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Computational modeling for clean, reliable, and affordable electricity  
Massachusetts Institute of Technology (MIT)

# *Medium-Term Stochastic Hydrothermal Coordination Model*

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# Learning objectives

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- To understand
  - What is a medium-term hydrothermal coordination model
    - Purpose
    - How to use
    - How an optimization model is used to support hydropower plant operation
  - How stochasticity is modeled
    - Scenario tree
    - Effect of uncertainties in hydro scheduling decisions
  - What techniques are used for solving the stochastic problem
    - Stochastic optimization

# Hydro-power in Portugal

A huge hydroelectric power plant 250 km southeast of Lisbon in Portugal uses Alstom technology to **combine and store wind and hydro energies**. This plant has a capacity of 260 MW and supplies energy to 175,000 households. The reduction in CO2 emissions is the equivalent of removing 185,000 cars from the road. The **biggest upper reservoir in Europe is** 85 km long with a surface of 250 square kilometers.

During the day water falls into the lower reservoir, moving the turbines and producing electricity. But at night – when energy consumption falls – **the turbine uses wind energy to pump water back into the upper reservoir** so the cycle can continue the next day without significant water loss.

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Alqueva  
reservoir



<http://www.youtube.com/watch?v=82efZBKBXSg>

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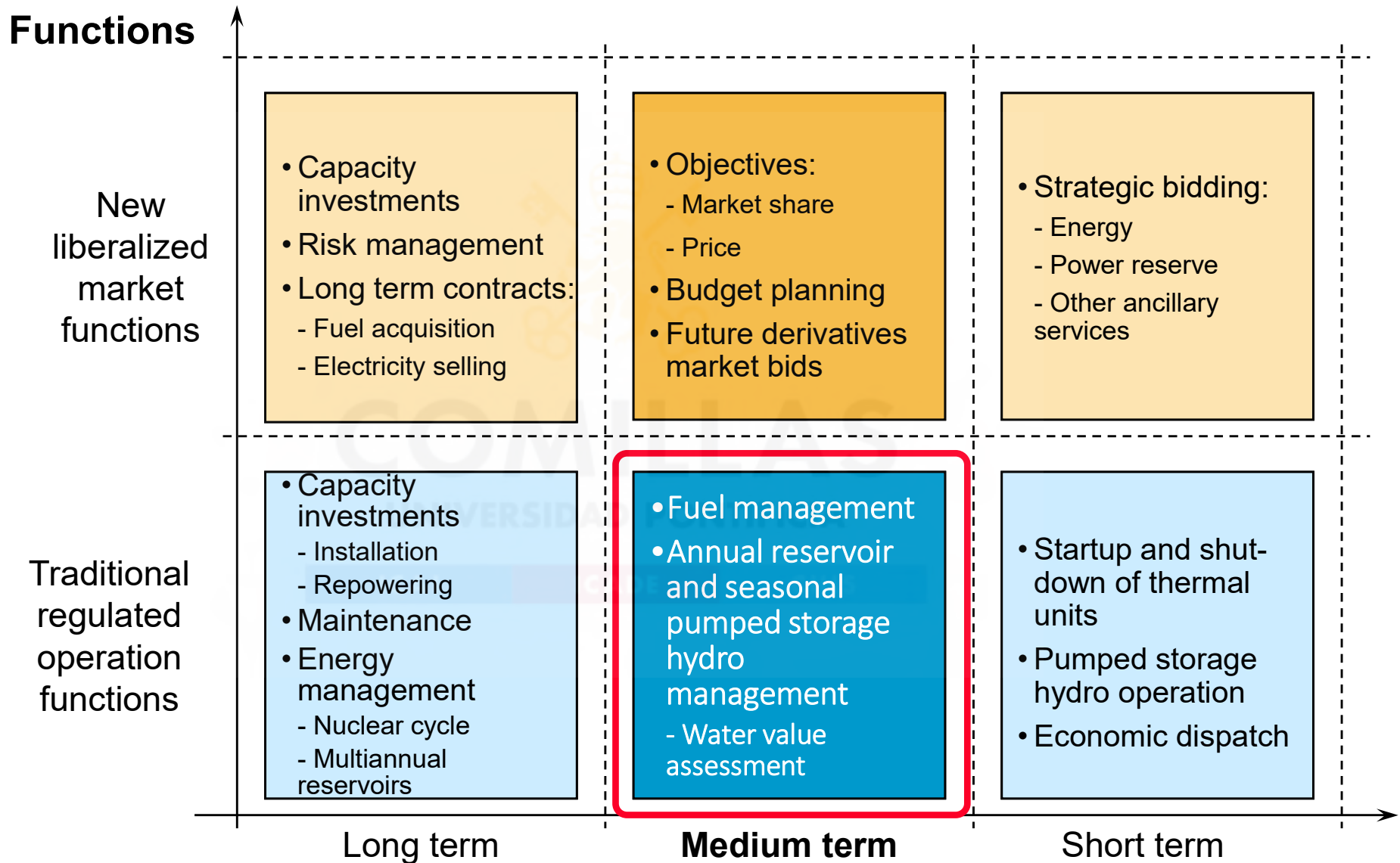


1. **Medium term stochastic hydrothermal coordination model**
2. Modeling issues
3. Stochastic optimization
4. Prototype SHTCM. Mathematical formulation
5. Prototype SHTCM. Computer implementation

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# Medium term stochastic hydrothermal coordination model

# Generation planning functions



# Hydrothermal operation

- Strong **interaction between hydro and thermal plants**. Must be scheduled simultaneously
- Possibility of storing water with **time dependencies** to represent water volume regulations
- **Long-term storage capability** allows better use of the water and, therefore, causes more extended optimization periods
- Existence of **cascading and water dependencies** between them
- **Pumped-storage hydro plants** introduce additional operational complexity
- **The operational cost** of hydro plants is **negligible**
- Hydro generation is **very flexible but uncertain**
  - Affordable way of storing energy at large scale
  - Inflows can have significant uncertainty



# Opportunities for optimal decision making

- Optimization of hydro generation results in the **replacement of expensive thermal generation**
- Optimization of pumped-storage hydro plants' operation is based on **pumping at cheap hours** and **turbining at the high-price ones**
- High penetration of intermittent generation will stress the electric system operation. **Storage hydro and pumped storage hydro plants** are going to play a much more important role due to their **flexibility and complementary use with intermittent generation**
- Besides, under a deregulated framework, electric companies manage their generation resources and need detailed operation planning tools

# Uses of hydro scheduling models

- Determine the operation of **complex basins**
- Determine **re-powering** alternatives
- Determine the power capacity of **new hydro investments**
- Determine the **risk (amount)** of spillage
- Determine **firm energy** generation
- Determine the **firm capacity**

# Medium term optimization model. Characteristics

- Hydroelectric vs. hydrothermal models
  - A hydroelectric model deals **only with hydro plants**
  - A hydrothermal model manages both hydro and thermal plants simultaneously
- Thermal units are considered **individually**. So rich marginal cost information for guiding hydro scheduling
- No **aggregation** or **disaggregation process** for hydro input and output is needed
- It isn't easy to obtain meaningful results for each hydro plant because:
  - It requires a considerable amount of data and
  - The complexity of hydro subsystems

# Medium-term optimization model. Main modeling assumptions

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- System characteristics and some data that are known with certainty (**deterministic**)
  - Technical characteristics of existing power plants
  - Multiple cascaded reservoirs
  - Net load demand (includes intermittent generation and imported/exported power)
  - Availability of generation units as a reduced-rated power
  - Fuel costs
- **Uncertain** or **stochastic** data
  - Unregulated (natural) hydro inflows
- Hydro plants are **limited** in both **energy** and **power output**
- **Transmission network** doesn't affect the optimal operation of the units (it is not represented)

# Optimization-simulation combination

- Use the model in an **open-loop** control mechanism with a **rolling horizon**
  1. First, planning by **stochastic optimization**
  2. Second, **simulation** of the random parameters
- Repeat the process
  1. Stochastic **optimization**
    - Determines **optimal scheduling policies** considering the uncertainty in a **simplified way**
  2. **Simulation (out-of-sample) (cross-validation)**
    - Evaluates **possible future outcomes** of random parameters given the optimal policies obtained previously
- Focus on **STOCHASTIC OPTIMIZATION MODELS**

# How to use of a medium term stochastic hydrothermal coordination model

- Run in a **rolling** mode (i.e., the model is **run each week** with a time scope of several months up to one year)
- Only decisions for the closest period are of interest (i.e., the next week). The **remaining decisions are ignored**

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# Medium term optimization model. Results

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- Operation planning
  - Fuel consumption, unit (thermal, storage hydro, and pumped storage hydro), and/or technology operation
  - CO2 Emissions
  - Reservoir management
  - Production targets for short-term models (water balance)
- Economic planning
  - Annual budget
  - Operational costs
  - System marginal costs
  - Economic targets for short-term models (water value)

1. Medium term stochastic hydrothermal coordination model
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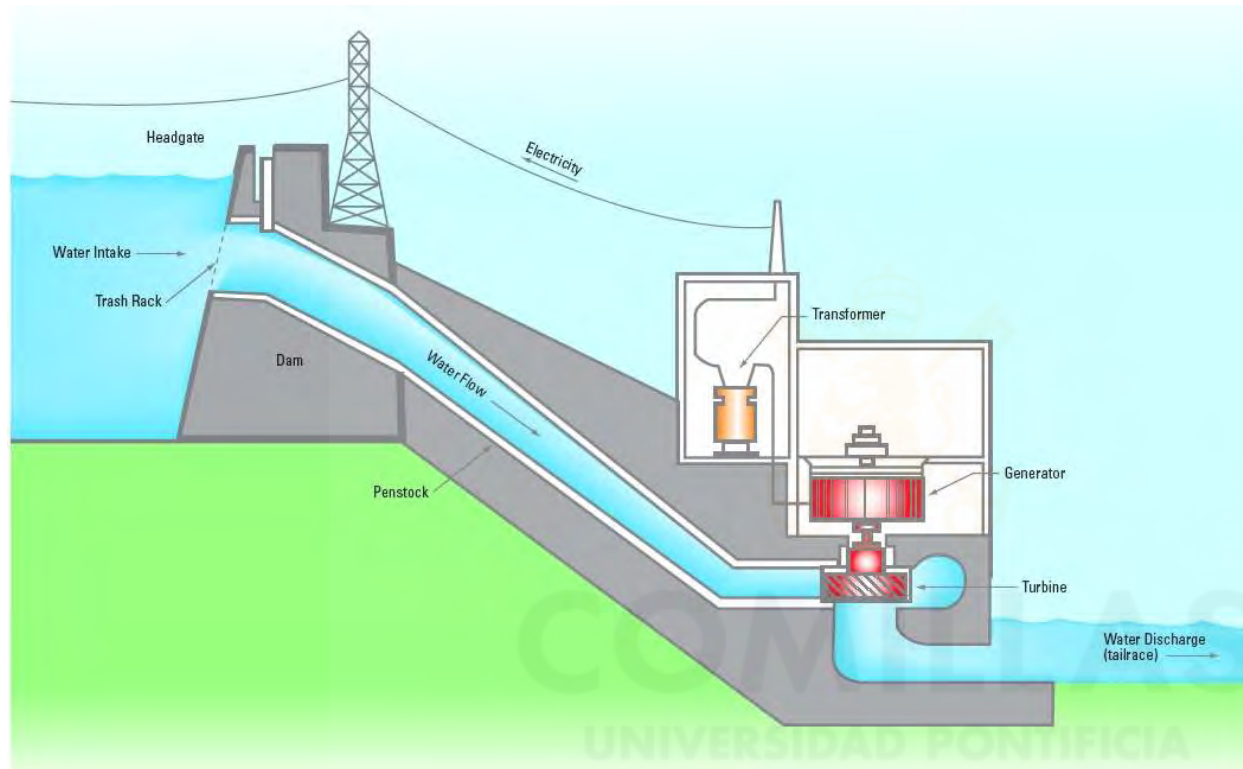
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## Modeling issues



# Hydroelectric Dam



Source: <http://etrical.wordpress.com/power-generation/>

Ricobayo hydroelectric power plant



# Variety of Spanish hydro subsystem

- Hydro reservoir volumes: 0.15 - 2433 hm<sup>3</sup>
- Hydro plant capacity: 1.5 - 934 MW
  - 375 units non UGH (small hydro)
  - 855 units UGH (large hydro), with 80 units > 50 MW



Almendra reservoir (2433 hm<sup>3</sup>)



Almendra (Nov-2017)

[https://www.lasexta.com/noticias/sociedad/las-desoladoras-imagenes-aereas-del-embalse-de-la-almendra-en-salamanca-totalmente-seco\\_201711235a1728080cf2f56e3eb493b2.html](https://www.lasexta.com/noticias/sociedad/las-desoladoras-imagenes-aereas-del-embalse-de-la-almendra-en-salamanca-totalmente-seco_201711235a1728080cf2f56e3eb493b2.html)

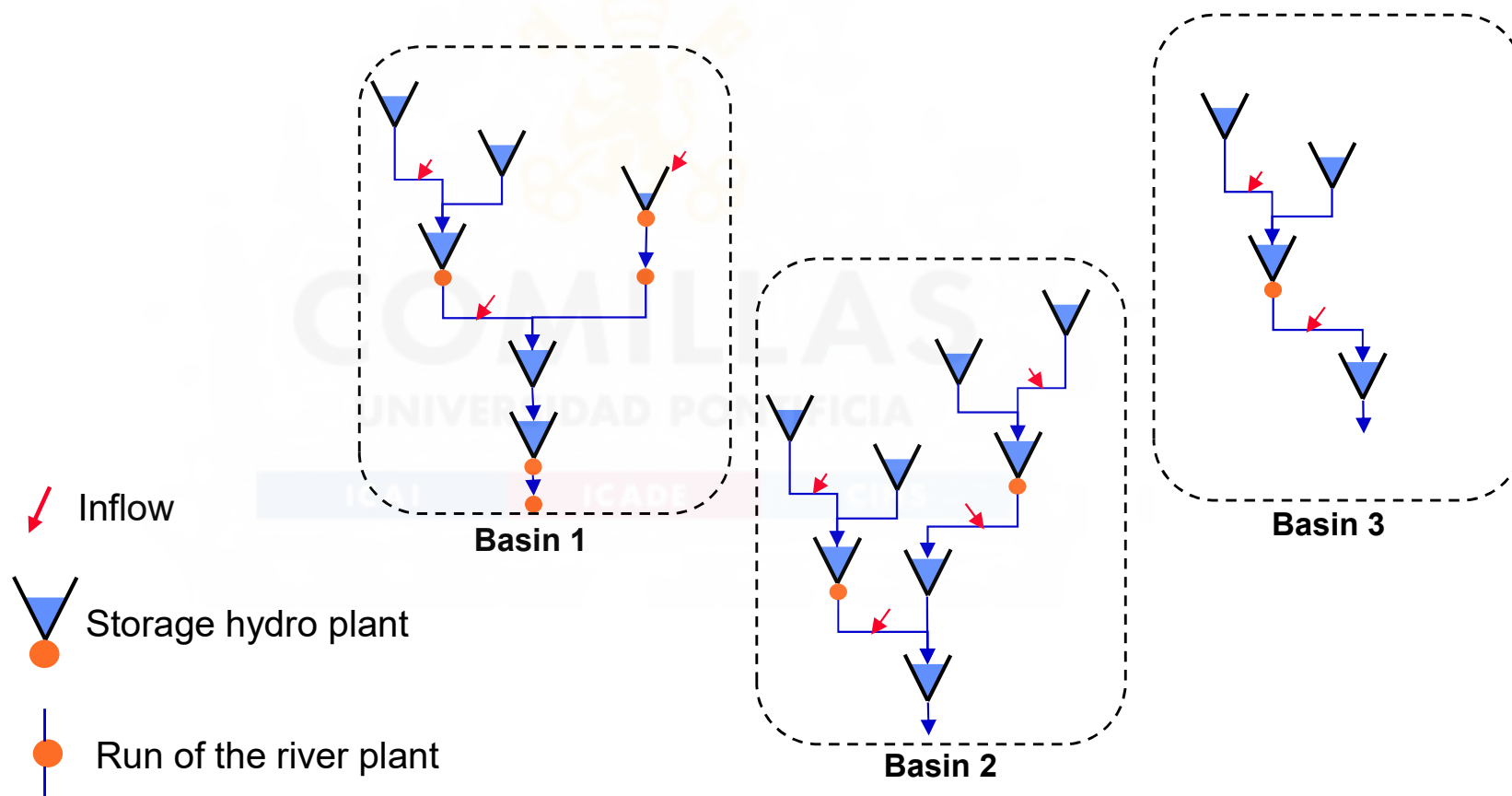
# Hydro subsystem modeling difficulties

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- **Topological complexities** in waterways
- **Nonlinearities** in the production function. **Head dependency**: energy production depends on the water reserve at the reservoir and on the water inflows
  - Important when changes in reservoir levels are significant for the time scope of the model
- **Stochasticity** in natural hydro inflows
- **Complex operation** constraints by other uses of water (irrigation, minimum and maximum river flow, minimum and maximum reservoir levels, sporting activities)

# Multiple basins

- The hydro subsystem is divided into a set of independent hydro basins:

