Mathematical programming approach to underground timetabling for maximizing the use of regenerative braking power

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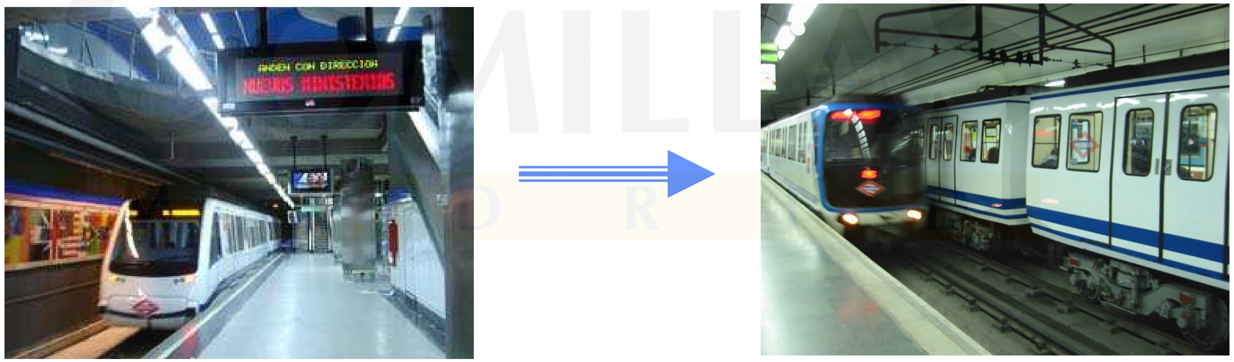
- Model description
- Case study
- Conclusions

Motivation

- Saving energy in underground operations is important. The energy cost represents 10% of the operational cost.
- Synchronization of speed-up and slow-down processes of two trains allows energy exchange among trains fed from the same electrical section



- In peak hours this process is more probable due to the high frequency of trains. In off-peak hours (11pm-2am) train synchronization is achieved by changing the timetable

Train timetabling problem

- Objective
  - Determine a new timetable to maximize overlapping time during off-peak hours (11pm-2am)
- Highly combinatorial nature
  - Time coincidence detected for every train with every other train located in the same electrical section “at any time”
- Solving techniques used:
  - Mathematical programming (Lagrangean relaxation, direct solution)
  - Metaheuristic techniques (genetic algorithms)
  - Hybrid approach
  - Constraint programming

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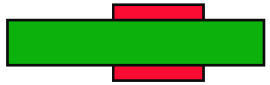
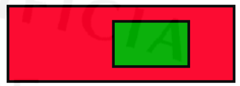
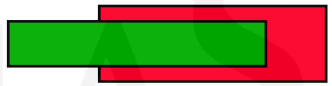
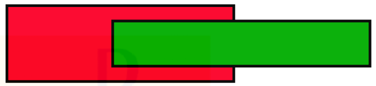
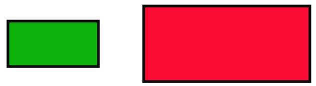
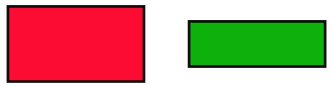
Optimization problem

- Objective function
  - Maximize overlapping time between speed-up and slow-down actions of any train pair fed from the same electrical section
- Constraints
  - Stopping time at each platform is upper and lower bounded
  - Run time between consecutive platforms is bounded
  - Upper bound for delays from the commercial schedule
  - Departure time of any train in any platform must be greater than the departure time published as the commercial schedule
  - Computation of coincidence time
- Variables
  - Arrival and departure time of the trains
  - Overlapping detection (binary)

Coincidence time detection

- Six possibilities (slow-down, speed-up)
  - Slow-down process while speeding-up 
  - Speed-up process while slowing-down 
  - Departure before arrival 
  - Arrival before departure 
  - Beginning of slow-down process after departure 
  - Beginning of speed-up process after arrival 

Genetic algorithm

- A solution is represented as a matrix with entry 1 if the trains considered share energy in two given platforms
  - The algorithm includes an LP phase where the timetable is obtained as the local optimum with these binary variables
- Population at time 0
  - The algorithm starts with a solution provided by MIP (it is very difficult to obtain feasible solutions with genetic algorithms). Other initial solutions are obtained by mutation
- Operators
  - The mutation operator takes the best timetable obtained and changes some entries.
  - The crossover operator is applied to two timetables (randomly selected from a population of ten timetables)
- Parameters
  - Number of mutations and probability of mutation

Model implementation

- Spreadsheet-based graphical input/output interface
- GAMS-based optimization model
- LP/MIP solver CPLEX 11.0
- Hybrid approach
  - MIP solver to obtain the initial population
  - Genetic algorithm to obtain new solutions for the LP solver
  - LP solver to evaluate each solution

