ESD.S30

Electric Power System Modeling for a Low Carbon Economy

Medium-Term Market Equilibrium Model

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Introduction
Cournot model – conjectural variations
Bushnell model
Model based on the complementarity problem
Some real models

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Introduction

Objectives

- To understand:
  - How imperfect competition (oligopoly) is modeled
  - How to address technical and market constraints
  - How to formulate and solve a realistic model

Bibliography

- M. Ventosa, A. Baíllo, A. Ramos, M. Rivier *Electricity Market Modeling Trends* Energy Policy 33 (7): 897-913 May 2005

- M. Rivier, M. Ventosa, A. Ramos, F. Martínez-Córcoles, A. Chiarri *"A Generation Operation Planning Model in Deregulated Electricity Markets based on the Complementarity Problem"* in the book M.C. Ferris, O.L. Mangasarian and J-S. Pang (eds.) *Complementarity: Applications, Algorithms and Extensions* pp. 273-295 Kluwer Academic Publishers 2001 ISBN 0792368169
- J. Bushnell (1998) *"Water and Power: Hydroelectric Resources in the Era of Competition in the Western US"* (http://www.ucei.berkeley.edu/PDF/pwp056.pdf)
- J. Barquín, E. Centeno, J. Reneses, *"Medium-term generation programming in competitive environments: A new optimization approach for market equilibrium computing"*, IEE Proceedings-Generation Transmission and Distribution. vol. 151, no. 1, pp. 119-126, January 2004.

Why a market equilibrium model?

- Electricity generation business
  - Electricity production **market**
- Generating companies have new roles and responsibilities
  - **New** decision-making **tools and models** that take into account the **market**
  - Markets generally having only few companies
  - Companies' **decisions** are **mutually dependent**

Generation planning functions

Functions	Long term	**Medium term**	Short term
New liberalized market functions	• Capacity investments • Risk management • Long term contracts: - Fuel acquisition - Electricity selling	• Objectives: - Market share - Price • Budget planning • Future derivatives market bids	• Strategic bidding: - Energy - Power reserve - Other ancillary services
Traditional regulated operation functions	• Capacity investments - Installation - Repowering • Maintenance • Energy management - Nuclear cycle - Multiannual reservoirs	**• Fuel management** **• Annual reservoir and seasonal pumped storage management** **- Water value assessment**	• Startup and shut-down of thermal units • Pumped storage operation • Economic dispatch

Scope

Why an equilibrium model based on the complementarity problem?

- Modeling the electricity market by a complementarity problem approach provides
  - A **flexible** representation of the market and its **medium- and long-term** operation
    - Modeling large-scale **electricity schedules**
  - A **technically feasible** solution
  - **Actual, unique** market equilibrium (in realistic conditions)
- Methods for **solving** complementarity problems (MCP)
  - Allow **realistic sizes: 10,000** variables
  - Although solution time is greater than in linear optimization
- Alternative formulations and numerical solutions exist, based on the **equivalent quadratic problem** (QP)
  - Same optimality conditions as the equilibrium problem
  - Iterative solution of a linear problem

Nonlinear Optimization Optimality Conditions

- Unconstrained optimization
- Lagrangian
- Constrained optimization

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Cournot model – conjectural variations

Session outline

Cournot model

- ✓ Thermal generation
- ✓ Single period

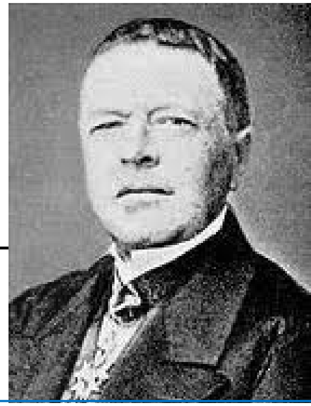
Bushnell model

- ✓ Hydro thermal generation
- ✓ Multi-period

Model based on the complementarity problem

- ✓ Means of production
  - Fuel stock management
  - Pumped storage hydro plants
- ✓ Market aspects
  - Contracts for differences
  - Take-or-pay contracts

Cournot model (1838)



French philosopher and mathematician (1801 –1877)

- Pioneer model to study companies' **strategic** behavior
- Simple model
  - Single generating plant
    - All the generating plants of each company are grouped
  - Inter-period constraints not considered
    - **Single period** equilibrium
  - Assumes perfect information
- Applicable to **medium-** and **long-term** analyses of thermal systems

Cournot model: approach

- Main characteristics
  - It explicitly considers
    - Each company's objective is to **maximize profits**
    - Company decisions are **interdependent**
    - Consumer behavior
  - **Nash** equilibrium in **quantity** strategies: each firm chooses an output quantity to maximize its profit. The Nash-Cournot market equilibrium defines a set of outputs such that no firm, taking its competitors' output as given, wishes to change its own output unilaterally
  - Price is derived from the inverse demand function
- These components are enough to interpret it as the origin of more complex models, such as the model based on the complementarity problem

Cournot model: formulation

(https://pascua.iit.comillas.edu/aramos/StarMrkLite_CournotEn.gms)

e	Company
$e*$	Other companies
B_e	Profits
p	Price
q_e	Output
C_e	Variable costs
MC_e	Marginal cost
MR_e	Marginal revenue
p-MC_e	**Price mark-up**

- Objective function: profits

$$B_e = p \cdot q_e - C_e$$

- Inverse demand function

$$p = f\left(\sum_e q_e\right)$$

- Cournot conjecture: vertical supply

$$\frac{\partial p}{\partial q_e} = \frac{\partial p}{\partial q}\frac{\partial (q_e + q_{e*})}{\partial q_e} = p' \leftrightarrow \frac{\partial q_{e*}}{\partial q_e} = 0$$

Own output decision will not have an effect on the decisions of the competitors

- Optimality conditions

$$\frac{\partial B_e}{\partial q_e} = 0 \rightarrow MR_e = p + q_e p' = MC_e\left(q_e\right) \rightarrow q_e = \frac{p - MC_e\left(q_e\right)}{-p'}$$

System of equations

Cournot model: conclusion

- In equilibrium, for each company (utility) *e*, marginal cost and marginal revenue must be equal

$$MR_e = p + p' q_e = MC_e\left(q_e\right)$$

- Marginal revenue has two components:
  - 1 additional MWh earns the market price *p*
  - But because of the greater production, market price decreases an amount p'. The price fall impacts on all the energy sold in the market, that we assume to be the total generation.
- Price mark-up $p - MC_e\left(q_e\right)$
  - Small mark-up means competitive behavior
  - Large mark-up implies strategic behavior

Cournot model with contracts

- Objective function

$$B_e = p\left(q_e - L_e\right) - C_e + p_c L_e$$

- Optimality conditions

$$\frac{\partial B_e}{\partial q_e} = 0$$

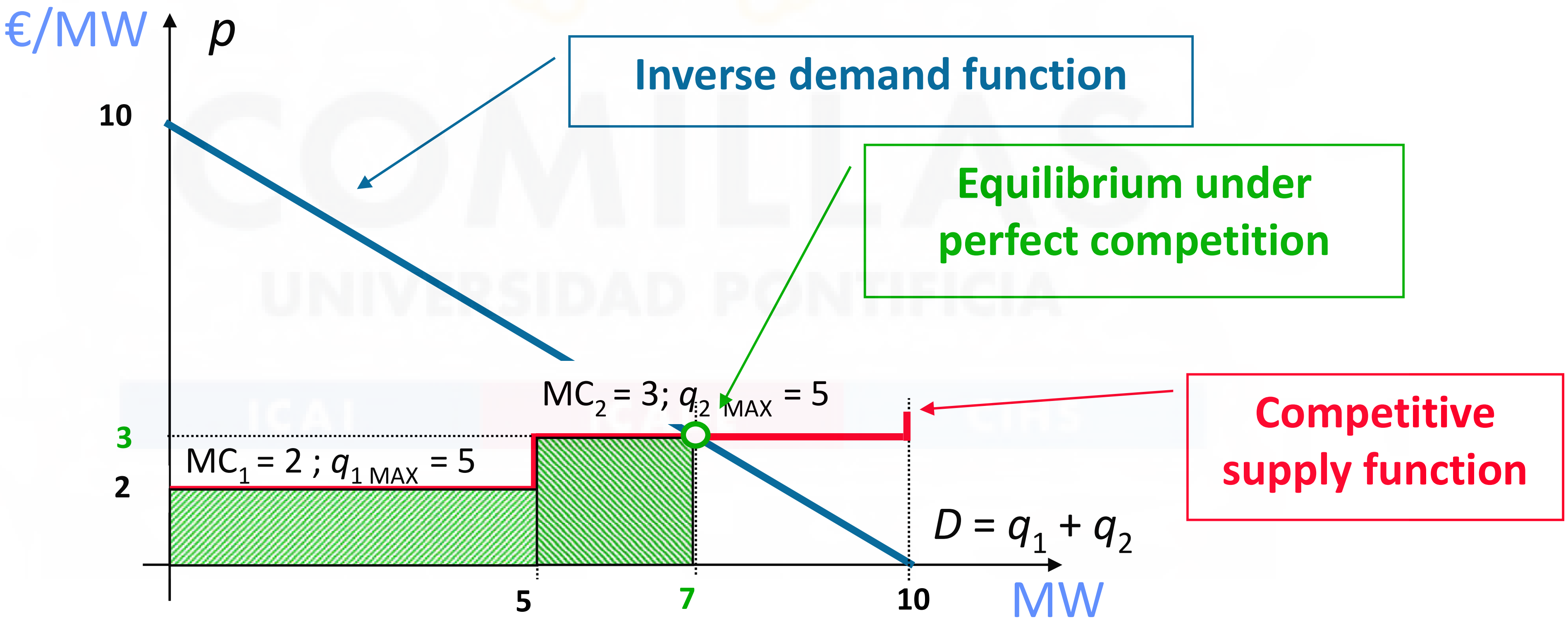
$$MR_e = p + \left(q_e - L_e\right)p' = MC_e\left(q_e\right)$$

