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Node Aggregation in Stochastic Nested Benders Decomposition Applied to Hydrothermal Coordination

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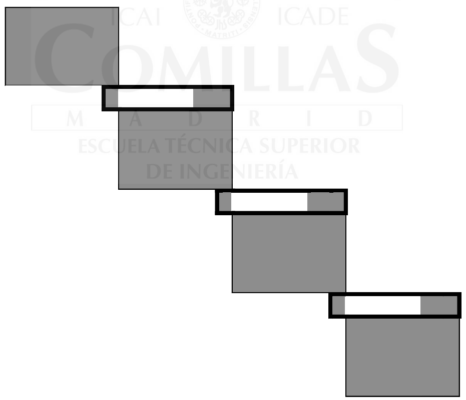
Introduction (I)

- Hydrothermal coordination problem may be formulated as a large scale multistage stochastic linear problem.
- Stochasticity appears when considering water inflows as random variables.
- Stochastic problem that in an equivalent deterministic way can be presented as a problem over a tree, scenario tree.
- The scenario tree intrinsically considers the non-anticipativity nature of the decisions.

Introduction (II)

- Problem size can reach million of equations.
- It is solved using nested Benders decomposition.
- Staircase structure of the constraint matrix.



Overview Benders Decomposition (I)

Problem

$$\min \quad c_1 x_1 + p^1 c_2 x_2^1 + p^2 c_2 x_2^2$$
$$A_1 x_1 = b_1$$
$$B_1 x_1 + A_2^1 x_2^1 = b_2^1$$
$$B_1 x_1 + A_2^2 x_2^2 = b_2^2$$

Monocut decomposition

$$\min \quad c_1 x_1 + \theta(x_1) \quad with \quad \theta(x_1) = \min \quad p^1 c_2 x_2^1 + p^2 c_2 x_2^2$$
$$A_1 x_1 = b_1 \qquad B_1 x_1 + A_2^1 x_2^1 = b_2^1$$
$$B_1 x_1 + A_2^2 x_2^2 = b_2^2$$

Multicut decomposition

$$\min \quad c_1 x_1 + p^1 \theta^1(x_1) + p^2 \theta^2(x_1) \quad with \quad \theta^i(x_1) = \min \quad c_2 x_2^i$$
$$A_1 x_1 = b_1 \qquad B_1 x_1 + A_2^i x_2^i = b_2^i$$

Overview Benders Decomposition (II)

- The recourse function is expressed as

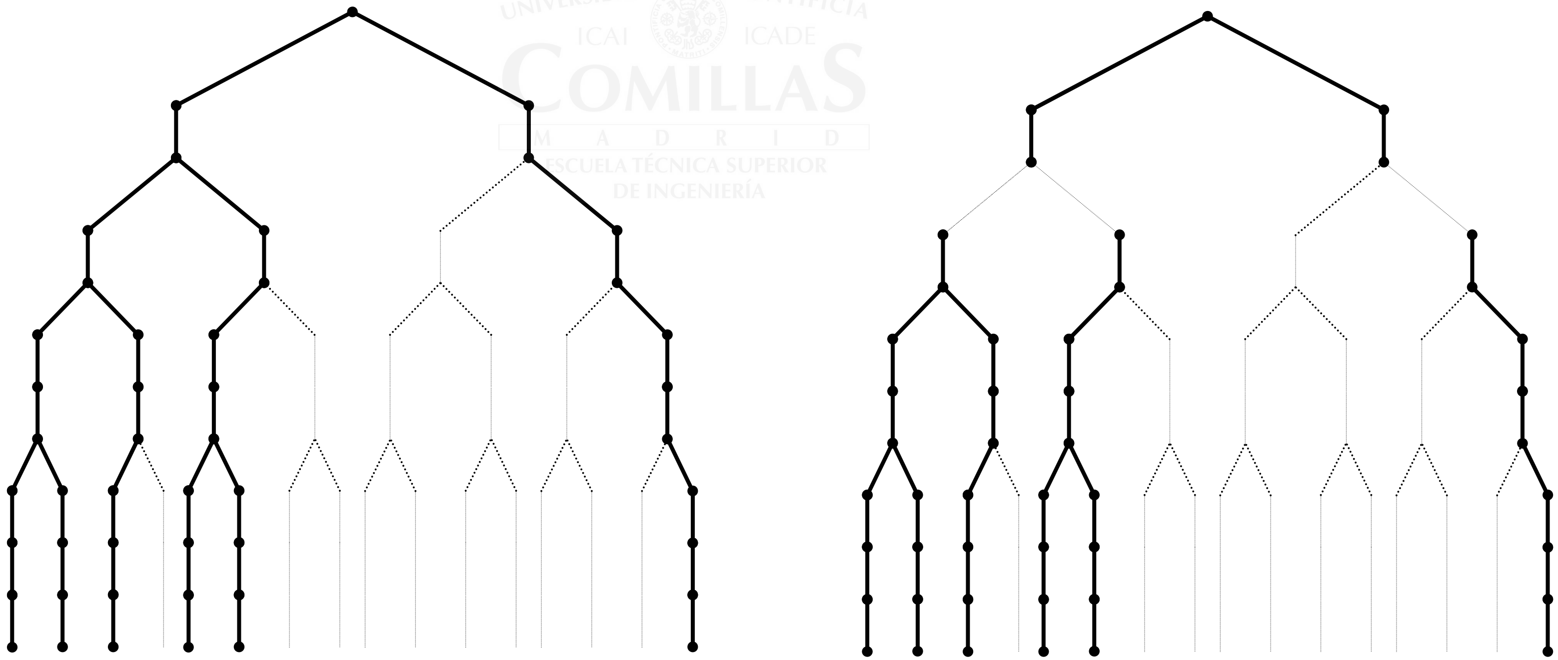
$$\theta^{i}(x_1) = max\{\, (b_2^i - B_1 x_1)\pi \ / \ \pi A_2^i \le c_2 \}$$