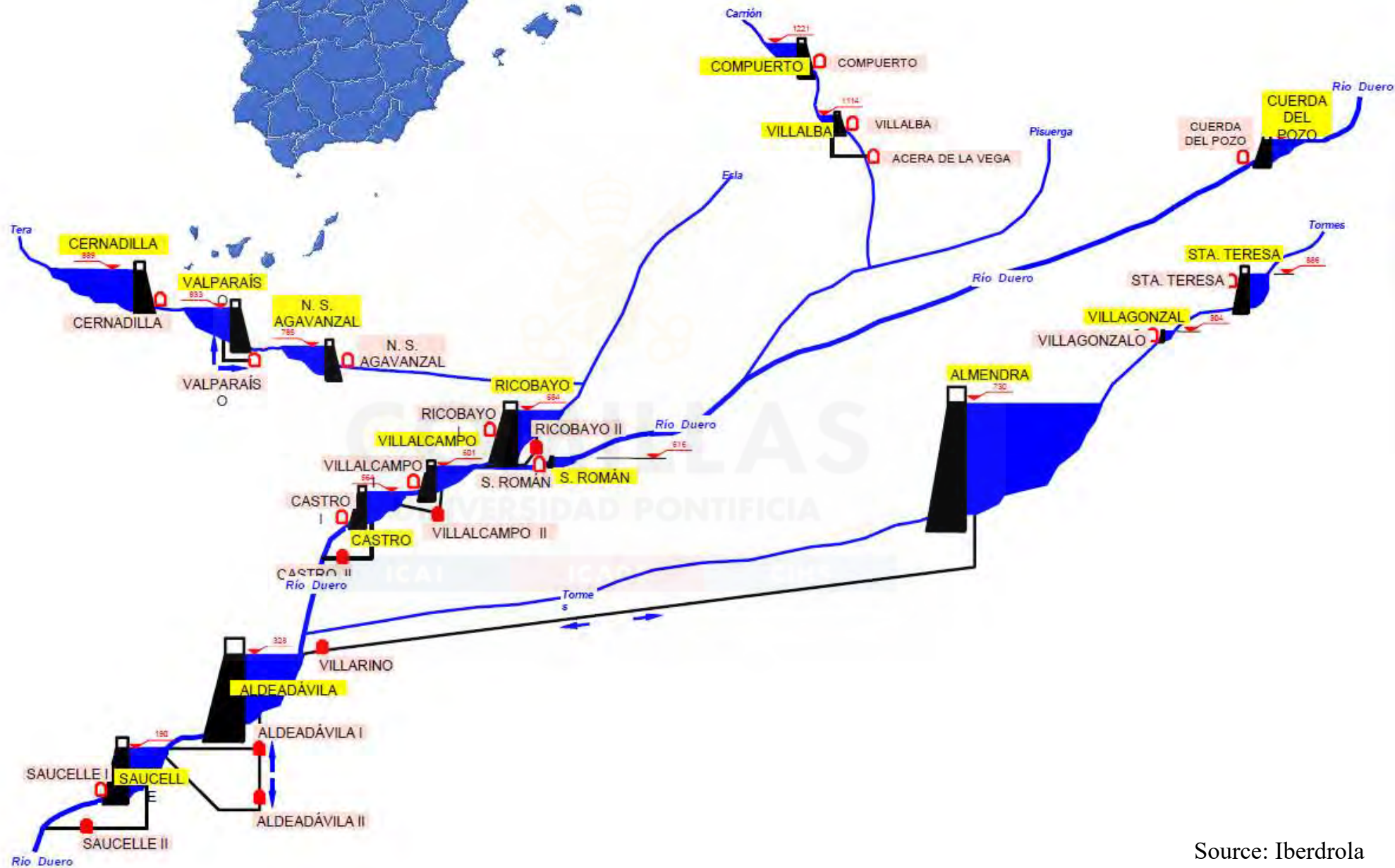


# Tajo

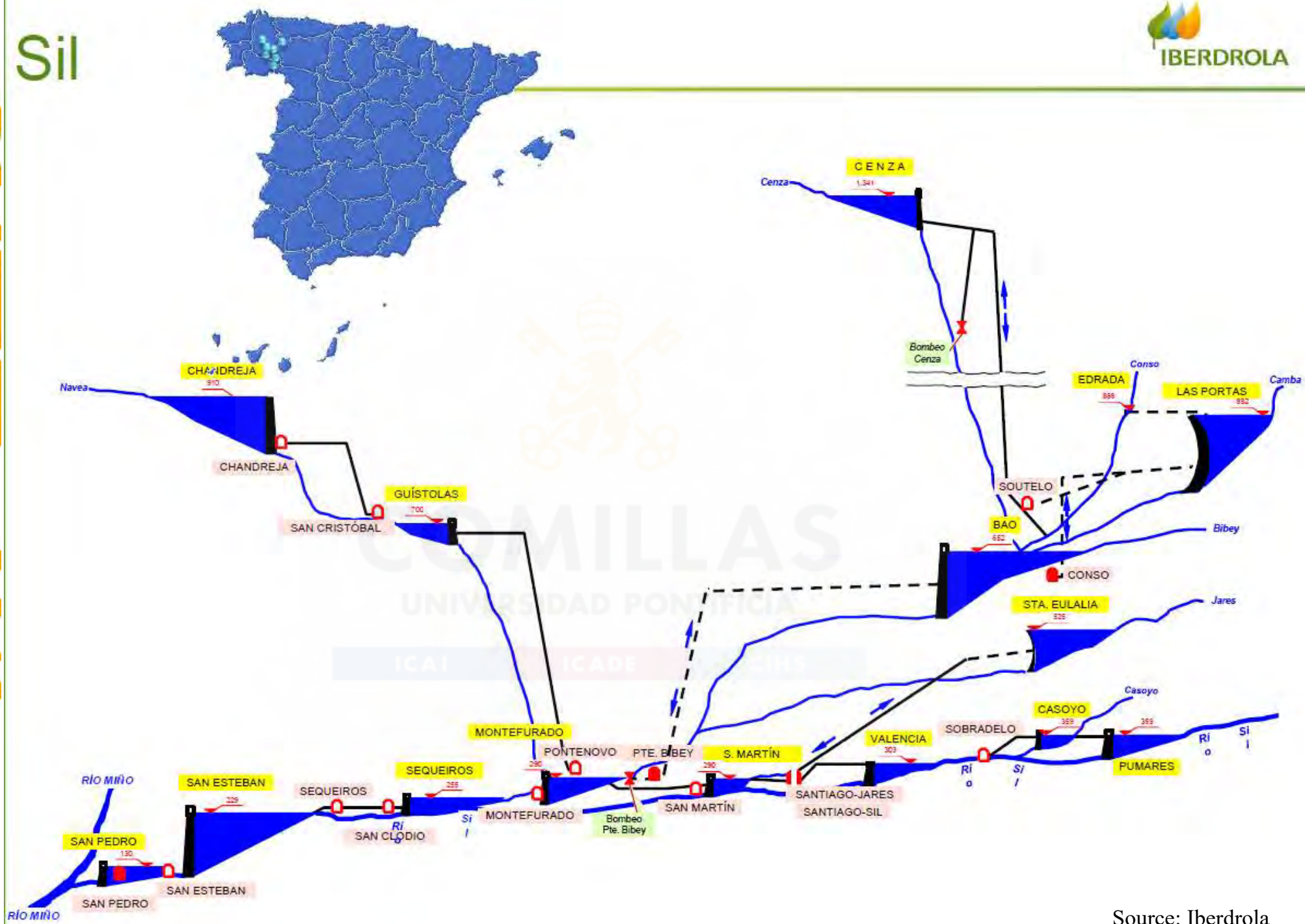


Source: Iberdrola

# Duero

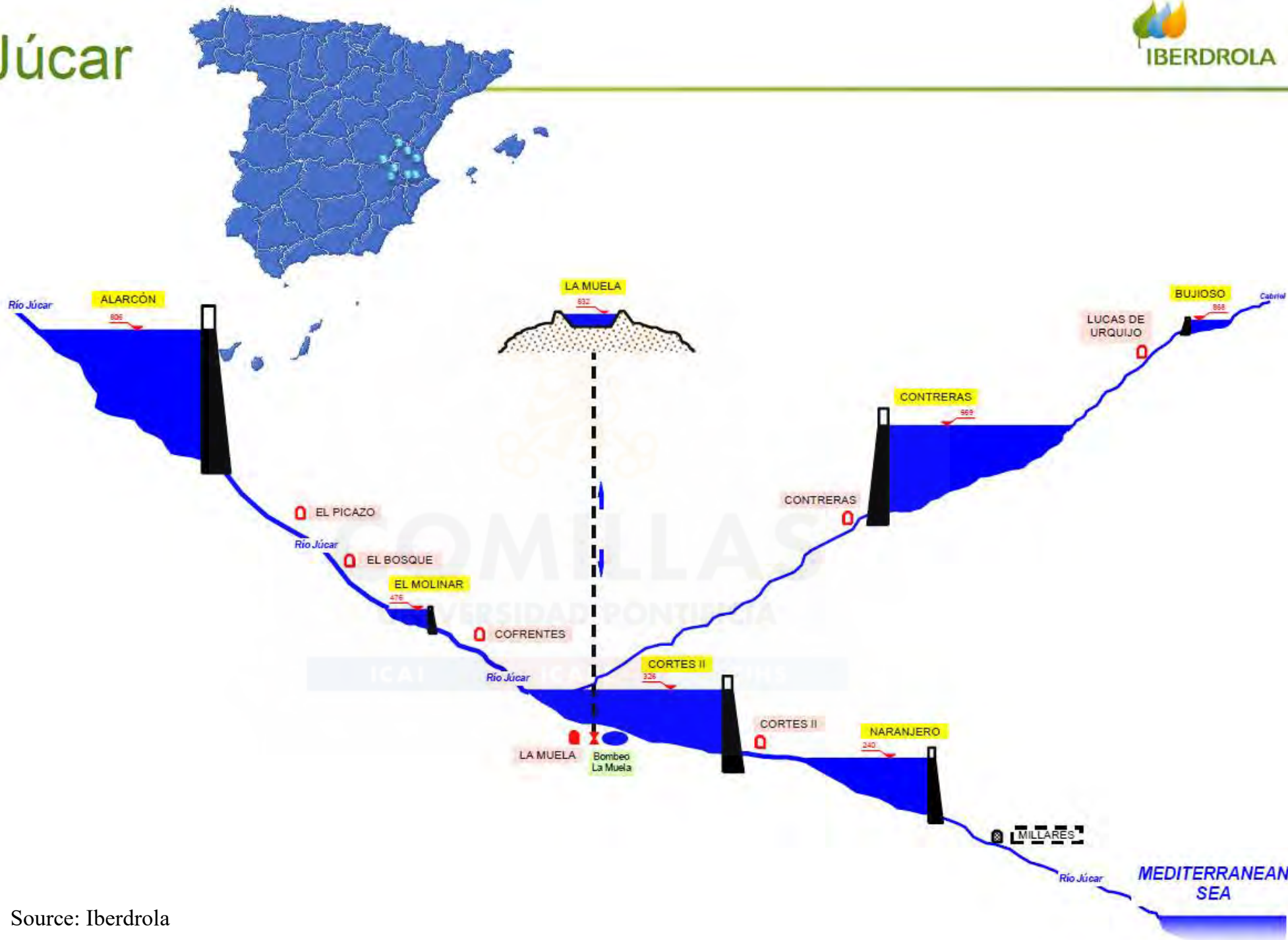


Source: Iberdrola



Source: Iberdrola

# Júcar



Source: Iberdrola



# Alto Tâmega hydroelectric complex

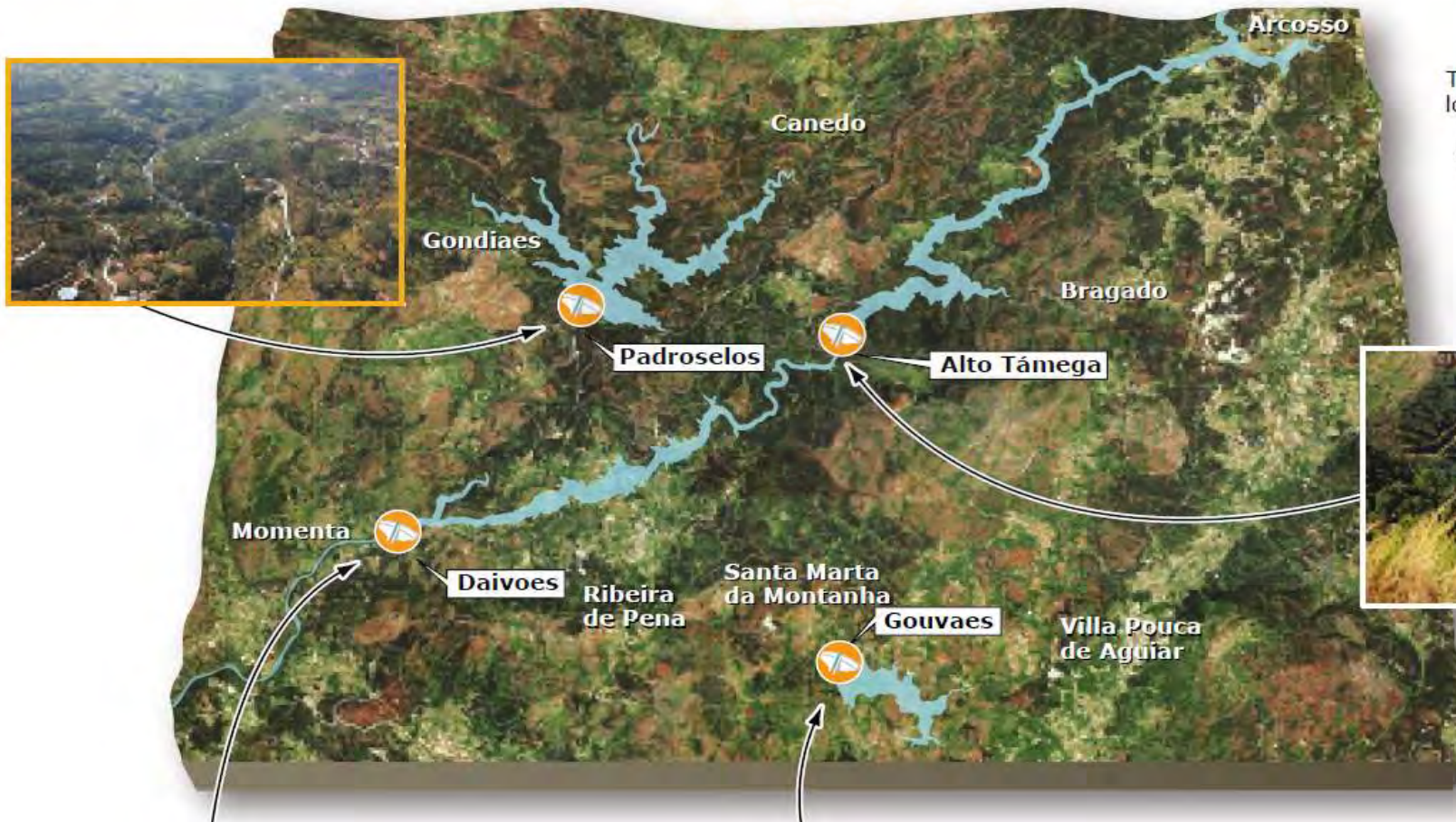
## IBERDROLA

### One of Europe's largest hydro projects over the last 25 years

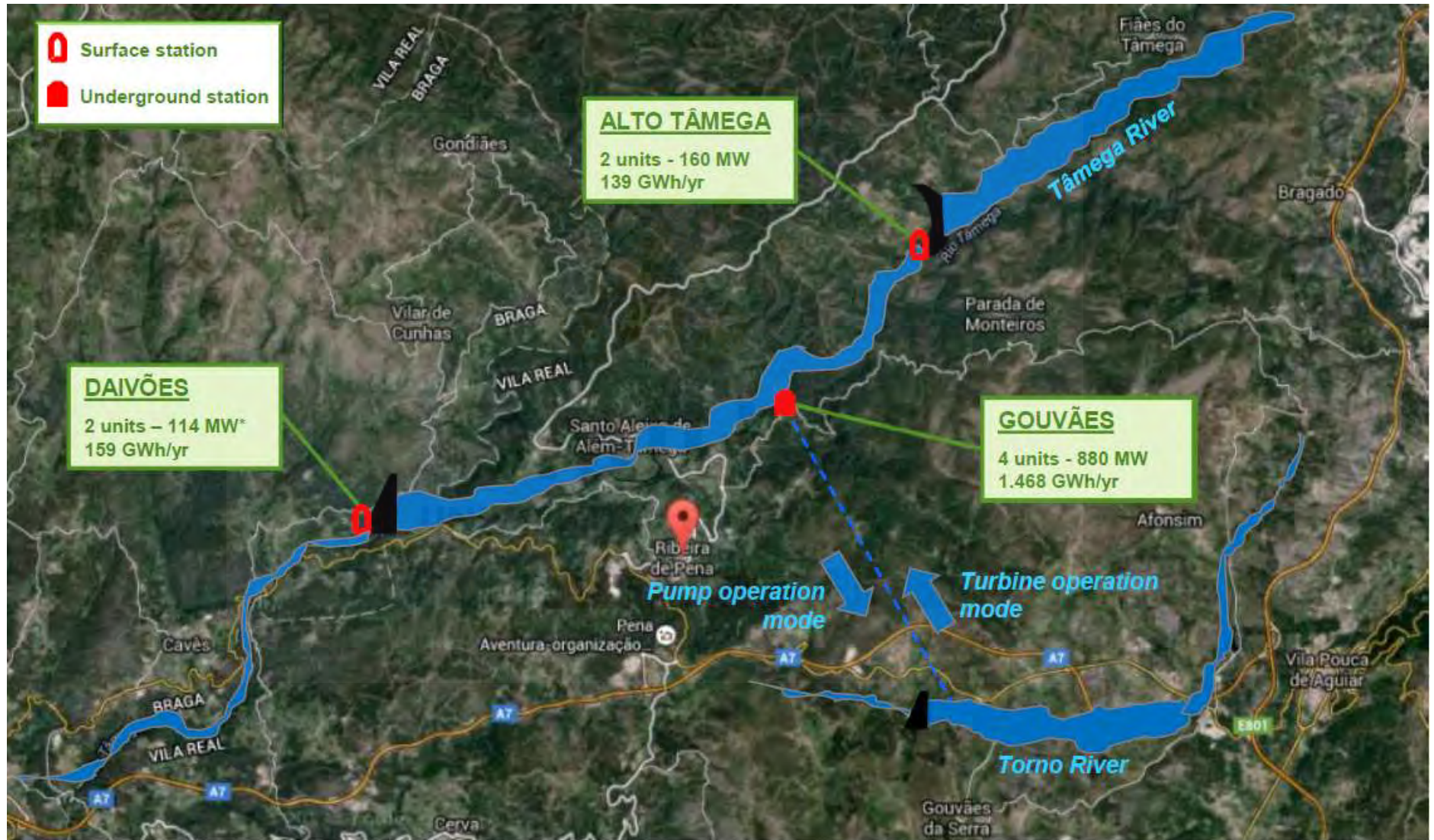
Iberdrola will develop the Alto Tâmega hydroelectric complex in Portugal, one of the largest projects of its kind in Europe to have been carried out in the last 25 years. This initiative, which underlines the company's commitment to cleaner generation technology and to progress in Portugal, envisages exploiting hydroelectricity infrastructure at Gouvaes, Padroselos, Alto Tâmega and Daivoes for 65 years.



The new installations will be located in close proximity to Galicia, where there are plans to upgrade electricity interconnections between Spain and Portugal, and near the Duero and Sil hydro power stations in Spain.



# Alto Tâmega hydroelectric complex



(\*) Does not include 4 MW of capacity installed in "Natural flow" circuit

# Alto Tâmega Project



<https://www.iberdrola.com/about-us/lines-business/tamega>

# La Muela-Cortes hydroelectric complex (> 2000 MW)

La Muela upper reservoir

Pumped storage hydro La Muela  
1722 MW

Hydro plant Cortes II  
290 MW

Cortes reservoir



# Stochastic hydro inflows

- Natural hydro inflows (clearly the most critical annual factor in the Spanish electric system)

- Changes in reservoir volumes are significant because of:

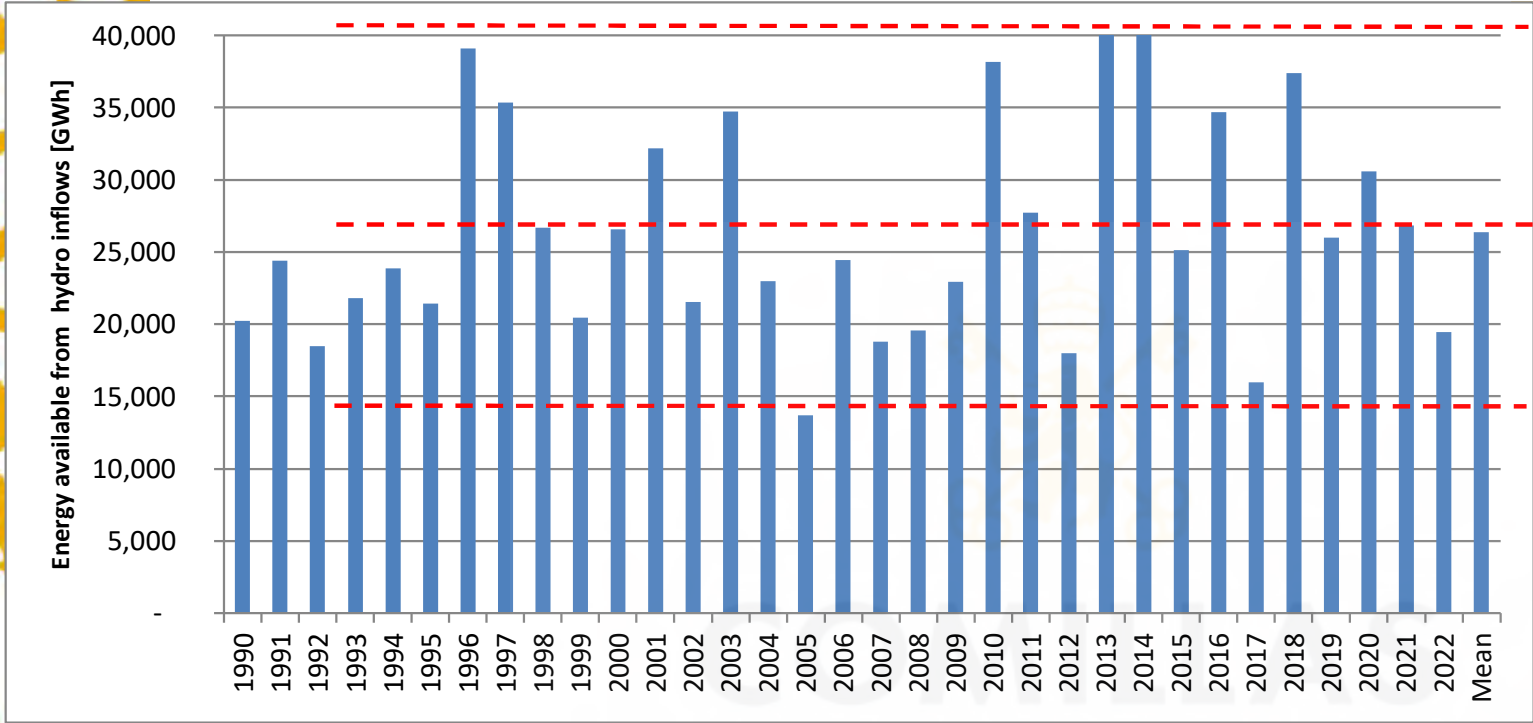
- stochasticity in hydro inflows
- chronological pattern of inflows and
- capacity of the reservoir with respect to the inflows

Year	Hydro energy TWh	Index	% of being exceeded
1990	20.3	0.98	98%
1991	24.4	0.98	49%
1992	18.5	0.74	83%
1993	21.8	0.88	64%
1994	23.9	0.96	52%
1995	21.4	0.86	66%
1996	39.1	1.57	5%
1997	35.4	1.42	10%
1998	26.7	1.07	37%
1999	20.4	0.82	72%
2000	26.6	1.07	38%
2001	32.2	1.29	17%
2002	21.6	0.87	65%
2003	34.8	1.40	11%
2004	23.0	0.92	58%
2005	13.7	0.55	97%
2006	24.4	0.98	49%
2007	18.8	0.75	81%
2008	19.6	0.78	77%
2009	23.0	0.92	57%
2010	38.2	1.53	6%
2011	27.7	0.95	55%
2012	18.0	0.61	97%
2013	41.0	1.41	9%
2014	40.3	1.35	15%
2015	25.1	0.82	79%
2016	34.7	1.12	37%
2017	16.0	0.53	99%
2018	37.4	1.28	17%
2019	26.0	0.88	64%
2020	30.6	1.03	44%
2021	26.9	0.91	60%
2022	19.5	0.67	89%

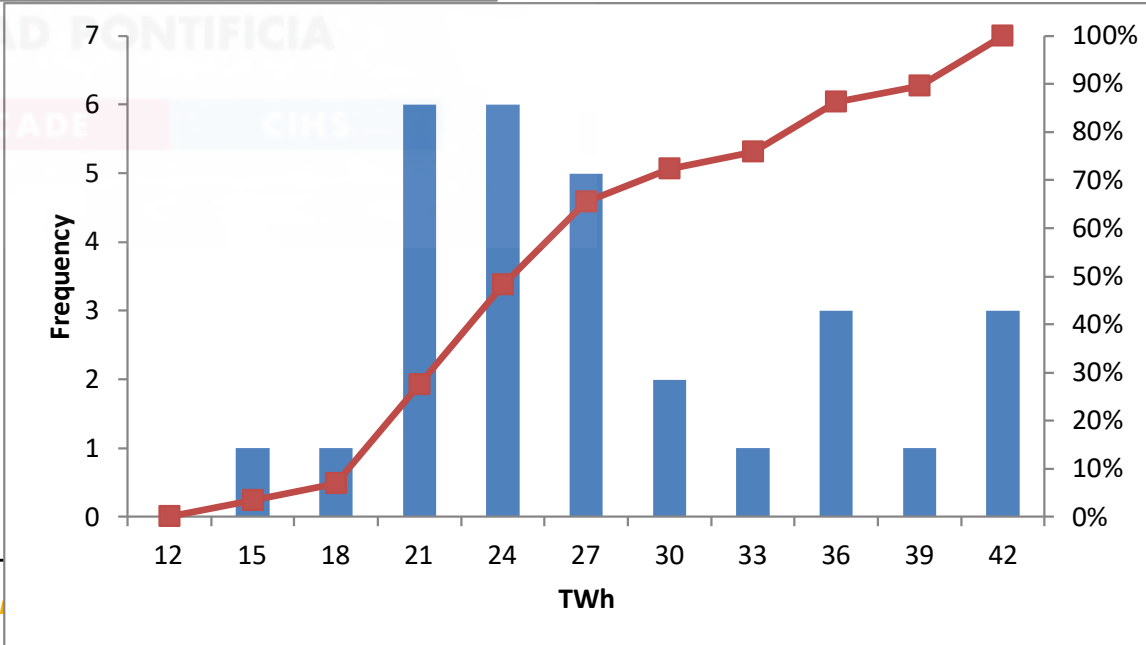
Source: REE



# How uncertain are annual hydro inflows in Spain?



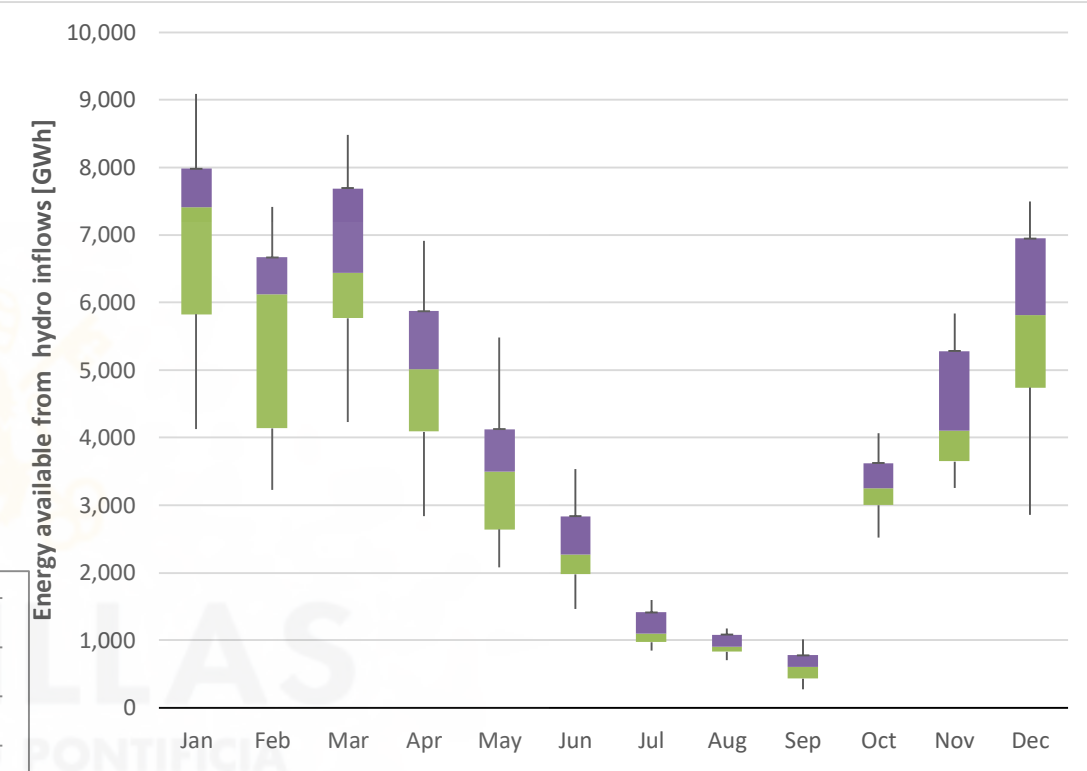
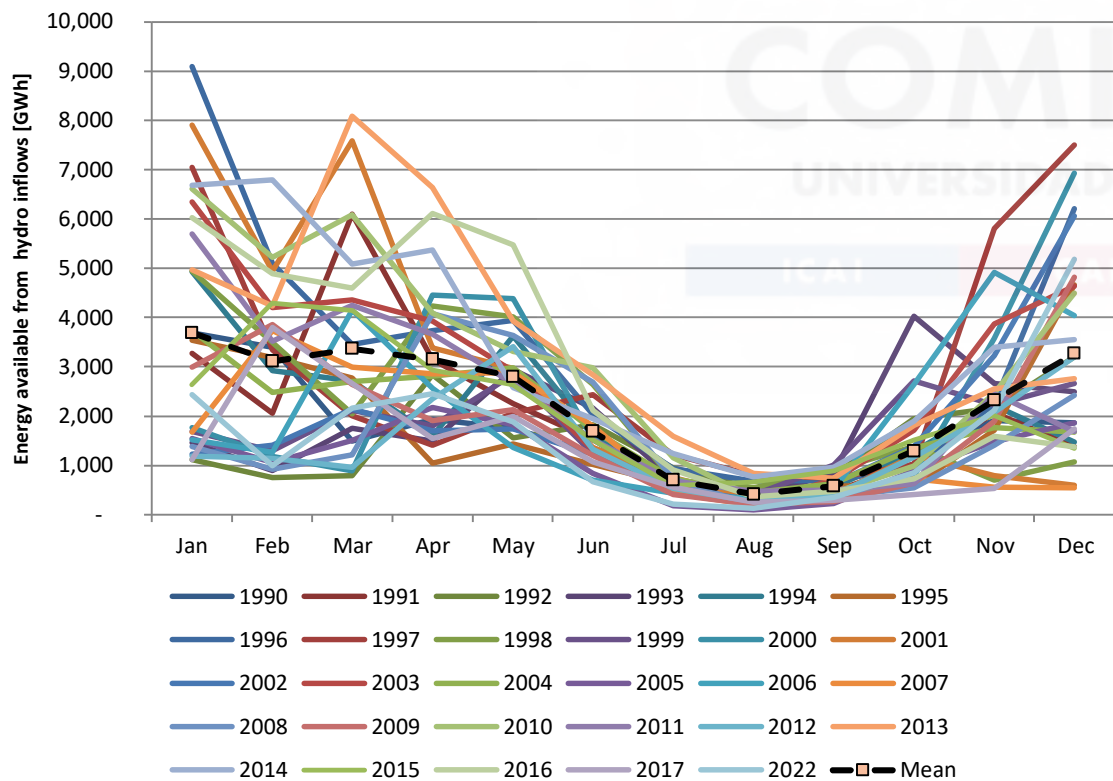
- Mean values
- 26382 GWh
- 3012 MW



Source: REE



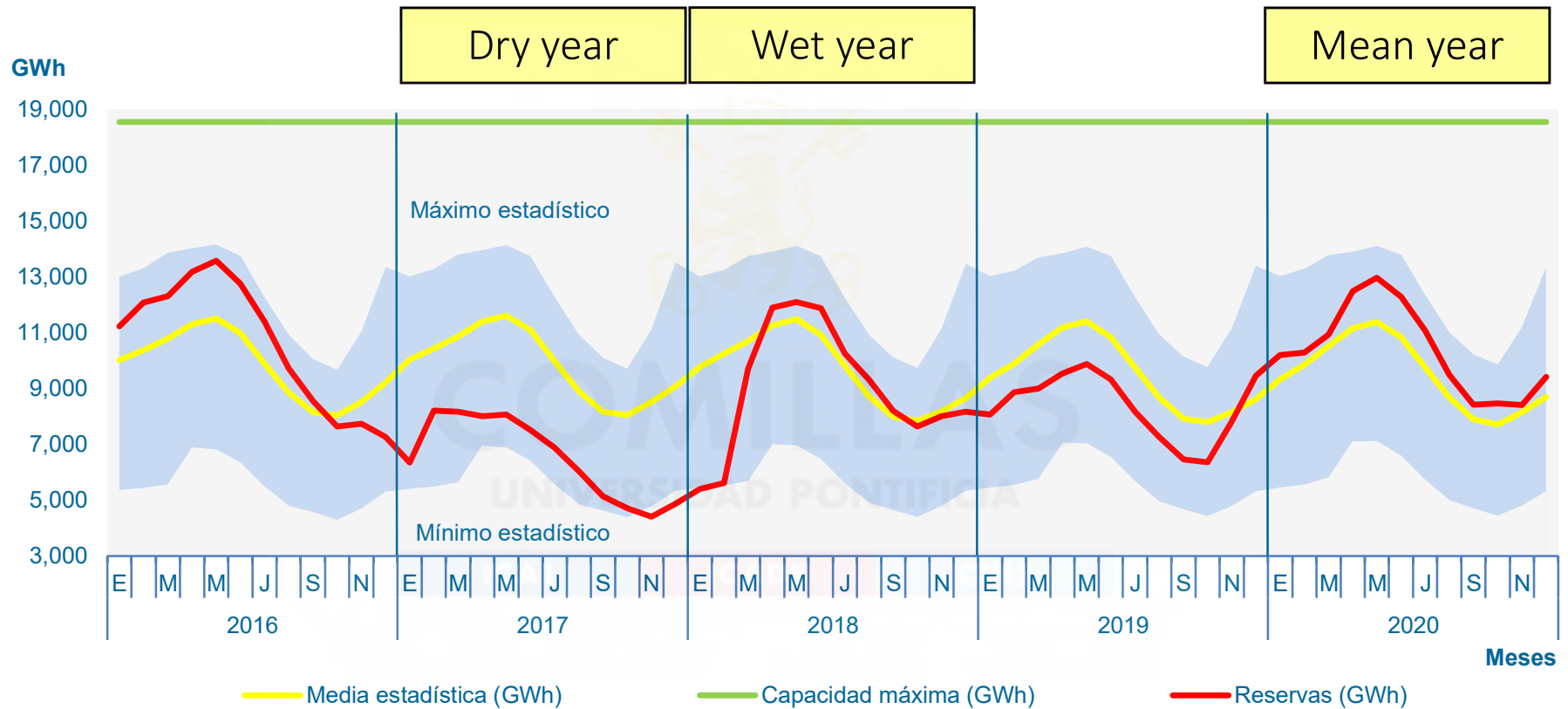
# How uncertain are monthly hydro inflows in Spain?



Source: REE



# Historical, max and min reservoir levels

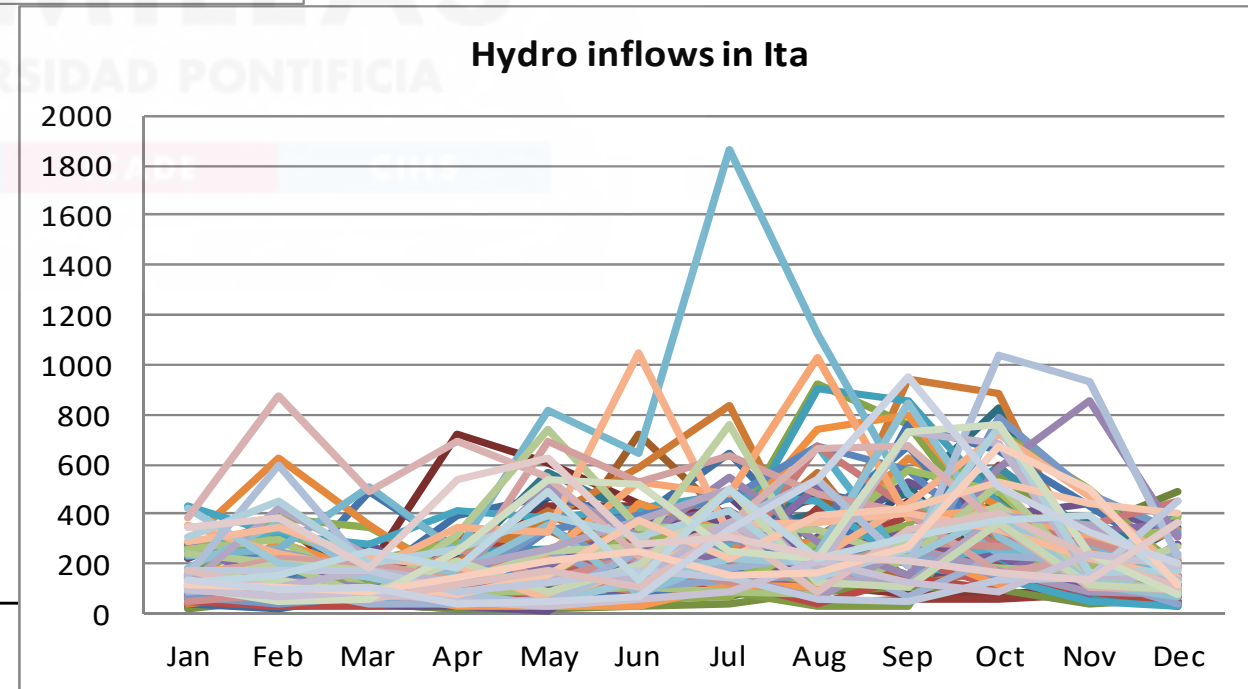
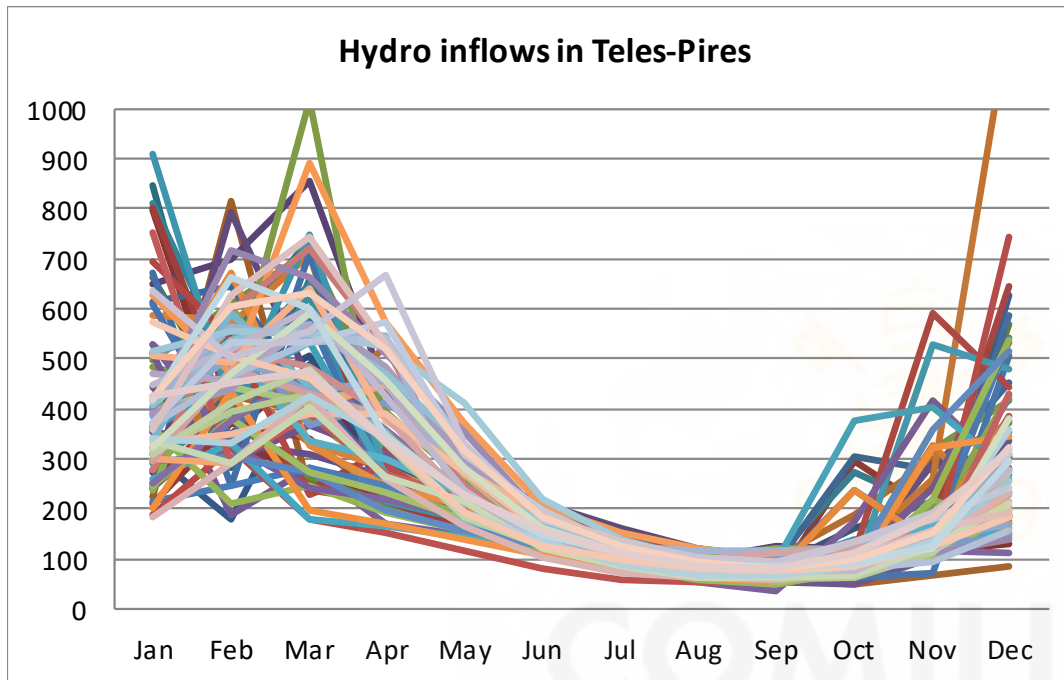


Máximo y mínimo estadístico: media de los valores máximos y mínimos de los últimos 20 años.