$$q_e = \frac{p - MC_e\left(q_e\right)}{-p'} + L_e$$

e	Company
B_e	Profits
p	Price
q_e	Output
L_e	Contracted output
p_c	Contract price
C_e	Variable costs
MC_e	Marginal cost
MR_e	Marginal revenue

Cournot model: example (I)

- Perfect competition
  - Company 1: $MC_1 = 2$ €/MW $\quad q_{1\ MAX} = 5$ MW
  - Company 2: $MC_2 = 3$ €/MW $\quad q_{2\ MAX} = 5$ MW
  - Inverse demand function: $p = 10 - (q_1 + q_2)$



Cournot model: example (II)

- Duopoly
  - Company 1: $MC_1 = 2$ €/MW
  - Company 2: $MC_2 = 3$ €/MW
  - IDF: $p = 10 - (q_1 + q_2)$; $p' = -1$

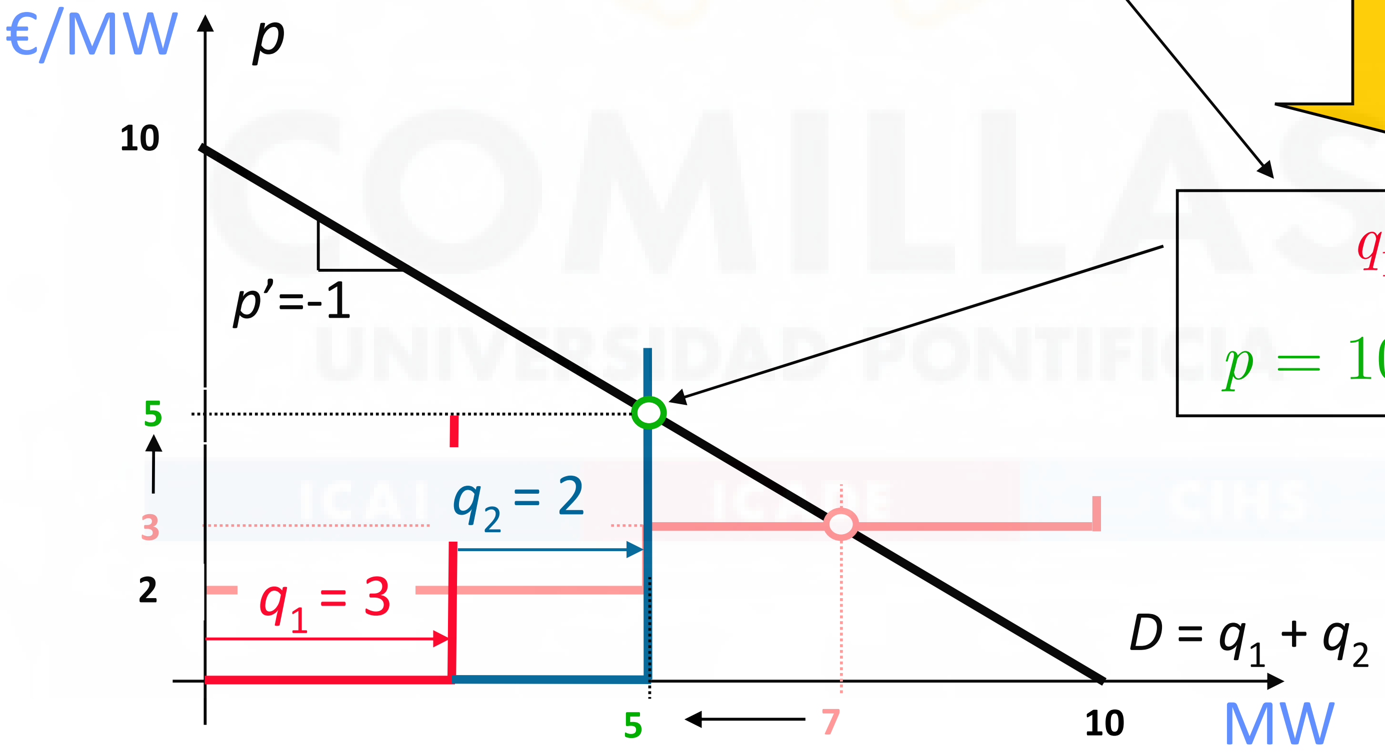
Cournot

$$\begin{cases} \dfrac{\partial B_1}{\partial q_1} = 0 \rightarrow p + q_1 \cdot (-1) = 2 \\ \dfrac{\partial B_2}{\partial q_2} = 0 \rightarrow p + q_2 \cdot (-1) = 3 \end{cases}$$

Solving

$$q_1 = 3, \; q_2 = 2$$

$$p = 10 - (q_1 + q_2) = 5$$



From Cournot to conjectural variations (CV)

e	Company
B_e	Profits
p	Price
q_e	Output
C_e	Variable costs
MC_e	Marginal cost
MR_e	Marginal revenue

- Objective function

$$\max B_e = p \cdot q_e - C_e$$

- Inverse demand function

$$p = f\left(\sum_e q_e\right)$$

- Conjecture: **every company sees its residual demand**

$$\frac{\partial p}{\partial q_e} = p'_e \quad \rightarrow$$ **generalization of the model based on CV**

- Optimality conditions

$$\frac{\partial B_e}{\partial q_e} = 0 \rightarrow MR_e = p + q_e p'_e = MC_e\left(q_e\right)$$

Conjectural variation (CV)

- Other names:
  - Cross elasticity of demand between firms
  - Strategic parameter
  - Implicit residual demand slope
  - Conjectured price response
- It is a measure of the interdependence between firms. It captures the extent to which one firm reacts to changes in strategic variables (quantity) made by other firms
- The CV approach considers the reaction of competitors when a firm is deciding its optimal production. This reaction comes from firm's demand curve and supply functions (residual demand function). This curve is different for each firm and relates the market price with the firm's production
- Values of 0-10 c€/MWh/MW can be sensible

Some Comillas-IIT publications on computing CV

- Optimization approach
  - S. López, P. Sánchez, J. de la Hoz-Ardiz, J. Fernández-Caro, "*Estimating conjectural variations for electricity market models*", European Journal of Operational Research. vol. 181, no. 3, pp. 1322-1338, September 2007.
- Econometric approach
  - A. García, M. Ventosa, M. Rivier, A. Ramos, G. Relaño, "*Fitting electricity market models. A conjectural variations approach*", 14th PSCC Conference, Session 12-3, pp. 1-8. Sevilla, Spain, 24-28 June 2002 (http://www.pscc-central.org/uploads/tx_ethpublications/s12p03.pdf)

Cournot model: from system of equations to NLP

$$MC_e = MR_e = p + p'_e q_e$$

$$D = D_0 - D_1 p$$

$$\sum_e q_e = D$$

Marginal cost = Marginal revenue

Inverse demand function

Balance between generation and demand

Cournot model: NLP equivalent problem

- It is easy to check that previous system of equations are just the KKT optimality conditions of the problem

$$\max_{q_e, D} U(D) - \sum_e \overline{C}_e(q_e)$$
$$\sum_e q_e = D \quad \perp p$$

