Mathematical programming approach to underground timetabling problem for maximizing time synchronization

A. Ramos, M.T. Peña, A. Fernández, P. Cucala

ETSI de Ingeniería - ICAI

Universidad Pontificia Comillas

Contents

Motivation

- Saving energy in underground operations is important. Regenerative brakes generate electricity when slowing-down
- Synchronization of speed-up and slow-down processes of two trains allows energy exchange among trains fed from the same electrical section

- In peak hours this process is more probable due to the high train frequency. In off-peak hours train synchronization is made by changing the timetable

Train timetabling problem

- Objective
  - Determine a new timetable to maximize overlapping time
- Highly combinatorial nature
  - Time coincidence detected for every train with every other train located in the same electrical section “at any time”
- Solving techniques used:
  - Mathematical programming (Lagrangean relaxation, direct solution)
  - Metaheuristic techniques (genetic algorithms)
  - Hybrid approach
  - Constraint programming

Three possible uses

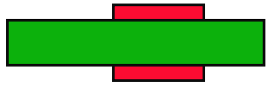
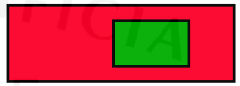
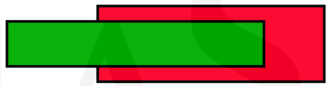
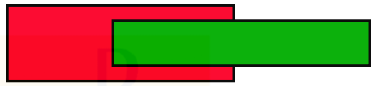
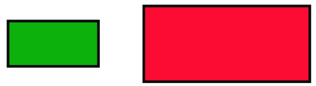
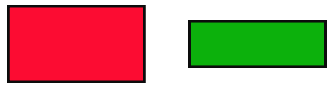
- Evaluation of overlapping time for the initial schedule
- Maximize overlapping time while keeping current advertised timetable (only with departure times)
- Maximize overlapping time allowing changes in arrival and departure times

Contents

Optimization problem

- Objective function
  - Maximize overlapping time between speed-up and slow-down actions of any train pair fed from the same electric section
- Constraints
  - Change in stopping time at each platform wrt initial schedule upper and lower bounded
  - Change in traveling time between consecutive platforms wrt initial schedule upper and lower bounded
  - Change in total trip time in each way wrt initial schedule upper bounded
  - Computation of coincidence time
- Variables
  - Arrival and departure time of the trains
  - Overlapping detection (binary)

Coincidence time detection

- Six possibilities (slow-down, speed-up)
  - Slow-down process while speeding-up 
  - Speed-up process while slowing-down 
  - Departure before arrival 
  - Arrival before departure 
  - Beginning of slow-down process after departure 
  - Beginning of speed-up process after arrival 

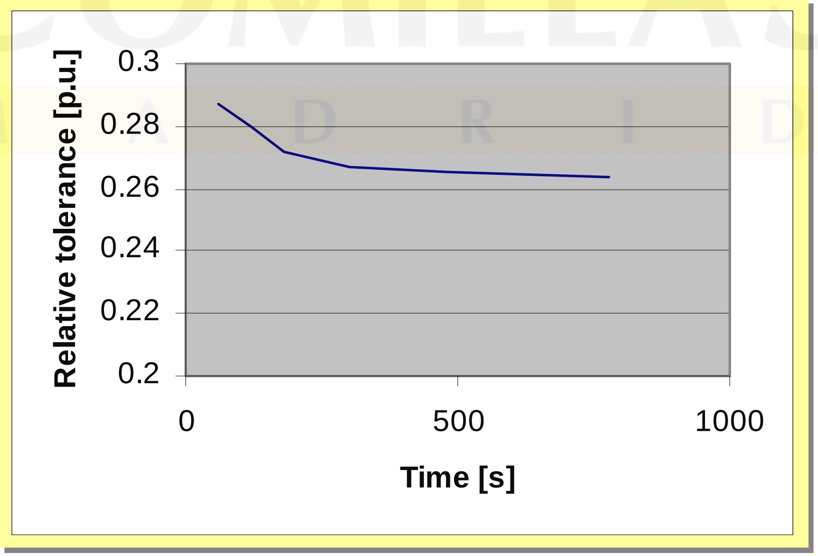
Model implementation

- Spreadsheet-based graphical input/output interface
- GAMS-based optimization model
- MIP solver CPLEX 10.1