- The recourse function is replaced by a partial description during the algorithm.
- Master problem.
- Subproblem / subproblems

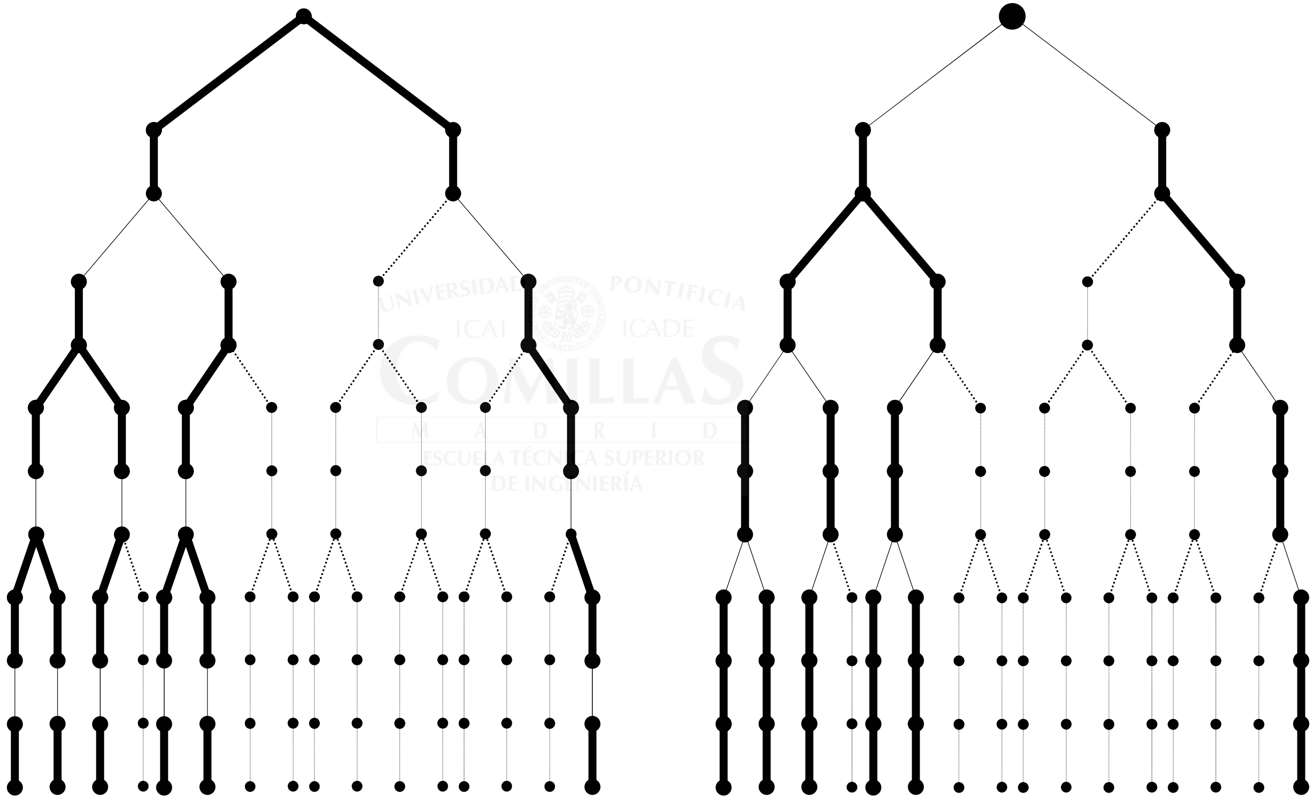
- Nested decomposition

Subtree Decomposition (I)

- Generalization of previous ideas leads to decompose the problem forming arbitrary subproblems.
- These subproblems are related to the subtrees that appear when the scenario tree nodes are aggregated.



Subtree Decomposition (II)



Decomposition algorithm features (I)