Price is the multiplier

- Demand utility: $U(D) = \int_0^D p \cdot dD = \frac{1}{D_1}\left(D \cdot D_0 - \frac{D^2}{2}\right)$
- Effective cost: $\overline{C}_e(q_e) = C_e(q_e) - \frac{p'_e}{2} q_e^2$

Cournot model and CV: conclusions

- **Solving Cournot** equilibrium or equilibrium based on **conjectural variations** requires solving a **system** of **equations (MCP)**
- This system of equations is **linear** if:
  - **Inverse demand function** is linear
  - **Marginal cost** function is linear
- It can also be solved as a **nonlinear programming problem (NLP)**

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Bushnell model

Session outline

Cournot model
- ✓ Thermal generation
- ✓ Single period

Bushnell model
- ✓ Hydrothermal generation
- ✓ Multi-period

Model based on the complementarity problem
- ✓ Means of production
  - Fuel stock management
  - Pumped storage hydro plants
- ✓ Market aspects
  - Contracts for differences
  - Take-or-pay contracts

Bushnell model (1998)

- Extensions of the Cournot model to
  - Electricity markets
- Representation of **storage hydro plants**
  - Constraint on available hydro energy
    - Optimization problem with some **constraints**
  - **Multi-period equilibrium** when the available energy constraint involves several periods
    - Increased solution time
- Definition of the **water value in electricity markets**

Bushnell model: formulation I

(https://pascua.iit.comillas.edu/aramos/StarMrkLite_BushnellEn.gms)

- Objective function

$$B_e = \sum_p \left[p_p \left(q^T_{p,e} + q^H_{p,e} \right) - C^T_{p,e} \right]$$

- Available hydro energy:

$$\sum_p q^H_{p,e} \le Q^H_e \qquad \perp \lambda^H_e$$

- Inverse demand function

$$p_p = f_p \left(\sum_e \left(q^T_{p,e} + q^H_{p,e} \right) \right)$$

e	Company
p	Period (duration 1)
B_e	Profits
p	Price
q^T	Thermal output
q^H	Hydro output
C_e	Variable costs
Q_e^H	Available energy
λ_e	Water value

Bushnell model: formulation II

- Lagrangian function **(equivalent optimization problem but without constraints)**

$$\mathcal{L}_e = \sum_p \left[p_p \left(q^T_{p,e} + q^H_{p,e} \right) - C^T_{p,e}\left(q^T_{p,e} \right) \right] + \lambda^H_e \left[\sum_p q^H_{p,e} - Q^H_e \right]$$

- Optimality conditions

$$\frac{\partial \mathcal{L}_e}{\partial q^T_{p,e}} = 0 \rightarrow MR_{p,e} = p_p + \left(q^T_{p,e} + q^H_{p,e} \right) p'_p = MC^T_{p,e}\left(q^T_{p,e} \right)$$

$$\frac{\partial \mathcal{L}_e}{\partial q^H_{p,e}} = 0 \rightarrow MR_{p,e} = p_p + \left(q^T_{p,e} + q^H_{p,e} \right) p'_p = -\lambda^H_e$$

e	Company
p	Period (duration 1)
B_e	Profits
p	Price
q^T	Thermal output
q^H	Hydro output
C_e	Variable costs
Q_e^H	Available energy
λ_e	Water value
MC_e	Marginal cost
MR_e	Marginal revenue

Bushnell model: comments

- The "**Bushnell conjecture**" is the Cournot conjecture extended to all the periods (*e** = all other companies)

$$\frac{\partial \sum_{e^*} q_{p',e^*}}{\partial q_{p,e}} = 0$$

- **Qualitative conclusions** are drawn from the **optimality conditions**
  – Total optimal output (as in Cournot)

$$\frac{\partial \mathcal{L}_{\mathrm{e}}}{\partial q^T_{p,e}} = 0 \qquad \rightarrow q^T_{p,e} + q^H_{p,e} = \frac{p_p - MC^T_{p,e}\left(q^T_{p,e}\right)}{-p'_p}$$

  – **Water value**

$$\frac{\partial \mathcal{L}_{\mathrm{e}}}{\partial q^T_{p,e}} = 0; \frac{\partial \mathcal{L}_{\mathrm{e}}}{\partial q^H_{p,e}} = 0 \rightarrow \boxed{MR_{p,e} = MC^T_{p,e} = -\lambda^H_e}$$

Water value in electricity markets

- Very important concept in hydrothermal coordination
  - **Final decision** on **hydro** output
- The objective function changes when the amount of hydro energy available increases
  - In centralized planning
    - Reduces **system** operating **costs**
  - In electricity markets
    - Increases in each **company's profits**
- Calculated as the **dual variable** of the **available** hydro **energy** constraint

Water value: Bushnell model

- Hydro output attempts to equal inter-period marginal revenues

$$\frac{\partial \mathcal{L}_e}{\partial q^T_{p,e}} = 0; \frac{\partial \mathcal{L}_e}{\partial q^H_{p,e}} = 0 \rightarrow MR_{p,e} = MC^T_{p,e} = -\lambda^H_e$$

- Each company regards its water to be marginal revenue, whose value coincides with the marginal cost of **its** thermal plant

- Intuitively, when a company replaces **1 MW of thermal** output with **1 MW of hydro** generation, the savings equal its marginal cost

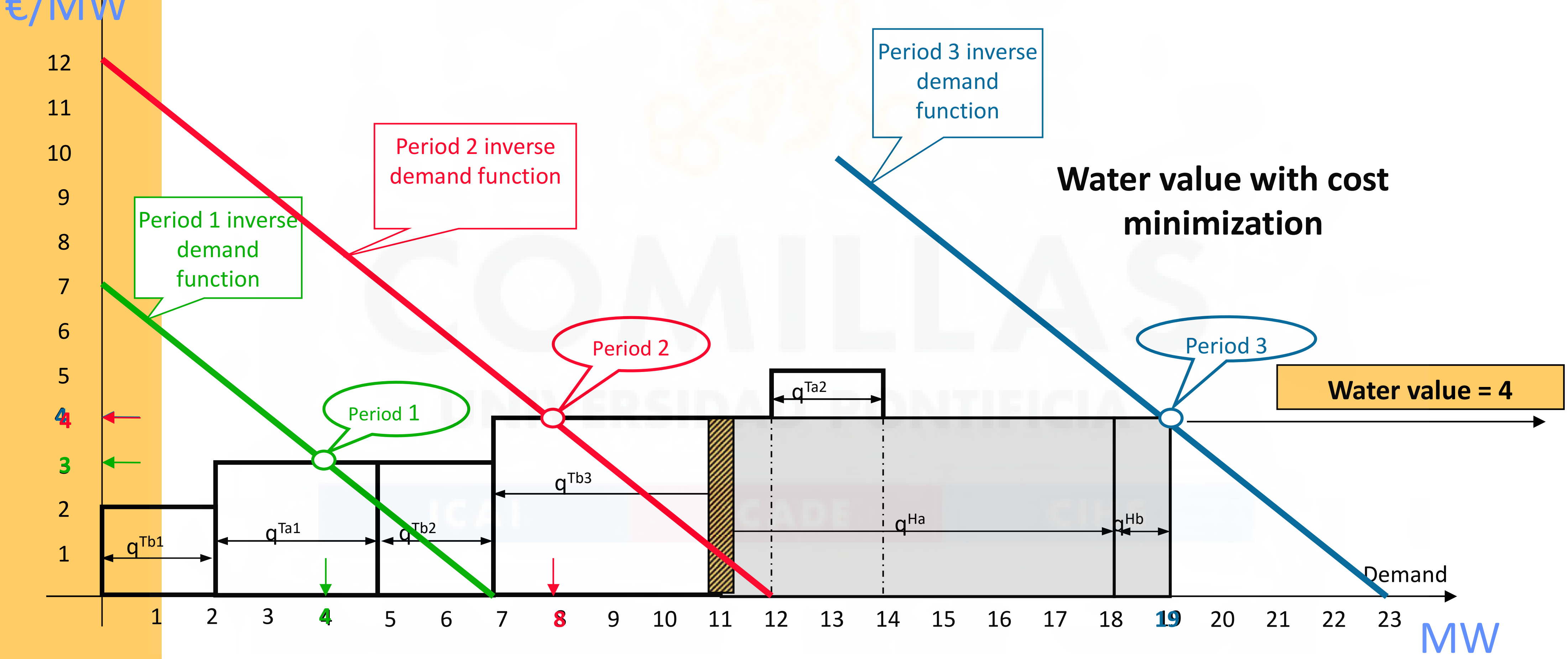
$$q^T_{p,e} + q^H_{p,e} = \frac{p_p - MC^T_{p,e}\left(q^T_{p,e}\right)}{-p'_p}$$