Source: REE



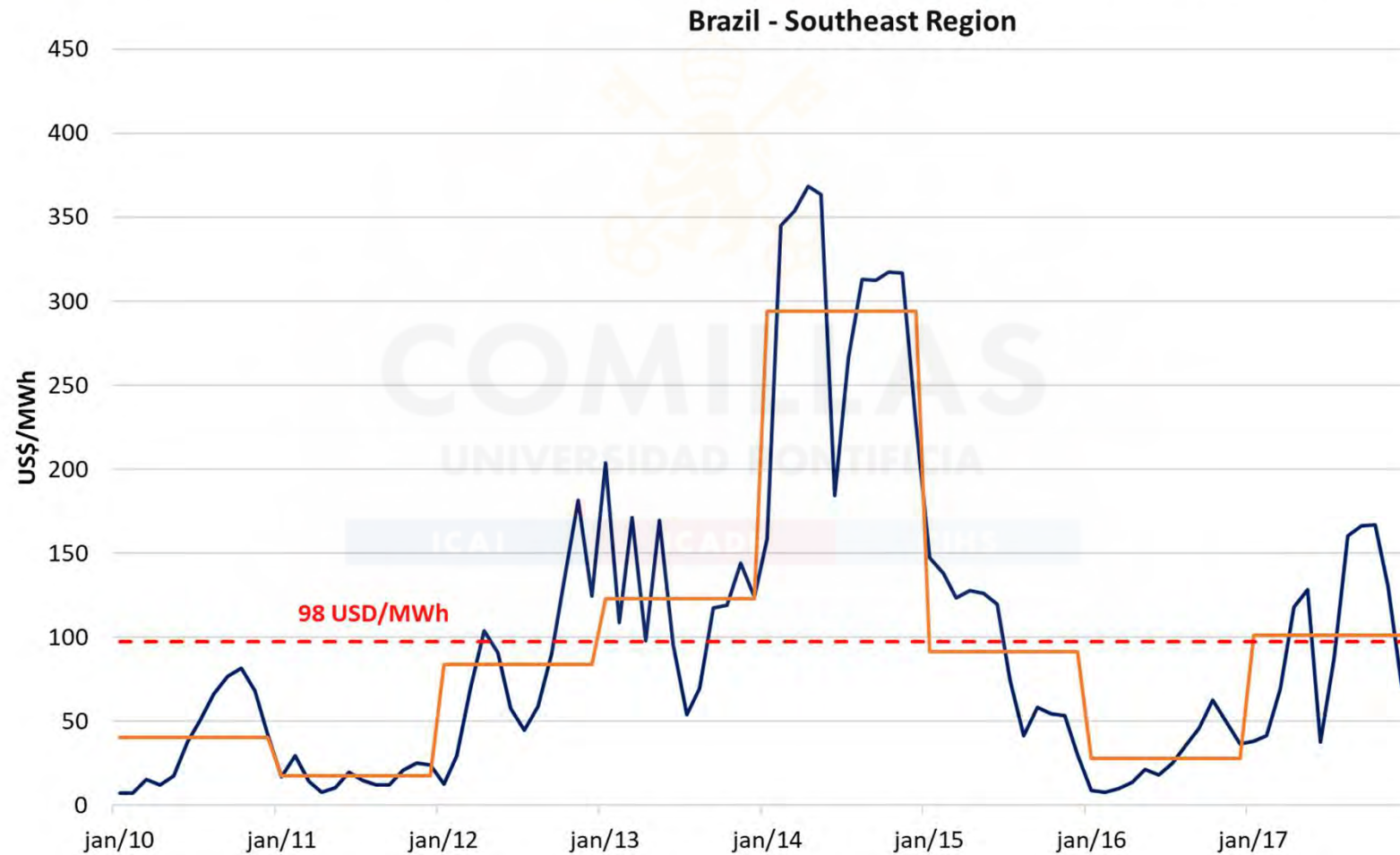
# How uncertain are hydro inflows in Brazil?



# Spot prices strongly driven by hydrology

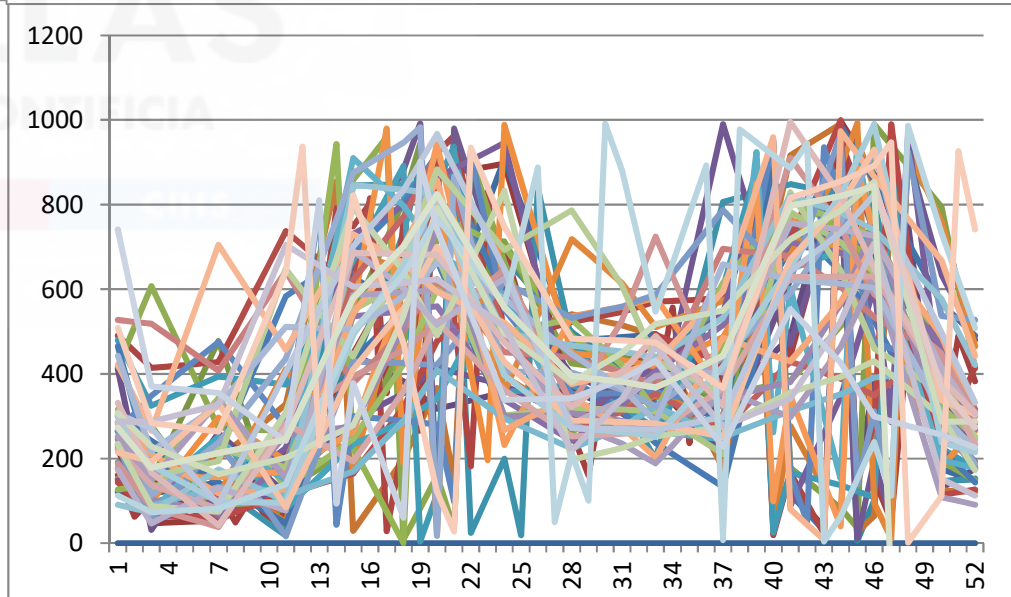
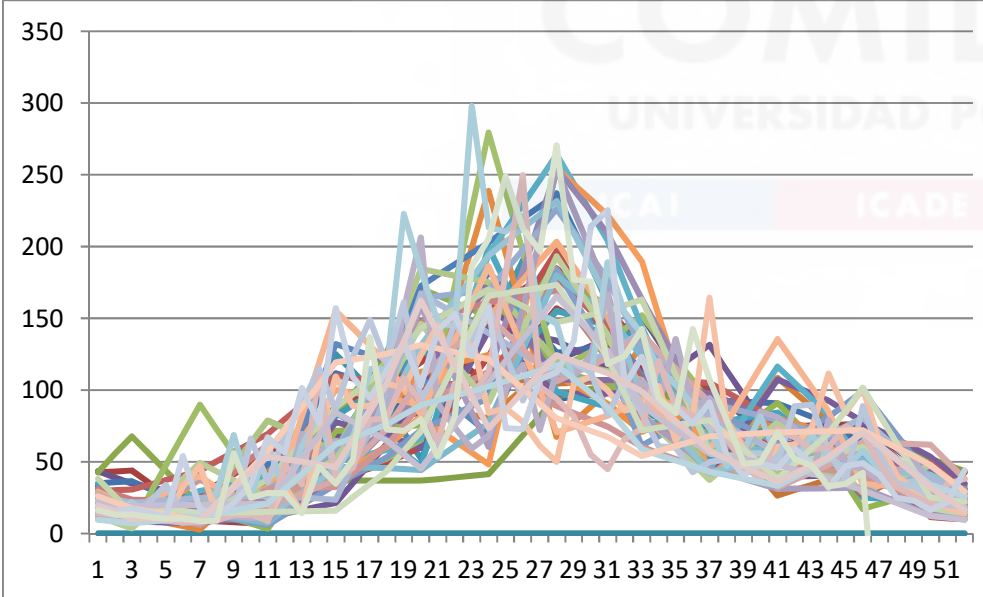
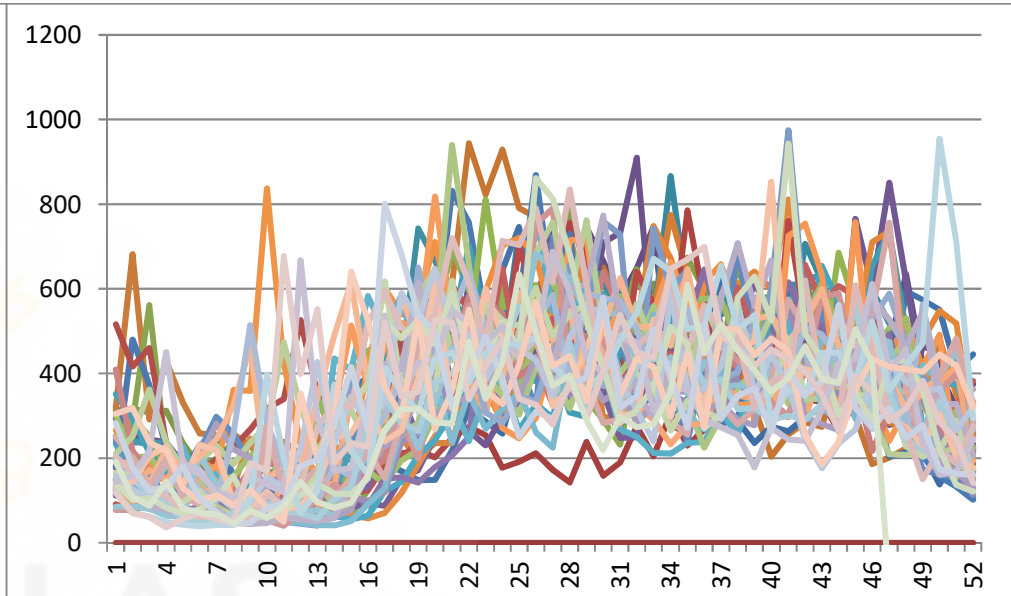
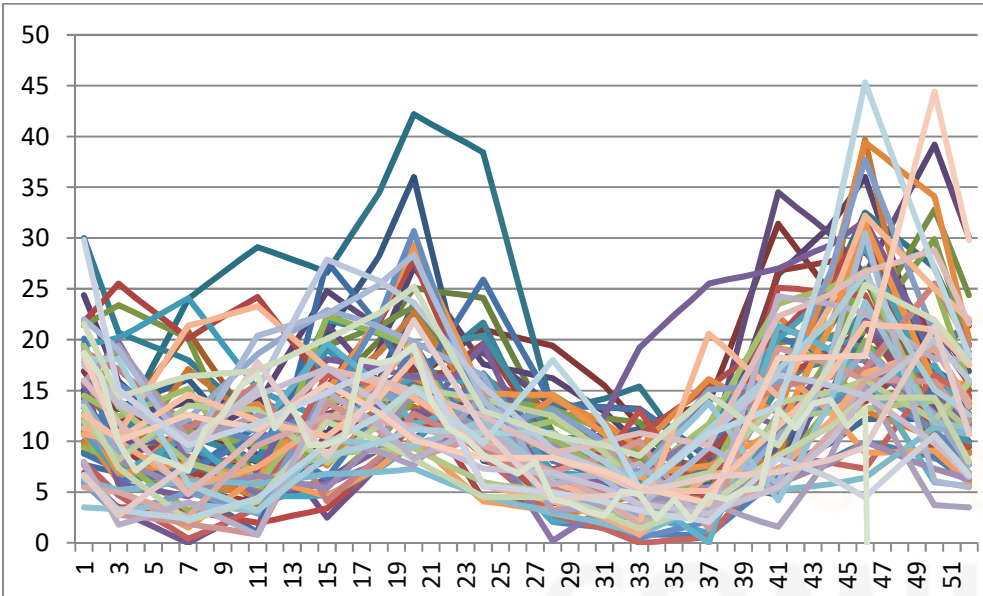


— Spot prices (USD/MWh)    — Yearly average    - - Average 2010-2017



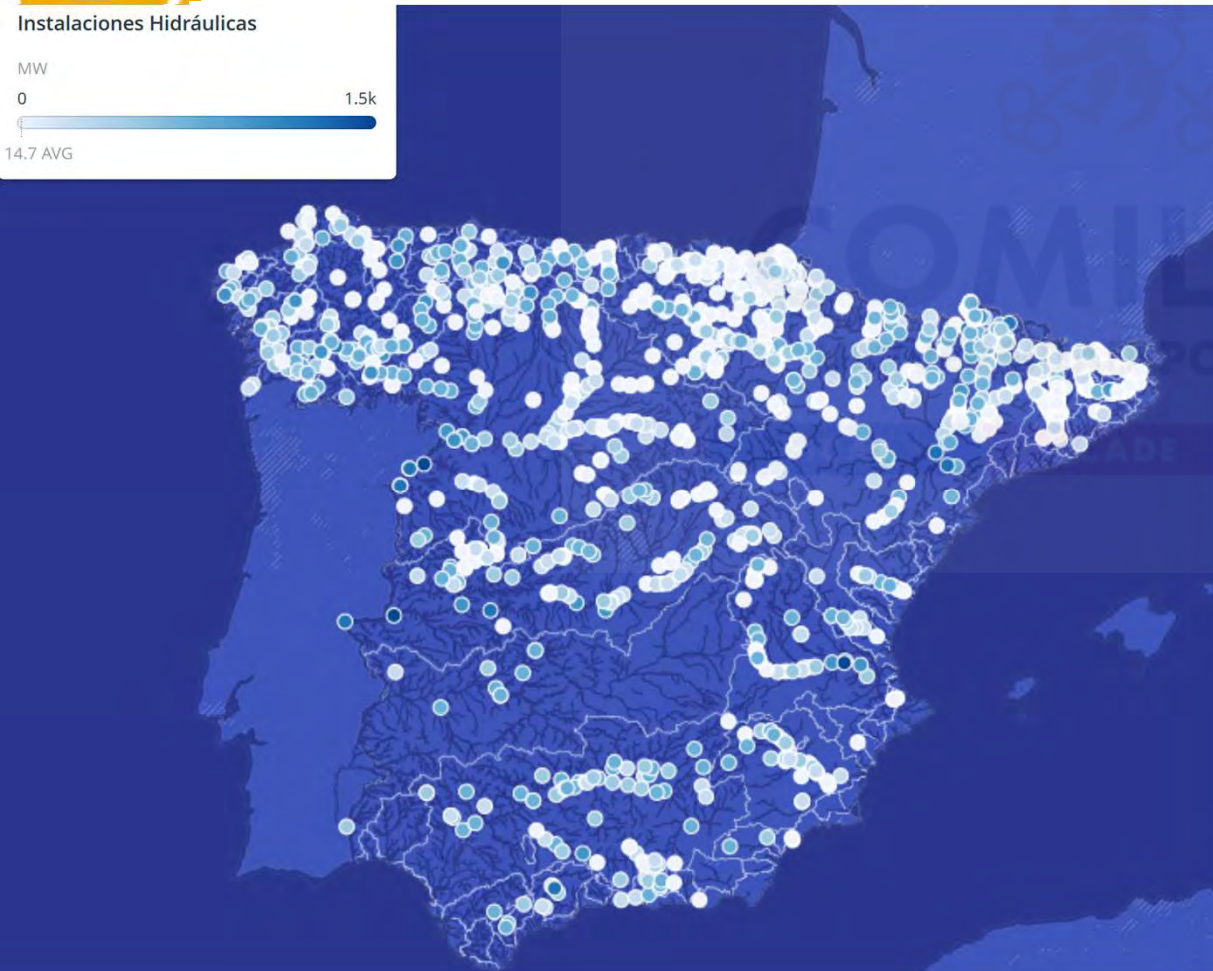
Source: L. Barroso. PSR

# How uncertain are hydro inflows in Colombia?



# Stochastic hydro inflows

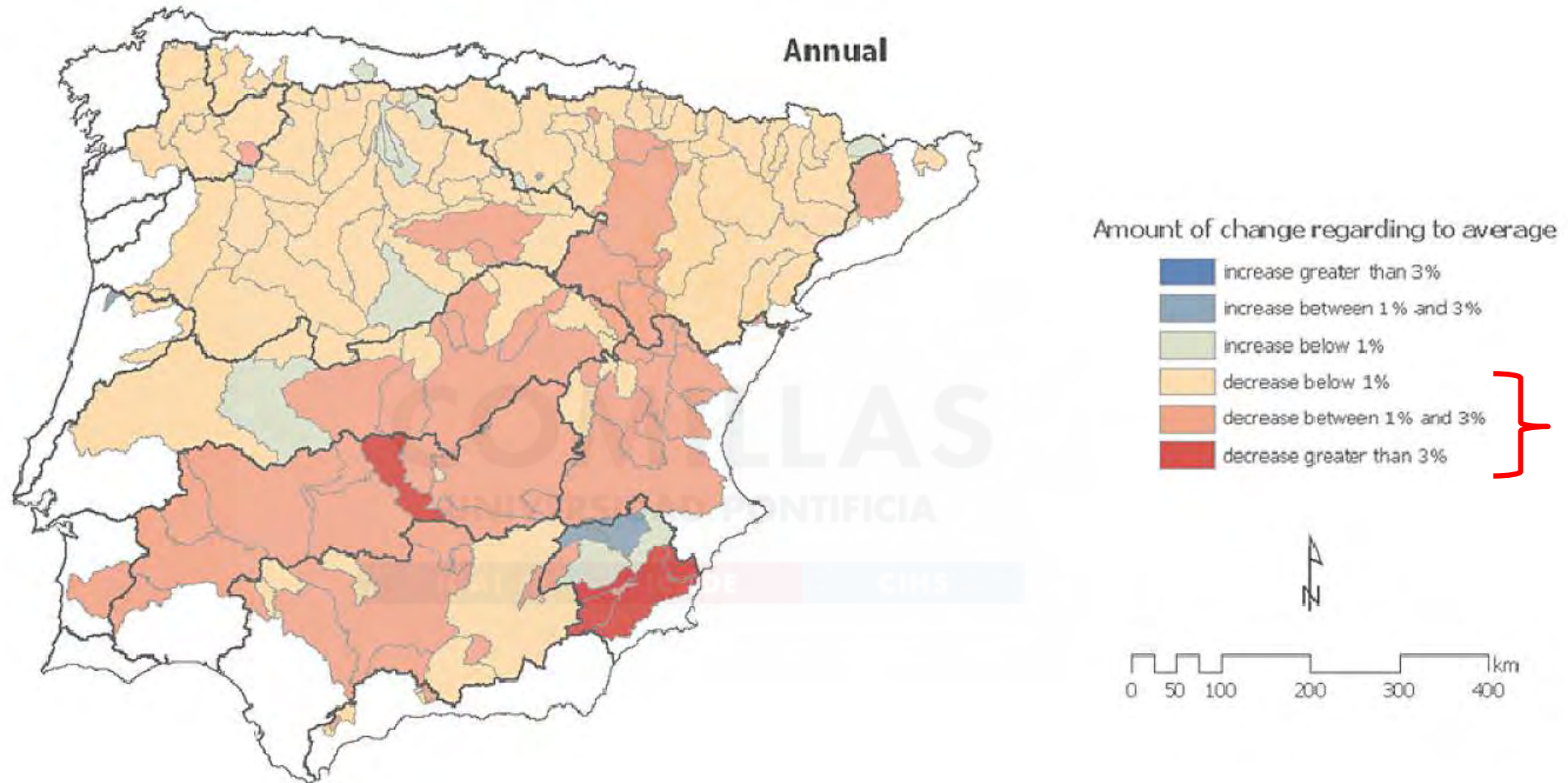
- Several measurement points in main different river basins
- Partial spatial correlation among them
- Temporal correlation in each one



<https://www.esios.ree.es/en/interesting-maps/hydraulic-installations-map>



# Inflows have decreased in Iberian rivers in the last 60 years



J. Lorenzo-Lacruz, S.M. Vicente-Serrano, J.I. López-Moreno, E. Morán-Tejeda, J. Zabalza. Recent trends in Iberian streamflows (1945–2005). *Journal of Hydrology*. Vol 414–415, Jan 2012, pp. 463-475

1. Medium term stochastic hydrothermal coordination model
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## Stochastic optimization

# Decision under uncertainty

- **DETERMINISTIC** optimization
  - Best decision when the future is known

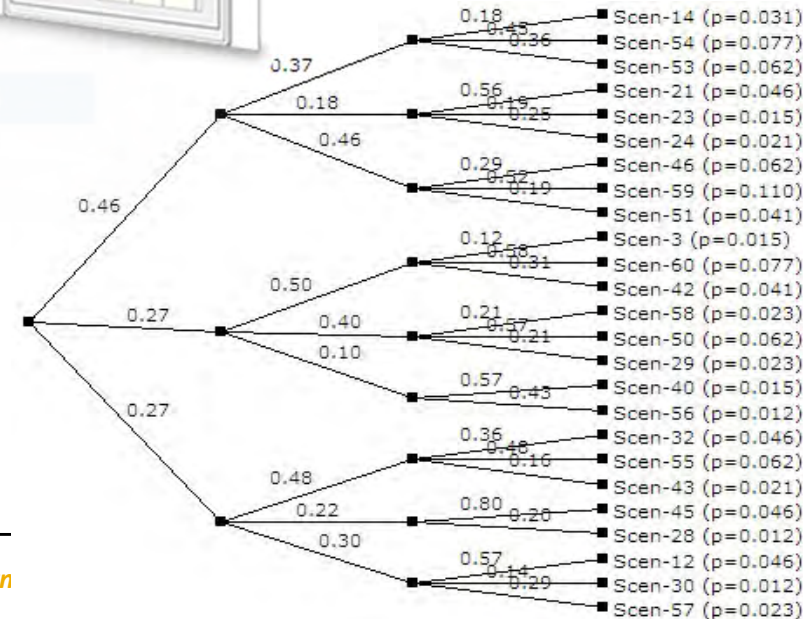


- Simulation. Scenario analysis
  - What could happen if ...?



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- **STOCHASTIC** optimization
  - Best decision when the future is uncertain (with a known probability)



# Alternatives for modeling the uncertainty (i)

- Wait and see or scenario analysis or what-if analysis
  - Decisions are taken once solved the uncertainty
  - The problem is solved independently for each scenario
  - The scenario with a mean value of the parameters is just a particular case
  - A priori, decisions will be different for each scenario (**anticipative, clairvoyant, non-implementable**)
  - Solution of a scenario can be infeasible in the others

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# Alternatives for modeling the uncertainty (iii)

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- Here and now
  - Decisions must be taken **before solving uncertainty**
  - **Non-anticipative decisions** (only the available information so far can be used, no future information)
  - The only **relevant decisions** are those of the **first stage**, given that they are the only ones to be taken immediately
  - Stochastic solution **considers the stochasticity distribution**
  - It allows including **risk-averse** attitudes, penalizing the worst cases
  - **STOCHASTIC OPTIMIZATION**

# Example: hydrothermal coordination problem

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- Scenario analysis
  - Run the model supposing that the natural hydro inflows will be the same as **any of the previous historical inflows** (i.e., the year 1989 or 2004, etc.) for the time scope
  - Run the model supposing that the natural hydro inflows for each period will be exactly the **mean of the historical values** (i.e., average year) for the time scope
- Stochastic optimization
  - Run the model considering that the **distribution of future natural hydro inflows** will be the same as it has been in the past

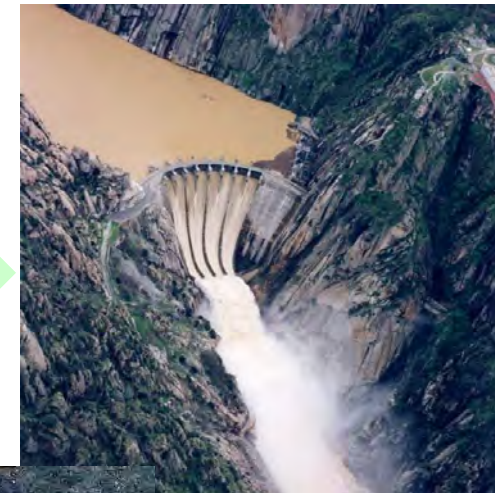
# What is a stochastic optimization model?