- The subtree partition is determined just choosing a set of nodes as root nodes for the different subtrees (aggregation methodologies).
- Each subproblem is solved using
  - Simplex (< 10000x10000)
  - Barrier (> 10000x10000)
- Multicut or monocut when available. (Numerical results presented have been obtained with multicut methodology.)
- Benders cut is expressed as the linearization of the recourse function around the decision variable.

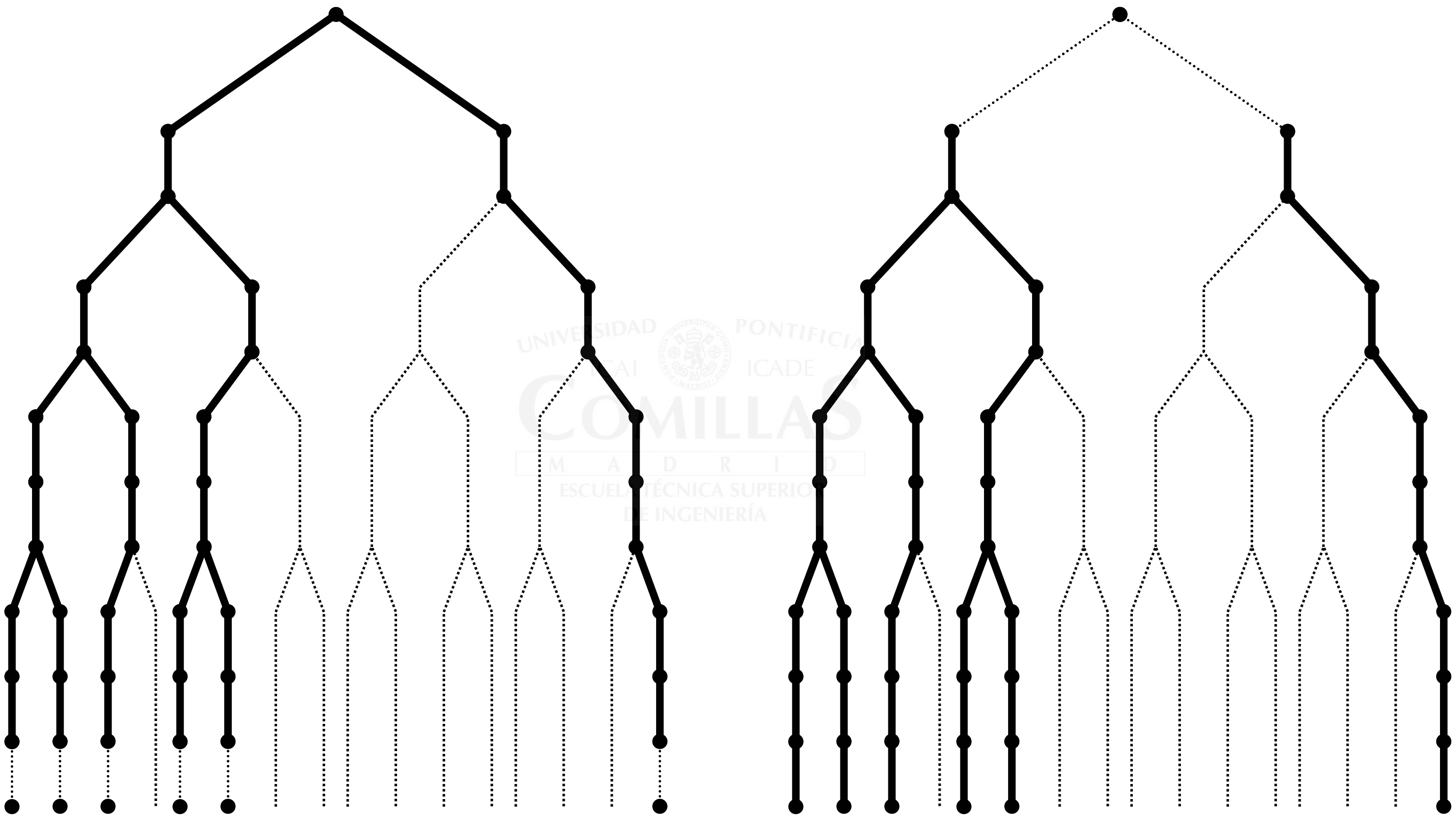
Decomposition algorithm features (II)