Very difficult to solve MIP optimization problem

- 7700 constraints, 4200 continuous variables and 600 binary variables

	60 s	120 s	180 s	300 s	480 s	780 s
MIP solution [s]	4909.645	4932.533	4957.533	4972.661	4972.587	4972.661
LP Relaxation [s]	6320.811	6312.254	6305.047	6298.265	6290.912	6285.199
Relative Tolerance [p.u.]	0.287427	0.279718	0.271811	0.266578	0.265119	0.263951
Iterations	136564	283620	452945	708080	1171531	1886725
Nodes	18701	38101	60701	95301	157401	247801



Contents

Case study: line 1 of Metro de Madrid

- Overlapping time for the initial schedule
  - 0 seconds
- Overlapping time keeping advertised timetable (departure times)
  - 1.30 hours
- Overlapping time optimizing arrival and departure times
  - 1.36 hours

Coincident trains

Calculated Schedule		Stations											
Trains	**Events**	**JA1**	**S1**	**TM1**	**AM1**	**AT1**	**AR1**	**AR2**	**AT2**	**AM2**	**TM2**	**S2**	**JA2**
N1	Arrival	23:19:00	23:19:55	23:21:50	23:22:45	23:24:40	23:25:35	24:06:40	24:07:35	24:08:30	24:10:40	24:12:35	24:14:00
	Departure	23:19:10	23:20:05	23:22:00	23:22:55	23:24:50	23:25:45	24:06:50	24:07:45	24:08:55	24:10:50	24:13:15	24:14:10
N2	Arrival	23:26:00	23:27:55	23:28:50	23:30:45	23:31:39	23:32:34	24:14:05	24:15:00	24:16:55	24:17:50	24:19:45	24:20:40
	Departure	23:26:10	23:28:05	23:29:00	23:30:55	23:31:50	23:33:08	24:14:15	24:15:10	24:17:05	24:18:00	24:19:55	24:20:50
N3	Arrival	23:34:28	23:35:24	23:37:18	23:38:13	23:40:09	23:41:04	24:28:41	24:30:06	24:32:01	24:32:56	24:34:51	24:35:46
	Departure	23:34:39	23:35:34	23:37:29	23:38:24	23:40:19	23:41:19	24:29:21	24:30:15	24:32:10	24:33:06	24:35:01	24:35:55
N4	Arrival	23:41:29	23:43:24	23:44:19	23:46:14	23:47:09	23:48:04	24:43:48	24:45:13	24:47:08	24:48:03	24:49:58	24:50:53
	Departure	23:41:39	23:43:34	23:44:29	23:46:24	23:47:19	23:48:19	24:44:27	24:45:23	24:47:18	24:48:13	24:50:08	24:51:03
N5	Arrival	23:49:59	23:50:55	23:52:55	23:53:49	23:55:45	23:56:40	24:57:58	24:59:23	25:01:30	25:02:25	25:04:20	25:05:15
	Departure	23:50:09	23:51:10	23:53:05	23:53:59	23:55:55	23:56:50	24:58:38	24:59:45	25:01:40	25:02:35	25:04:30	25:05:25