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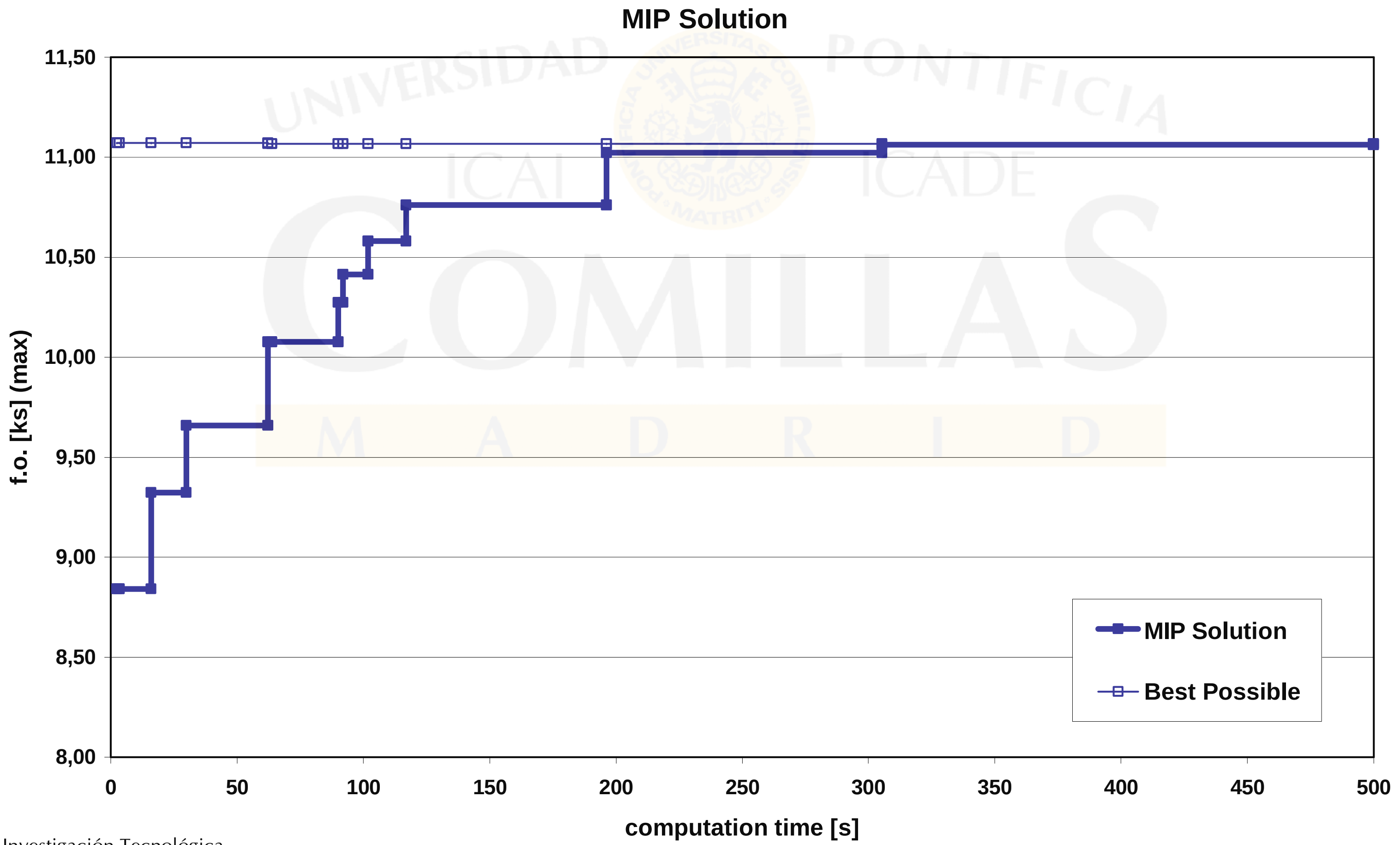
Case study: line 3 of Metro de Madrid

- Overlapping time for the initial schedule
  - 2.0 hours (53% slow-downs)
- Overlapping time keeping published timetable (maximum delay bounded) (computation time: 100s)
  - 2.9 hours (78% slow-downs)

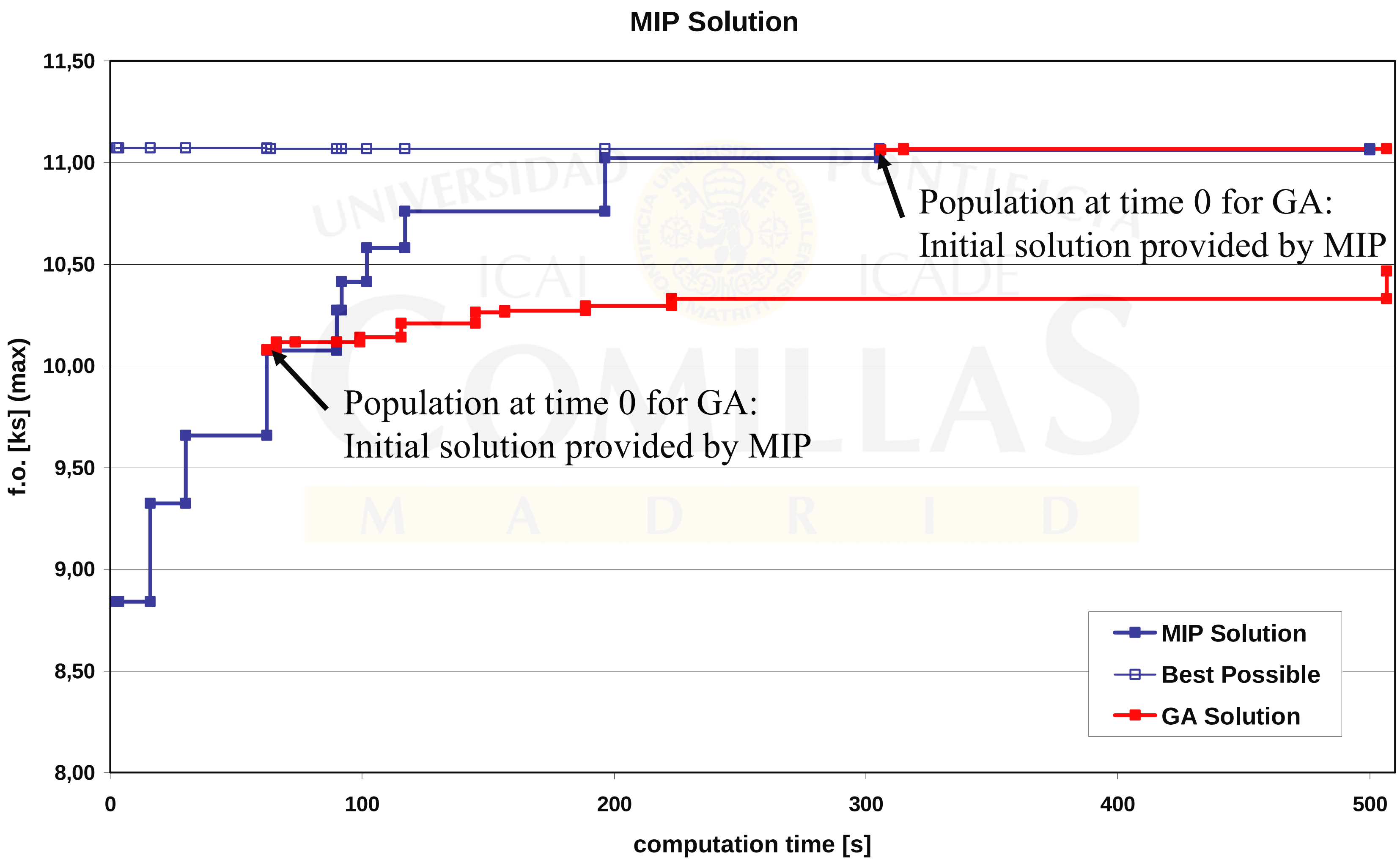
Difficult to solve MIP optimization problem