Bushnell model: Example data

- Demand data (3 periods lasting 1 h each):
  - *period* 1 $p_1 = 7 - Dem_1$
  - *period* 2 $p_2 = 12 - Dem_2$
  - *period* 3 $p_3 = 23 - Dem_3$
- Company A data:
  - $q^{Ta1} = 3$ MW $MC^{Ta1} = 3$ €/MW
  - $q^{Ta2} = 2$ MW $MC^{Ta2} = 5$ €/MW
  - $q^{Ha} = 7$ MW $Q^{Ha} = 7$ MWh
- Company B data:
  - $q^{Tb1} = 2$ MW $MC^{Tb1} = 2$ €/MW
  - $q^{Tb2} = 2$ MW $MC^{Tb2} = 3$ €/MW
  - $q^{Tb3} = 5$ MW $MC^{Tb3} = 4$ €/MW
  - $q^{Hb} = 1$ MW $Q^{Hb} = 1$ MWh

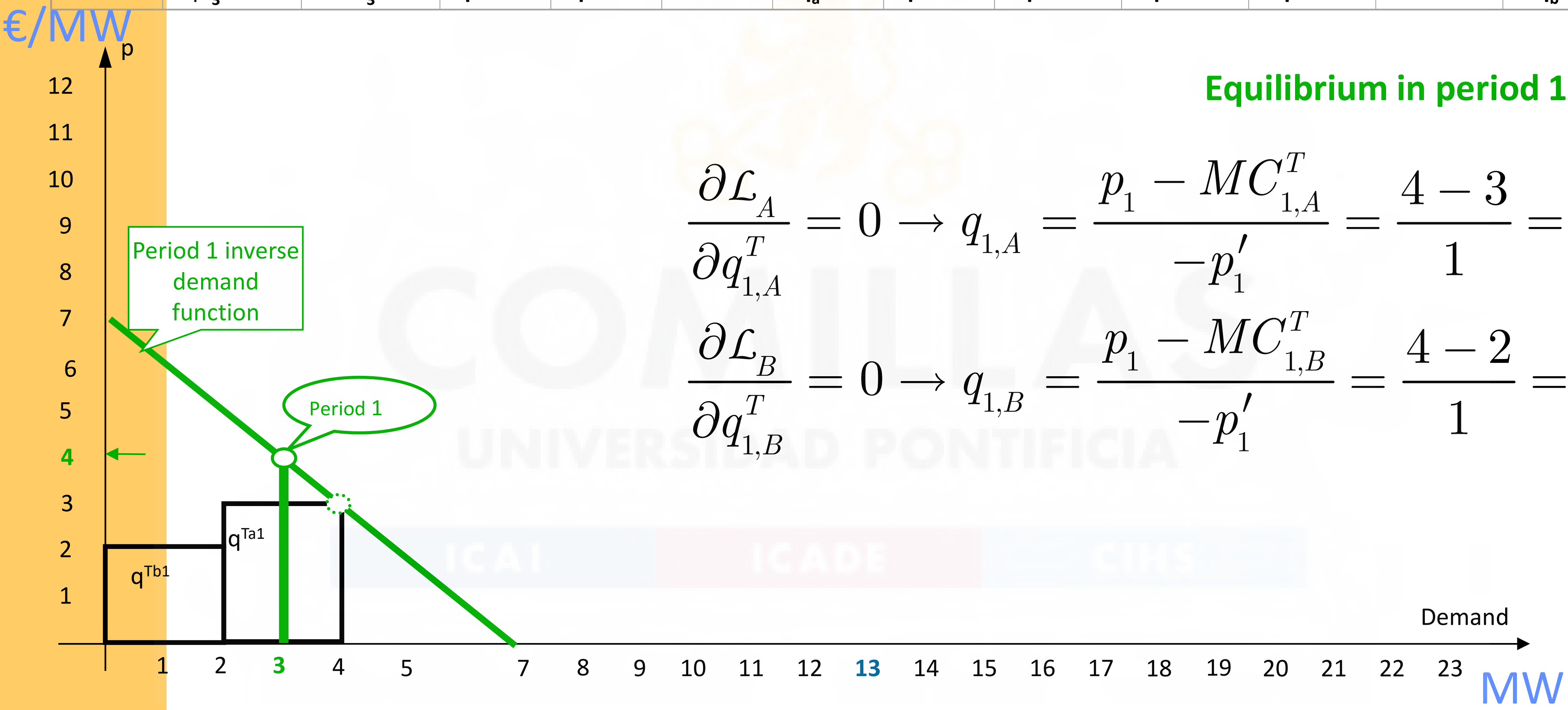
Example: Perfect competition or minimum cost dispatch

Period 1	**$p_1 = 3$**	$Dem_1 = 4$	**$q^{Ta1} = 2$**	$q^{Ta2} = 0$	$q^{Ha} = 0$	$q^{Tb1} = 2$	$q^{Tb2} = 0$	$q^{Tb3} = 0$	$q^{Hb} = 0$
Period 2	**$p_2 = 4$**	$Dem_2 = 8$	$q^{Ta1} = 3$	$q^{Ta2} = 0$	$q^{Ha} = 0$	$q^{Tb1} = 2$	$q^{Tb2} = 2$	**$q^{Tb3} = 1$**	$q^{Hb} = 0$
Period 3	**$p_3 = 4$**	$Dem_3 = 19$	$q^{Ta1} = 3$	$q^{Ta2} = 0$	$q^{Ha} = 7$	$q^{Tb1} = 2$	$q^{Tb2} = 2$	**$q^{Tb3} = 4$**	$q^{Hb} = 1$



Example: dispatch under the Bushnell model

Period 1	$p_1 = 4$	$Dem_1 = 3$	$q^{Ta1} = 1$		$MC^a = 3$	$q_a = 1$	$q^{Tb1} = 2$				$MC^b = 2$	$q_b = 2$
Period 2	$p_2 = 6$	$Dem_2 = 6$	$q^{Ta1} = 1$	$q^{Ha} = 2$	$MC^a = 3$	$q_a = 3$	$q^{Tb1} = 2$	$q^{Tb2} = 1$			$MC^b = 3$	$q_b = 3$
Period 3	$p_3 = 10$	$Dem_3 = 13$	$q^{Ta1} = 2$	$q^{Ha} = 5$	$MC^a = 3$	$q_a = 7$	$q^{Tb1} = 2$	$q^{Tb2} = 2$	$q^{Tb3} = 1$	$q^{Hb} = 1$	$MC^b = 4$	$q_b = 6$



