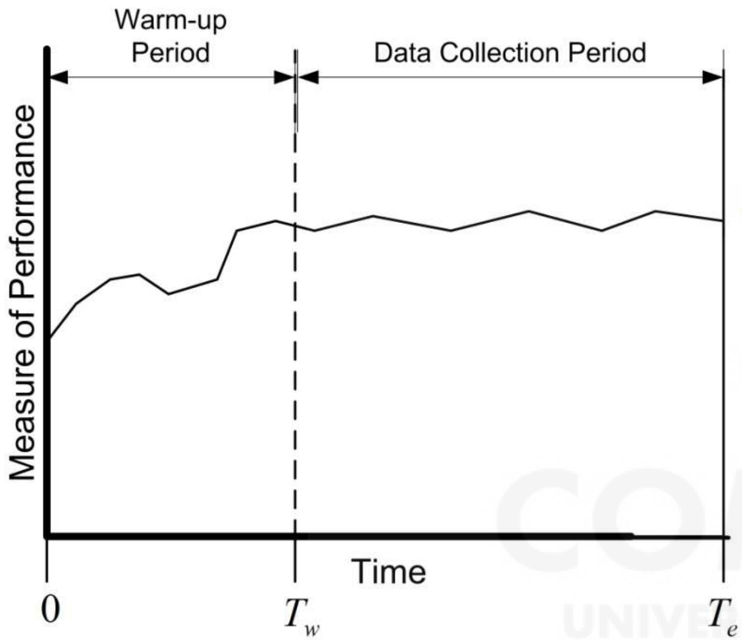
- A warm-up time (the time it takes the system to reach a steady state) must be determined within the simulation tool. In ARENA: (Run>Setup>Warm-up Period)

- Specifying a warm-up period in Arena causes to schedule a warm-up event for time T_w. At that time, all the accumulated statistical counters are cleared so that the net effect is that statistics are only collected over the period from T_w to T_e



Analysis of Infinite-Horizon Simulations

Assessing the Effect of Initial Conditions



- Let $F(x/I)$ be the conditional cumulative distribution function of X where I represents the initial conditions to start the simulation at time 0

- If $F(x/I) \rightarrow F(x)$ when $t \rightarrow \infty$ for all initial conditions, then $F(x)$ is called the steady-state distribution of the output process

- Unless the system is initialized using the steady-state distribution, there is no way to directly observe the steady-state distribution

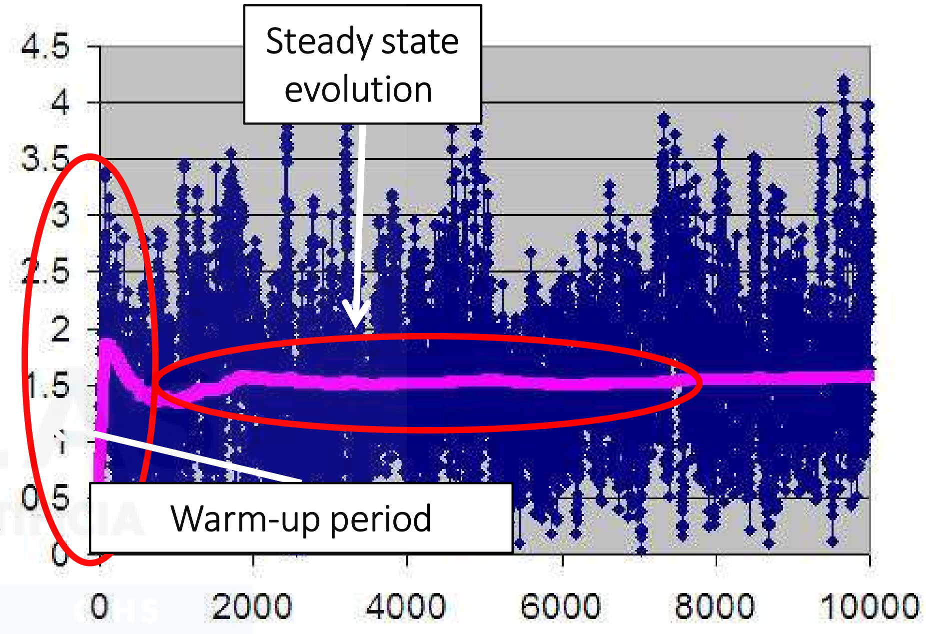
- It should be decided on how long to run the simulations and how to handle the effect of the initial conditions on the estimates

- Rule of thumb: the simulation length should be at least 10 times the warm-up period

Analysis of Infinite-Horizon Simulations

Warm-up period (cont.):

- To determine the warm-up period, a **visual method** proposed by **Welch** (1983) consists of:



1. Make R replications
2. Let Y_{ri} be the i-*th* observation within replication r
3. Compute the averages across replications
4. Plot $\overline{Y_{\blacksquare i}}$ for each i
5. Apply smoothing techniques to this plot
6. Visually assess where the plots start to converge

$$\overline{Y}_{\bullet i} = \frac{\sum_{r=1}^{R} Y_{ri}}{R}$$

Example: On sheet *10Replications* at Excel file *MM1-QueueingSimulation.xls*

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