- 9370 constraints, 6420 continuous variables and 2120 binary variables (real case of line 3, Metro de Madrid)
- Synchronization time obtained for different computation times:



Little improvements of the solutions with GA

- Local improvements of the solutions (only mutation operator)

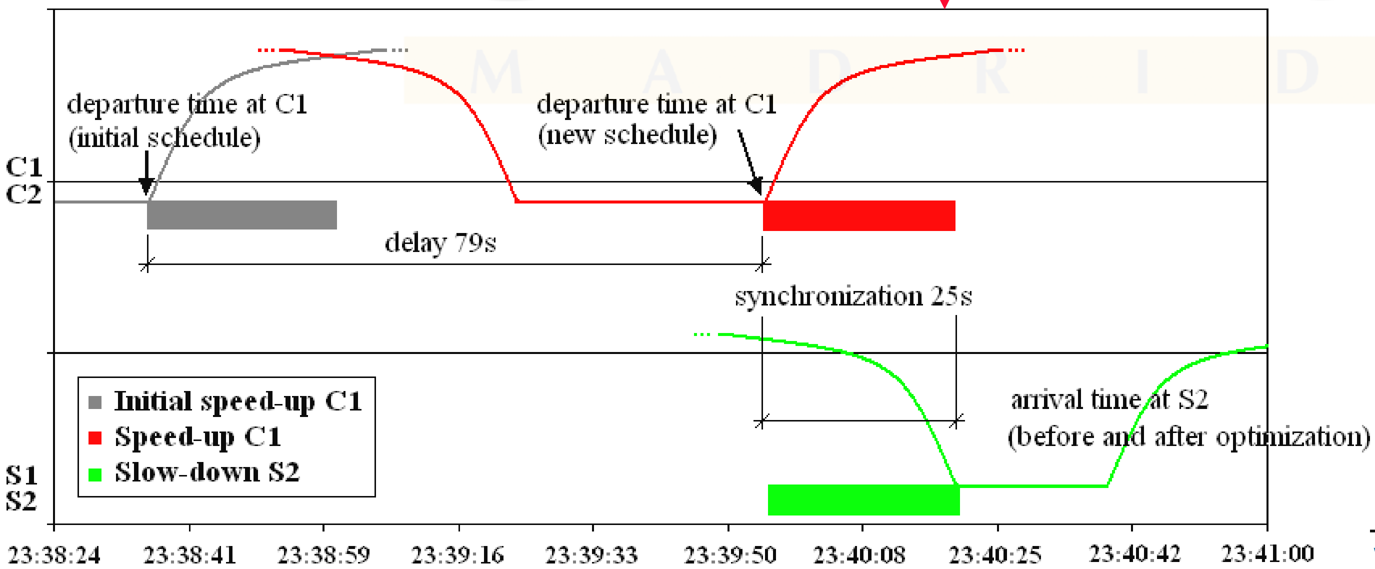


Coincident trains

non-synchronized processes (white)