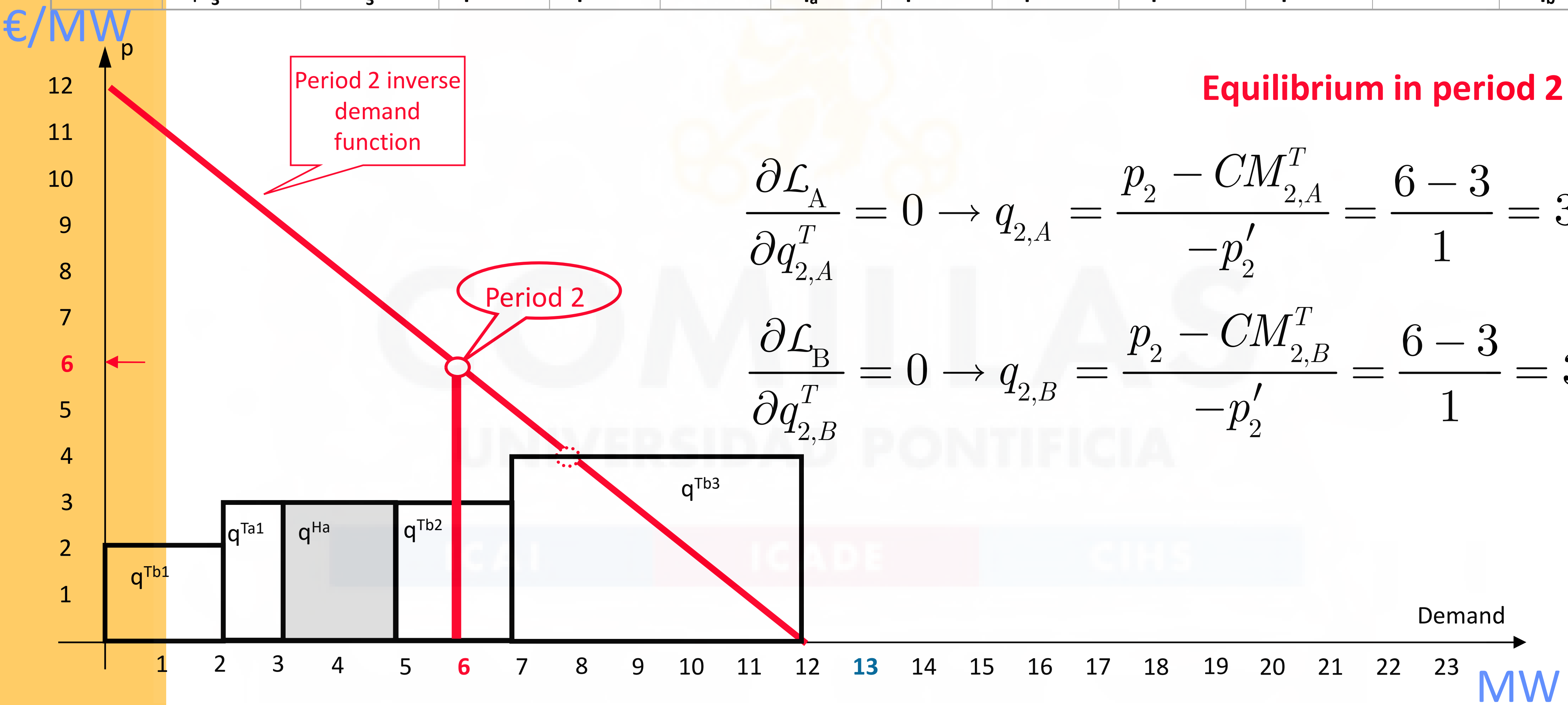
Equilibrium in period 1

$$\frac{\partial \mathcal{L}_A}{\partial q^T_{1,A}} = 0 \rightarrow q_{1,A} = \frac{p_1 - MC^T_{1,A}}{-p_1'} = \frac{4-3}{1} = 1$$

$$\frac{\partial \mathcal{L}_B}{\partial q^T_{1,B}} = 0 \rightarrow q_{1,B} = \frac{p_1 - MC^T_{1,B}}{-p_1'} = \frac{4-2}{1} = 2$$

Example: dispatch under the Bushnell model

Period 1	$p_1 = 4$	$Dem_1 = 3$	$q^{Ta1} = 1$		$MC^a = 3$	$q_a = 1$	$q^{Tb1} = 2$				$MC^b = 2$	$q_b = 2$
Period 2	$p_2 = 6$	$Dem_2 = 6$	$q^{Ta1} = 1$	$q^{Ha} = 2$	$MC^a = 3$	$q_a = 3$	$q^{Tb1} = 2$	$q^{Tb2} = 1$			$MC^b = 3$	$q_b = 3$
Period 3	$p_3 = 10$	$Dem_3 = 13$	$q^{Ta1} = 2$	$q^{Ha} = 5$	$MC^a = 3$	$q_a = 7$	$q^{Tb1} = 2$	$q^{Tb2} = 2$	$q^{Tb3} = 1$	$q^{Hb} = 1$	$MC^b = 4$	$q_b = 6$



Equilibrium in period 2

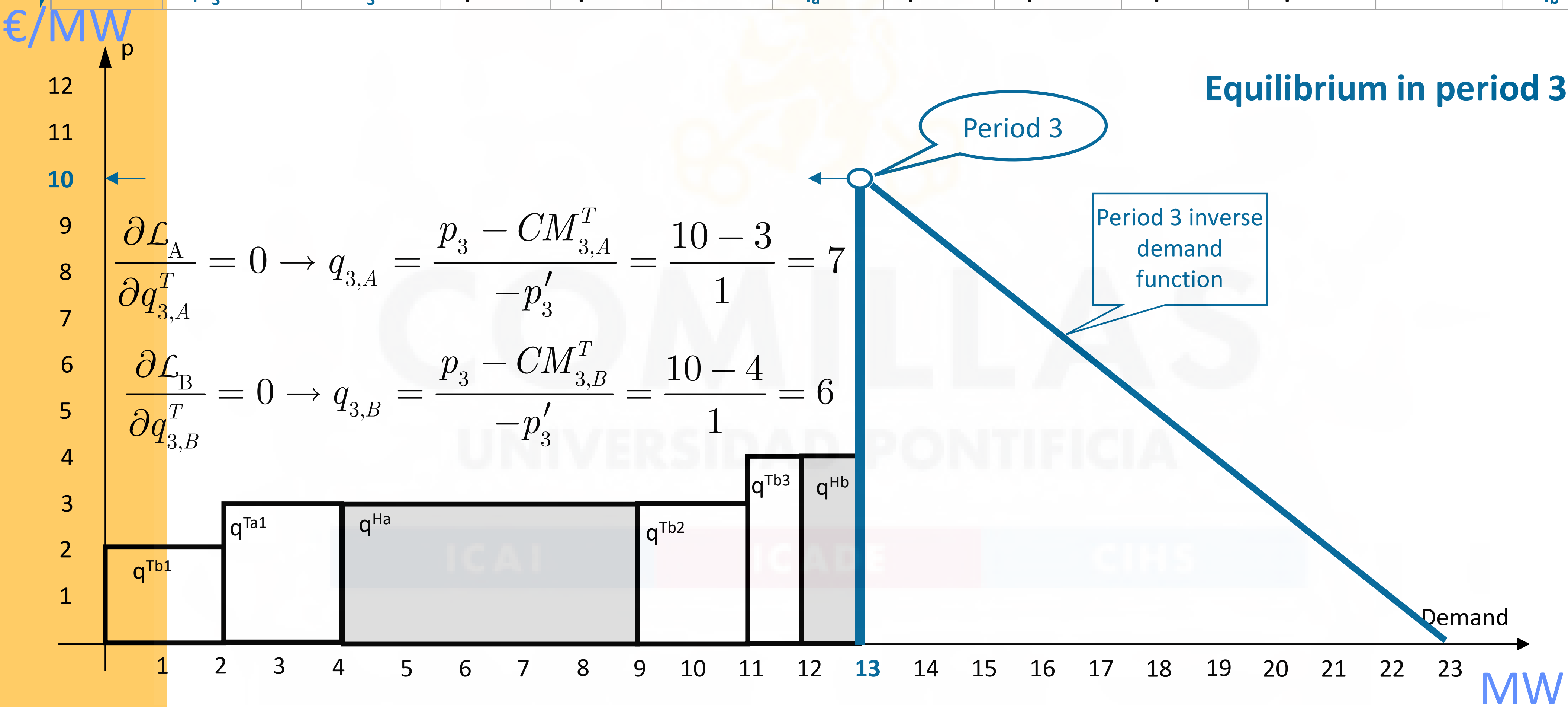
$$\frac{\partial \mathcal{L}_A}{\partial q^T_{2,A}} = 0 \rightarrow q_{2,A} = \frac{p_2 - CM^T_{2,A}}{-p'_2} = \frac{6-3}{1} = 3$$

$$\frac{\partial \mathcal{L}_B}{\partial q^T_{2,B}} = 0 \rightarrow q_{2,B} = \frac{p_2 - CM^T_{2,B}}{-p'_2} = \frac{6-3}{1} = 3$$

Example: dispatch under the Bushnell model

Period 1	$p_1 = 4$	$Dem_1 = 3$	$q^{Ta1} = 1$		$MC^a = 3$	$q_a = 1$	$q^{Tb1} = 2$				$MC^b = 2$	$q_b = 2$
Period 2	$p_2 = 6$	$Dem_2 = 6$	$q^{Ta1} = 1$	$q^{Ha} = 2$	$MC^a = 3$	$q_a = 3$	$q^{Tb1} = 2$	$q^{Tb2} = 1$			$MC^b = 3$	$q_b = 3$
Period 3	$p_3 = 10$	$Dem_3 = 13$	$q^{Ta1} = 2$	$q^{Ha} = 5$	$MC^a = 3$	$q_a = 7$	$q^{Tb1} = 2$	$q^{Tb2} = 2$	$q^{Tb3} = 1$	$q^{Hb} = 1$	$MC^b = 4$	$q_b = 6$