N6	Arrival	23:56:40	23:58:35	23:59:30	24:01:25	24:02:20	24:03:20	25:13:15	25:14:40	25:16:36	25:17:31	25:19:26	25:20:21
	Departure	23:56:50	23:58:45	23:59:40	24:01:35	24:02:35	24:03:30	25:13:55	25:14:50	25:16:45	25:17:41	25:19:36	25:20:30
N7	Arrival	24:04:20	24:05:15	24:07:10	24:08:05	24:10:00	24:11:10	23:20:35	23:22:15	23:23:10	23:25:05	23:27:00	23:28:20
	Departure	24:04:25	24:05:25	24:07:20	24:08:15	24:10:25	24:11:50	23:21:30	23:22:25	23:23:20	23:25:15	23:27:35	23:28:30
N8	Arrival	24:11:10	24:13:35	24:14:30	24:16:25	24:17:20	24:18:15	23:28:00	23:29:13	23:31:09	23:32:03	23:33:58	23:34:54
	Departure	24:11:50	24:13:45	24:14:40	24:16:35	24:17:30	24:18:50	23:28:29	23:29:24	23:31:19	23:32:14	23:34:09	23:35:04
N9	Arrival	24:26:50	24:28:45	24:29:40	24:31:35	24:32:30	24:33:25	23:36:18	23:37:42	23:38:37	23:40:34	23:42:28	23:43:50
	Departure	24:27:00	24:28:55	24:29:49	24:31:44	24:32:40	24:33:34	23:36:58	23:37:53	23:38:49	23:40:44	23:43:04	23:44:00
N10	Arrival	24:41:57	24:43:52	24:44:47	24:46:42	24:47:37	24:48:32	25:28:30	25:29:47	25:31:42	25:32:37	25:34:32	25:35:27
	Departure	24:42:06	24:44:01	24:44:57	24:46:52	24:47:47	24:48:42	25:29:01	25:29:57	25:31:52	25:32:47	25:34:42	25:35:37
N11	Arrival	24:56:08	24:58:03	24:58:58	25:00:53	25:01:55	25:02:55	25:49:50	25:51:15	25:52:10	25:54:14	25:56:09	25:57:04
	Departure	24:56:18	24:58:13	24:59:08	25:01:10	25:02:10	25:03:35	25:50:30	25:51:25	25:52:29	25:54:24	25:56:18	25:57:14
N12	Arrival	25:11:25	25:13:20	25:14:15	25:16:10	25:17:05	25:18:00	23:43:34	23:44:44	23:46:39	23:47:34	23:49:28	23:50:25
	Departure	25:11:35	25:13:30	25:14:24	25:16:19	25:17:15	25:18:09	23:43:59	23:44:54	23:46:49	23:47:44	23:49:39	23:50:35
N13	Arrival	25:27:27	25:29:22	25:30:17	25:32:12	25:33:07	25:34:02	23:51:30	23:52:25	23:53:19	23:55:15	23:57:10	23:58:05
	Departure	25:27:36	25:29:31	25:30:27	25:32:22	25:33:17	25:34:12	23:51:40	23:52:35	23:53:29	23:55:25	23:57:19	23:58:15
N14	Arrival	25:47:45	25:48:45	25:50:45	25:51:45	25:53:45	25:54:44	23:58:45	23:59:45	24:01:45	24:02:45	[illegible]	24:05:45
	Departure	25:48:00	25:49:00	25:51:00	25:52:00	25:54:00	25:55:00	23:59:00	24:00:00	24:02:00	24:03:00	24:05:00	24:06:00