trains synchornized in the same colour

Calculated Schedule		Platform													
Trains	Event					VR1	AR1	M1	M2	AR2	VR2	PE2	C2	S2	LV2
N1	Arrival	2				3:27:55	23:29:27	23:30:49	23:04:34	23:05:37	23:06:48	23:07:57	23:09:15	23:11:05	23:13:15
	Departure	2				3:28:31	23:29:58	23:31:00	23:04:53	23:05:56	23:07:07	23:08:16	23:10:14	23:11:49	23:13:34
N2	Arrival	23:28:29	23:30:27	23:31:55	23:33:23	23:34:49	23:36:10	23:37:29	23:10:39	23:12:14	23:13:44	23:15:16	23:16:49	23:18:49	23:20:40
	Departure	23:28:56	23:31:00	23:32:19	23:33:56	23:35:16	23:36:38	23:38:03	23:11:28	23:12:50	23:14:24	23:15:48	23:17:59	23:19:14	23:21:00
N3	Arrival	23:35:58	23:38:12	23:39:24	23:40:59	[illegible]	23:43:46	23:45:03	23:18:49	23:19:58	23:21:25	23:22:42	23:24:13	23:25:20	23:27:21
	Departure	23:36:43	23:38:31	23:39:55	23:41:40	23:42:52	23:44:12	23:45:15	23:19:14	23:20:31	23:21:50	23:23:14	23:24:32	23:25:53	23:28:31
N4	Arrival	23:43:17	23:45:56	23:47:32	23:48:55	23:50:24	23:51:55	23:53:31	23:26:18	23:27:40	23:28:56	23:30:05	23:31:25	23:33:28	23:35:23
	Departure	23:44:26	23:46:37	23:47:51	23:49:31	23:50:59	23:52:40	23:53:42	23:26:56	23:28:04	23:29:15	23:30:24	23:32:38	23:33:55	23:35:45
N5	Arrival	23:50:59	23:53:10	23:54:37	23:56:04	23:57:26	23:58:45	23:59:55	23:33:28	23:34:30	23:35:41	23:37:12	23:38:53	23:40:20	23:42:05
	Departure	23:51:38	23:53:42	23:54:59	23:56:35	23:57:51	23:59:04	23:59:54	23:33:47	23:34:49	23:36:20	23:37:54	23:39:32	23:40:39	23:43:17
N6	Arrival	23:58:21	24:01:00	24:02:16	24:03:37	24:05:31	24:06:45	24:07:55	23:40:59	23:42:19	23:43:42	23:44:51	23:46:32	23:47:58	23:49:56
	Departure	23:59:30	24:01:21	24:02:35	24:04:40	24:05:50	24:07:04	24:08:23	23:41:33	23:42:50	23:44:01	23:45:31	23:47:07	23:48:30	23:51:02
N7	Arrival	24:06:05	24:07:55	24:09:07	24:10:47	24:12:10	24:13:32	24:14:43	23:48:16	23:50:13	23:51:24	23:52:33	23:54:07	23:56:17	23:58:02
	Departure	24:06:24	24:08:14	24:09:43	24:11:19	24:12:36	24:13:51	24:14:42	23:49:29	23:50:32	23:51:43	23:53:06	23:55:28	23:56:36	23:58:21



Gantt Diagram

(detail of the synchronization achieved for the train coloured in blue)

Time differences among timetables

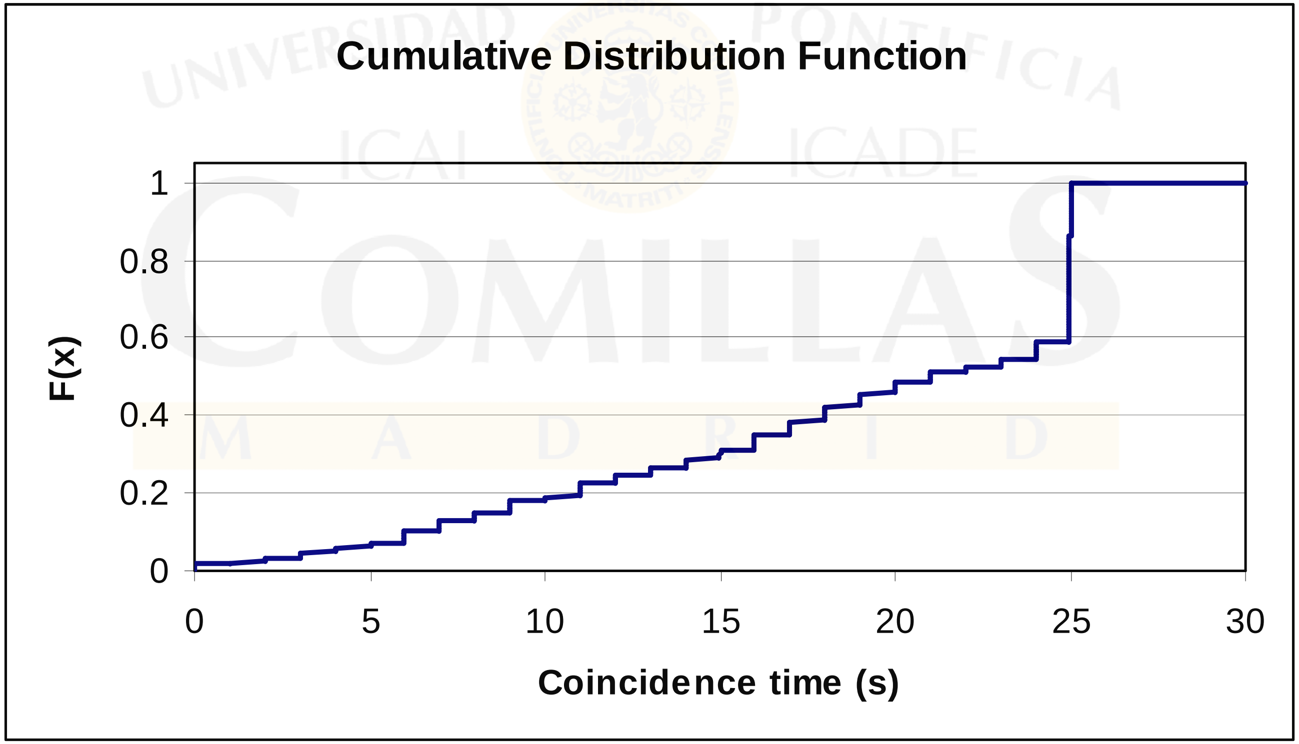
- Delays of the designed timetable from commercial schedule
- Train with speed-ups or slow-downs synchronized are coloured