- Initial point is obtained solving a problem with deterministic inflows
- First iteration of Benders algorithm consists of a backward pass from the last stage to the first.
- Fast-pass tree-traversing strategy.
- Use of previous basis.

Node aggregation (I)

- Implementation in GAMS (General Algebraic Modeling Language).
- Use of dynamic sets that activate a group of sets and equations when necessary.
- Provide the possibility of testing different ways of splitting up the scenario tree.
- Objective: speed up the convergence toward the optimum.

Two stage decomposition



Two stage Decomposition

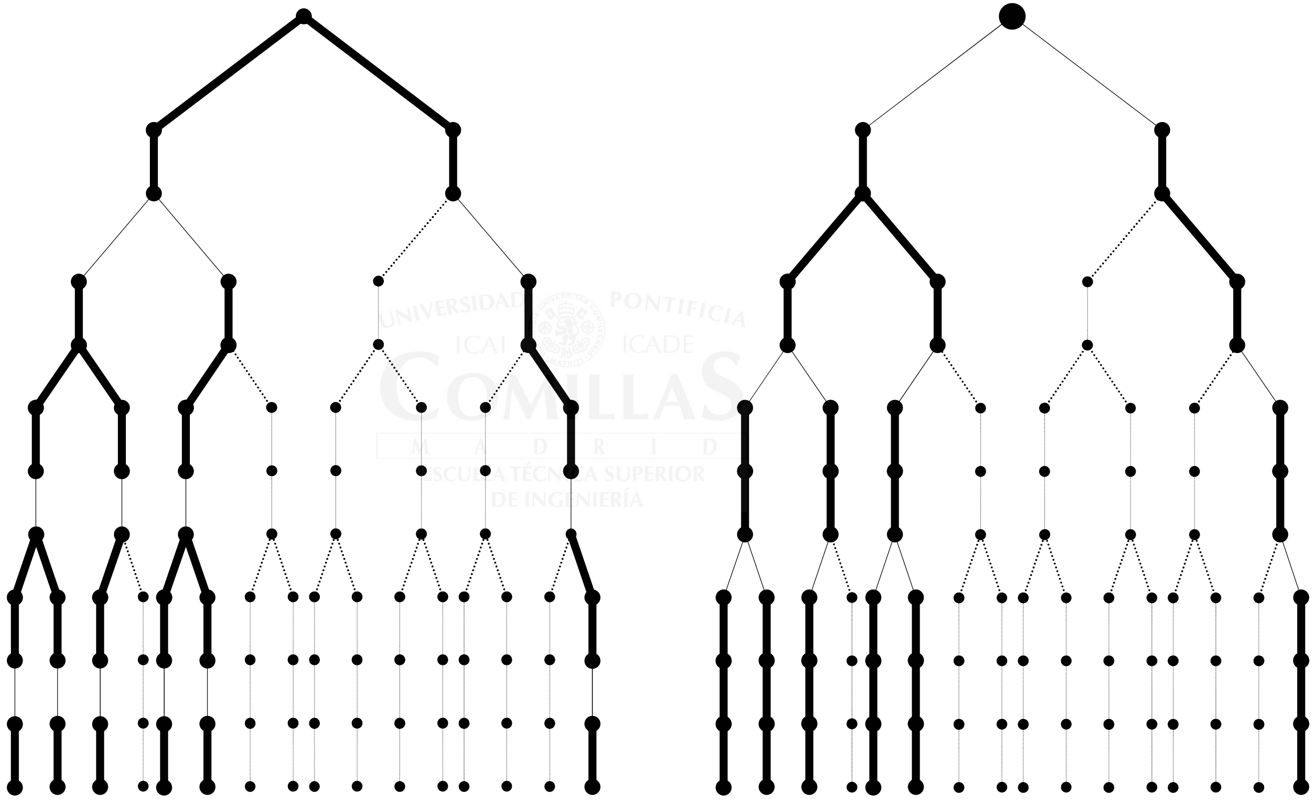
Numerical results

Name	Periods	# Subpr	# iter	Converg	Size Master Problem (r, c, e)			Size Biggest Subproblem (r, c, e)			Sol. Time	Total Time
A/1	1	1	1	0				70623	104892	332336	351	467
A/1/2	1/ 2	3	1	0.000036	1591	2349	7454	52642	78194	247747	592	779
A/1/3	1/ 3	3	2	0.000053	4512	6704	21060	51182	76017	240853	639	869
A/1/4	1/ 4	4	2	0.000052	7470	11104	34940	30137	44779	141864	383	628
A/1/5	1/ 5	4	4	0.000020	11980	178311	56245	28634	42537	134721	545	926
A/1/6	1/ 6	5	4	0.000045	16352	24327	76724	16597	24670	78113	449	848
A/1/7	1/ 7	5	5	0.000051	22485	33428	105634	15064	22395	70886	526	995
A/1/8	1/ 8	5	7	0.000039	28569	42484	134350	13543	20131	63665	608	1248
A/1/9	1/ 9	7	8	0.000078	34645	51496	162958	5998	8949	28216	507	1312
A/1/10*	1/10	7	10	0.000132	43773	65082	206036	4477	6685	21037	512	1440
A/1/11*	1/11	7	10	0.000524	52641	78360	247904	2999	4472	14059	784	1714
A/1/12*	1/12	7	10	0.001372	61653	91770	290342	1497	2237	6985	963	1888

Node Aggregation (II)

- Previous results suggest dividing the scenario tree into a small number of subtrees.
- Forming big subtrees.
- Aggregation methodologies that allow forming subtrees whose associated subproblem do not exceed a certain number of equations.