Equilibrium in period 3

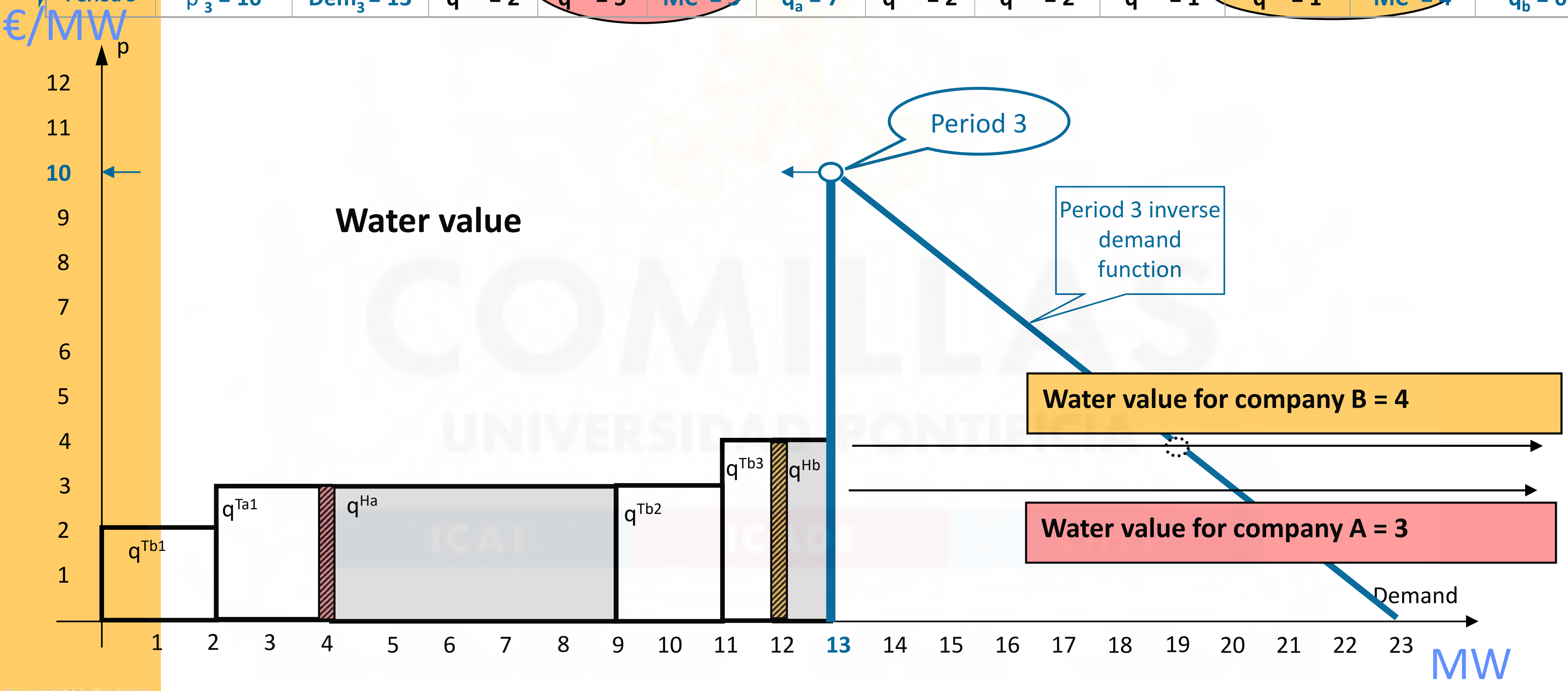


$$\frac{\partial \mathcal{L}_A}{\partial q^T_{3,A}} = 0 \rightarrow q_{3,A} = \frac{p_3 - CM^T_{3,A}}{-p'_3} = \frac{10-3}{1} = 7$$

$$\frac{\partial \mathcal{L}_B}{\partial q^T_{3,B}} = 0 \rightarrow q_{3,B} = \frac{p_3 - CM^T_{3,B}}{-p'_3} = \frac{10-4}{1} = 6$$

Example: dispatch under the Bushnell model

Period 1	$p_1 = 4$	$Dem_1 = 3$	$q^{Ta1} = 1$		$MC^a = 3$	$q_a = 1$	$q^{Tb1} = 2$				$MC^b = 2$	$q_b = 2$
Period 2	$p_2 = 6$	$Dem_2 = 6$	$q^{Ta1} = 1$	$q^{Ha} = 2$	$MC^a = 3$	$q_a = 3$	$q^{Tb1} = 2$	$q^{Tb2} = 1$			$MC^b = 3$	$q_b = 3$
Period 3	$p_3 = 10$	$Dem_3 = 13$	$q^{Ta1} = 2$	$q^{Ha} = 5$	$MC^a = 3$	$q_a = 7$	$q^{Tb1} = 2$	$q^{Tb2} = 2$	$q^{Tb3} = 1$	$q^{Hb} = 1$	$MC^b = 4$	$q_b = 6$



Bushnell model: conclusions

- **Solving Bushnell** equilibrium (without less than or equal to constraints) entails solving a **multi-period system** of **equations**
- The system of equations is **linear** if:
  - **Inverse demand function** is linear
  - **Marginal cost** function is linear

Optimization problems for both companies (i)

$$\max B_A = \sum_p \left[p_p \left(\sum_t q^t_{p,A} + \sum_h q^h_{p,A} \right) - C^T_{p,A} \right]$$

$$\sum_p q^h_{p,A} \le Q^h_A \quad \perp \lambda^h_A$$

$$q^t_{p,A} \ge 0 \quad \perp \mu^t_{p,A}$$

$$q^h_{p,A} \ge 0 \quad \perp \mu^h_{p,A}$$

$$q^t_{p,A} \le \overline{q}^t \quad \perp \nu^t_{p,A}$$

$$q^h_{p,A} \le \overline{q}^h \quad \perp \nu^h_{p,A}$$

$$\max B_B$$

$$p_p = \alpha_p + \beta_p \left[\sum_t q^t_{p,A} + \sum_h q^h_{p,A} + \sum_t q^t_{p,B} + \sum_h q^h_{p,B} \right]$$

Minimization

Optimization problems for both companies (ii)

$$\mathcal{L}_A = -\sum_p \left[p_p \left(\sum_t q^t_{p,A} + \sum_h q^h_{p,A} \right) - C^T_{p,A} \right] + \lambda^h_A \left[\sum_p q^h_{p,A} - Q^h_A \right] +$$
$$+ \mu^t_{p,A} q^t_{p,A} + \mu^h_{p,A} q^h_{p,A} + \nu^t_{p,A} \left[q^t_{p,A} - \overline{q}^t \right] + \nu^h_{p,A} \left[q^h_{p,A} - \overline{q}^h \right]$$
$$\lambda^h_A, \nu^t_{p,A}, \nu^h_{p,A} \geq 0; \mu^t_{p,A}, \mu^h_{p,A} \leq 0$$

$$\mathcal{L}_B$$

$$p_p = \alpha_p + \beta_p \left[\sum_t q^t_{p,A} + \sum_h q^h_{p,A} + \sum_t q^t_{p,B} + \sum_h q^h_{p,B} \right]$$

Optimization problems for both companies (iii)

$$\frac{\partial \mathcal{L}_A}{\partial q^t_{p,A}} = -p_p - p'_p\left(\sum_t q^t_{p,A} + \sum_h q^h_{p,A}\right) + MC^T_{p,A} + \mu^t_p + \nu^t_p = 0$$

$$\frac{\partial \mathcal{L}_A}{\partial q^h_{p,A}} = -p_p - p'_p\left(\sum_t q^t_{p,A} + \sum_h q^h_{p,A}\right) + \lambda^h_A + \mu^t_p + \nu^t_p = 0$$

$$\lambda^h_A\left[\sum_p q^h_{p,A} - Q^h_A\right] = 0$$

$$\mu^t_p q^t_{p,A} = 0; \quad \mu^h_p q^h_{p,A} = 0$$

$$\nu^t_p\left[q^t_{p,A} - \overline{q}^t\right] = 0; \quad \nu^h_p\left[q^h_{p,A} - \overline{q}^h\right] = 0$$

$$p_p = \alpha_p + \beta_p\left[\sum_t q^t_{p,A} + \sum_h q^h_{p,A} + \sum_t q^t_{p,B} + \sum_h q^h_{p,B}\right]$$

$$\lambda^h, \nu^t_p, \nu^h_p \geq 0; \mu^t_p, \mu^h_p \leq 0$$

4

Model based on the complementarity problem

Session outline

Cournot model

- ✓ Thermal generation
- ✓ Single period

Bushnell model

- ✓ Hydrothermal generation
- ✓ Multi-period

Model based on the complementarity problem

- ✓ **Means of production**
  - Fuel stock management
  - Pumped storage hydro plants
- ✓ **Market aspects**
  - Contracts for differences
  - Take-or-pay contracts