Coincidence			
20	20	15	16
15	20	15	20
20	14	15	20
15	20	15	10
20	20	20	20
15	20	19	16
5	15	20	15
20	16	20	15
15	17	16	15
15	20	16	20
14	20	13	16
20	19	14	20
20	20	13	20
15	20	15	20
15	20	20	15
20	17	20	20
19	17	20	20
19	20	20	8
19	20	13	20
15	16	19	15
10	20	10	20
20	15	16	10
20	20	13	16

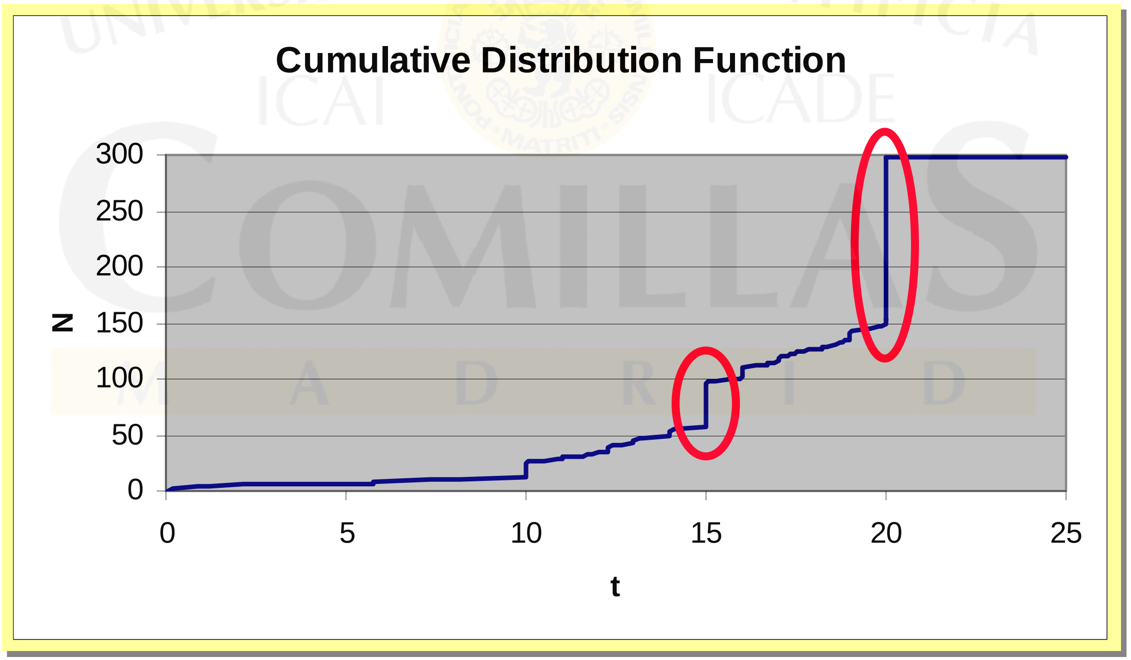
Time differences among timetables

Trains	Events	PC	VA1	TE1	E1	AL1	CC1	RR1	I1	B1	T1	JA1	S1	TM1	AM1	AT1	AR1	MP1	P1	V1	NN1	PO1	BA1	AA1	MH1	SG1	VV1	COM	CO	VV2
N1	Arrival			-5	-11	-15	-20	-25	-30	-35	-40	-45	-50	-55	-60	-65	-70	-75	-81	-56	-62	-67	-71	-71	-76	-50	-26			
	Departure		-5	-10	-16	-20	-25	-30	-35	-40	-45	-50	-55	-60	-65	-70	-75	-80	-56	-61	-66	-71	-70	-75	-50	-25				25
N2	Arrival			-5	-10	-15	-20	-25	-30	-35	-40	-45	-50	-56	-60	-66	-71	-52	-53	-28	-32	-37	-42	-47	-51	-29	-25			
	Departure		-5	-10	-15	-20	-25	-30	-35	-40	-45	-50	-55	-60	-65	-70	-52	-52	-27	-32	-37	-42	-47	-51	-29	-25				-5
N3	Arrival			-5	-10	-15	-20	-25	-31	-35	-41	-17	-21	-27	-32	-36	-41	-41	-46	-26	-31	-37	-41	-25	-31	-35	-25			
	Departure		-5	-10	-15	-20	-25	-30	-35	-40	-16	-21	-26	-31	-36	-41	-41	-46	-26	-31	-36	-42	-25	-30	-36	-25				-5
N4	Arrival			-6	-10	-16	7	4	-2	-7	-11	-16	-21	-26	-31	-36	-41	-41	-46	-31	-35	-35	-11	-16	-20	5				
	Departure		-5	-10	-15	8	4		-6	-11	-16	-21	-26	-31	-36	-41	-41	-46	-30	-36	-35	-11	-16	-20	5					-5
N5	Arrival			-6	-11	-11	-16	8	20	24	19	14	10	10	4		-5	-10	-15	-20	-25	-30	-35	-40	-45	-50	-25			
	Departure		-5	-10	-11	-16	8	20	24	19	15	9	10	5	-1	-5	-10	-15	-20	-25	-30	-35	-40	-45	-50	-25				-6