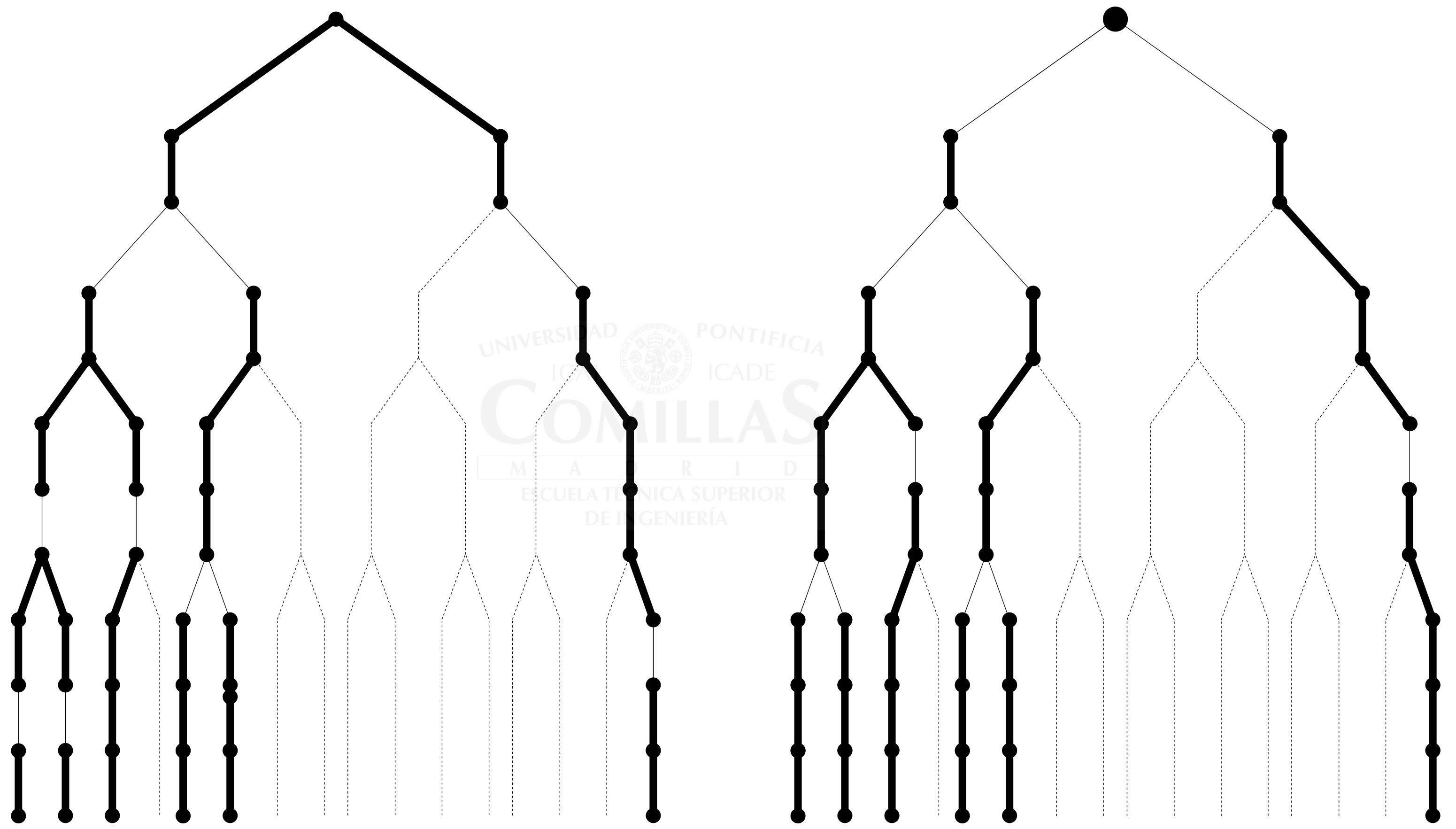
Protocols 1 and 2. Period Aggregation



Period Aggregation. Numerical Results

	Agregación desde el primer periodo		Agregación desde el último periodo	
	Convergencia	Tiempo (s)	Convergencia	Tiempo (s)
1ª iteración	0.013210	120	0.003575	176
2º iteración	0.007030	151	0.002068	218
3ª iteración	0.003648	182	0.001261	261
4ª iteración	0.001864	205.9	0.000707	297.2
5ª iteración	0.001245	229.8	0.000467	326.4
6ª iteración	0.000847	257.1	0.000205	355.7
7ª iteración	0.000802	306.2	0.000171	382.5
8ª iteración	0.000525	306.7	0.000106	408.2
9ª iteración	0.000385	330.6	0.000067	434.1
10ª iteración	0.000290	358.2		
Tiempo total (s)	2378		2117	
Tiempo solución (s)	413		489	