- If I knew the future (I am clairvoyant, anticipative, deterministic), I would run my system as cheapest as possible
  - I am not going to crash my car this year
  - It is going to rain enough in Brazil this year
  - No generating unit is going to fail in the next 24 hours
- If I don't know the future (it is uncertain, and I am not clairvoyant), I must hedge against it, and therefore I take a stochastic decision
  - I am going to buy a car insurance policy
  - I must sign some gas contracts for running CCGT units
  - N-1 dispatch criterion
- Stochastic decision will be cheaper than some of the (catastrophic) scenarios but more expensive than others

# Stochastic hydro scheduling

- Determine the optimal operation of a hydro system subject to uncertainty in future hydro inflows
- Taking optimal decisions in different stages in the presence of random parameters with known distributions
- A decision tree represents uncertainty

High outflows (Spring 2010)



Minimum outflow



Aldeadávilla hydro plant

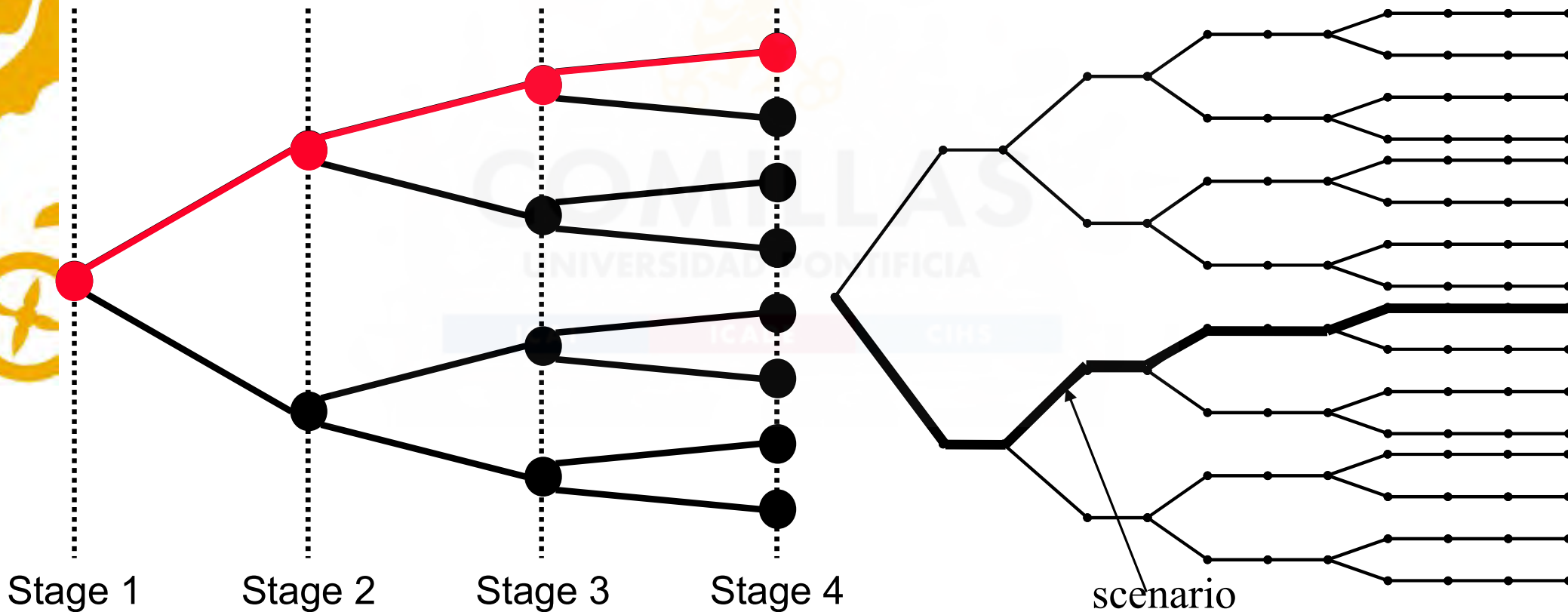
# Probability tree or scenario tree

- **Tree**: represents how the **stochasticity is revealed over time**, i.e., the different states of the random parameters and simultaneously the non-anticipative decisions over time. **Correlation** among parameters should be considered
- **Scenario**: any path going from the root to the leaves
- The scenarios that share the information until a certain period do the same into the tree (non-anticipative decisions)

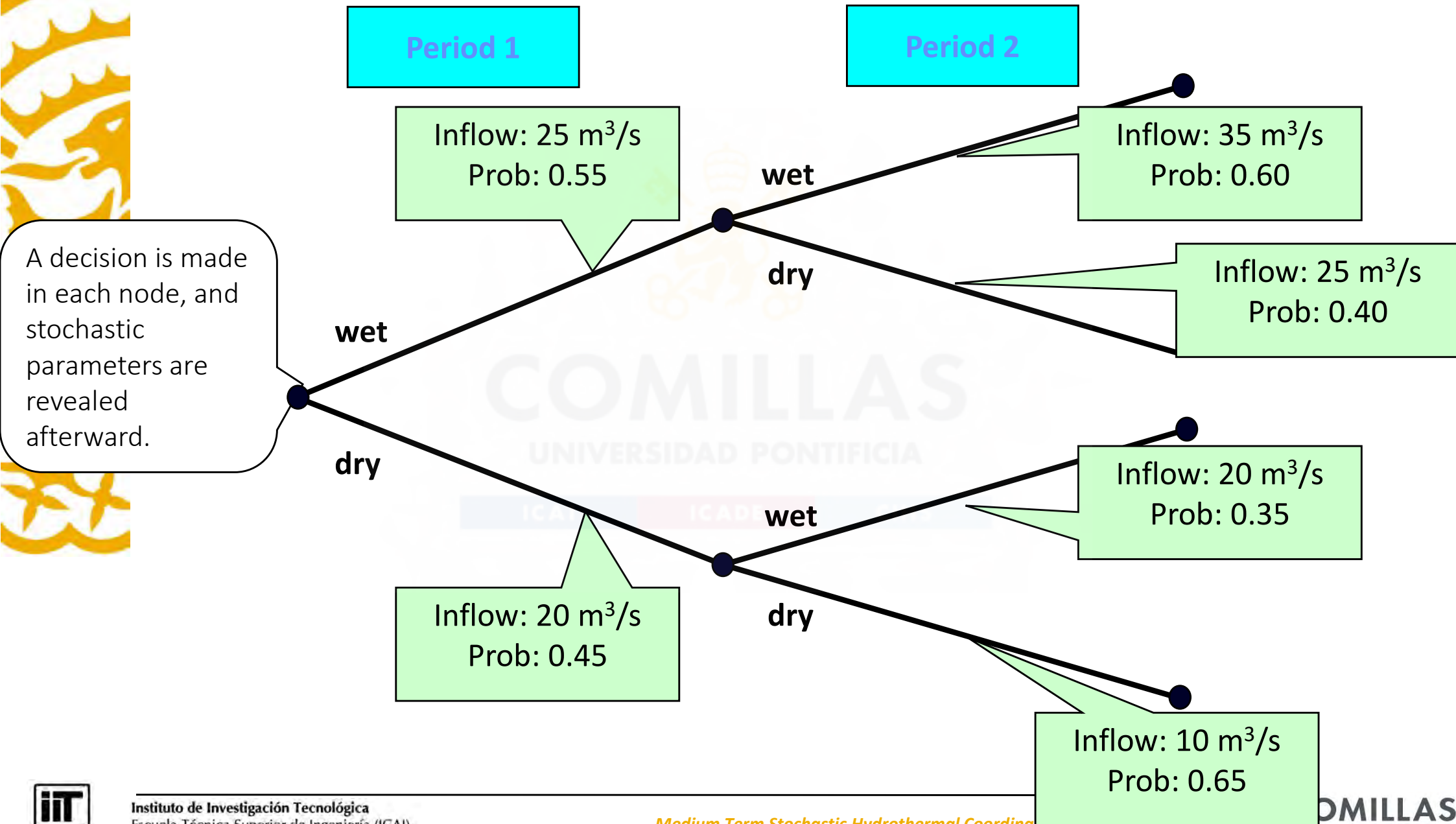
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# Scenario tree

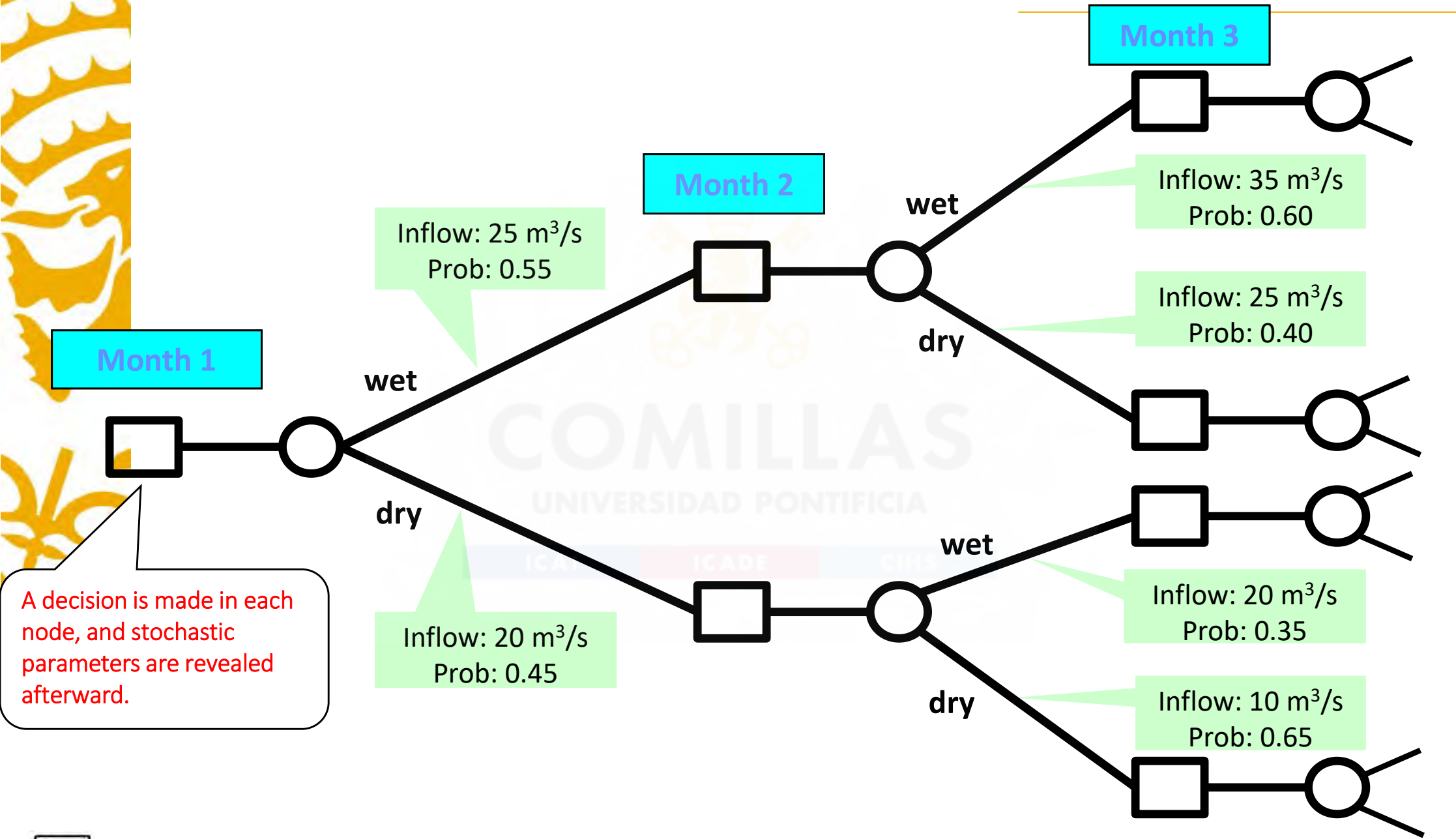
- **Nodes:** where decisions are taken.
- **Scenarios:** instances of the random process.



# Scenario tree example

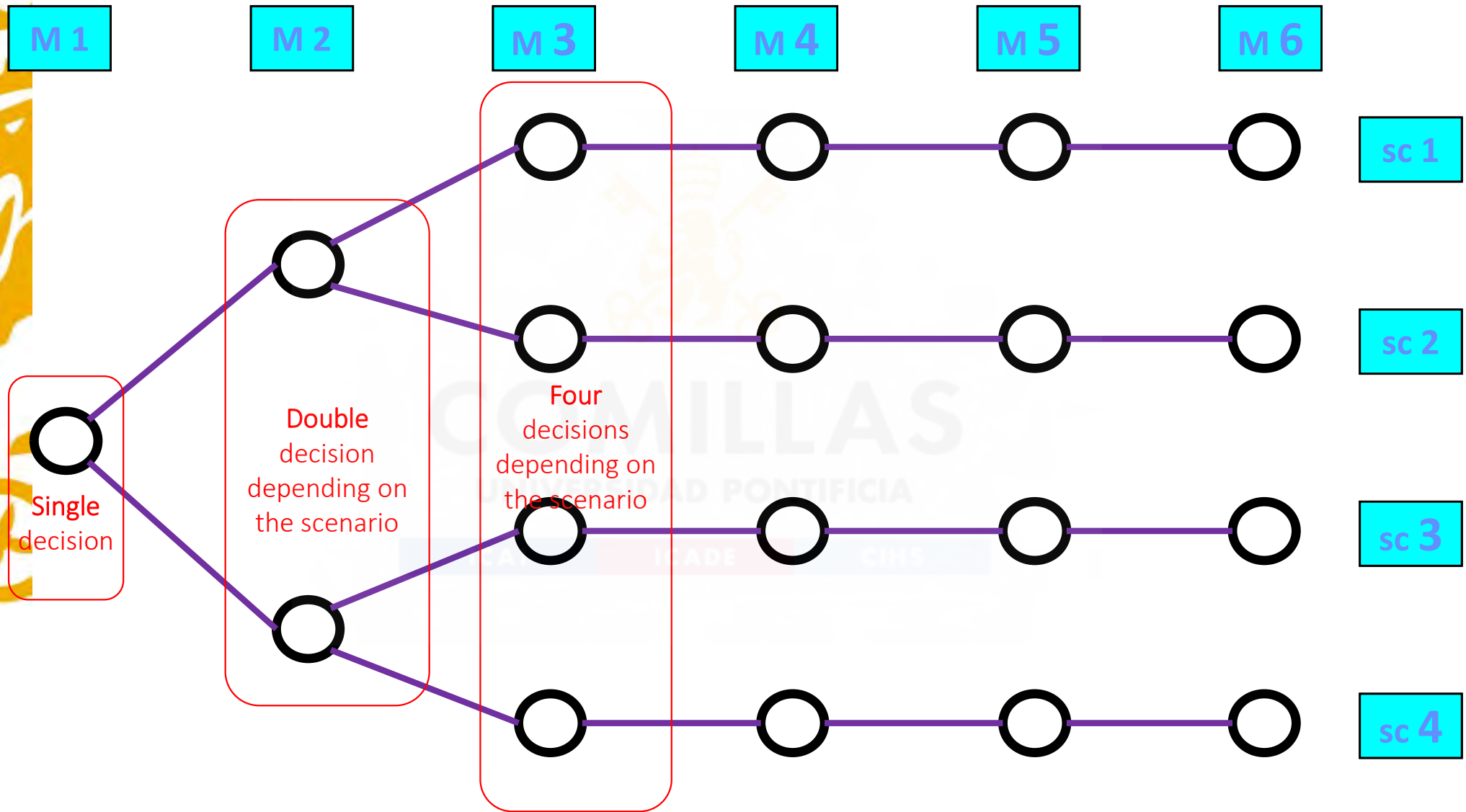


# Decision tree for stochastic hydro scheduling





# Scenario tree for 6 months



# Scenario tree trade-off

- Big scenario tree and simplified electric system operation problem
  - Where do we branch the tree?
- Small scenario tree and realistic electric system operation problem

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# Where is it important to branch the tree?

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- Where there is a **huge variety of stochastic values**
  - Winter and spring in hydro inflows
- **Short-term future** will affect more than long-term future
  - If the scope of the model is from January to December, branching in winter and spring will be more relevant than branching in autumn

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# Scenario tree generation (i)

- **Univariate** series (**one** hydro inflow/reservoir)
  - Distance from the cluster centroid to each series from a period to the last one
- **Multivariate** series (**several** hydro inflows/reservoirs)
  - Distance from the multidimensional cluster centroid to each series of each variable from a period to the last one

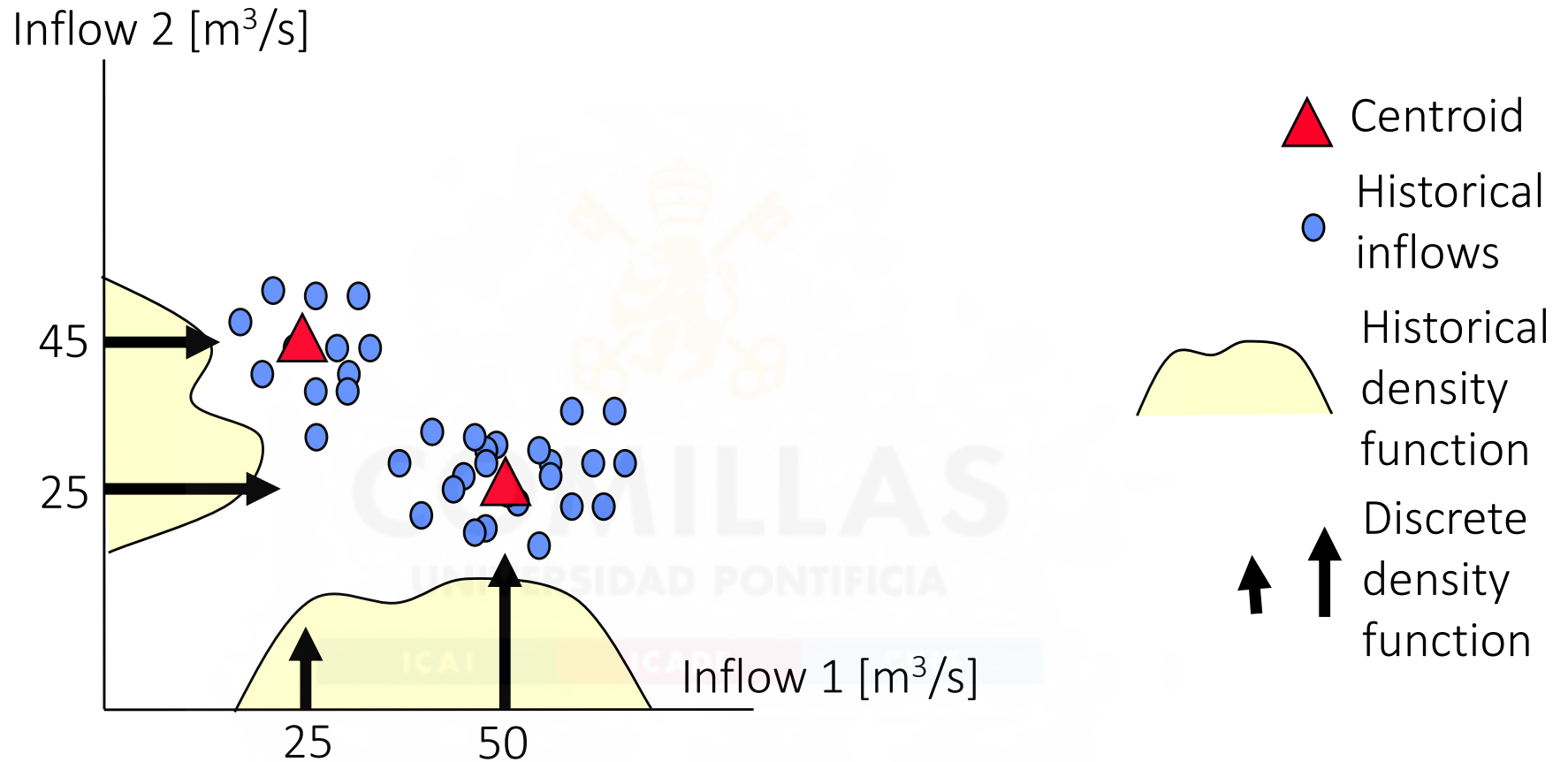
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## Scenario tree generation (ii)

- There is no established method to obtain a unique scenario tree
- A **multivariate scenario tree** is obtained by the **neural gas clustering** technique that simultaneously considers the main stochastic series and their **spatial and temporal dependencies**.
- **Contamination: very extreme scenarios** can be artificially introduced with a very low probability
- Number of scenarios generated enough for yearly operation planning

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# Clustering in two dimensions



Centroids have the **minimum distance** to their corresponding points

Their **probability** is proportional to the number of points represented by the centroid

# Common approach for tree generation

- Process divided into **two phases**:
  - **Generation** of a scenario tree.  
Neural gas method.
  - **Reduction** of a scenario tree.  
Using probabilistic distances.

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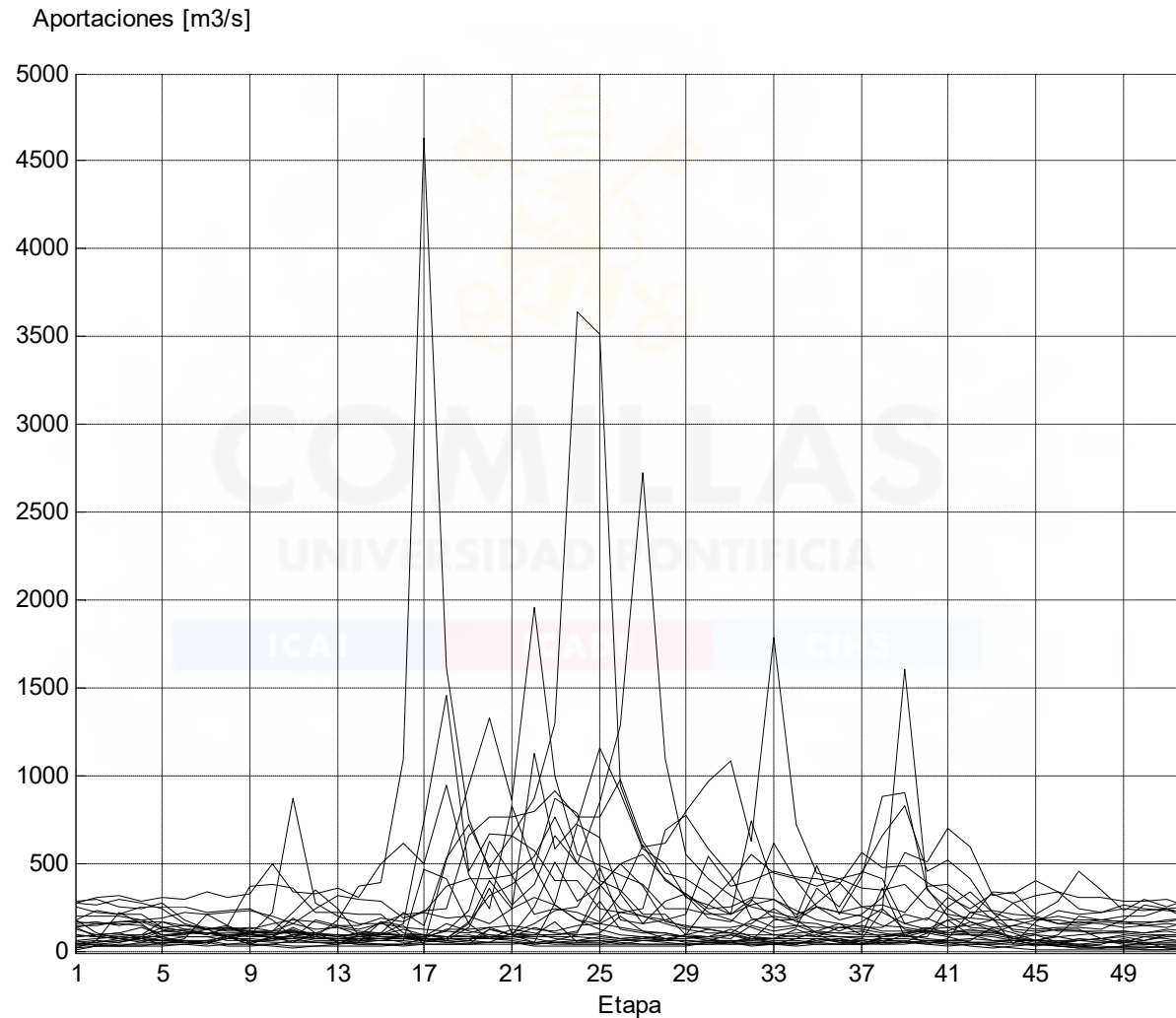
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# Natural hydro inflows (V)

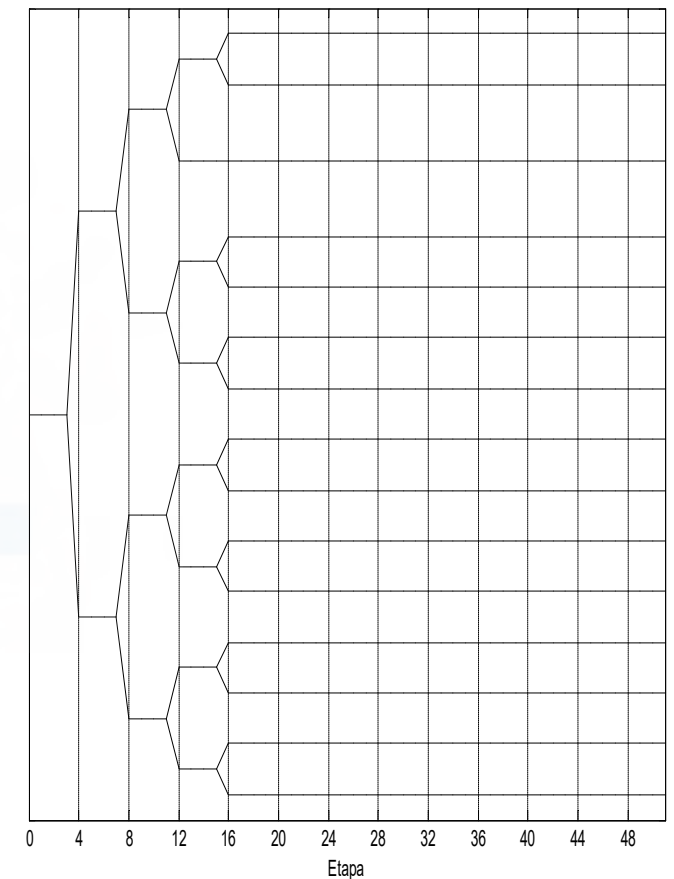
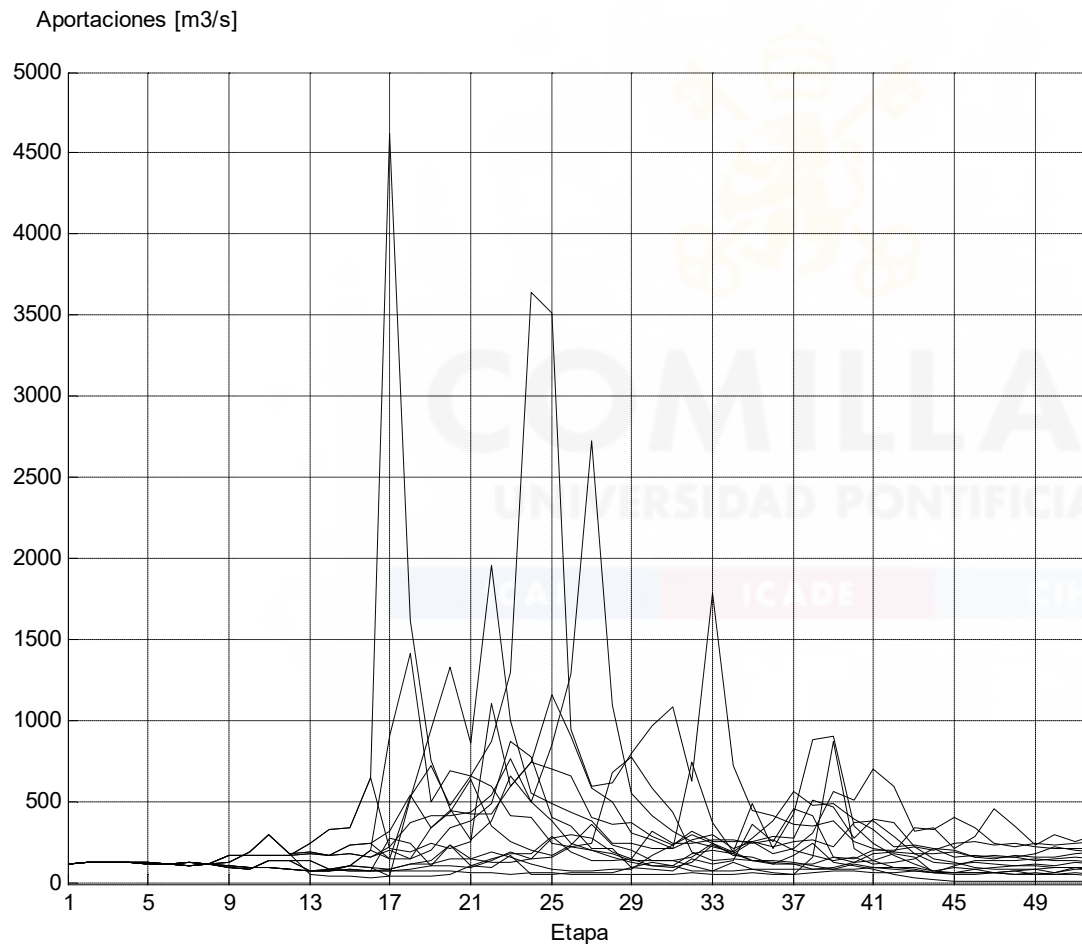
- Data series for one hydro inflow:





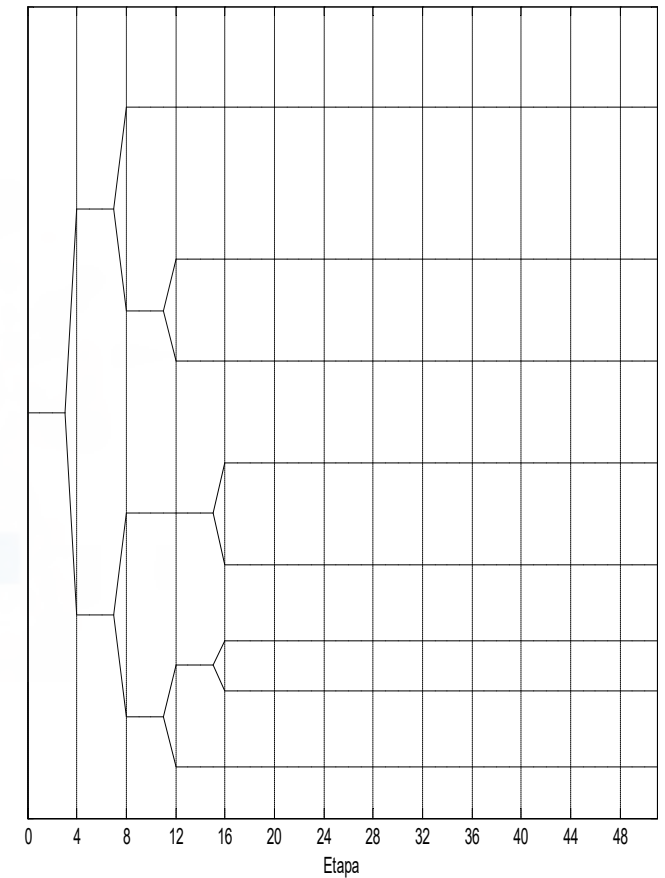
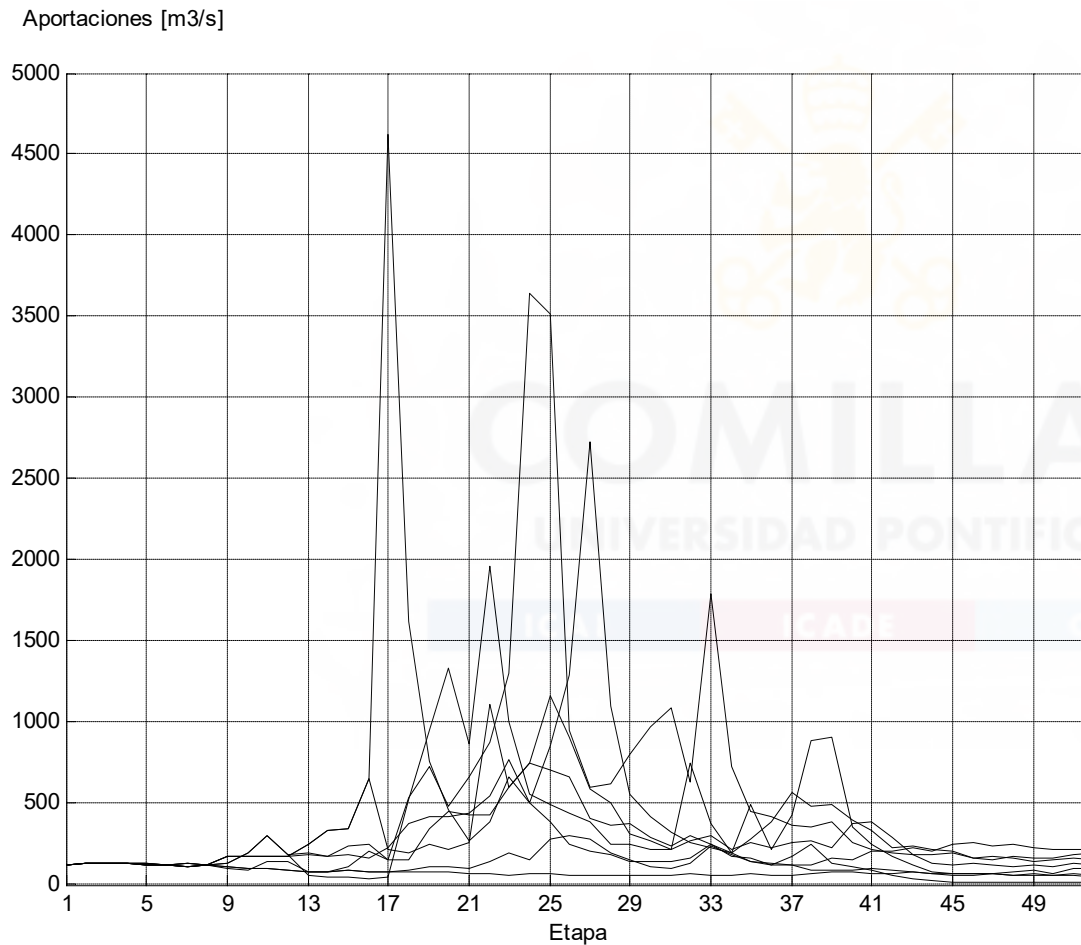
# Natural hydro inflows (VI)