Stating the problem

Company 1's optimization problem

$$\max z^1\left(x^1\right)$$
subject to
$$g_k^1 \le 0$$
$$h_j^1 = 0$$

…

Company *e*'s optimization problem

$$\max z^e\left(x^e\right)$$
subject to
$$g_k^e \le 0$$
$$h_j^e = 0$$

…

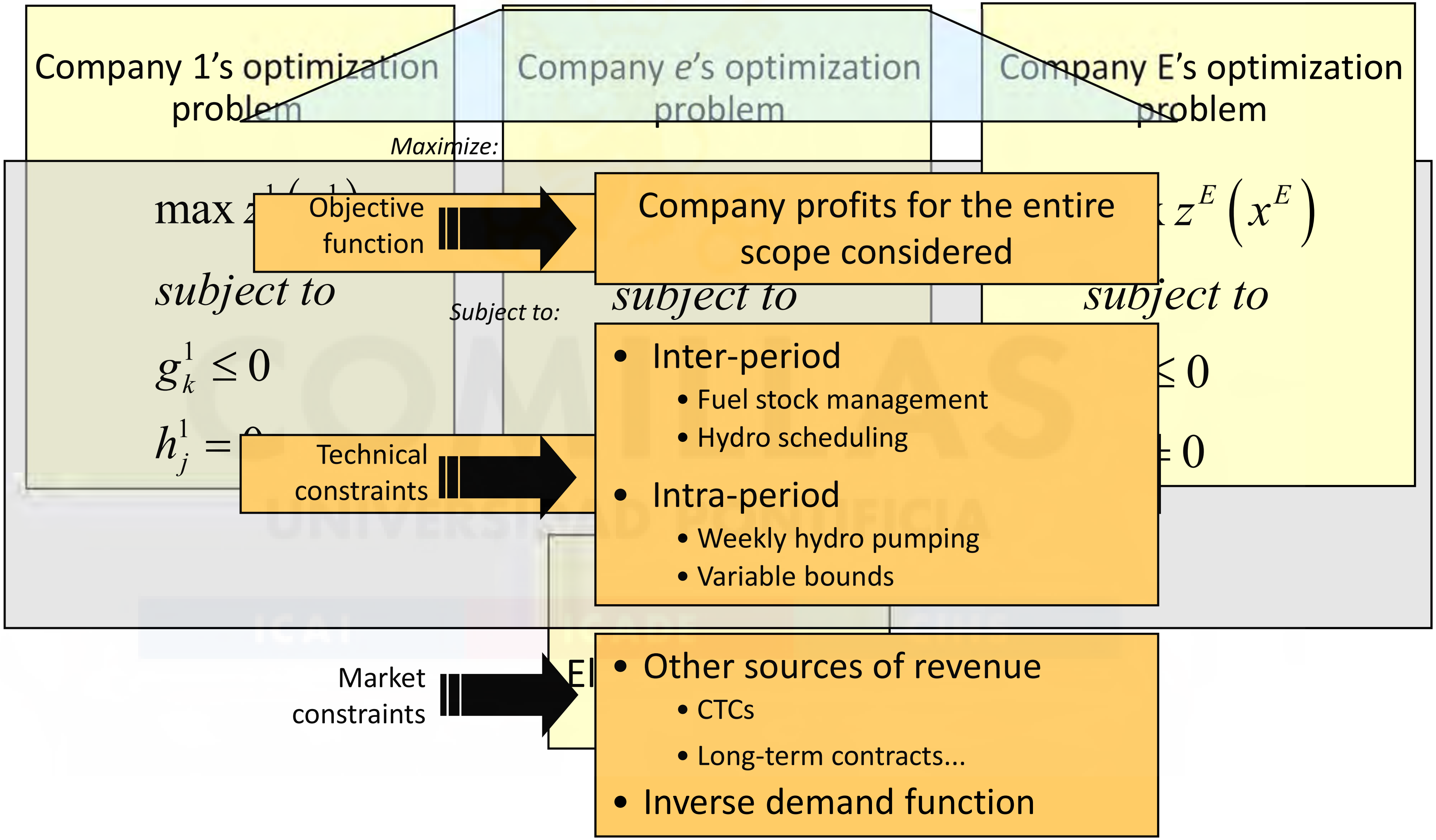
Company E's optimization problem

$$\max z^E\left(x^E\right)$$
subject to
$$g_k^E \le 0$$
$$h_j^E = 0$$

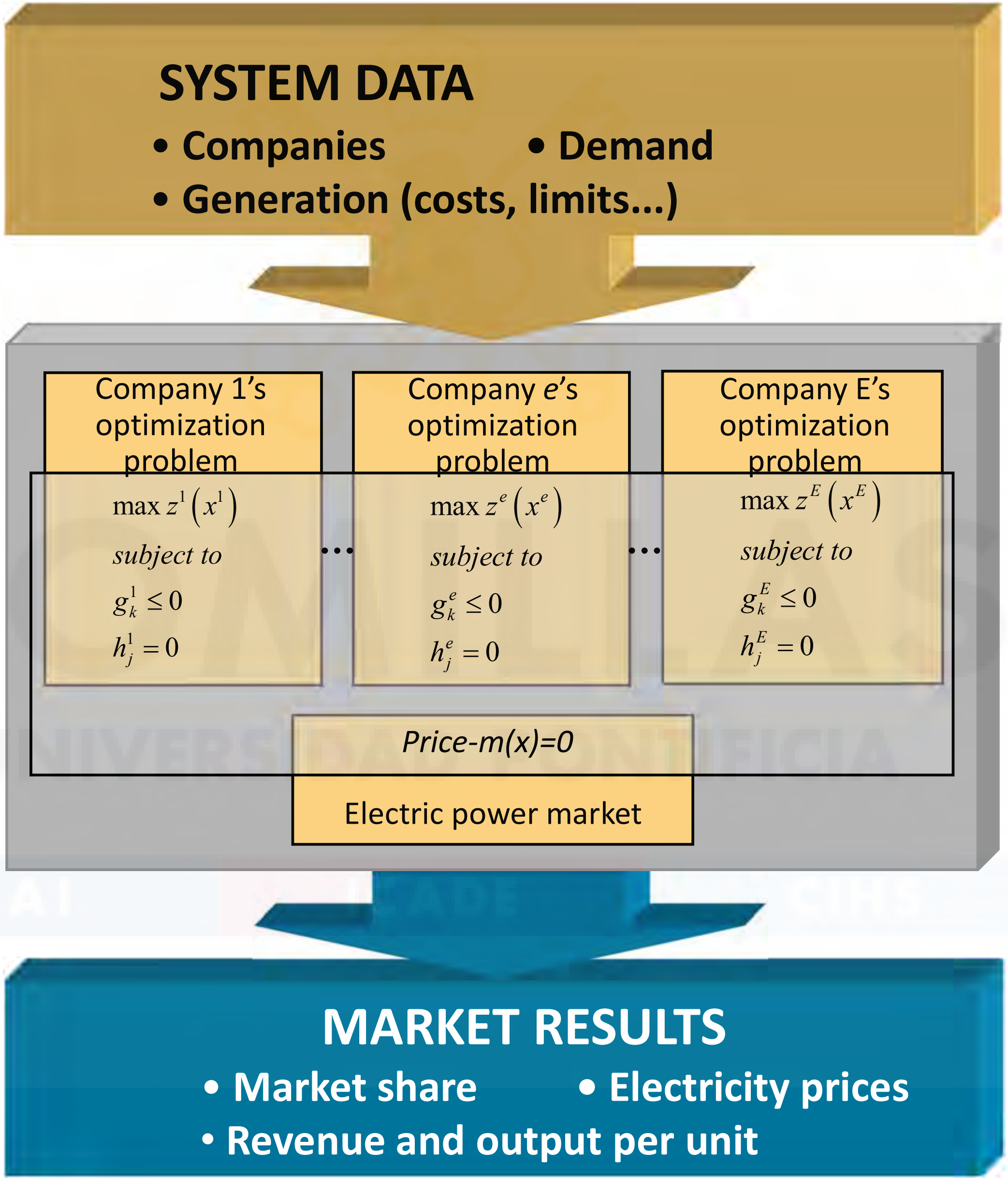
Price-m(x)=0

Electric power market

Stating the problem: Each company's problem



Stating the problem: Using the model



Practical difficulties

- The foregoing approach is impeccable, **theoretically speaking**
- And yet, **there is** no solver or algorithm **able** to **solve** the above mathematical problem:
  - Several optimization problems inter-connected by the inverse demand function
- An equivalent problem must be sought
  - With the same solution for its variables
  - Numerically **solvable**