Differences		Platforms																																			
Trains	Events	VA1	SC1	VC1	CI1	SF1	DO1	AL1	L1	DL1	PF1	E1	LV1	S1	C1	PE1	VR1	AR1	M1	M2	AR2	VR2	PE2	C2	S2	LV2	E2	PF2	DL2	L2	AL2	DO2	SF2	CI2	VC2	SC2	VA2
N1	Arrival																																				
	Departure								118	119	84		75	74	88	62	91	118	120	113	116	67	76	74	109	94		92	84	73	68						120
N2	Arrival																																				
	Departure							76	79		117	90							63	88		94	108	119	74	60	77	115	72		71		78	79	77		75
N3	Arrival																																				
	Departure					72			69	100	63		103	91	79			72	75	74	91		74			91	62	104	63	73	68						95
N4	Arrival																																				
	Departure						73	63	64		102	77	86	97	111	91		100	102	116	64	75	84	98			64										120
N5	Arrival																																				
	Departure		77	83	71	72							98	102				64								77		92	97	73	68						120
N6	Arrival																																				
	Departure							65	68		99	83	90	81	95	100		64	83	93		61	91	67		62	105	86			96	99	119	116	114	95	103
N7	Arrival																																				
	Departure											65	84	74						89	92		66	88	96	81		113	72	100	95	82					120
N8	Arrival																																				
	Departure							87	90	67	88	63						88	84	77		71	95	88			90	71		63			63	64			74
N9	Arrival																																				
	Departure		118	120	107	98			98	77			69				63	85	110	106	75	91		63						78	75	91					119
N10	Arrival																																				
	Departure		63	68								73	92	80				70	88	83									62	100	97	111	72	61			120
N11	Arrival																																				
	Departure								98	72	85	60	79	67			62	84	111	114		99		71				99	120	94	89	106	67				120
N12	Arrival																																				
	Departure														63		63	85	106	99		89		60						66	62	82					119
N13	Arrival																																				
	Departure													11	63				66	80		71								73	75	89					118
N14	Arrival																																				
	Departure													84					84	87		67						71	90	64							119
N15	Arrival																																				
	Departure	61	93	76															120									68									68

- Coincidences < 10s
- Coincidences between 10 and 15s
- Coincidences between 15 and 20s
- Coincidences > 20s

Cumulative distribution function of overlapping time

- Numerous overlaps of 25 seconds (bound defined by the user). Further improvement can be achieved.



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Conclusions

- Decision support tool that can be used for maximizing the overlapping time between the slow-down and speed-up processes
- Large potential energy savings
- Difficult to solve MIP optimization problem
- Little improvements of the solutions with GA
- Optimal or quasioptimal solutions can be obtained in a reasonable amount of time
- Most of the overlapping time can be obtained with no changes in the published timetable
- Model can be extended to consider several lines or a wider time scope

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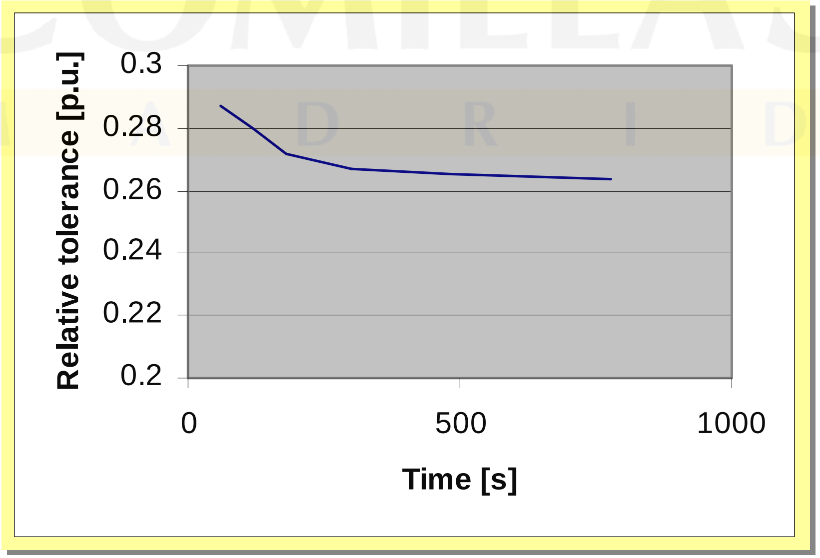
Three possible uses

- Evaluation of overlapping time for the initial schedule
- Maximize overlapping time while keeping current commercial timetable
- Maximize overlapping time allowing changes in arrival and departure times

Very difficult to solve MIP optimization problem

- 7700 constraints, 4200 continuous variables and 600 binary variables (study case of line 1, Metro de Madrid)

	60 s	120 s	180 s	300 s	480 s	780 s
MIP solution [s]	4909.645	4932.533	4957.533	4972.661	4972.587	4972.661
LP Relaxation [s]	6320.811	6312.254	6305.047	6298.265	6290.912	6285.199
Relative Tolerance [p.u.]	0.287427	0.279718	0.271811	0.266578	0.265119	0.263951
Iterations	136564	283620	452945	708080	1171531	1886725
Nodes	18701	38101	60701	95301	157401	247801