- Initial scenario tree (15 scenarios) for one hydro inflow

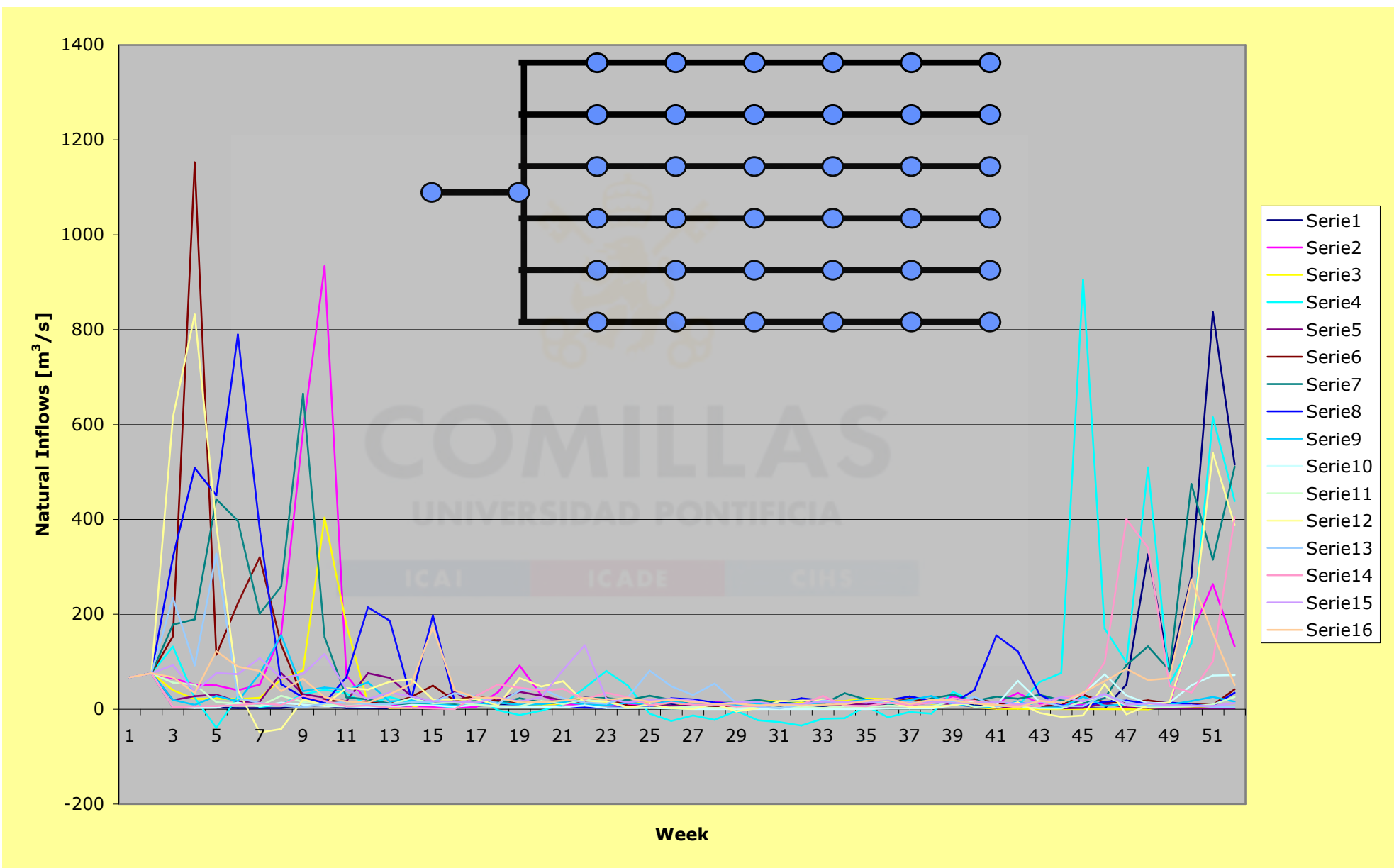


# Natural hydro inflows (VII)

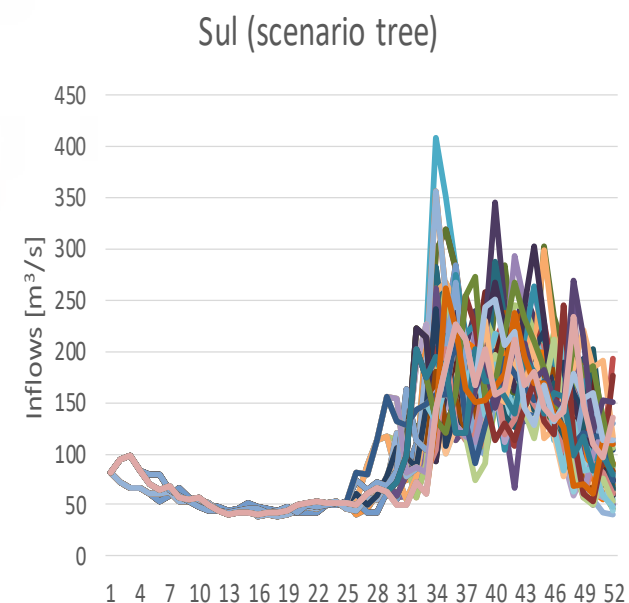
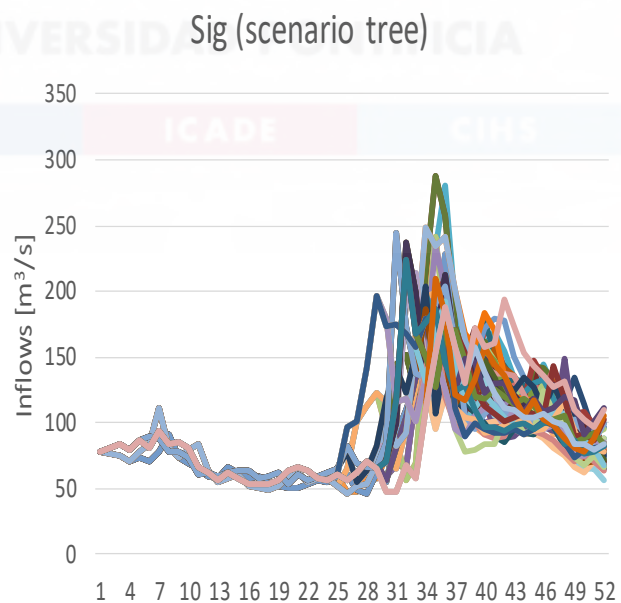
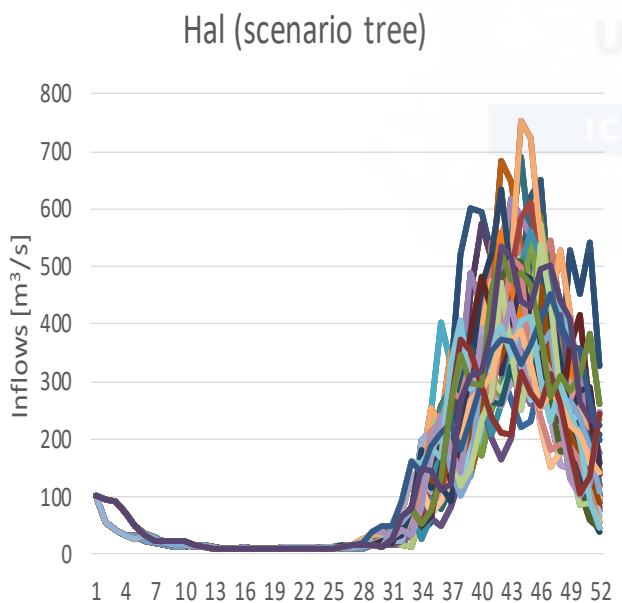
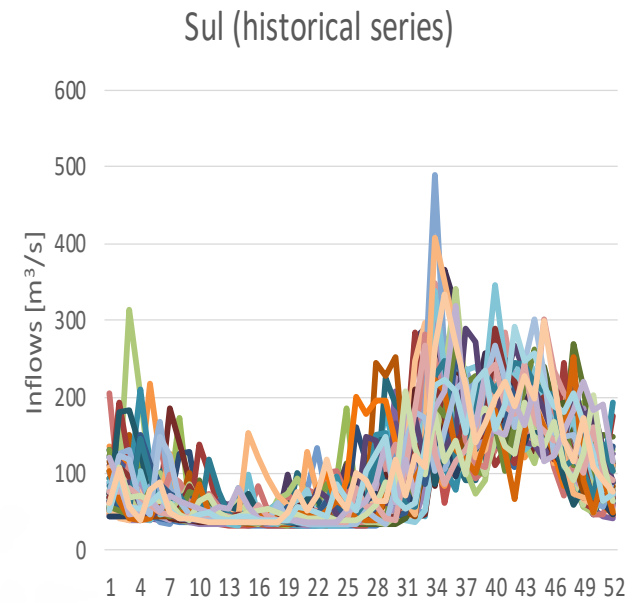
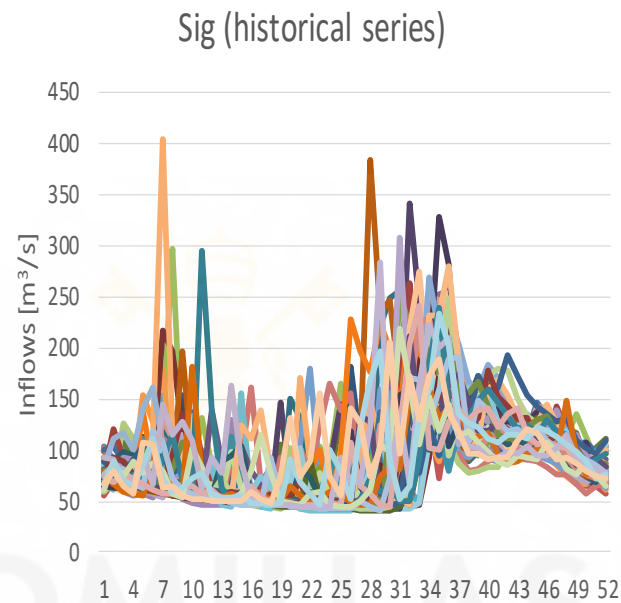
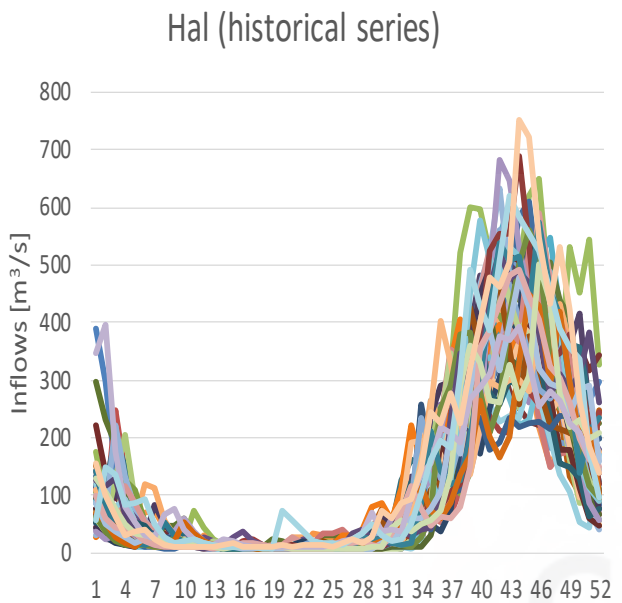
- Reduced scenario tree (8 scenarios) for one hydro inflow



# Natural hydro inflows: scenario tree



# Historical hydro inflows and scenario trees in Iceland



1. Medium term stochastic hydrothermal coordination model
2. Modeling issues
3. Stochastic optimization
- 4. Prototype SHTCM. Mathematical formulation**
5. Prototype SHTCM. Computer implementation



4



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Prototype SHTCM.

Mathematical formulation

# Prototype. Mathematical formulation

- Objective function
  - Minimize the total **expected** variable costs plus penalties for energy and power not served
- Variables
  - **BINARY**: Commitment, startup, and shutdown of thermal units
  - Thermal, storage hydro, and pumped-storage hydro output
  - Reservoir levels
- Operation constraints
  - Inter-period
    - Storage hydro and pumped-storage hydro scheduling
    - Water balance with stochastic inflows*
  - Intra-period
    - Load balance and operating reserve
    - Detailed hydro basin modeling
    - Thermal, storage hydro, and pumped-storage hydro operation constraints
- **Mixed integer linear programming (MIP)**

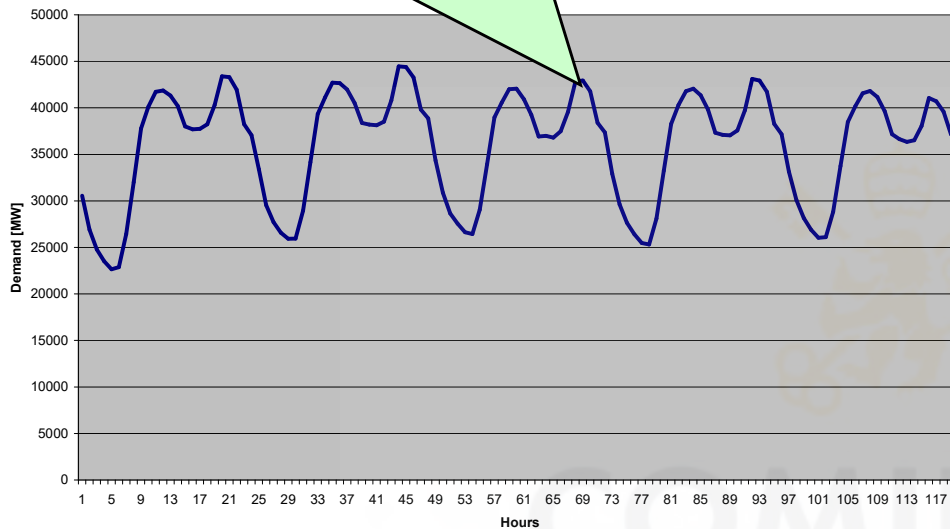
# Indices

- Time scope
  - 1 year
- Period
  - 1 month
- Subperiod
  - weekdays and weekends
- Load level
  - peak, shoulder, and off-peak

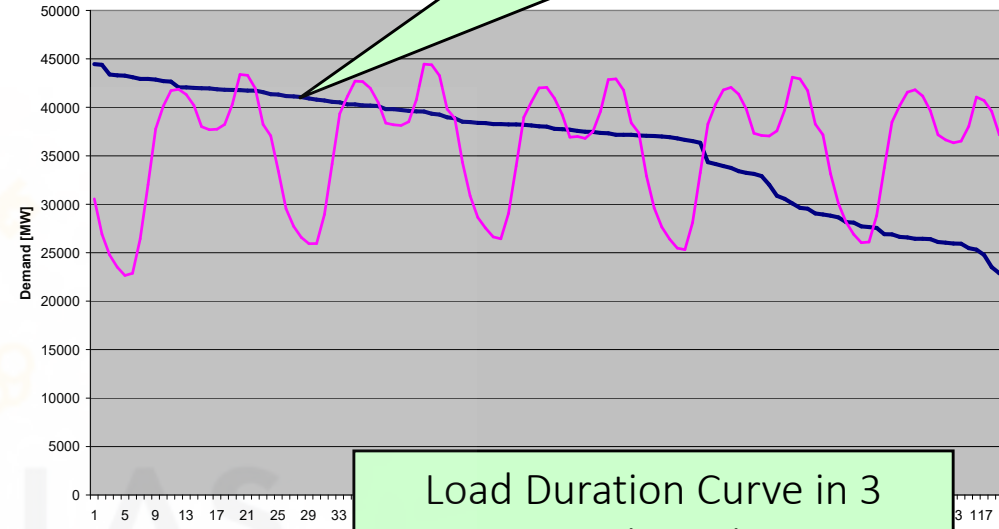
<i>Period</i>	<i>p</i>
<i>Subperiod</i>	<i>s</i>
<i>Load level</i>	<i>n</i>

# Demand (5 weekdays)

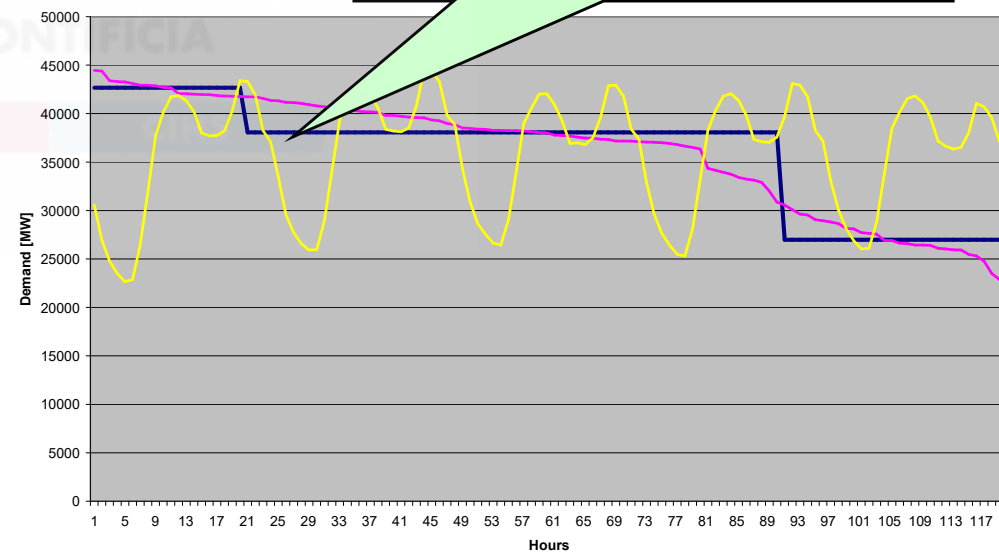
Chronological Load Curve



Load Duration Curve



Load Duration Curve in 3 Load Levels

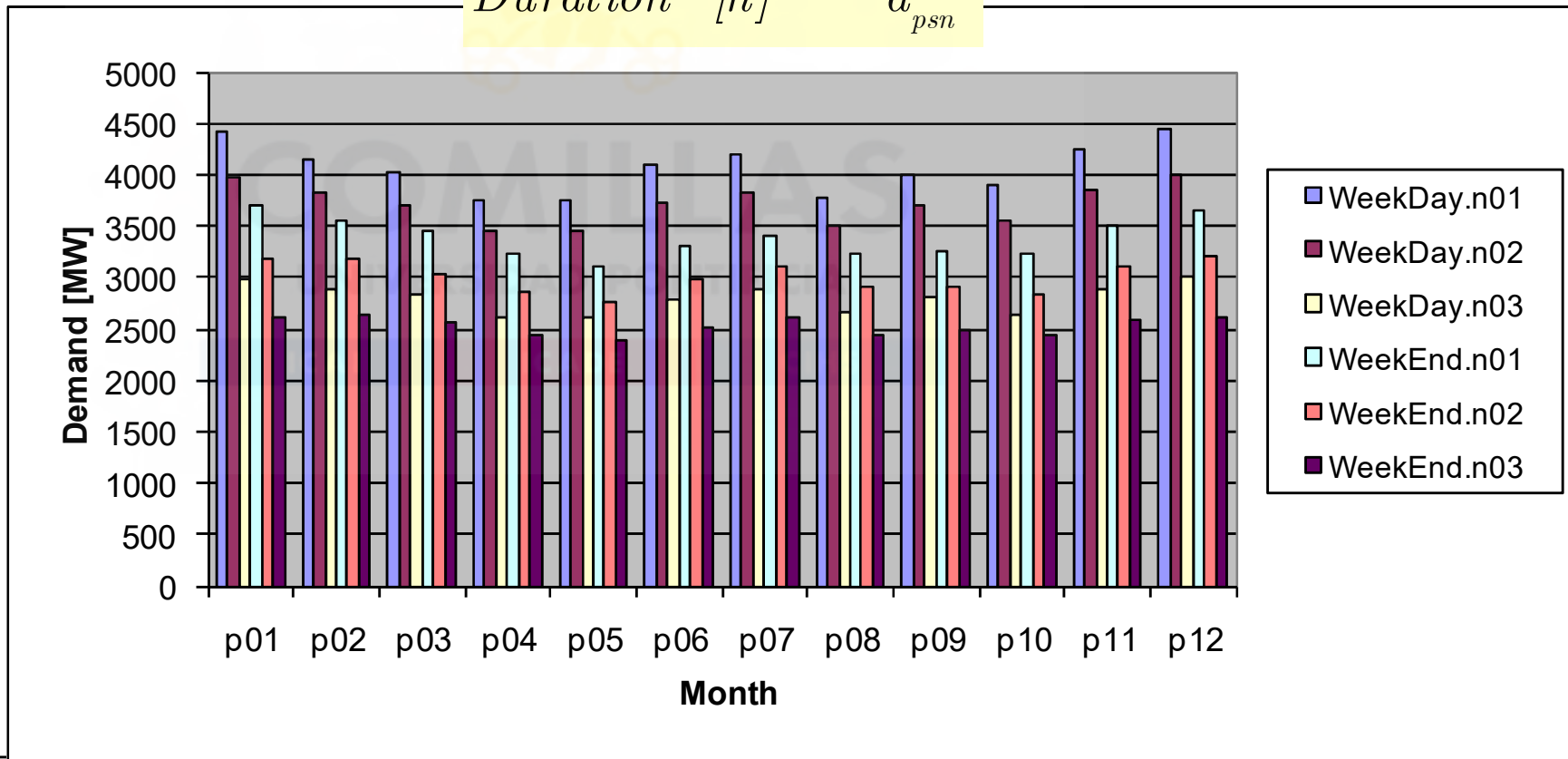




# Demand

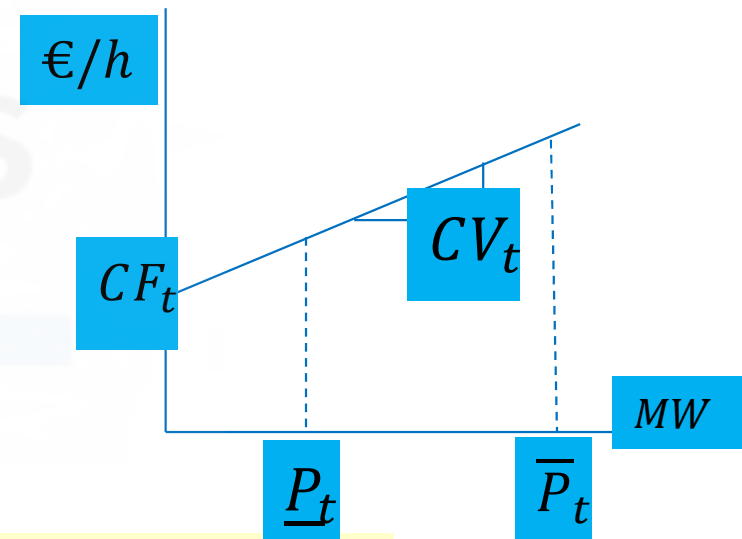
- Monthly demand with several load levels
  - Peak, shoulder, and off-peak for weekdays and weekends
- All the weekdays of the same month are similar (same for weekends)

$$\begin{array}{l}
 \text{Demand [MW]} \quad D_{psn} \\
 \text{Duration [h]} \quad d_{psn}
 \end{array}$$



# Technical characteristics of thermal units ( $t$ )

- Maximum and minimum output
- Fuel cost
- Slope and intercept of the heat rate straight line
- Operation and maintenance (O&M) variable cost
  - No load cost = fuel cost x heat rate intercept
  - Variable cost = fuel cost x heat rate slope + O&M cost
- Cold startup and shutdown cost
- Up and down ramps



Max and min output	[MW]	$\bar{P}_t, \underline{P}_t$
No load cost	[€/h]	$CF_t$
Variable cost	[€/MWh]	$CV_t$
Startup cost	[€]	$CSU_t$
Shutdown cost	[€]	$CSD_t$

Ramp up	[MW/h]	$RU_t$
Ramp down	[MW/h]	$RD_t$

# Technical characteristics of hydro plants ( $h$ )

- Maximum and minimum output
- Production function (efficiency for conversion of water release in  $m^3/s$  to electric power MW)
- Round-trip efficiency of pumped storage hydro plants
  - Only this ratio of the energy consumed to pump the water is recovered by turbinning it

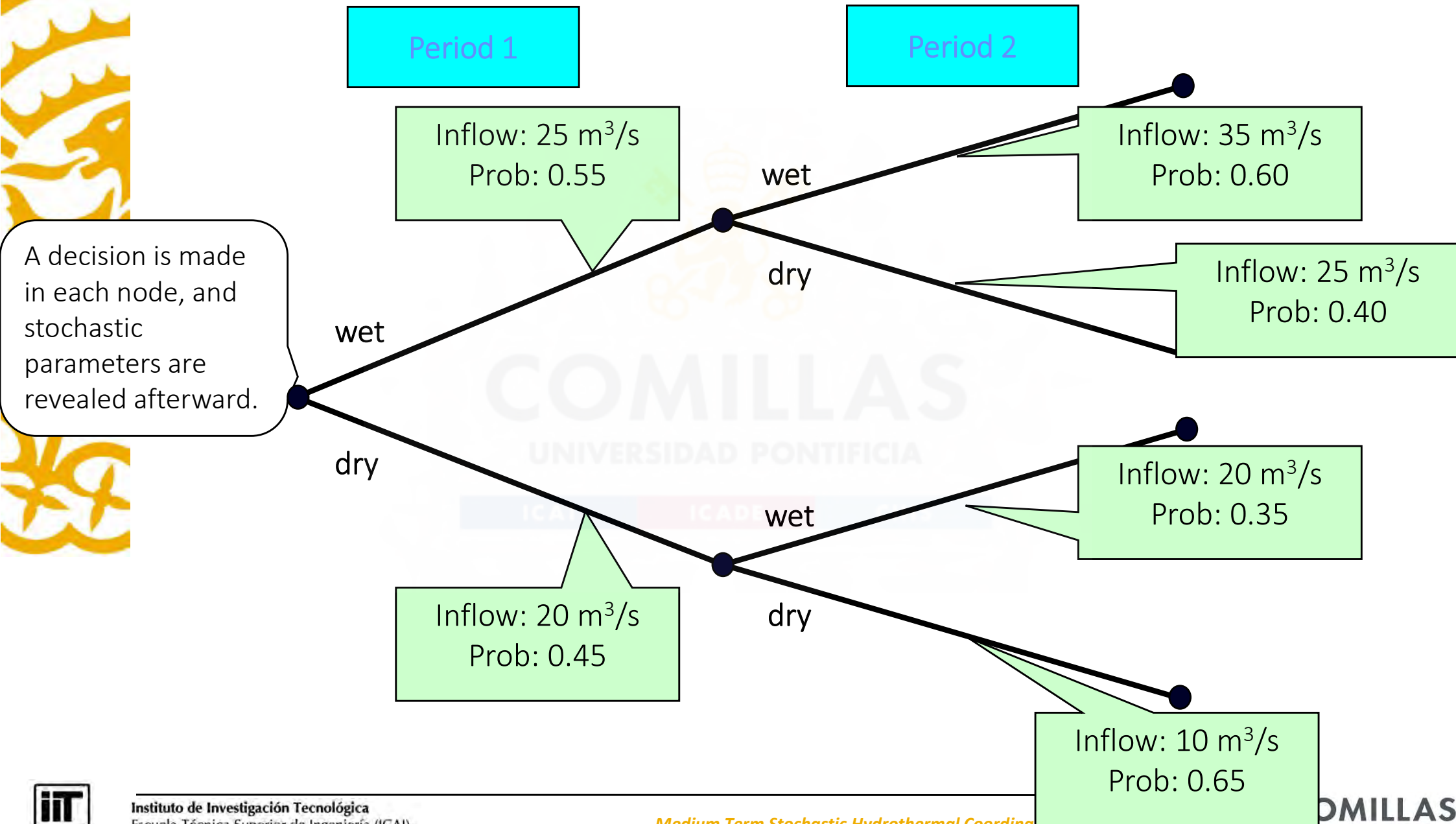
<i>Max and min output</i>	<i>[MW]</i>	$\overline{P}_h, \underline{P}_h$
<i>Production function</i>	<i>[kWh / m<sup>3</sup>]</i>	$C_h$
<i>Efficiency</i>	<i>[p.u.]</i>	$\eta_h$