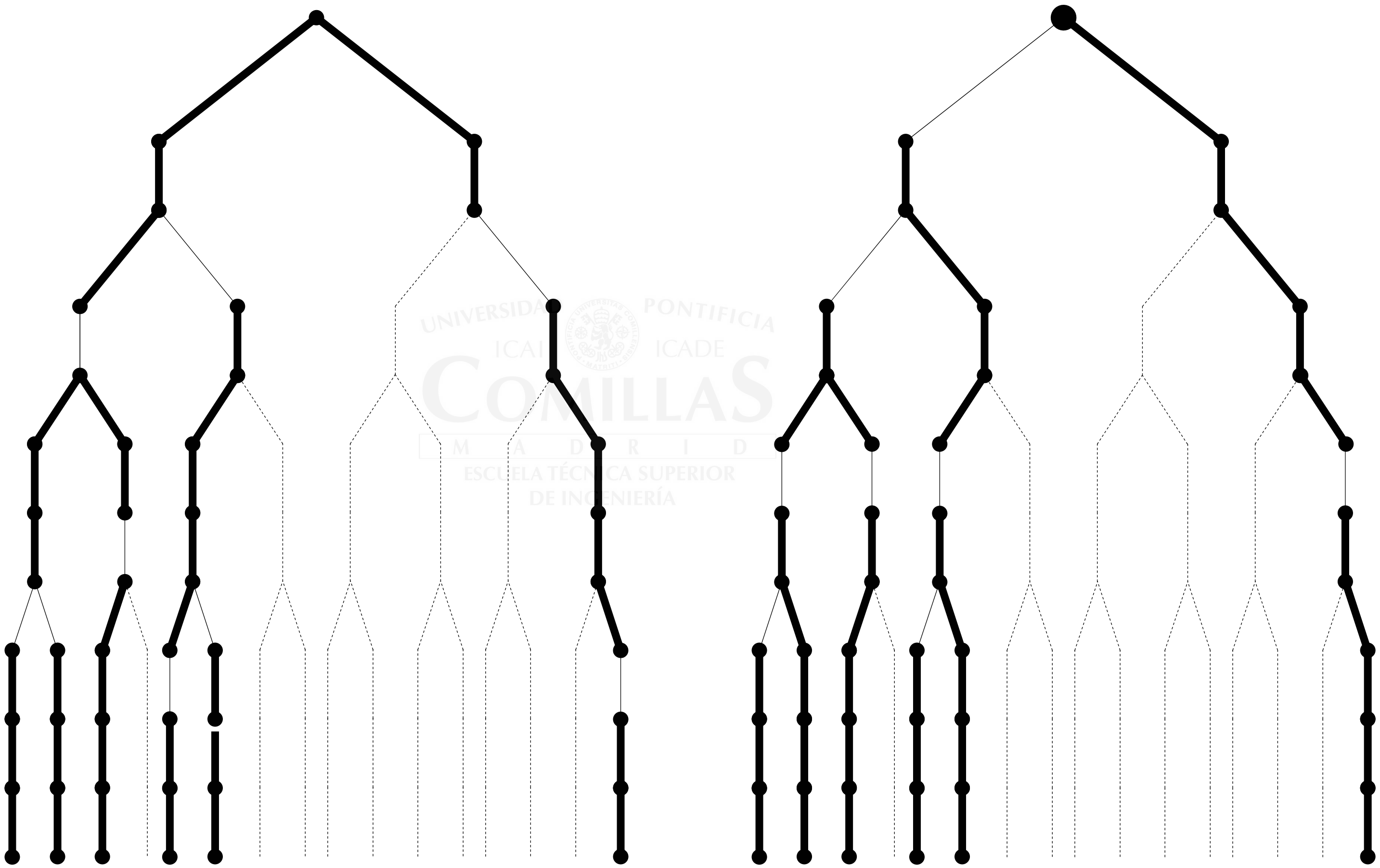
Protocols 3 and 4. Aggregation allowing monocut



Protocols 3 and 4. Numerical results

	Agregación desde el primer periodo		Agregación desde el último periodo	
	Convergencia	Tiempo (s)	Convergencia	Tiempo (s)
1ª iteración	0.008248	155.6	0.004031	194.2
2ª iteración	0.004115	195.6	0.002213	251.3
3ª iteración	0.001620	240.2	0.000719	305.7
4ª iteración	0.001271	278.5	0.000200	341.1
5ª iteración	0.000868	317.1	0.000192	371.3
6ª iteración	0.000824	355.2	0.000079	398.0
7ª iteración	0.000561	388.4		
8ª iteración	0.000522	426.8		
9ª iteración	0.000336	460.6		
10ª iteración	0.000253	500.5		
Tiempo total (s)	2484		1562	
Tiempo solución (s)	555		453	