Case study: line 1 of Metro de Madrid

- Overlapping time for the initial schedule
  - 0 seconds
- Overlapping time keeping the advertised/ published timetable (departure times)
  - 1.30 hours
- Overlapping time optimizing arrival and departure times
  - 1.36 hours

Coincident trains

Calculated Schedule		Stations											
Trains	Events	JA1	S1	TM1	AM1	AT1	AR1	AR2	AT2	AM2	TM2	S2	JA2
N1	Arrival	23:19:00	23:19:55	[illegible]	23:22:45	23:24:40	23:25:35	24:06:40	24:07:35	24:08:30	24:10:40	24:12:35	24:14:00
	Departure	23:19:10	23:2?:05	23:22:00	23:22:55	23:24:50	23:25:45	24:06:50	24:07:45	24:08:55	[illegible]	24:13:15	24:14:10
N2	Arrival	23:26:00	23:27:3?	[illegible]	23:30:45	23:31:39	23:32:34	24:14:05	24:15:00	24:1?:55	24:17:50	24:19:45	24:20:40
	Departure	23:26:10	23:28:05	23:29:00	23:30:55	23:31:50	23:33:08	24:14:15	24:15:10	24:17:0?	[illegible]	24:19:55	24:20:50
N3	Arrival	23:34:28	23:35:24	23:37:18	23:38:13	23:40:09	23:41:04	24:28:41	24:30:06	24:32:01	24:32:56	24:34:51	24:35:46
	Departure	23:34:39	23:35:34	23:37:29	23:38:24	23:40:19	23:41:19	24:29:21	24:30:15	24:32:10	24:33:06	24:35:01	24:35:55
N4	Arrival	23:41:29	23:43:24	23:44:19	23:46:14	23:47:09	23:48:04	24:43:48	24:45:13	24:47:08	24:48:03	24:49:58	24:50:53
	Departure	23:41:39	23:43:34	23:44:29	23:46:24	23:47:19	23:48:19	24:44:27	24:45:23	24:47:18	24:48:13	24:50:08	24:51:03
N5	Arrival	23:49:59	23:50:55	23:52:55	23:53:49	23:55:45	23:56:40	24:57:58	24:59:23	25:01:30	25:02:25	25:04:20	25:05:15
	Departure	23:50:09	23:51:10	23:53:05	23:53:59	23:55:55	23:56:50	24:58:38	24:59:45	25:01:40	25:02:35	25:04:30	25:05:25
N6	Arrival	23:56:40	23:58:35	23:59:30	24:01:25	24:02:20	24:03:20	25:13:15	25:14:40	25:16:36	25:17:31	25:19:26	25:20:21
	Departure	23:56:5?	[illegible]	23:59:40	24:01:35	24:02:35	24:03:30	25:13:55	[illegible]	25:16:45	25:17:41	25:19:36	25:20:30
N7	Arrival	24:0?:20	24:05:15	24:07:10	24:08:05	24:10:00	24:11:10	23:20:?5	23:22:15	23:?3:10	23:25:05	23:27:00	23:28:20
	Departure	24:04:2?	[illegible]	24:07:20	24:08:15	24:10:25	24:11:50	23:21:30	[illegible]	23:23:20	23:25:15	23:27:35	23:28:30
N8	Arrival	24:11:10	24:13:35	24:14:30	24:16:25	24:17:20	24:18:15	23:28:00	23:29:13	23:31:09	23:32:03	23:33:58	23:34:54
	Departure	24:11:50	24:13:45	24:14:40	24:16:?5	24:17:30	2?:18:50	23:28:29	23:29:24	23:31:19	23:32:14	23:34:09	23:35:04
N9	Arrival	24:26:50	24:28:45	24:29:40	24:31:3?	24:32:30	24:33:25	23:36:18	23:37:42	23:38:37	23:40:34	23:42:28	23:43:50
	Departure	24:27:00	24:28:55	24:29:49	24:31:44	24:32:40	24:33:34	23:36:58	23:37:53	23:38:49	23:40:44	23:43:04	23:44:00
N10	Arrival	24:41:57	24:43:52	24:44:47	24:46:42	24:47:37	24:48:32	25:28:30	25:29:47	25:31:42	25:32:37	25:34:32	25:35:27
	Departure	24:42:06	24:44:01	24:44:57	24:46:52	24:47:47	24:48:42	25:29:01	25:29:57	25:31:52	25:32:47	25:34:42	25:35:37
N11	Arrival	24:56:08	24:58:03	24:58:58	25:00:53	25:01:55	25:02:55	25:49:50	25:51:15	25:52:10	25:54:14	25:56:09	25:57:04
	Departure	24:56:18	24:58:13	24:59:08	25:01:10	25:02:10	25:03:35	25:50:30	25:51:25	25:52:29	25:54:24	25:56:18	25:57:14
N12	Arrival	25:11:25	25:13:20	25:14:15	25:16:10	25:17:05	25:18:00	23:43:34	23:44:44	23:46:39	23:47:34	23:49:28	23:50:25
	Departure	25:11:35	25:13:30	25:14:24	25:16:19	25:17:15	25:18:09	23:43:59	23:44:54	23:46:49	23:47:44	23:49:39	23:50:35
N13	Arrival	25:27:27	25:29:22	25:30:17	25:32:12	25:33:07	25:34:02	23:51:30	23:52:25	23:53:19	23:55:15	23:57:10	23:58:05
	Departure	25:27:36	25:29:31	25:30:27	25:32:22	25:33:17	25:34:12	23:51:40	23:52:35	23:53:29	23:55:25	23:57:19	23:58:15
N14	Arrival	25:47:45	25:48:45	25:50:45	25:51:45	25:53:45	25:54:44	23:58:45	23:59:45	24:01:45	24:02:45	24:0?:45	24:05:45
	Departure	25:48:00	25:49:00	25:51:00	25:52:00	25:54:00	25:55:00	23:59:00	24:00:00	24:02:00	24:0?:00	24:05:00	?4:06:00