# Technical characteristics of hydro reservoirs ( $r$ )

- Maximum and minimum water reserve
- Initial water reserve
  - Final reserve = initial reserve
- Natural hydro inflows

<i>Max and min water reserve</i>	$[hm^3]$	$\overline{R}_r, \underline{R}_r$
<i>Initial and final water reserve</i>	$[hm^3]$	$R'_r$
<i>Stochastic natural hydro inflows</i>	$[m^3 / s]$	$I_{pr}^\omega$

# Scenario tree example

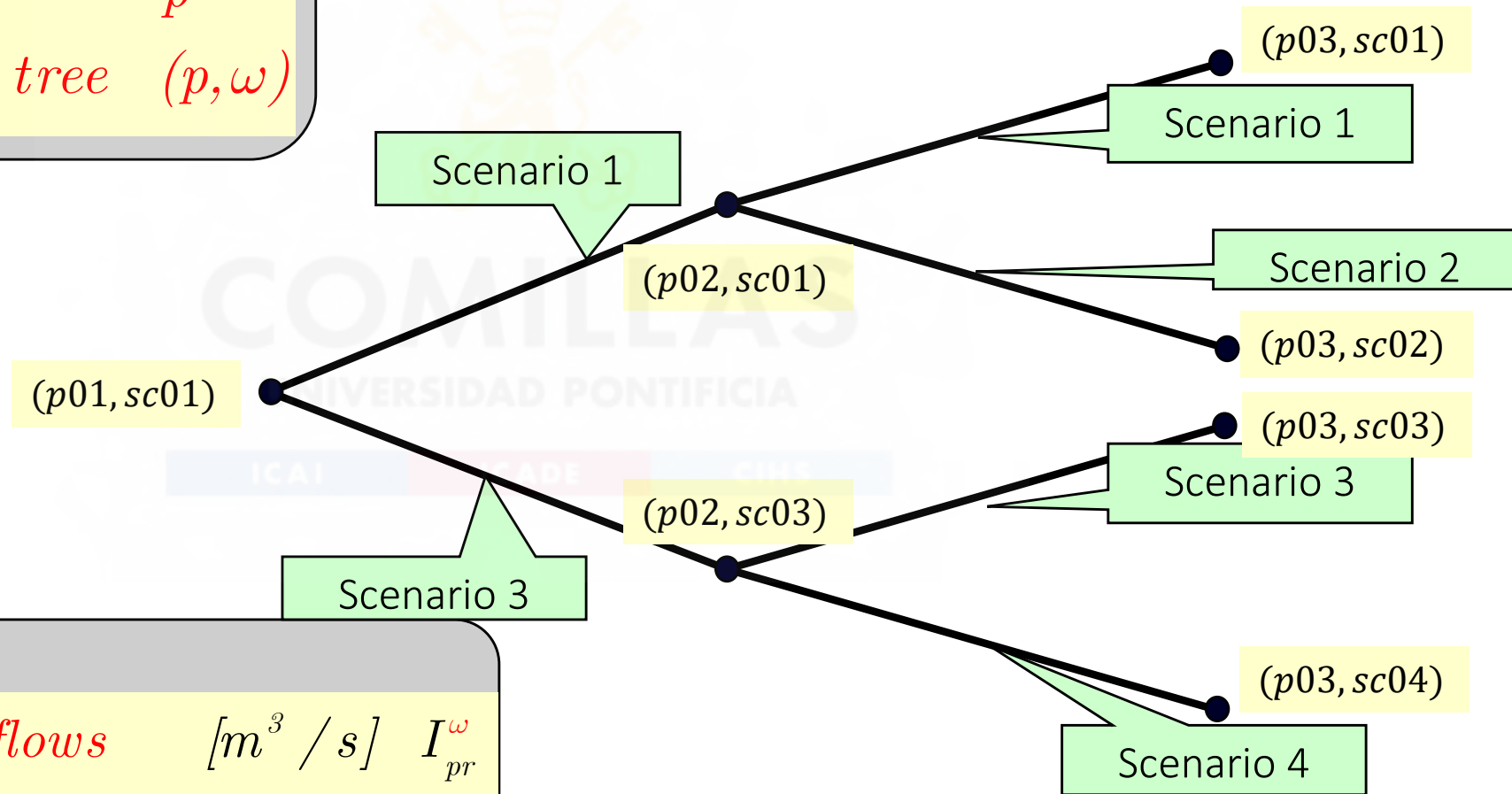


# Scenario tree. Ancestor and descendant

**Tree structure**

<i>Scenario</i>	$\omega$
<i>Period</i>	$p$
<i>Scenario tree</i>	$(p, \omega)$

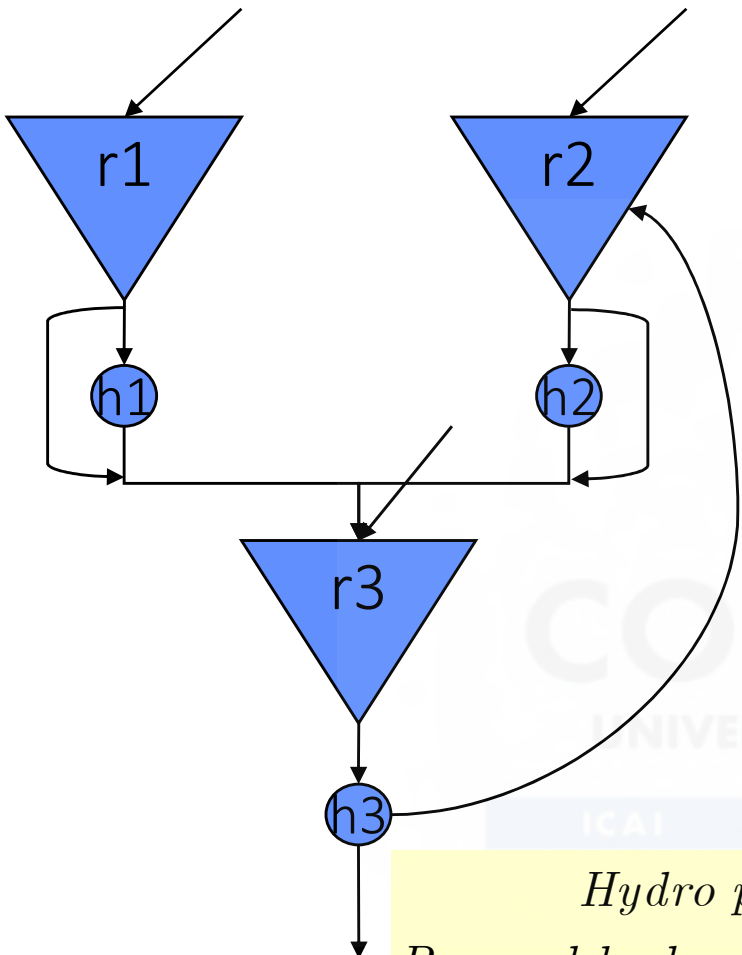
**Tree relations**

$$\omega' \in a(\omega) \quad (p02, sc03) \in a[(p03, sc03)]$$


**Tree data**

<i>Stochastic inflows</i>	$[m^3 / s]$	$I_{pr}^\omega$
<i>Scenario probability</i>	$[p.u.]$	$P_p^\omega$

# Hydro topology



Only one spillage per reservoir can be considered

Written in Math	Written in GAMS	Example
-----------------	-----------------	---------

<i>Hydro plant upstream of reservoir</i>	$h \in up(r)$	$hur(h, r)$	$(h1, r3)$
<i>Pumped hydro plant upstream of reservoir</i>	$h \in up(r)$	$hpr(h, r)$	$(h3, r2)$
<i>Reservoir upstream of hydro plant</i>	$h \in dw(r)$	$ruh(r, h)$	$(r2, h2)$
<i>Reservoir upstream of pumped hydro plant</i>	$h \in dw(r)$	$rph(r, h)$	$(r3, h3)$
<i>Reservoir upstream of reservoir</i>	$r' \in up(r)$	$rur(r, r)$	$(r1, r3)$

## Other system parameters

- Energy not served cost
- Operating power reserve not served cost
- Operating power reserve

<i>Energy not served cost</i>	$[\text{€} / \text{MWh}]$	$CV'$
<i>Operating power reserve not served cost</i>	$[\text{€} / \text{MW}]$	$CV''$
<i>Operating reserve</i>	$[\text{MW}]$	$O_{ps1}$

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# Variables

- Commitment, startup, and shutdown of thermal units (BINARY)

*Commitment, startup and shutdown*  $\{0,1\}$   $uc_{pst}^\omega, su_{pst}^\omega, sd_{pst}^\omega$

- Production of thermal units and hydro plants

*Production of a thermal or hydro unit* [MW]  $p_{psnt}^\omega, p_{psnh}^\omega$

- Consumption of pumped storage hydro plants

*Consumption of a hydro plant* [MW]  $c_{psnh}^\omega$

- Reservoir levels (at the beginning of the period)

*Reservoir level* [hm<sup>3</sup>]  $r_{pr}^\omega$

- Energy and power not served

*Energy and power not served* [MW]  $ens_{psn}^\omega, pns_{ps}^\omega$

# Constraints: Operating power reserve

*Committed output of thermal units*

+ *Maximum output of hydro plants*

+ *Power not served*

≥ *Demand*

+ *Operating reserve* *for peak load level, subperiod, period, and scenario [MW]*

$$\sum_t \bar{P}_t uc_{pst}^\omega + \sum_h \bar{P}_h + pns_{ps}^\omega \geq D_{ps1} + O_{ps1} \quad \forall \omega ps$$

# Constraints: Generation and load balance

*Generation of thermal units*

+ *Generation of storage hydro plants*

– *Consumption of pumped storage hydro plants*

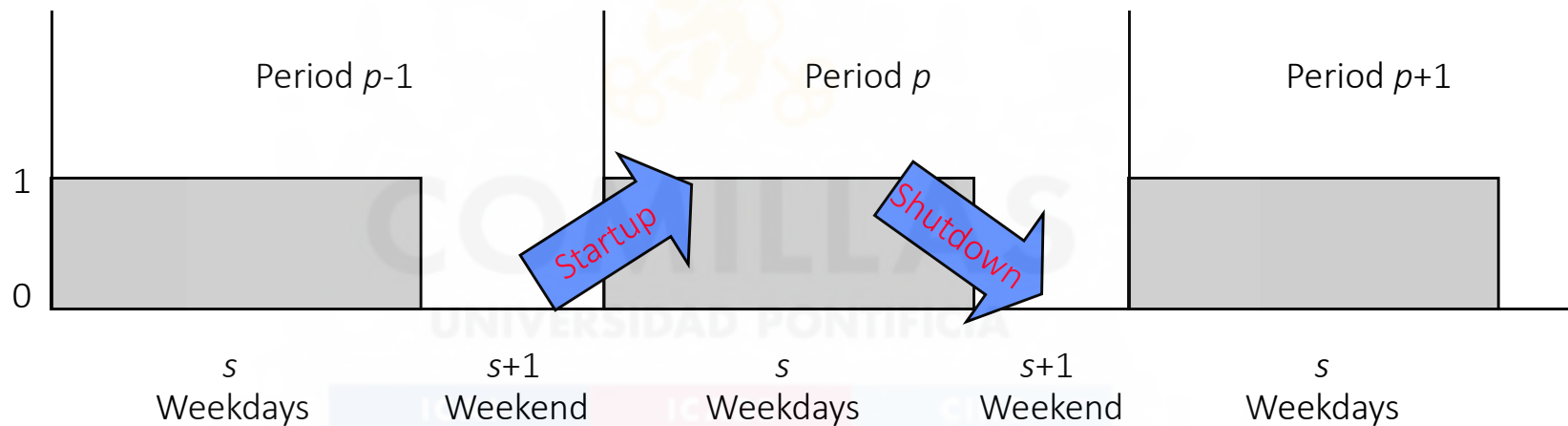
+ *Energy not served*

= *Demand* for each load level, subperiod, period, and scenario [MW]

$$\sum_t p_{psnt}^\omega + \sum_h p_{psnh}^\omega - \sum_h c_{psnh}^\omega + ens_{psn}^\omega = D_{psn} \quad \forall \omega_{psn}$$

# Constraints: Commitment, startup, and shutdown

- All the weekdays of the same month are similar (same for weekends)
- Commitment decision of a thermal unit



# Constraints: Commitment, startup, and shutdown

- **Startup** of thermal units can only be made in the transition between consecutive weekend and weekdays

*Commitment of a thermal unit in a weekday*

– *Commitment of a thermal unit in the weekend of previous period*

= *Startup of a thermal unit in this weekday*

– *Startup of a thermal unit in this weekday* [p.u.]

$$uc_{pst}^{\omega} - uc_{p-1s+}^{\omega'} = su_{pst}^{\omega} - sd_{pst}^{\omega} \quad \forall \omega pst \quad \omega' \in a(\omega)$$

- **Shutdown** only in the opposite transition

*Commitment of a thermal unit in a weekend*

– *Commitment of a thermal unit in the previous weekday*

= *Startup of a thermal unit in this weekend*

– *Shutdown of a thermal unit in this weekend* [p.u.]

$$uc_{ps+1t}^{\omega} - uc_{pst}^{\omega} = su_{ps+1}^{\omega} - sd_{ps+1}^{\omega} \quad \forall \omega pst$$

# Constraints: Commitment and production

*Production of a thermal unit*

$\leq$  Commitment of a thermal unit x the maximum output  
reduced by availability rate [MW]

*Production of a thermal unit*

$\geq$  Commitment of a thermal unit x the minimum output  
reduced by availability rate [MW]

$$uc_{pst}^{\omega} P_t(1 - Q_t) \leq p_{psnt}^{\omega} \leq uc_{pst}^{\omega} \bar{P}_t(1 - Q_t) \quad \forall \omega psnt$$

- If the thermal unit is committed ( $uc_{pst}^{\omega} = 1$ ) it can produce between its minimum and maximum output
- If the thermal unit is not committed ( $uc_{pst}^{\omega} = 0$ ) it can't produce

# Constraints: Water balance for each reservoir

- Reservoir volume at the beginning of the period*
- *Reservoir volume at the end of the period*
- + *Natural hydro inflows*
- *Spills from this reservoir*
- + *Spills from upstream reservoirs*
- + *Turbined water from upstream storage hydro plants*
- *Turbined and pumped water from this reservoir*
- + *Pumped water from upstream pumped hydro plants = 0* *for each reservoir,*
- period, and scenario*
- [hm<sup>3</sup>]*

$$\begin{aligned}
 & r_{p-1r}^{\omega'} - r_{pr}^{\omega} + I_{pr}^{\omega} - S_{pr}^{\omega} + \sum_{r' \in \text{up}(r)} S_{pr'}^{\omega} \\
 & + \sum_{\substack{sn \\ h \in \text{up}(r)}} d_{psn} p_{psnh}^{\omega} / C_h - \sum_{\substack{sn \\ h \in \text{dw}(r)}} d_{psn} p_{psnh}^{\omega} / C_h \\
 & + \sum_{\substack{sn \\ h \in \text{up}(r)}} d_{psn} c_{psnh}^{\omega} \eta_h / C_h - \sum_{\substack{sn \\ h \in \text{dw}(r)}} d_{psn} c_{psnh}^{\omega} \eta_h / C_h = 0 \quad \forall \omega_{pr} \quad \omega' \in a(\omega)
 \end{aligned}$$

# Constraints: Operation limits

Reservoir volumes between limits for each hydro reservoir [hm<sup>3</sup>]

$$\begin{aligned} \underline{R}_r &\leq r_{pr}^\omega \leq \bar{R}_r & \forall \omega pr \\ R_{0r} &= R_{pr}^\omega = R'_r & \forall \omega r \end{aligned}$$

Power output between limits for each unit [MW]

$$\begin{aligned} 0 &\leq p_{psnt}^\omega \leq \bar{P}_t(1 - Q_t) & \forall \omega psnt \\ 0 &\leq p_{psnh}^\omega, c_{psnh}^\omega \leq \bar{P}_h & \forall \omega psnh \end{aligned}$$

Commitment, startup, and shutdown for each unit [p.u.]

$$uc_{pst}^\omega, su_{pst}^\omega, sd_{pst}^\omega \in \{0,1\} \quad \forall \omega pst$$



# Multiobjective function

- Minimize
  - Expected thermal variable costs [€]

$$\sum_{\omega p s t} P_p^\omega S U_t s u_{p s t}^\omega + \sum_{\omega p s t} P_p^\omega S D_t s d_{p s t}^\omega + \sum_{\omega p s n t} P_p^\omega d_{p s n} C F_t u c_{p s t}^\omega + \sum_{\omega p s n t} P_p^\omega d_{p s n} C V_t P_{p s n t}^\omega$$

- Expected penalties introduced in the objective function for energy and power not served [€]

$$\sum_{\omega p s n} P_p^\omega d_{p s n} C V' e n s_{p s n}^\omega + \sum_{\omega p s} P_p^\omega C V'' p n s_{p s}^\omega$$

# Short Run Marginal Cost (SRMC)

- Dual variable of generation and load balance [€/MW]
  - Change in the objective function due to a marginal increment in the demand when binary variables (commitment, startup, and shutdown) are fixed

$$\sum_t p_{psnt}^\omega + \sum_h p_{psnh}^\omega - \sum_h c_{psnh}^\omega + ens_{psn}^\omega = D_{psn} \quad : \sigma_{psn}^\omega \quad \forall \omega_{psn}$$

- Short Run Marginal Cost = dual variable / load level duration / scenario probability. Expressed in [€/MWh]

$$SRMC_{psn}^\omega = \sigma_{psn}^\omega / d_{psn} / P_p^\omega \quad \forall \omega_{psn}$$

# Water value

- **Dual variable** of water balance for each reservoir [ $\text{€}/\text{hm}^3$ ]
  - Change in the objective function due to a marginal increment in the reservoir inflow

$$\begin{aligned}
 & r_{p-1r}^{\omega'} - r_{pr}^{\omega} + I_{pr}^{\omega} - S_{pr}^{\omega} + \sum_{r' \in \text{up}(r)} S_{pr'}^{\omega} \\
 & + \sum_{\substack{sn \\ h \in \text{up}(r)}} d_{psn} p_{psnh}^{\omega} / C_h - \sum_{\substack{sn \\ h \in \text{dw}(r)}} d_{psn} p_{psnh}^{\omega} / C_h \\
 & + \sum_{\substack{sn \\ h \in \text{up}(r)}} d_{psn} c_{psnh}^{\omega} \eta_h / C_h - \sum_{\substack{sn \\ h \in \text{dw}(r)}} d_{psn} c_{psnh}^{\omega} \eta_h / C_h = 0 \quad : \pi_{pr}^{\omega} \quad \forall \omega_{pr} \quad \omega' \in a(\omega)
 \end{aligned}$$

- Turbining water has no variable cost. However, an additional  $\text{hm}^3$  turbined allows to substitute energy produced by thermal units with the corresponding variable cost (this is called **water value**)

1. Medium term stochastic hydrothermal coordination model
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4. Prototype SHTCM. Mathematical formulation
5. **Prototype SHTCM. Computer implementation**



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Prototype SHTCM.

Computer implementation

# StarGenLite\_SHTCM Medium Term Stochastic Hydrothermal Coordination Model

([https://pascua.iit.comillas.edu/aramos/StarGenLite\\_SHTCM.zip](https://pascua.iit.comillas.edu/aramos/StarGenLite_SHTCM.zip))

- Files

- Microsoft Excel interface for input data and output results  
StarGenLite\_SHTCM.xlsm
- GAMS file StarGenLite\_SHTCM.gms

- How to run it from Windows

- Save the Excel workbook if data have changed

- Run the model

Run



- The model creates

- tmp\_StarGenLite\_SHTCM.xlsx with the output results
- tmp\_StarGenLite\_SHTCM.gdx with the output results
- StarGenLite\_SHTCM.lst as the listing file of the GAMS execution

- Load the results into the Excel interface

Load results

# StarGenLite\_SHTCM Medium Term Stochastic Hydrothermal Coordination Model

([https://pascua.iit.comillas.edu/aramos/StarGenLite\\_SHTCM.zip](https://pascua.iit.comillas.edu/aramos/StarGenLite_SHTCM.zip))

- Files

- Text files for input data
- GAMS file `StarGenLite_SHTCM.gms`

- How to run it from MacOS

- Run the model from GAMS Studio with these parameters
  - `u1=StarGenLite_SHTCM u2=1 u3=1`
- The model creates
  - `tmp_StarGenLite_SHTCM.gdx` with the output results
  - `StarGenLite_SHTCM.lst` as the listing file of the GAMS execution



# Menu

StarGen Lite Medium Term Stochastic Hydrothermal Coordination Model

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<https://www.iit.comillas.edu/aramos/>  
[andres.ramos@comillas.edu](mailto:andres.ramos@comillas.edu)

Run  
Load results

Menu Indices Parameters DemandDuration Generation Inflows UC GrUC Output GrOutput Energy GrEnergy WtrRe ...

# Input Data. Indices (i)

The screenshot displays an Excel spreadsheet with the following content:

Row	Category	Item	Separator	Item	Separator	Item	Separator
5	p	periods	/	p01	*	p12	/
6	s	subperiods	/	WeekDay	,	WeekEnd	/
7	n	load levels	/	n01	*	n03	/
8	sc	scenarios	/	sc01	*	sc03	/
10	g	thermal units	/				
11				Nuclear			
12				DomesticCoal_Anthracite			
13				BrownLignite			
14				ImportedCoal_SubBituminous			
15				ImportedCoal_Bituminous			
16				CCGT_1			
17				CCGT_2			
18				CCGT_3			
19				CCGT_4			
20				OCGT_1			
21				OCGT_2			
22				OCGT_3			
23				FuelOilGas			
25	*	hydro plants					
26				RunOfRiver			
27				StorageHydro1_Basin1			
28				StorageHydro2_Basin1			
29				StorageHydro3_Basin1			
30				PumpedStorageHydro			
33	r	reservoirs	/				
34			/				
35				RunOfRiver			
36				Reservoir1_Basin1			
37				Reservoir2_Basin1			
38				Reservoir3_Basin1			



# Input Data. Indices (ii)

Row	Description	Components
34		RunOfRiver
35		Reservoir1_Basin1
36		Reservoir2_Basin1
37		Reservoir3_Basin1
38		PumpedStorageHydro
41	ruh(r,g) reservoir upstream of hydro plant	RunOfRiver, Reservoir1_Basin1, Reservoir2_Basin1, Reservoir3_Basin1, PumpedStorageHydro
44		RunOfRiver, Reservoir1_Basin1, Reservoir2_Basin1, Reservoir3_Basin1, RunOfRiver, StorageHydro1_Basin1, StorageHydro2_Basin1, StorageHydro3_Basin1
49	rph(r,g) reservoir upstream of pumped hydro plant	RunOfRiver, Reservoir1_Basin1, Reservoir2_Basin1, Reservoir3_Basin1, PumpedStorageHydro, PumpedStorageHydro
54	hur(g,r) hydro plant upstream of reservoir	StorageHydro1_Basin1, StorageHydro2_Basin1, Reservoir3_Basin1, Reservoir3_Basin1
59	hpr(g,r) pumped hydro plant upstream of reservoir	PumpedStorageHydro, PumpedStorageHydro
64	rur(r,r) reservoir 1 upstream of reservoir 2	Reservoir1_Basin1, Reservoir2_Basin1, Reservoir3_Basin1, Reservoir3_Basin1

# Input Data. Cost of energy and power not served

StarGenLite\_SHTCMxslm - Excel

Archivo Inicio Insertar Diseño de página Fórmulas Datos Revisar Vista ¿Qué desea hacer? Andrés Ramos Galán Compartir

A1

\* parameters

\* cost of energy n [€/MWh]  
pENSCost = 10000 ;

\* cost of power n [€/MW]  
pPNSCost = 30000 ;

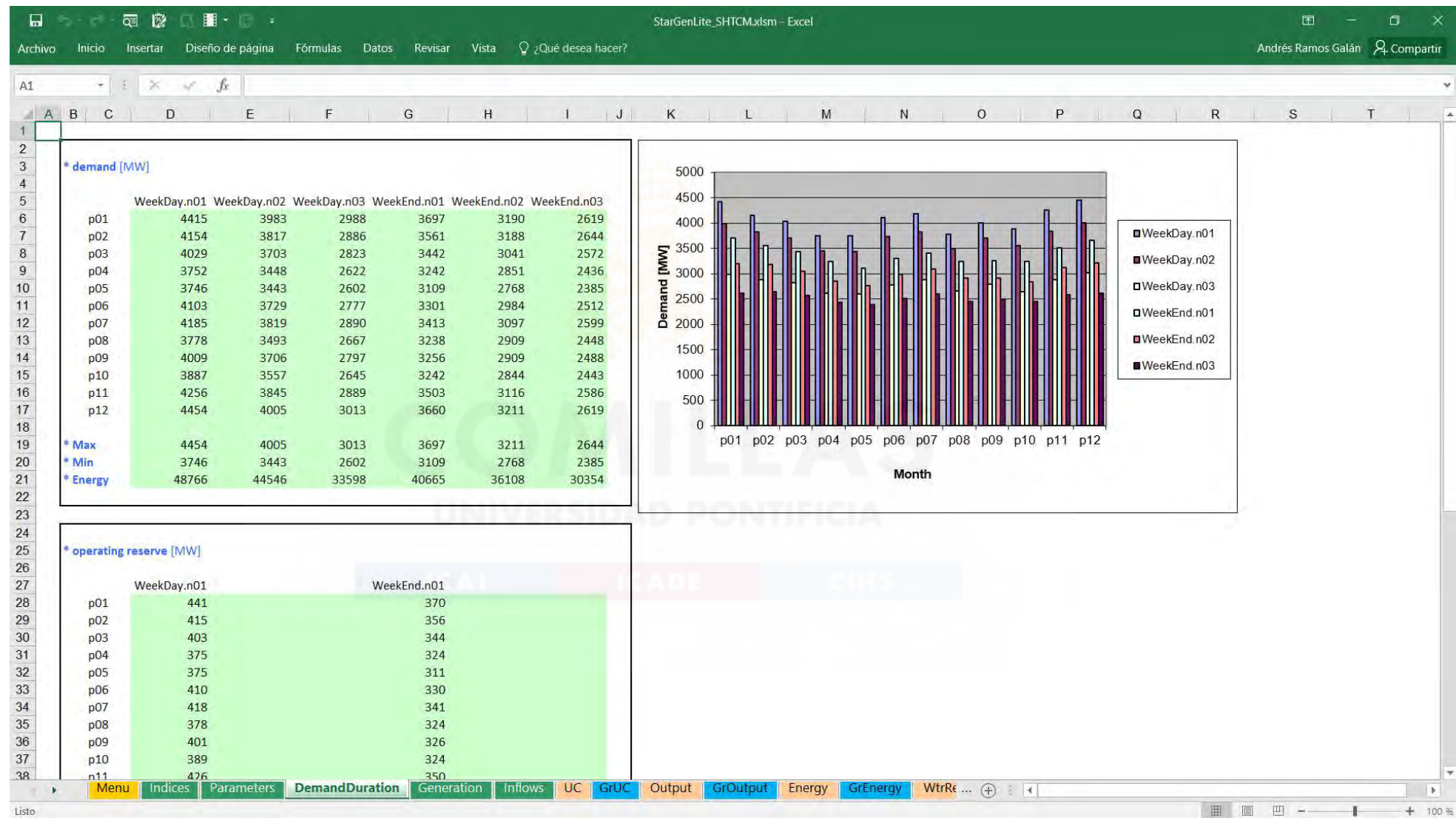
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Menu Indices Parameters DemandDuration Generation Inflows UC GRUC Output GrOutput Energy GrEnergy WtrRe ...