Potocols 5 and 6. Aggregation without posibility of using monocut



Protocols 5 and 6. Numerical results

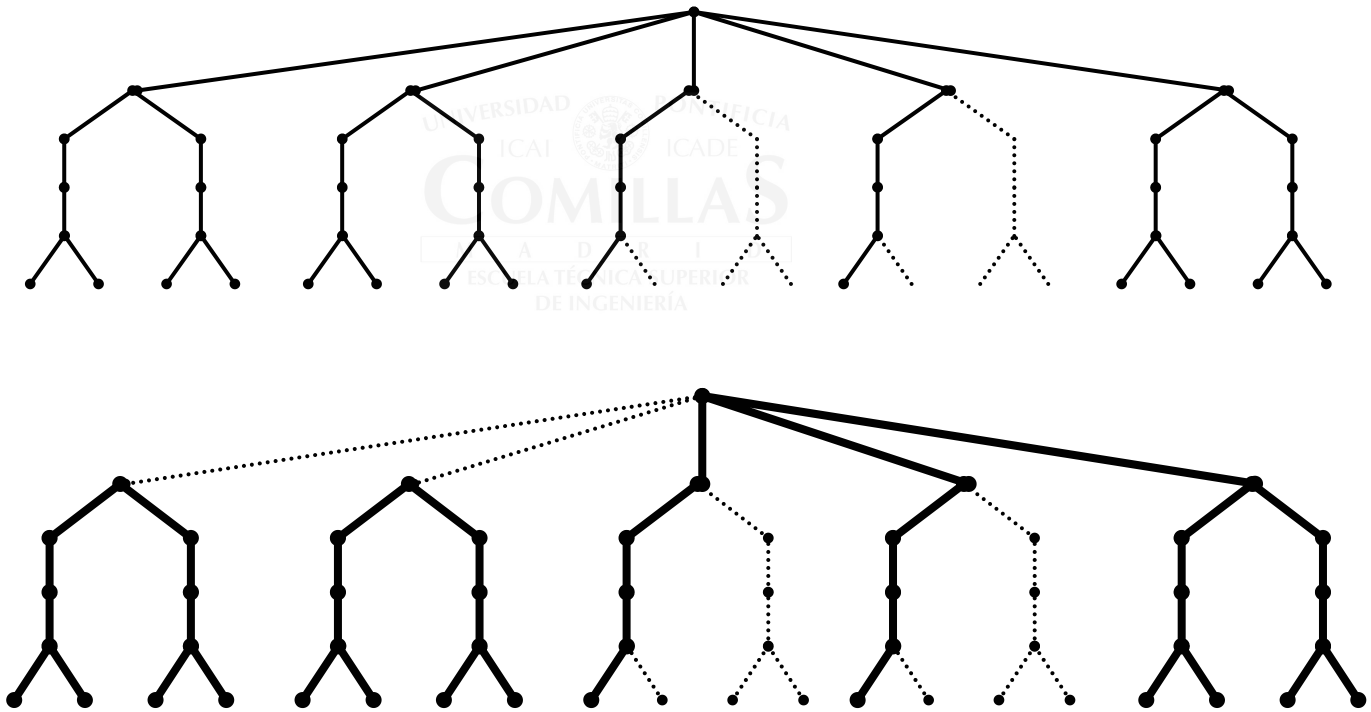
	Agregación desde el primer periodo		Agregación desde el último periodo	
	Convergencia	Tiempo (s)	Convergencia	Tiempo (s)
1ª iteración	0.041237	190.8	0.001612	210.9
2ª iteración	0.033394	254.1	0.000871	272.2
3ª iteración	0.009788	316.4	0.000384	333.3
4ª iteración	0.003276	370.0	0.000141	386.3
5ª iteración	0.001381	412.0	0.000131	431.1
6ª iteración	0.000935	448.4	0.000074	471.1
7ª iteración	0.000662	482.4		
8ª iteración	0.000433	518.6		
9ª iteración	0.000359	551.8		
10ª iteración	0.000227	582.3		
Tiempo total (s)	2542		1662	
Tiempo solución (s)	637		526	

Different Protocols Results

Protocol	#Subtrees	#iter	Converg	Sol. time	Total time
1	14	10	0.000470	442	2923
2	13	8	0.000094	406	1966
3	11	10	0.000206	420	2207
4	11	8	0.000041	428	1801
5	10	10	0.000198	499	2130
6	9	6	0.000060	449	1450

Example V

- Deterministic equivalent Problem presents 324121 rows, 432322 columns y 1313216 nonzero elements.



Results using protocol 6

- Node Aggregation limiting the size subtree up to different number of equations.

Maximum equations	#Sub	#iter	Convergence	Exec. time (s)	Total time (s)
10000	44	8	0.000089	1384	11595
20000	18	5	0.000098	1109	4055
40000	10	3	0.000037	1105	2275
80000	4	2	0.000031	1260	1889
160000	3	1	0.000037	1121	1583
240000	OUT OF MEMORY				

Computer characteristics

- Pentium II Processor at 233 MHz
- 128 MB of RAM memory

Conclusions

- Model of a stochastic large-scaled problem solved via nested Benders decomposition algorithm.
- Flexibility of GAMS provides us the possibility of testing different ways of decomposing the problem.
- Subproblems can be any subtree of the scenario tree.
- We observe the necessity of forming big subtrees or equivalent dividing the scenario tree into the smallest possible number of subtrees.
- We have tested different aggregation methodologies for the nodes of the scenario tree.
- Resolution time improves when considering aggregation protocols that begin with those nodes at the bottom of the tree.