N6	Arrival			-5	-10	10	14	14	9	3		-5	-10	-15	-20	-25	-30	-35	-40	-45	-50	-55	-60	-35	-15	10	5			
	Departure		-5	-10	10	15	13	9	4	-1	-5	-10	-15	-20	-25	-30	-35	-40	-45	-50	-55	-60	-35	-15	10	5				-5
N7	Arrival			11	5		-4	-9	-5	-10	-15	-20	-25		5	5	5		-5	-10	-15	-20	-25	-30	-35	-40	-25			
	Departure		11	6		-4	-9	-5	-10	-15	-20	-26		5	5	5		-5	-10	-15	-20	-25	-30	-35	-40	-25				-5
N8	Arrival			-5	-10	-15	-20	-24	-29	-25	-30	-35	-35	-40	-45	-20	-25		25	20	15	10	5		-5	10	5			
	Departure		-5	-10	-15	-20	-25	-29	-25	-30	-35	-35	-40	-45	-20	-25		25	20	15	9	5		-6	9	4				-5
N9	Arrival			4		25	20	25	20	15	9	5		-5	-10	-15	-20	-25		-6	8	3	-2	-7	-12	10	5			
	Departure		4		25	20	25	20	15	10	5		-5	-11	-16	-20	-26	-2	-7	8	3	-3	-8	-12	10	5				-5
N10	Arrival			-4	-9	-14	-19		25	20	16	12	7	2	-3	-8	-13	-18		-5	-10	-15	-20	-25	-15	10	5			
	Departure		-4	-10	-15	-19		24	19	16	11	6	1	-3	-8	-13	-18		-5	-10	-15	-20	-25	-15	10	5				-5
N11	Arrival			-5	-10	-9	-14	-19	-24	-27	-32	-37	-42	-47	-52	-50	-50	-25		-5	-10	-15	-20	-25	-15	10	5			
	Departure		-6	-10	-9	-14	-19	-24	-27	-32	-37	-42	-47	-52	-50	-50	-25		-5	-10	-16	-20	-25	-16	9	4				25
N12	Arrival			-5	-10	-2	-7	-12	-5	-10	-16	-20	-25	-30	-35	-40	-45	-50	-55	-60	-65	-70	-75	-75	-50	-25	-25			
	Departure		-5	-10	-2	-7	-12	-5	-10	-15	-20	-25	-30	-36	-41	-45	-51	-56	-61	-65	-70	-76	-76	-50	-25	-25				
N13	Arrival			20	45	40	35	30	27	52	47	42	37	32	27	22	17	12	7	2	-3	-8	-13	-18	-23	-28	-3			
	Departure		20	44	39	35	30	26	51	47	41	36	31	27	22	17	12	7	2	-3	-8	-13	-18	-23	-28	-3				-5
N14	Arrival																													
	Departure																													