Listo 100%

# Input Data. Demand, operating reserve and duration



# Input Data. Hydro and thermal parameters

StarGenLite\_SHTCM.xlsx - Excel

Archivo Inicio Insertar Diseño de página Fórmulas Datos Revisar Vista ¿Qué desea hacer? Andrés Ramos Galán Compartir

A1

* thermal generation									
	MaxProd	MinProd	FuelCost	SlopeVarCost	InterVarCost	OMVarCost	StartupCost	EFOR	Aux
	[MW]	[MW]	[€/Mcal]	[Mcal/MWh]	[Mcal/h]	[€/MWh]	[Mcal]	[p.u.]	
Nuclear	771.6	771.6	1.00	15				0.00	15.00
DomesticCoal_Anthracite	588.0	235.2	0.02	2400	50000	6	2000000	0.00	49.70
BrownLignite	203.1	81.2	0.02	2300	50000	6	2000000	0.00	50.92
ImportedCoal_SubBituminous	150.4	60.2	0.02	2300	50000	6	2000000	0.00	52.65
ImportedCoal_Bituminous	194.4	77.8	0.02	2200	50000	6	2000000	0.00	49.14
CCGT_1	500.0	100.0	0.03	800	300000	6	1000000	0.00	42.00
CCGT_2	500.0	100.0	0.03	900	300000	4	1000000	0.00	45.00
CCGT_3	500.0	100.0	0.03	1000	300000	4	1000000	0.00	48.00
CCGT_4	667.5	133.5	0.03	800	300000	4	1000000	0.00	37.48
OCGT_1	400.0		0.03	2000	100000	4		0.00	67.50
OCGT_2	400.0		0.03	2100	100000	4		0.00	70.50
OCGT_3	400.0		0.03	2200	100000	4		0.00	73.50
FuelOilGas	441.8		0.06	2000	300000	3	1000000	0.00	160.74

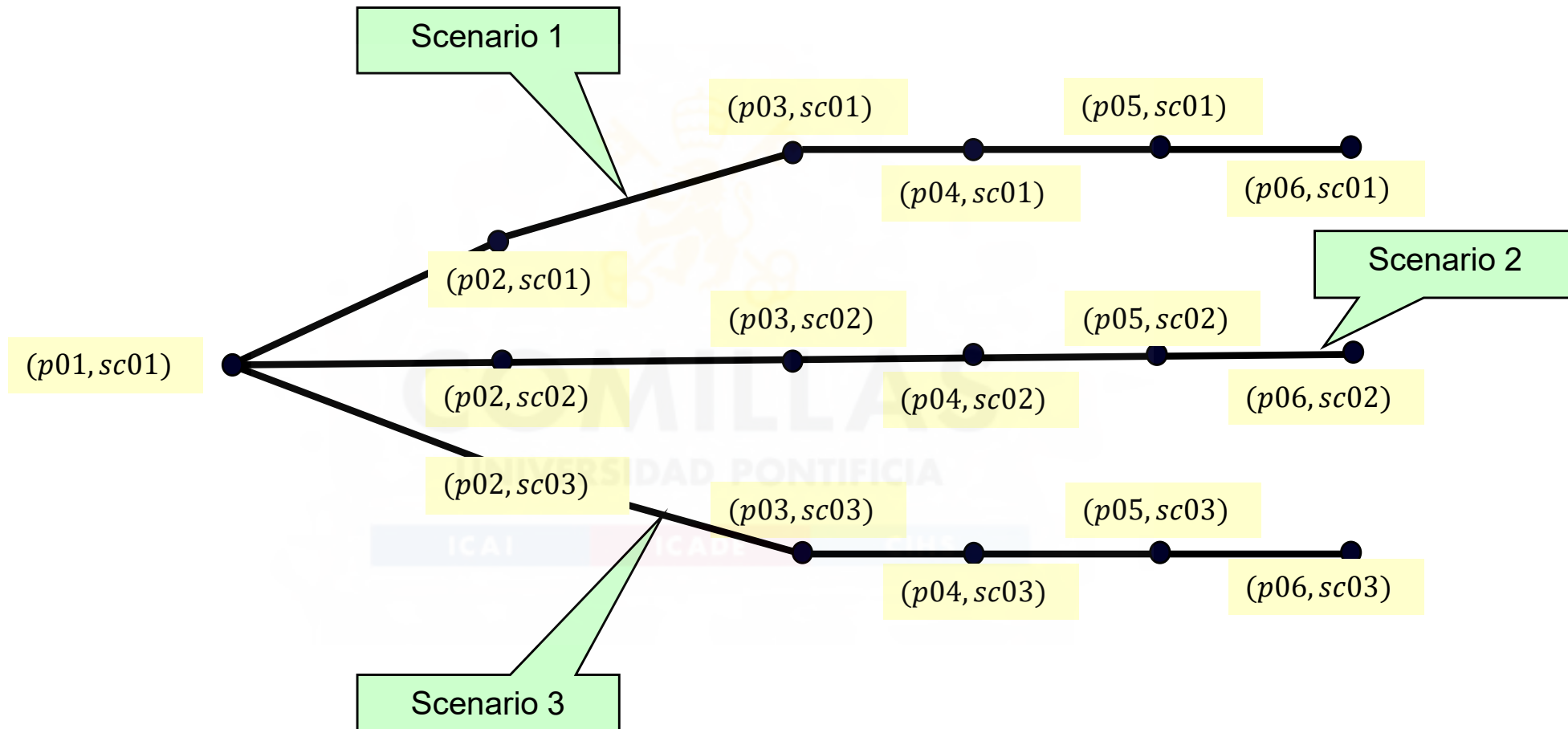
* hydro generation					
	MaxProd	MinProd	ProdFunct	Efficiency	MaxCons
	[MW]	[MW]	[kWh/m <sup>3</sup> ]	[p.u.]	[MW]
RunOfRiver	150.0				
StorageHydro1_Basin1	200.0		0.30		
StorageHydro2_Basin1	200.0		0.30		
StorageHydro3_Basin1	200.0		0.30		
PumpedStorageHydro	200.0			0.70	200.0

* reservoir
-------------

Menu Indices Parameters DemandDuration Generation Inflows UC GRUC Output GrOutput Energy GrEnergy WtrRe ...

# Scenario tree



# Input Data. Inflows and scenario tree

StarGenLite\_SHTCM.xlsx - Excel

Archivo Inicio Insertar Diseño de página Fórmulas Datos Revisar Vista ¿Qué desea hacer? Andrés Ramos Galán Compartir

A1

			p01	p02	p03	p04	p05	p06	p07	p08	p09	p10	p11	p12	
* natural hydro inflows [m <sup>3</sup> /s]															
RunOfRiver	. sc01	40.0	40.0	40.0	35.0	30.0	20.0	20.0	20.0	30.0	35.0	40.0	40.0		
RunOfRiver	. sc02	40.0	38.0	38.0	33.3	28.5	19.0	19.0	19.0	28.5	33.3	38.0	38.0		
RunOfRiver	. sc03	40.0	42.0	42.0	36.8	31.5	21.0	21.0	21.0	31.5	36.8	42.0	42.0		
*	RunOfRiver	. sc01	40.0	39.4	39.4	34.5	29.6	19.7	19.7	19.7	29.6	34.5	39.4		
Reservoir1_Basin1	. sc01	34.5	48.7	66.2	76.1	33.6	10.4	5.4	15.3	14.6	16.1	62.7	157.1		
Reservoir1_Basin1	. sc02	34.5	34.1	46.3	53.3	23.5	7.3	3.8	10.7	10.2	11.3	43.9	110.0		
Reservoir1_Basin1	. sc03	34.5	65.7	89.4	102.7	45.4	14.0	7.3	20.7	19.7	21.7	84.6	212.1		
*	Reservoir1_Basin1	. sc01	34.5	44.6	60.6	69.6	30.7	9.5	4.9	14.0	13.4	14.7	57.4	143.7	
Reservoir2_Basin1	. sc01	28.8	6.9	11.9	4.4	16.6	4.6	5.3	8.8	6.1	10.7	36.3	60.0		
Reservoir2_Basin1	. sc02	28.8	5.2	8.9	3.3	12.5	3.5	4.0	6.6	4.6	8.0	27.2	45.0		
Reservoir2_Basin1	. sc03	28.8	9.7	16.7	6.2	23.2	6.4	7.4	12.3	8.5	15.0	50.8	84.0		
*	Reservoir2_Basin1	. sc01	28.8	6.5	11.2	4.1	15.6	4.3	5.0	8.3	5.7	10.1	34.1	56.4	
Reservoir3_Basin1	. sc01	113.8	54.9	101.0	21.2	29.6	54.9	25.3	16.8	21.2	21.2	33.7	168.4		
Reservoir3_Basin1	. sc02	113.8	32.9	60.6	12.7	17.8	32.9	15.2	10.1	12.7	12.7	20.2	101.0		
Reservoir3_Basin1	. sc03	113.8	76.8	141.4	29.7	41.5	76.8	35.4	23.6	29.7	29.7	47.1	235.7		
*	Reservoir3_Basin1	. sc01	113.8	48.3	88.9	18.7	26.1	48.3	22.2	14.8	18.7	18.7	29.6	148.1	
* scenario tree															
	Ancestor	FirstPeriod	Prob												
	sc01	-1	1	0.500											
	sc02	1	2	0.400											
	sc03	1	2	0.100											

Menu Indices Parameters DemandDuration Generation Inflows UC GRUC Output GrOutput Energy GrEnergy WtrRe ...

Ancestor scenario  
-1 means root node

First period where this  
scenario branches

# StarGenLite\_SHTCM (i)

```
$Title StarGen Lite Medium Term Stochastic Hydrothermal Coordination Model (SHTCM)
```

```
$OnText
```

*Developed by*

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*October 23, 2017*

```
$OffText
```

```
$OnEmpty OnMulti OffListing
```

*\* options to skip or not the Excel input/output  
\* if you want to skip it put these values to 1  
\* in such a case input files must be already in the directory created by any other means  
\* output file will be the tmp.gdx that can be exported to Excel manually*

```
$ifthen.OptSkipExcelInput %gams.user2% == ""  
$ setglobal OptSkipExcelInput 0  
$else.OptSkipExcelInput  
$ setglobal OptSkipExcelInput %gams.user2%  
$endif.OptSkipExcelInput
```

```
$ifthen.OptSkipExcelOutput %gams.user3% == ""  
$ setglobal OptSkipExcelOutput 0  
$else.OptSkipExcelOutput  
$ setglobal OptSkipExcelOutput %gams.user3%  
$endif.OptSkipExcelOutput
```

*\* solve the optimization problems until optimality*

```
option OptcR = 0
```

Model name

Authorship and version

Allow declaration of empty sets and multiple declaration. Suppress listing

Obtain the optimal solution

# StarGenLite\_SHTCM (ii)

\* *definitions*

## sets

p period  
p1(p) first period  
pn(p) last period  
s subperiod  
s1(s) first subperiod  
n load level  
n1(n) first load level  
sc scenario  
sca (sc ) scenario  
scp (sc,p ) tree defined as scenario and period  
scscp(sc,p,sc) ancestor sc2 of node (sc1 p)  
scsch(sc,sc,p) descendant (sc2 p) of node sc1  
scscr(sc,p,sc) representative sc2 of node (sc1 p)  
spsn(sc,p,s,n) active load levels for each scenario  
psn ( p,s,n) active load levels

g generating unit  
t (g) thermal unit  
h (g) hydro plant  
r reservoir  
rs(r) storage reservoir  
ruh(r,g) reservoir upstream of hydro plant  
rph(r,g) reservoir upstream of pumped hydro plant  
hur(g,r) hydro plant upstream of reservoir  
hpr(g,r) pumped hydro plant upstream of reservoir  
rur(r,r) reservoir 1 upstream of reservoir 2

Set definition

alias (sc,scc,sccc), (r,rr)



# StarGenLite\_SHTCM (iii)

## parameters

pDemand	( p,s,n)	hourly load	[GW]
pOperReserve	( p,s,n)	hourly operating reserve	[GW]
pDuration	( p,s,n)	duration	[h]
pCommitt	(sc,g,p,s )	commitment of the unit	[0-1]
pProduct	(sc,g,p,s,n)	production of the unit	[GW]
pEnergy	(sc,g,p,s,n)	energy of the unit	[GWh]
pReserve	(sc,r,p )	reservoir level	[hm <sup>3</sup> ]
pSRMC	(sc, p,s,n)	short run marginal cost	[M€ per GWh]
pWValue	(sc,r,p )	water value	[M€ per hm <sup>3</sup> ]
pEFOR	(g)	EFOR	[p.u.]
pMaxProd	(g)	maximum output	[GW]
pMinProd	(g)	minimum output	[GW]
pMaxCons	(g)	maximum consumption	[GW]
pSlopeVarCost	(g)	slope variable cost	[M€ per GWh]
pInterVarCost	(g)	intercept variable cost	[M€ per h]
pStartupCost	(g)	startup cost	[M€]
pMaxReserve	(r)	maximum reserve	[km <sup>3</sup> ]
pMinReserve	(r)	minimum reserve	[km <sup>3</sup> ]
pIniReserve	(r)	initial reserve	[km <sup>3</sup> ]
pProdFunct	(g)	production function	[GWh per km <sup>3</sup> ]
pEffic	(g)	pumping efficiency	[p.u.]
pInflows	(r,sc,p)	inflows	[km <sup>3</sup> ]
pInflOrg	(r,sc,p)	inflows original	[km <sup>3</sup> ]
pENSCost		energy non-served cost	[M€ per GWh]
pPNSCost		power non-served cost	[M€ per GW ]
pProbsc	(sc,p)	probability of a given node	
lag(p)		backward counting of period	
scaux		scenario number	

Parameter definition

# StarGenLite\_SHTCM (iv)

## variables

vTotalIVCost                      total system variable cost                      [M€]

## binary variables

vCommitt (sc,p,s, g)                      commitment of the unit                      [0-1]  
 vStartup (sc,p,s, g)                      startup of the unit                      [0-1]  
 vShutdown (sc,p,s, g)                      shutdown of the unit                      [0-1]

## positive variables

vProduct (sc,p,s,n,g)                      production of the unit                      [GW]  
 vConsump (sc,p,s,n,g)                      consumption of the unit                      [GW]  
 vENS (sc,p,s,n )                      energy non served                      [GW]  
 vPNS (sc,p,s )                      power non served                      [GW]  
 vWtReserve(sc,p, r)                      water reserve at end of period [km3]  
 vSpillage (sc,p, r)                      spillage                      [km3]

## equations

eTotalIVCost                      total system variable cost                      [M€]  
 eOpReserve(sc,p,s,n )                      operating reserve                      [GW]  
 eBalance (sc,p,s,n )                      load generation balance                      [GW]  
 eMaxOutput(sc,p,s,n,g)                      max output of a committed unit [GW]  
 eMinOutput(sc,p,s,n,g)                      min output of a committed unit [GW]  
 eProdctPer(sc,p,s,n,g)                      unit production in same period [GW]  
 eStrtUpPer(sc,p,s, g)                      unit startup in same period  
 eStrtUpNxt(sc,p,s, g)                      unit startup in next period  
 eWtReserve(sc,p, r)                      water reserve                      [km3] ;

Variables

Equation definition

# StarGenLite\_SHTCM (v)

```

* mathematical formulation

eTotalVCost .. vTotalVCost =e= sum[(spsn(sc,p ,s,n) ), pProbSc(sc,p)*pDuration(p,s,n)*pENSCost      *vENS      (sc,p,s,n )] +
sum[(scp (sc,p),s      ), pProbSc(sc,p)      *pPNSCost      *vPNS      (sc,p,s      )] +
sum[(scp (sc,p),s, t), pProbSc(sc,p)      *pStartupCost (t)*vStartup(sc,p,s, t)] +
sum[(spsn(sc,p ,s,n),t), pProbSc(sc,p)*pDuration(p,s,n)*pInterVarCost(t)*vCommitt(sc,p,s, t)] +
sum[(spsn(sc,p ,s,n),t), pProbSc(sc,p)*pDuration(p,s,n)*pSlopeVarCost(t)*vProduct(sc,p,s,n,t)] ;

eOpReserve(spsn(sc,p,s,n1(n))) .. sum[t, pMaxProd(t)*vCommitt(sc,p,s, t)] + sum[h, pMaxProd(      h)] + vPNS(sc,p,s ) =g= pDemand(p,s,n) + pOperReserve(p,s,n) ;
eBalance (spsn(sc,p,s, n )) .. sum[g,      vProduct(sc,p,s,n,g)] - sum[h, vConsump(sc,p,s,n,h)] + vENS(sc,p,s,n) =e= pDemand(p,s,n) ;

eMaxOutput(spsn(sc,p,s,n),t) $pMaxProd(t) .. vProduct(sc,p,s,n,t) / pMaxProd(t) =l= vCommitt(sc,p,s,t) ;
eMinOutput(spsn(sc,p,s,n),t) $pMinProd(t) .. vProduct(sc,p,s,n,t) / pMinProd(t) =g= vCommitt(sc,p,s,t) ;

eProdtPer(spsn(sc,p,s1(s),n),g) .. vProduct(sc,p,s+1,n,g) =l= vProduct(sc,p,s,n,g) ;

eStrtUpPer(scp(sc,p),s1(s),t) $[card(s) > 1      ] .. vCommitt(sc,p,s+1,t) =e=      vCommitt(sc ,p ,s ,t) + vStartup(sc,p,s+1,t) -
vShutdown(sc,p,s+1,t) ;
eStrtUpNxt(scp(sc,p),s1(s),t) $[card(s) > 1 and not p1(p)] .. vCommitt(sc,p,s ,t) =e= sum[scscp(sc,p,scc), vCommitt(scc,p-1,s+1,t)] + vStartup(sc,p,s ,t) - vShutdown(sc,p,s ,t) ;

eWtReserve(scp(sc,p), r) .. sum[scscp(sc,p,scc), vWtReserve(scc,p-1,r)] + pIniReserve(r) $p1(p) - vWtReserve(sc,p,r) +
pInflows(r,sc,p) - vSpillage(sc,p,r) + sum[rur(rr,r), vSpillage(sc,p,rr)] +
sum{(s,n), pDuration(p,s,n)*sum[hur(h,r), vProduct(sc,p,s,n,h)/pProdFunc(h)]} -
sum{(s,n), pDuration(p,s,n)*sum[ruh(r,h), vProduct(sc,p,s,n,h)/pProdFunc(h)]} +
sum{(s,n), pDuration(p,s,n)*sum[hpr(h,r), vConsump(sc,p,s,n,h)/pProdFunc(h)*pEffic(h)]} -
sum{(s,n), pDuration(p,s,n)*sum[rph(r,h), vConsump(sc,p,s,n,h)/pProdFunc(h)*pEffic(h)]} =e= 0 ;

model mSHTCM / all / ;
mSHTCM.SolPrint = 1 ; mSHTCM.HoldFixed = 1 ;
    
```

Reduced solution output

Model includes all the equations

Eliminate fixed variables

Mathematical formulation of equations



# StarGenLite\_SHTCM (vi)

*\* read input data from Excel and include into the model*

```
file TMP / tmp_%gams.user1%.txt /
$OnEcho > tmp_%gams.user1%.txt
r1= indices
o1=tmp_indices.txt
r2= param
o2=tmp_param.txt
r3= demand
o3=tmp_demand.txt
r4= oprres
o4=tmp_oprres.txt
r5= duration
o5=tmp_duration.txt
r6= thermalgen
o6=tmp_thermalgen.txt
r7= hydrogen
o7=tmp_hydrogen.txt
r8= reservoir
o8=tmp_reservoir.txt
r9= inflows
o9=tmp_inflows.txt
r10= tree
o10=tmp_tree.txt
```

\$OffEcho

*\* Mac OS X and Linux users must comment the following call and copy and paste the named ranges of the Excel interface into the txt files*

```
$ifthen.OptSkipExcelInput '%OptSkipExcelInput%' == '0'
$call xls2gms m i="%gams.user1%.xlsm" @"tmp_%gams.user1%.txt"
$else.OptSkipExcelInput
$ log Excel input skipped
$endif.OptSkipExcelInput
```

sets

```
$include tmp_indices.txt
;
$include tmp_param.txt
table pDemand(p,s,n)
$include tmp_demand.txt
table pOperReserve(p,s,n)
$include tmp_oprres.txt
table pDuration(p,s,n)
$include tmp_duration.txt
table pThermalGen(g,*)
$include tmp_thermalgen.txt
table pHydroGen(g,*)
$include tmp_hydrogen.txt
table pReservoir(r,*)
$include tmp_reservoir.txt
table pInflows(r,sc,p)
$include tmp_inflows.txt
table pScnTree(sc,*)
$include tmp_tree.txt
;
```

*\* Mac OS X and Linux users must comment the following execute*

```
execute 'del "%gams.user1%.txt tmp_indices.txt tmp_param.txt tmp_demand.txt tmp_oprres.txt tmp_duration.txt tmp_thermalgen.txt tmp_hydrogen.txt tmp_reservoir.txt tmp_inflows.txt tmp_tree.txt' ;
```

Read input from Excel  
named ranges and  
write into text files

Input from text files  
into GAMS

Delete read text files

# StarGenLite\_SHTCM (vii)

\* *determine the first and last period and the first subperiod*

```
p1(p)      $[ord(p) = 1] = yes ;  
s1(s)      $[ord(s) = 1] = yes ;  
n1(n)      $[ord(n) = 1] = yes ;  
pn(p)      $[ord(p) = card(p)] = yes ;  
psn(p,s,n) $pDuration(p,s,n) = yes ;  
lag(p) = card(p) - 2*ord(p) + 1 ;
```

First period, first subperiod  
first load level, ...

\* *assignment of thermal units, storage hydro and pumped storage hydro plants*

```
t (g) $[pThermalGen(g, 'MaxProd' ) and pThermalGen(g, 'FuelCost' )] = yes ;  
h (g) $[pHydroGen (g, 'MaxProd' ) ] = yes ;  
rs(r) $[pReservoir (r, 'MaxReserve' ) > 0 ] = yes ;
```

Defining thermal and hydro  
units and reservoirs

# StarGenLite\_SHTCM (viii)

*\* scaling of parameters*

```

pDemand      (p,s,n) = pDemand      (p,s,n)      * 1e-3 ;
pOperReserve(p,s,n) = pOperReserve(p,s,n)      * 1e-3 ;
pENSCost     = pENSCost              * 1e-3 ;
pPNSCost     = pPNSCost              * 1e-3 ;

pEFOR        (t) = pThermalGen(t, 'EFOR'        ) ;
pMaxProd     (t) = pThermalGen(t, 'MaxProd'     ) * 1e-3 * [1-pEFOR(t)] ;
pMinProd     (t) = pThermalGen(t, 'MinProd'     ) * 1e-3 * [1-pEFOR(t)] ;
pSlopeVarCost(t) = pThermalGen(t, 'OMVarCost'   ) * 1e-3 +
                  pThermalGen(t, 'SlopeVarCost' ) * 1e-3 * pThermalGen(t, 'FuelCost' ) ;
pInterVarCost(t) = pThermalGen(t, 'InterVarCost') * 1e-6 * pThermalGen(t, 'FuelCost' ) ;
pStartupCost (t) = pThermalGen(t, 'StartupCost' ) * 1e-6 * pThermalGen(t, 'FuelCost' ) ;

pMaxProd     (h) = pHydroGen  (h, 'MaxProd'     ) * 1e-3 ;
pMinProd     (h) = pHydroGen  (h, 'MinProd'     ) * 1e-3 ;
pMaxCons     (h) = pHydroGen  (h, 'MaxCons'     ) * 1e-3 ;
pProdFunct   (h) = pHydroGen  (h, 'ProdFunct'   ) * 1e+3 ;
pEffic       (h) = pHydroGen  (h, 'Efficiency'   ) ;
pMaxReserve  (r) = pReservoir (r, 'MaxReserve'  ) * 1e-3 ;
pMinReserve  (r) = pReservoir (r, 'MinReserve'  ) * 1e-3 ;
pIniReserve  (r) = pReservoir (r, 'IniReserve'  ) * 1e-3 ;

pInflows(r,sc,p) = pInflows  (r,sc,p          ) * 1e-6 * 3.6*sum[(s,n), pDuration(p,s,n)] ;
pInflOrg(r,sc,p) = pInflows  (r,sc,p          ) ;

* if the production function of a hydro plant is 0, it is changed to 1 and scaled to 1000
* if the efficiency of a hydro plant is 0, it is changed to 1

pProdFunct(h) $[pProdFunct(h) = 0] = 1e3 ;
pEffic      (h) $[pEffic      (h) = 0] = 1 ;
    
```

Parameter scaling

# StarGenLite\_SHTCM (ix)

\* *bounds on variables*

```
vProduct.up (sc,p,s,n,g) = pMaxProd(g) ;  
vConsump.up (sc,p,s,n,h) = pMaxCons(h) ;  
  
vENS.up (sc,p,s,n ) = pDemand(p,s,n) ;  
  
vWtReserve.up(sc,p,r) = pMaxReserve(r) ;  
vWtReserve.lo(sc,p,r) = pMinReserve(r) ;  
vWtReserve.fx(sc,p,r) $pn(p) = pIniReserve(r) ;
```

Bounds on variables

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# StarGenLite\_SHTCM (x)

*\* define the nodes of the scenario tree and determine ancestor sc2 of node (sc1 p) and descendant (sc2 p) of node sc1*

```
scp ( sc,p ) $[ord(p) >= pScnTree(sc,'FirstPeriod')] = yes ;
scscp(sc,sc,p,scc) $[ord(p) > pScnTree(sc,'FirstPeriod') and ord(scc) = ord(sc)] = yes ;
scscp(sc,sc,p,scc) $[ord(p) = pScnTree(sc,'FirstPeriod') and ord(scc) = pScnTree(sc,'Ancestor')] = yes ;
scsch(sc,scp(scc,p)) $scscp(scc,p,sc) = yes ;
```

```
pProbSc(sc,pn(p)) = pScnTree(sc,'Prob')/sum[scc, pScnTree(scc,'Prob')] ;
loop ( p $[not p1(p)],
  pProbSc(scp(sc,p+lag(p))) = sum[scsch(sc,scc,p+(lag(p)+1)), pProbSc(scc,p+(lag(p)+1))] ;
) ;
```

*\* delete branches with probability  $\theta$  and define the active load levels*

```
scp ( sc,p ) $[pProbSc(sc,p) =  $\theta$ ] = no ;
scscp( sc,p ,scc) $[pProbSc(sc,p) =  $\theta$  or pProbSc(scc,p-1) =  $\theta$ ] = no ;
scsch(sc,scc,p ) = yes $scscp(scc,p,sc) ;
spsn (scp(sc,p),s,n) $psn (p,s,n) = yes ;
```

*\* determine the representative sc2 of node (sc1 p) for non-existing scenarios in the tree*

```
loop (sc $sum[p, pProbSc(sc,p)],
  scaux = ord(sc) ;
  loop (p,
    scscr(sc,p+lag(p),scc) $[ord(scc) = scaux] = yes ;
    SCA(scc) $[ord(scc) = scaux] = yes ;
    scaux = sum[scscp(sca,p+lag(p),scc), ord(scc)] ;
    SCA(scc) = no ;
  ) ;
) ;
SCA(sc) $sum[p, pProbSc(sc,p)] = yes ;
```

Building the scenario tree



# StarGenLite\_SHTCM (xi)

*\* solve stochastic hydrothermal coordination model*

**solve** mSHTCM using MIP minimizing vTotalVCost ;

*\* scaling of the results*

```
pCommitt(sca,t, p,s ) = sum[scscr(sca,p,scc) , vCommitt.l (scc,p,s, t) ] + eps ;
pProduct(sca,g,psn(p,s,n) = sum[scscr(sca,p,scc) , vProduct.l (scc,p,s,n,g) ]*1e3 + eps ;
pEnergy (sca,g,psn(p,s,n) = sum[scscr(sca,p,scc) , vProduct.l (scc,p,s,n,g) *pDuration(p,s,n) ]*1e3 + eps ;
pReserve(sca,rs(r),p ) = sum[scscr(sca,p,scc) , vWtReserve.l(scc,p, r) ]*1e3 + eps ;
pWValue (sca,rs(r),p ) = sum[scscr(sca,p,scc) $pProbSc(scc,p), eWtReserve.m(scc,p, r)/sum[psn(p,s,n), pDuration(p,s,n)]/pProbSc(scc,p)]*1e3 + eps ;
pSRMC (sca, psn(p,s,n) = sum[scscr(sca,p,scc) $pProbSc(scc,p), eBalance.m (scc,p,s,n ) /pDuration(p,s,n) /pProbSc(scc,p)]*1e3 + eps ;
```