Coincidence			
20	20	15	16
15	20	15	20
20	14	15	20
15	20	15	10
20	20	20	20
15	20	19	16
5	15	20	15
20	16	20	15
15	17	16	15
15	20	16	20
14	20	13	16
20	19	14	20
20	20	13	20
15	20	15	20
15	20	20	15
20	17	20	20
19	17	20	20
19	20	20	8
19	20	13	20
15	16	19	15
10	20	10	20
20	15	16	10
20	20	13	16

Time differences between timetables

Differences		Stations																																																			
Trains	Events	PC	VA1	TE1	E1	AL1	CC1	RR1	I1	B1	T1	JA1	S1	TM1	AM1	AT1	AR1	MP1	P1	V1	NN1	PO1	BA1	AA1	MH1	SG1	VV1	COM	CO	VV2	SG2	MH2	AA2	BA2	PO2	NN2	V2	P2	MP2	AR2	AT2	AM2	TM2	S2	JA2	T2	B2	I2	RR2	CC2	AL2	E2	TE2	VA2	PCM
N1	Arrival			-5	-11	-15	-20	-25	-30	-35	-40	-45	-50	-55	-60	-65	-70	-75	-81	-56	-62	-67	-71	-71	-76	-50	-26				25	29	24	19	15	10	5		-5	-10	-15	-20	-25	-30	-10	-15	-20	-5		-5	-10	-16	-20	-25	
	Departure		-5	-10	-16	-20	-25	-30	-35	-40	-45	-50	-55	-60	-65	-70	-75	-80	-56	-61	-66	-71	-70	-75	-50	-25				25	29	24	19	14	10	5		-5	-10	-15	-20	-25	-30	-10	-15	-20	-5		-5	-10	-15	-20	-25		
N2	Arrival			-5	-10	-15	-20	-25	-30	-35	-40	-45	-50	-56	-60	-66	-71	-52	-53	-28	-32	-37	-42	-47	-51	-29	-25				-5	-10	-15	-5	-10	-15	-20	-20	-25	-30	-15	10	5		-5	-10	-16	-20	-25	-30	-35	-40	-45	-25	
	Departure		-5	-10	-15	-20	-25	-30	-35	-40	-45	-50	-55	-60	-65	-70	-52	-52	-27	-32	-37	-42	-47	-51	-29	-25				-5	-10	-15	-5	-10	-15	-20	-20	-25	-30	-15	10	5		-5	-10	-15	-20	-26	-31	-36	-40	-45	-26		
N3	Arrival			-5	-10	-15	-20	-25	-31	-35	-41	-17	-21	-27	-32	-36	-41	-41	-46	-26	-31	-37	-41	-25	-31	-35	-25				-5	-10	-15	5		-5	-10	-15	-19	-5	20	16	11	6	1	-4	-9	-14	-19	-24	-29	-34	-39	-14	
	Departure		-5	-10	-15	-20	-25	-30	-35	-40	-16	-21	-26	-31	-36	-41	-41	-46	-26	-31	-36	-42	-25	-30	-36	-25				-5	-10	-15	5		-5	-10	-15	-20	-5	20	15	10	6	1	-5	-10	-15	-19	-24	-29	-34	-39	-14		
N4	Arrival			-6	-10	-16	7	4	-2	-7	-11	-16	-21	-26	-31	-36	-41	-41	-46	-31	-35	-35	-11	-16	-20	5					-5	-10	-15	-20	-25	-30	-35	-40	-22	3	28	23	18	13	8	3	-2	-7	-12	-17	-22	-27	-32	-7	
	Departure		-5	-10	-15	8	4		-6	-11	-16	-21	-26	-31	-36	-41	-41	-46	-30	-36	-35	-11	-16	-20	5					-5	-11	-15	-21	-26	-30	-36	-40	-22	2	27	23	18	13	8	3	-2	-7	-12	-17	-22	-27	-32	-7		
N5	Arrival			-6	-11	-11	-16	8	20	24	19	14	10	10	4		-5	-10	-15	-20	-25	-30	-35	-40	-45	-50	-25				-5	-10	-15	-20	-25	-30	-35	-40	-45	-47	-22	-15	-20	-25	-30	-35	-41	-45	-50	-55	-46	-51	-50	-25	
	Departure		-5	-10	-11	-16	8	20	24	19	15	9	10	5	-1	-5	-10	-15	-20	-25	-30	-35	-40	-45	-50	-25				-6	-10	-15	-20	-25	-30	-35	-40	-45	-47	-22	-15	-20	-25	-30	-35	-40	-45	-51	-56	-47	-51	-50	-26		
N6	Arrival			-5	-10	10	14	14	9	3		-5	-10	-15	-20	-25	-30	-35	-40	-45	-50	-55	-60	-35	-15	10	5				-5	-10	-15	-20	-25	-30	-35	-40	-41	-30	-5	-9	-14	-19	-24	-29	-34	-39	-44	-49	-54	-59	-34	-25	
	Departure		-5	-10	10	15	13	9	4	-1	-5	-10	-15	-20	-25	-30	-35	-40	-45	-50	-55	-60	-35	-15	10	5				-5	-10	-15	-20	-25	-30	-35	-40	-42	-30	-5	-10	-15	-19	-24	-30	-35	-40	-44	-49	-54	-59	-34	-25		