Trains	Events	SG2	MH2	AA2	BA2	PO2	NN2	V2	P2	MP2	AR2	AT2	AM2	TM2	S2	JA2	T2	B2	I2	RR2	CC2	AL2	E2	TE2	VA2	PCM
N1	Arrival	25	29	24	19	15	10	5		-5	-10	-15	-20	-25	-30	-10	-15	-20	-5		-5	-10	-16	-20	-25	
	Departure	29	24	19	14	10	5		-5	-10	-15	-20	-25	-30	-10	-15	-20	-5		-5	-10	-15	-20	-25		
N2	Arrival	-5	-10	-15	-5	-10	-15	-20	-20	-25	-30	-15	10	5		-5	-10	-16	-20	-25	-30	-35	-40	-45	-25	
	Departure	-10	-15	-5	-10	-15	-20	-20	-25	-30	-15	10	5		-5	-10	-15	-20	-26	-31	-36	-40	-45	-26		
N3	Arrival	-5	-10	-15	5		-5	-10	-15	-19	-5	20	16	11	6	1	-4	-9	-14	-19	-24	-29	-34	-39	-14	
	Departure	-10	-15	5		-5	-10	-15	-20	-5	20	15	10	6	1	-5	-10	-15	-19	-24	-29	-34	-39	-14		
N4	Arrival	-5	-10	-15	-20	-25	-30	-35	-40	-22	3	28	23	18	13	8	3	-2	-7	-12	-17	-22	-27	-32	-7	
	Departure	-11	-15	-21	-26	-30	-36	-40	-22	2	27	23	18	13	8	3	-2	-7	-12	-17	-22	-27	-32	-7		
N5	Arrival	-5	-10	-15	-20	-25	-30	-35	-40	-45	-47	-22	-15	-20	-25	-30	-35	-41	-45	-50	-55	-46	-51	-50	-25	
	Departure	-10	-15	-20	-25	-30	-35	-40	-45	-47	-22	-15	-20	-25	-30	-35	-40	-45	-51	-56	-47	-51	-50	-26		
N6	Arrival	-5	-10	-15	-20	-25	-30	-35	-40	-41	-30	-5	-9	-14	-19	-24	-29	-34	-39	-44	-49	-54	-59	-34	-25	
	Departure	-10	-15	-20	-25	-30	-35	-40	-42	-30	-5	-10	-15	-19	-24	-30	-35	-40	-44	-49	-54	-59	-34	-25		
N7	Arrival	-5	-10	-15	-20	-25	-30	-35	-40	-45	-50	-30	-35	-40	-45	-25	-30	-11	-17	-21	-26	-32	-36	-41	-25	
	Departure	-10	-15	-20	-26	-30	-35	-40	-45	-50	-30	-35	-40	-45	-25	-30	-11	-16	-21	-26	-31	-36	-41	-24		
N8	Arrival	-5	-10	-15	-20	-25	-30	-35	-35	-40	-45	-32	-36	-42	-47	-51	-57	-36	-41	-17	-5	-11	-15	-20	-25	
	Departure	-10	-15	-20	-25	-30	-35	-35	-40	-45	-31	-36	-41	-46	-51	-56	-36	-41	-16	-6	-10	-16	-20	-25		
N9	Arrival	-5	-10	-15	-20	-26	-26	-31	-37	-42	-27	-3	-8	-11	-17	5		-5	-10	-12	-16	-21	-25	-30	-25	
	Departure	-10	-15	-20	-25	-26	-31	-36	-41	-27	-2	-7	-11	-16	4		-6	-10	-12	-16	-20	-26	-30	-25		
N10	Arrival	-5	-10	-15	-20	-25	-26	-31	-35	-40	-23	2	-3	-8	-13	-18	-23	-28	-25	-30	-35	-40	-45	-50	-25	
	Departure	-11	-15	-21	-26	-26	-32	-35	-40	-24	1	-3	-8	-13	-18	-23	-28	-25	-30	-35	-40	-45	-50	-25		
N11	Arrival	25	25	20	15	17	12	7	2	-3	22	35	34	29	24	19	14	9	4		-6	-11	-16	-20	5	
	Departure	25	20	15	17	12	7	2	-3	22	35	34	29	24	18	14	9	3	-2	-7	-11	-16	-21	4		
N12	Arrival	-2	-7	18	14	14	8	3	-1	-6	-11		-6	-11	-17	-20	-25	-26	-30	-34	-30	-35	-40	-45	-20	
	Departure	-6	19	14	14	9	4		-6	-11		-6	-11	-16	-21	-25	-26	-30	-34	-31	-35	-40	-45	-20		
N13	Arrival	-6	-10	-10	-16	-16	-22		-5	-11	-15	-20	-26	-30	-35	-40	-45	-50	-45	-50	-55	-50	-55	-50	-25	
	Departure	-10	-10	-15	-17	-21	-1	-5	-10	-16	-20	-25	-31	-35	-41	-45	-50	-45	-50	-55	-50	-55	-50	-25		
N14	Arrival																									
	Departure																									

Cumulative distribution function of overlapping time

- Numerous overlaps of 20 seconds (bound defined by the user). Further improvement can be achieved.



Contents

Conclusions

- Decision support tool that can be used for maximizing the overlapping time between the slow-down and speed-up processes
- Large potential energy savings
- Very difficult to solve MIP optimization problem
- Quasioptimal solutions can be obtained in a reasonable amount of time
- Most of the overlapping time can be obtained with no changes in the advertised timetable
- Model can de extended to consider several lines or a wider time scope

Mathematical programming approach to underground timetabling problem for maximizing time synchronization

A. Ramos, M.T. Peña, A. Fernández, P. Cucala

Universidad Pontificia Comillas