*\* data output to xls file*

```
put TMP putclose 'par=pProduct rdim=2 rng=Output!a1' / 'par=pEnergy rdim=2 rng=Energy!a1' / 'par=pReserve rdim=2 rng=WtrReserve!a1' / 'par=pWValue rdim=2
rng=WtrValue!a1' / 'par=pSRMC rdim=1 rng=SRMC!a1' / 'par=pCommitt rdim=2 rng=UC!a1' /
'par=pCommitt rdim=2 rng=UC!a1' / 'text="Scen" rng=Output!a1' / 'text="Scen" rng=Energy!a1' / 'text="Scen" rng=WtrReserve!a1' / 'text="Scen"
rng=WtrValue!a1' / 'text="Scen" rng=SRMC!a1' / 'text="Scen" rng=UC!a1' /
'par=pCommitt rdim=2 rng=UC!a1' / 'text="Unit" rng=Output!b1' / 'text="Unit" rng=Energy!b1' / 'text="Reservoir" rng=WtrReserve!b1' / 'text="Reservoir"
rng=WtrValue!b1' /
'par=pCommitt rdim=2 rng=UC!b1' /
'text="Unit" rng=UC!b1' /
```

**execute\_unload** 'tmp\_%gams.user1%..gdx' pProduct pEnergy pReserve pWValue pSRMC pCommitt

*\*\$ifthen.OptSkipExcelOutput '%OptSkipExcelOutput%' == '0'*

**execute** 'gdxxrw tmp\_%gams.user1%..gdx SQ=n EpsOut=0 0=tmp\_%gams.user1%.xlsx @tmp\_%gams.user1%.txt'

**execute** 'del tmp\_%gams.user1%.gdx'

*\*\$else.OptSkipExcelOutput*

*\*\$ Log Excel output skipped*

*\*\$endif.OptSkipExcelOutput*

**execute** 'del

tmp\_%gams.user1%.txt'

\$OnListing

Solve the optimization problem

Scaling the results

Write output to Excel

# Medium term optimization model. Results

---

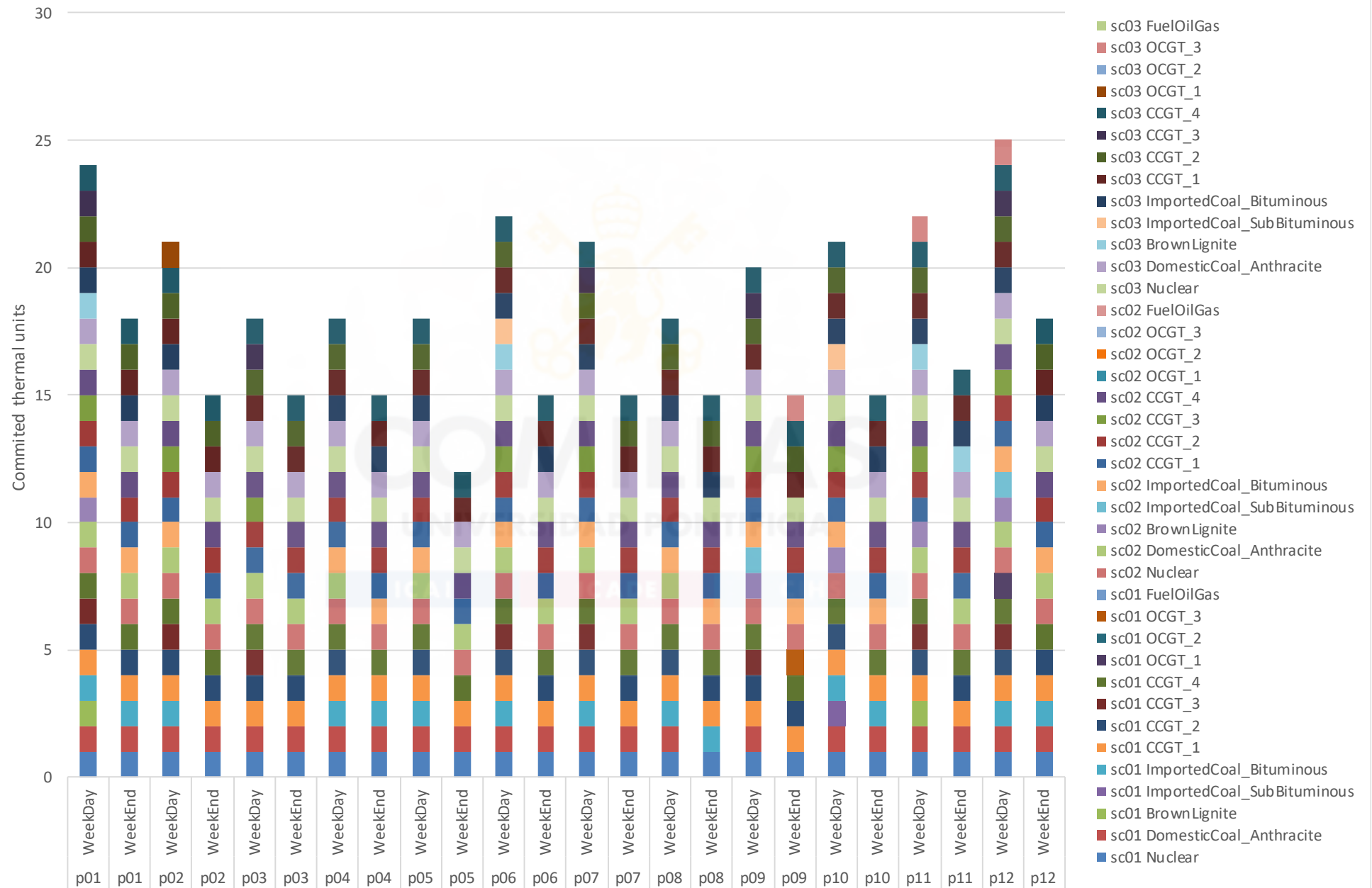
- Operation planning
  - Unit (thermal, storage hydro, and pumped-storage hydro) operation
  - Reservoir management
  - Targets for short-term models (water balance)
- Economic planning
  - System marginal costs
  - Targets for short-term models (water value)

ICAI

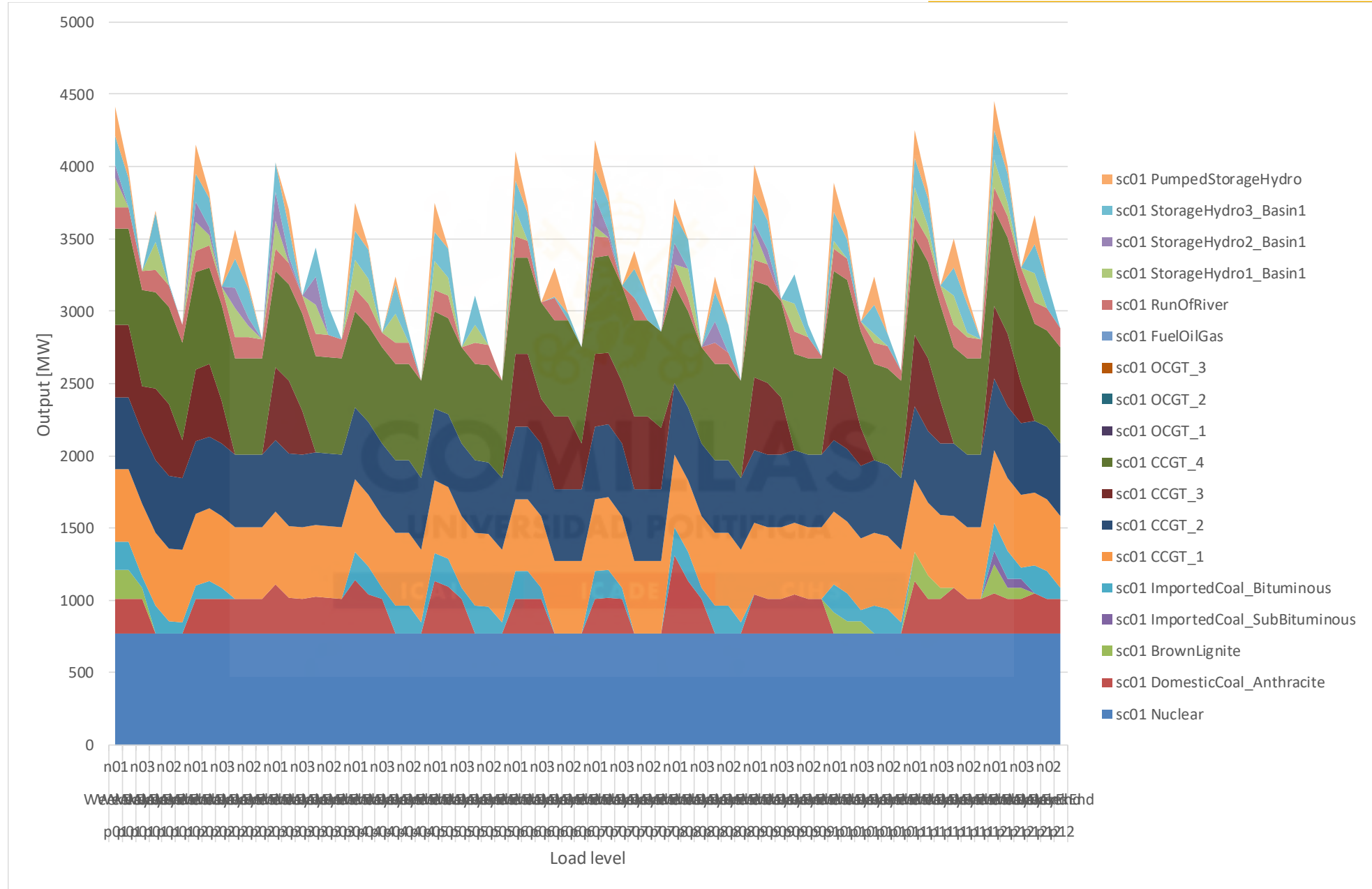
ICADE

CIHS

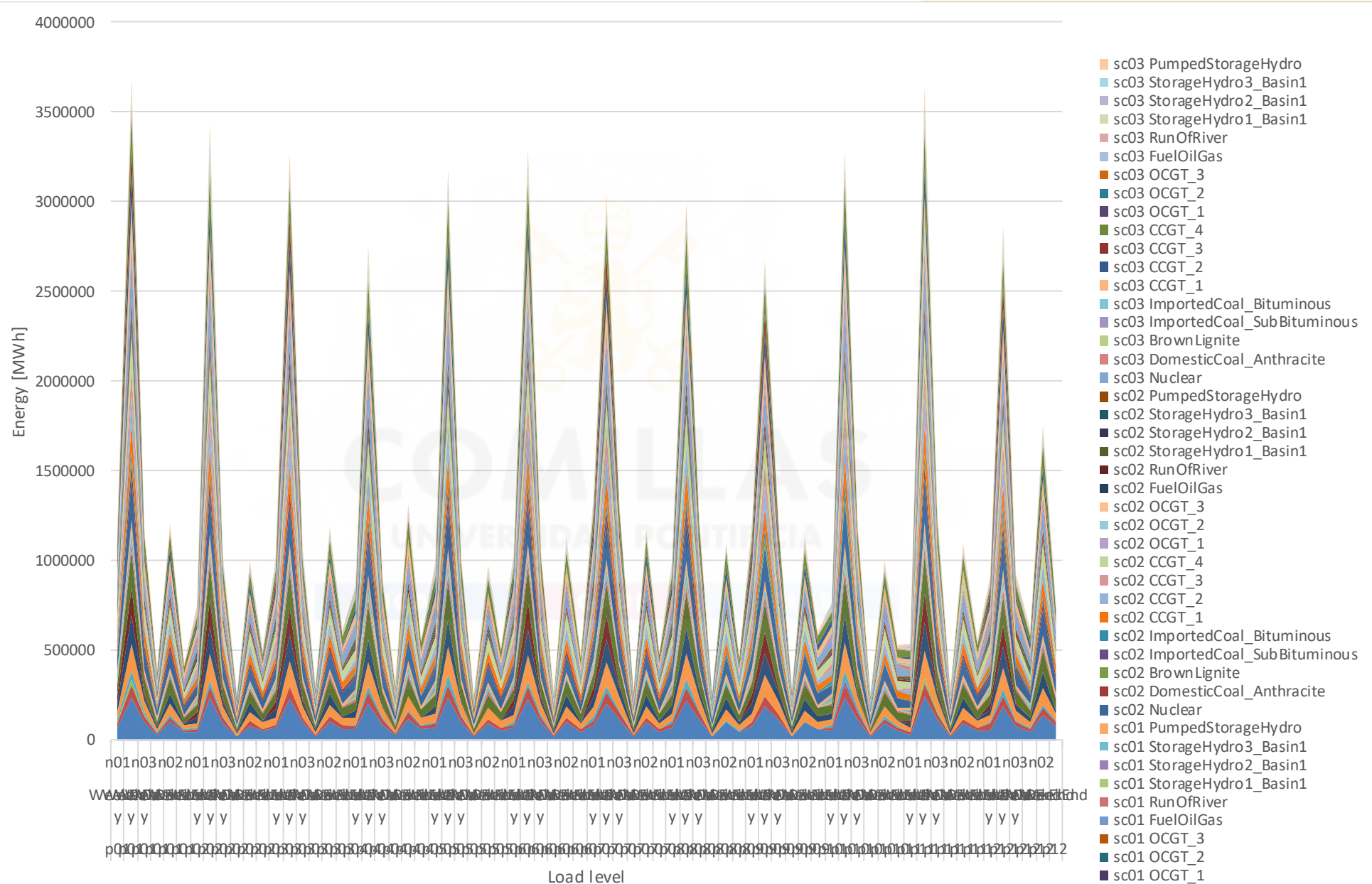
# Output Data. Thermal unit commitment



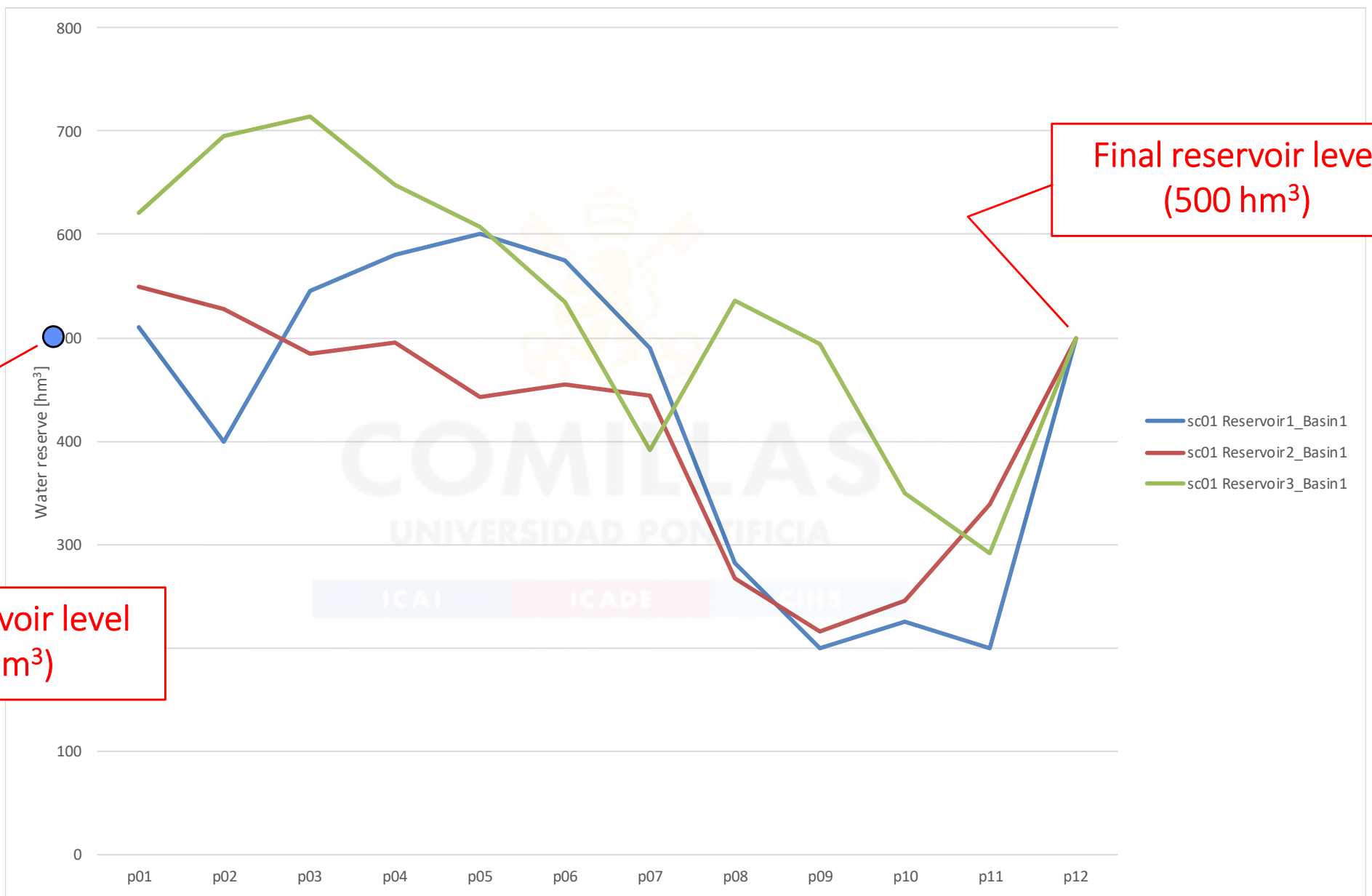
# Output Data. Production



# Output Data. Energy



# Reservoir level at the end of each period

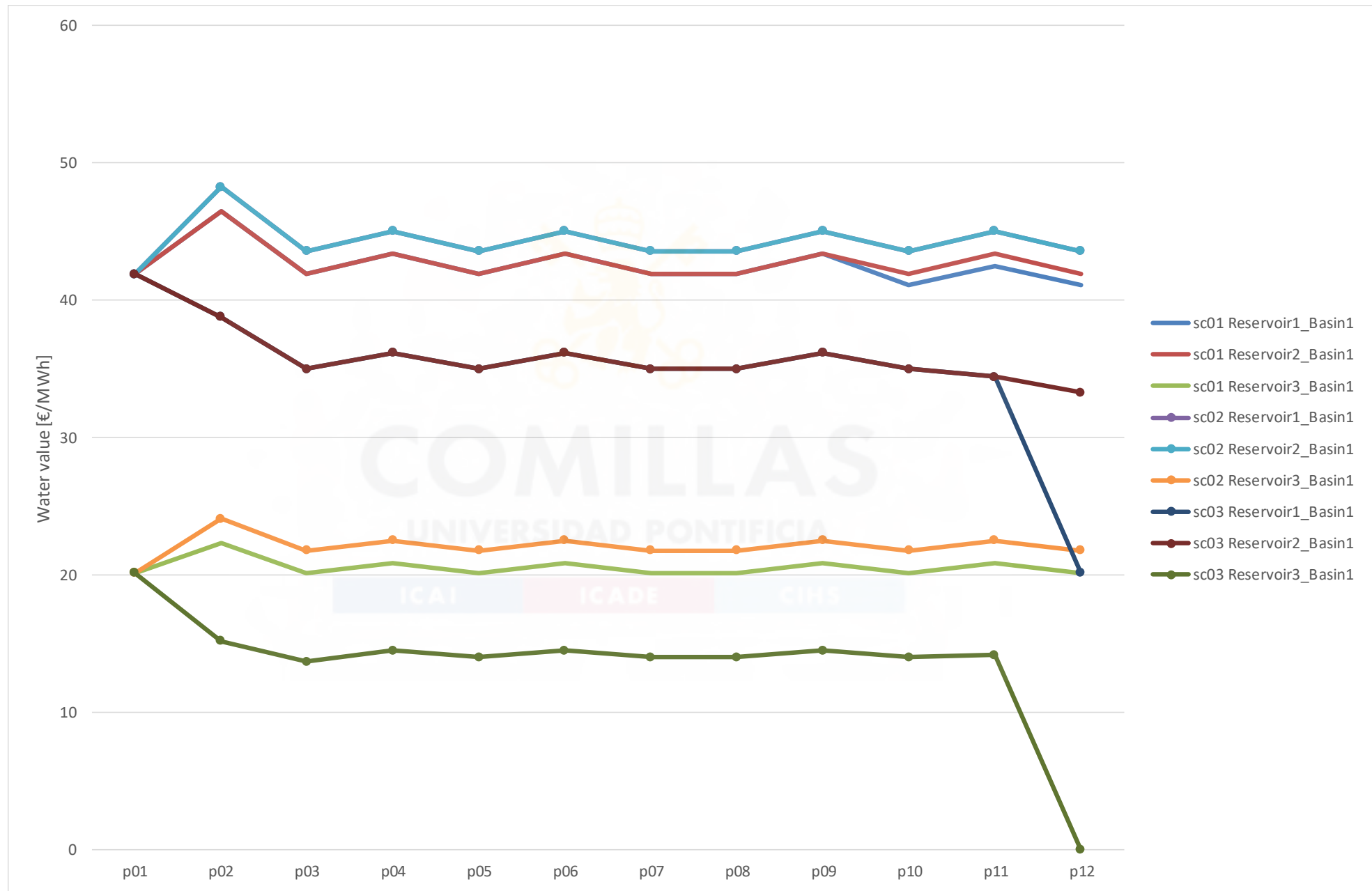


Initial reservoir level (500 hm³)

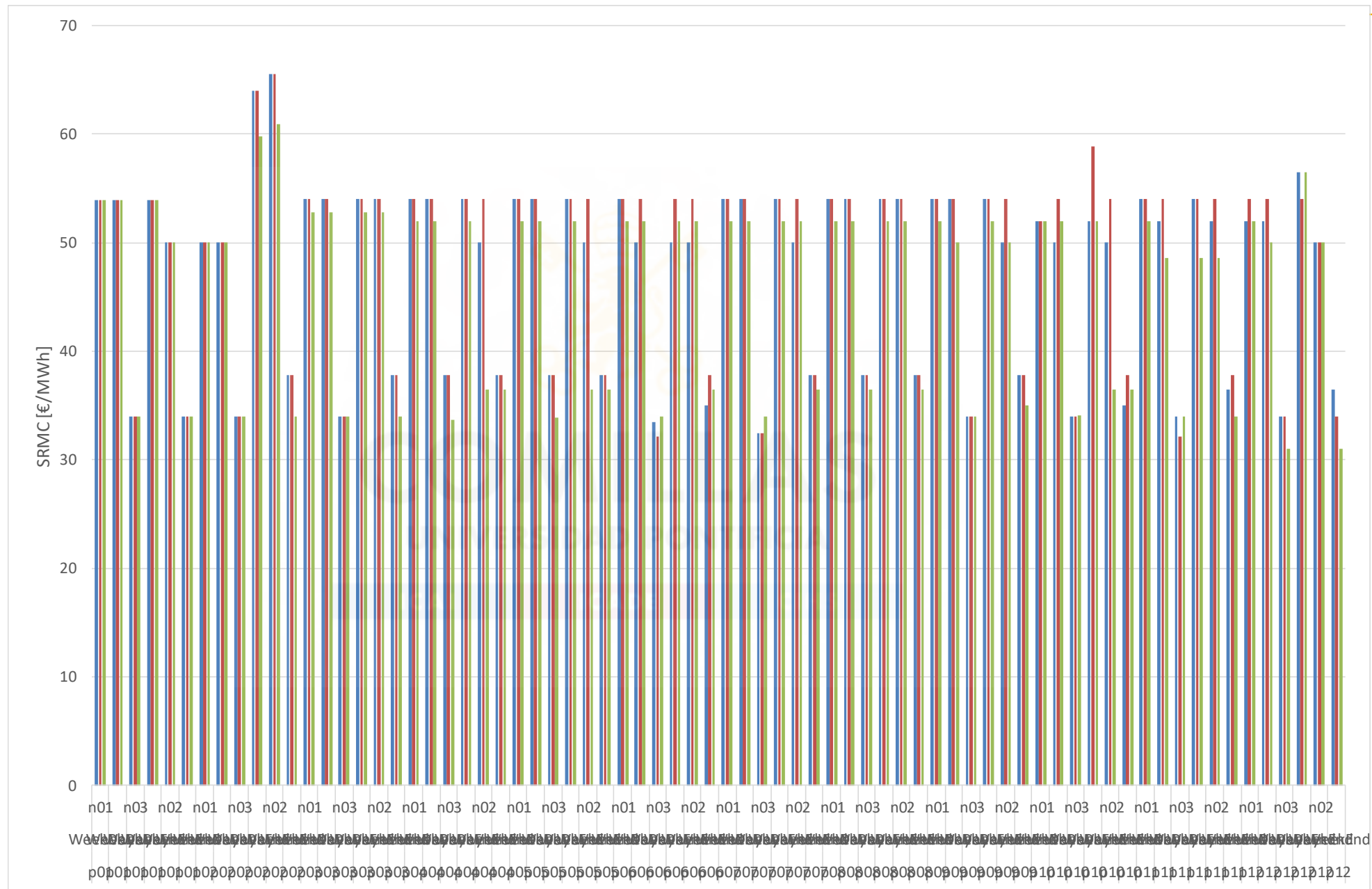
Final reservoir level (500 hm³)



# Water value



# Output Data. Short Run Marginal Cost (SRMC)



If the SHTCM model is solved with binary UC decisions, no marginal impact of those decisions is considered



# Stochastic measures

- *Expected value with perfect information* (EVWPI) o *Wait and See* (WS)
  - Weighted mean of the objective function of each scenario knowing what is going to happen (for minimization problems always  $\leq$  the objective function for the stochastic problem)
- *Value of the stochastic solution* (VSS)
  - Difference between the objective function of the expected value for the mean value solution of the stochastic parameters **EEV** and that of the stochastic problem **RP**
- *Expected value of perfect information* (EVPI) or mean regret
  - Weighted average of the difference between the stochastic solution for each scenario and the perfect information solution in this scenario (always positive for minimization)
  - How much are you willing to pay to have perfect information

- $EVPI = RP - WS$

- $VSS = EEV - RP$

- $WS \leq RP \leq EEV$

$$EVPI \geq 0$$

$$VSS \geq 0$$

Fortune-teller



# Stochastic measures

				sc01	sc02	sc03	Expected	Stochastic
Generation	RunOfRiver	in	p01	MWh	107136	107136	107136	107136
Generation	StorageHydro_Basin1	in	p01	MWh	79200	67356	82629	78741
Generation	StorageHydro_Basin2	in	p01	MWh	37466	17600	44903	12602
Generation	StorageHydro_Basin3	in	p01	MWh	124281	86400	148800	92787
Reserve	StorageHydro_Basin1	end	p01	hm <sup>3</sup>	328	368	317	328
Reserve	StorageHydro_Basin2	end	p01	hm <sup>3</sup>	452	518	427	535
Reserve	StorageHydro_Basin3	end	p01	hm <sup>3</sup>	779	800	734	800
Total	Hydro Generation	in	p01	MWh	348083	278492	383467	341912
Total	Reserve	end	p01	hm <sup>3</sup>	1560	1686	1478	1581
Total	System Variable Cost			M€	1123.997	1144.447	1103.624	1129.624

EWPI or WS	EEV	VSS	EVPI
1130.140	1130.360	0.077	0.144

- Stochasticity in hydro inflows is not relevant from the point of view of total variable cost
- But it is essential for defining the operation of the first period

# Task assignment

- Compute the water value numerically for a particular period and reservoir by running twice the hydrothermal model and comparing this water value with the dual variable of the water balance constraint. Apply it to one reservoir in period 1 scenario 1 and another in period 7 scenario 3.
  - Note that you need to take care of the change from  $\text{m}^3/\text{s}$  to  $\text{km}^3$  and of the scenario probability
- Introduce intermittent power (with curtailment) into the model
  - Play with this generation to observe the complementarity between hydro and intermittent generation
- Evaluate all the stochastic measures of considering the stochasticity of hydro inflows
- Introduce a take or pay gas contract into the model

# Takeaways

- Purpose of a medium-term stochastic hydrothermal coordination model
  - Characteristics
  - Overview
  - Results for operation planning and economic planning
  - Main modeling assumptions
- Prototype mathematical formulation
  - General structure
  - Parameters, variables, equations, objective function
  - Short-run marginal cost, water value
- Case study with *StarGenLite\_SHTCM*
  - Input data
  - Output data



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