N7	Arrival			11	5		-4	-9	-5	-10	-15	-20	-25		5	5	5		-5	-10	-15	-20	-25	-30	-35	-40	-25				-5	-10	-15	-20	-25	-30	-35	-40	-45	-50	-30	-35	-40	-45	-25	-30	-11	-17	-21	-26	-32	-36	-41	-25	
	Departure		11	6		-4	-9	-5	-10	-15	-20	-26		5	5	5		-5	-10	-15	-20	-25	-30	-35	-40	-25				-5	-10	-15	-20	-26	-30	-35	-40	-45	-50	-30	-35	-40	-45	-25	-30	-11	-16	-21	-26	-31	-36	-41	-24		
N8	Arrival			-5	-10	-15	-20	-24	-29	-25	-30	-35	-35	-40	-45	-20	-25		25	20	15	10	5		-5	10	5				-5	-10	-15	-20	-25	-30	-35	-35	-40	-45	-32	-36	-42	-47	-51	-57	-36	-41	-17	-5	-11	-15	-20	-25	
	Departure		-5	-10	-15	-20	-25	-29	-25	-30	-35	-35	-40	-45	-20	-25		25	20	15	9	5		-6	9	4				-5	-10	-15	-20	-25	-30	-35	-35	-40	-45	-31	-36	-41	-46	-51	-56	-36	-41	-16	-6	-10	-16	-20	-25		
N9	Arrival			4		25	20	25	20	15	9	5		-5	-10	-15	-20	-25		-6	8	3	-2	-7	-12	10	5				-5	-10	-15	-20	-26	-26	-31	-37	-42	-27	-3	-8	-11	-17	5		-5	-10	-12	-16	-21	-25	-30	-25	
	Departure		4		25	20	25	20	15	10	5		-5	-11	-16	-20	-26	-2	-7	8	3	-3	-8	-12	10	5				-5	-10	-15	-20	-25	-26	-31	-36	-41	-27	-2	-7	-11	-16	4		-6	-10	-12	-16	-20	-26	-30	-25		
N10	Arrival			-4	-9	-14	-19		25	20	16	12	7	2	-3	-8	-13	-18		-5	-10	-15	-20	-25	-15	10	5				-5	-10	-15	-20	-25	-26	-31	-35	-40	-23	2	-3	-8	-13	-18	-23	-28	-25	-30	-35	-40	-45	-50	-25	
	Departure		-4	-10	-15	-19		24	19	16	11	6	1	-3	-8	-13	-18		-5	-10	-15	-20	-25	-15	10	5				-5	-11	-15	-21	-26	-26	-32	-35	-40	-24	1	-3	-8	-13	-18	-23	-28	-25	-30	-35	-40	-45	-50	-25		
N11	Arrival			-5	-10	-9	-14	-19	-24	-27	-32	-37	-42	-47	-52	-50	-50	-25		-5	-10	-15	-20	-25	-15	10	5				25	25	20	15	17	12	7	2	-3	22	35	34	29	24	19	14	9	4		-6	-11	-16	-20	5	
	Departure		-6	-10	-9	-14	-19	-24	-27	-32	-37	-42	-47	-52	-50	-50	-25		-5	-10	-16	-20	-25	-16	9	4				25	25	20	15	17	12	7	2	-3	22	35	34	29	24	18	14	9	3	-2	-7	-11	-16	-21	4		
N12	Arrival			-5	-10	-2	-7	-12	-5	-10	-16	-20	-25	-30	-35	-40	-45	-50	-55	-60	-65	-70	-75	-75	-50	-25	-25				-2	-7	18	14	14	8	3	-1	-6	-11		-6	-11	-17	-20	-25	-26	-30	-34	-30	-35	-40	-45	-20	
	Departure		-5	-10	-2	-7	-12	-5	-10	-15	-20	-25	-30	-36	-41	-45	-51	-56	-61	-65	-70	-76	-76	-50	-25	-25					-6	19	14	14	9	4		-6	-11		-6	-11	-16	-21	-25	-26	-30	-34	-31	-35	-40	-45	-20		
N13	Arrival			20	45	40	35	30	27	52	47	42	37	32	27	22	17	12	7	2	-3	-8	-13	-18	-23	-28	-3				-6	-10	-10	-16	-16	-22		-5	-11	-15	-20	-26	-30	-35	-40	-45	-50	-45	-50	-55	-50	-55	-50	-25	
	Departure		20	44	39	35	30	26	51	47	41	36	31	27	22	17	12	7	2	-3	-8	-13	-18	-23	-28	-3				-5	-10	-10	-15	-17	-21	-1	-5	-10	-16	-20	-25	-31	-35	-41	-45	-50	-45	-50	-55	-50	-55	-50	-25		
N14	Arrival																																																						
	Departure																																																						

Cumulative distribution function of overlapping time

- Numerous overlaps of 20 seconds (bound defined by the user). Further improvement can be achieved.