Practical difficulties. Alternative approaches

- **Complementarity problem**
  - M. Rivier, M. Ventosa, A. Ramos *A Generation Operation Planning Model in Deregulated Electricity Markets based on the Complementarity Problem* 2nd International Conference on Complementarity Problems (ICCP 99) Madison, WI, USA June 1999
  - B.F. Hobbs. "*LCP Models of Nash – Cournot Competition in Bilateral and POOLCO–Based Power Markets*." In Proc. IEEE Winter Meeting, New York, 1999
- **Equivalent quadratic system**
  - J. Barquín, E. Centeno, J. Reneses, "*Medium-term generation programming in competitive environments: A new optimization approach for market equilibrium computing*", IEE Proceedings-Generation Transmission and Distribution. vol. 151, no. 1, pp. 119-126, Enero 2004.
  - B.F. Hobbs. "*Linear Complementarity Models of Nash–Cournot competition in Bilateral and POOLCO Power Markets*" IEEE Transactions on Power Systems, 16 (2), May 2001
- **Variational inequalities**
  - W. Jing-Yuan and Y. Streets, "*Spatial oligopolistic electricity models with Cournot generators and regulated transmission prices*," Operations Res., vol. 47, no. 1, pp. 102–112, 1999

KKT Optimality conditions for each company's problem

maximize $z^e(x)$

subject to:

$g_k^e(x) \le 0 \quad \perp \lambda_k^e$

$h_j^e(x) = 0 \quad \perp \mu_j^e$

Lagrange function

$$\mathcal{L}^e(x,\lambda,\mu) = z^e + \sum_k \lambda_k^e g_k^e + \sum_j \mu_j^e h_j^e$$

KKT optimality conditions

$$\nabla_x \mathcal{L}^e(x,\lambda,\mu) = \frac{\partial \mathcal{L}^e}{\partial x^e} = 0$$

$$\nabla_\mu \mathcal{L}^e(x,\lambda,\mu) = \frac{\partial \mathcal{L}^e}{\partial \mu_j^e} = h_j^e = 0$$

$$\lambda_k^e \cdot g_k^e = 0 \quad g_k^e \le 0 \quad \lambda_k^e \le 0$$

Mixed complementarity problem (MCP)

- Combining a **system of equations** with a **complementarity problem**
- Generalization of the complementarity problem

System of equations

$$x \text{ free}$$
$$\mu_j^e \text{ free}$$

$$\nabla_x \mathcal{L}^e(x, \lambda, \mu) = \frac{\partial \mathcal{L}^e}{\partial x^e} = 0$$

$$\nabla_\mu \mathcal{L}^e(x, \lambda, \mu) = \frac{\partial \mathcal{L}^e}{\partial \mu_j^e} = h_j^e = 0$$

$$\lambda_k^e \cdot g_k^e = 0 \quad g_k^e \leq 0 \quad \lambda_k^e \leq 0$$

Complementarity problem

Linear complementarity problem (CP)
Mixed linear complementarity problem (MCP)

LCP

$$x \geq 0$$
$$Ax - b \geq 0$$
$$x^T(Ax - b) = 0$$

MLCP

$$x_1 \geq 0$$
$$A_{11}x_1 + A_{12}x_2 - b_1 \geq 0$$
$$A_{21}x_1 + A_{22}x_2 - b_2 = 0$$
$$x_1^T(A_{11}x_1 + A_{12}x_2 - b_1) = 0$$

$$x \in \mathbb{R}^n$$
$$b \in \mathbb{R}^n$$
$$A \in \mathbb{R}^{n\times n}$$
$$x_1 \in \mathbb{R}^n, x_2 \in \mathbb{R}^m$$
$$b_1 \in \mathbb{R}^n, b_2 \in \mathbb{R}^m$$
$$A_{11} \in \mathbb{R}^{n\times n}, A_{22} \in \mathbb{R}^{n\times n}$$
$$A_{12} \in \mathbb{R}^{n\times m}, A_{21} \in \mathbb{R}^{m\times n}$$

Equivalent mixed complementarity problem for all the companies

Company 1's optimality conditions

$$\nabla_x \mathcal{L}^1(x,\lambda,\mu) = \frac{\partial \mathcal{L}^1}{\partial x^1} = 0$$

$$\nabla_\lambda \mathcal{L}^1(x,\lambda,\mu) = \frac{\partial \mathcal{L}^1}{\partial \mu_j^1} = h_j^1 = 0$$

$$\lambda_k^1 \cdot g_k^1 = 0 \quad g_k^1 \le 0 \quad \lambda_k^1 \le 0$$

…

Company *e*'s optimality conditions

$$\nabla_x \mathcal{L}^e(x,\lambda,\mu) = \frac{\partial \mathcal{L}^e}{\partial x^e} = 0$$

$$\nabla_\lambda \mathcal{L}^e(x,\lambda,\mu) = \frac{\partial \mathcal{L}^e}{\partial \mu_j^e} = h_j^e = 0$$

$$\lambda_k^e \cdot g_k^e = 0 \quad g_k^e \le 0 \quad \lambda_k^e \le 0$$

…

Company E's optimality conditions

$$\nabla_x \mathcal{L}^E(x,\lambda,\mu) = \frac{\partial \mathcal{L}^E}{\partial x^E} = 0$$

$$\nabla_\lambda \mathcal{L}^E(x,\lambda,\mu) = \frac{\partial \mathcal{L}^E}{\partial \mu_j^E} = h_j^E = 0$$

$$\lambda_k^E \cdot g_k^E = 0 \quad g_k^E \le 0 \quad \lambda_k^E \le 0$$

Price-m(y)=0

Electric power market

Detailed system modeling (I)

- **Market** modeling
  - **Demand**-side behavior
    - Price is a **linear function of demand**
    - Load-duration curve per period
  - **Cournot or CV** company competition
    - Simultaneous maximization of profits
    - Market revenues are a **quadratic function of price**
  - Other market characteristics
    - **Contracts** for differences (sales)
    - **Take-or-pay contracts** (purchase)

Detailed system modeling (II)

- **Thermal generation**
  - Output limits
  - Fuel **consumption** is **quadratic**
  - Scheduled maintenance
  - Deterministic modeling of unit outages
  - Linear **fuel stock management**
- **Hydro generation**
  - **Storage** hydro plants with reservoirs
  - Run-of-the-river hydro plants
  - **Pumped** storage hydro plants
  - Linear **reservoir management**

Mixed linear complementarity problem (MLCP)

- Medium-term problem formulated with
  - **linear constraints**
  - **quadratic objective function**

System of equations with a **mixed linear complementarity problem** structure

$$\max c^T x + \frac{1}{2} x^T Q x$$
$$Ax \le b \quad \perp \lambda$$
$$Cx = d \quad \perp \mu$$

$$\max \mathcal{L} = c^T x + \frac{1}{2} x^T Q x + \lambda^T (Ax - b) + \mu^T (Cx - d)$$

$$\begin{Bmatrix} c + Qx + \lambda^T A + \mu^T C = 0 \\ Cx = d \end{Bmatrix}$$
$$\begin{Bmatrix} \lambda \le 0 \\ Ax \le b \\ \lambda^T \left(Ax - b \right) = 0 \end{Bmatrix}$$

Existence and unicity

- In a medium-term model formulated with
  - **linear constraints**
  - **quadratic objective function**

System of equations with a **mixed linear complementarity problem** structure

- Sufficient conditions for **existence and unicity:**

An **increasing and** strictly monotonic **marginal cost** function and a **decreasing** linear **inverse demand** function

5

Some real models

MOES Stochastic

- **Purpose**
  - Medium-term generation operation
  - Market equilibrium model
  - Conjectural variations approach
  - Implicit elasticity of residual demand function
- **Main characteristics**
  - Market equilibrium model based on the complementarity problem (MCP)
- **References**

  - J. Cabero, Á. Baíllo, S. Cerisola, M. Ventosa, A. García, F. Perán, G. Relaño, "A Medium-Term Integrated Risk Management Model for a Hydrothermal Generation Company," *IEEE Transactions on Power Systems*. vol. 20, no. 3, pp. 1379-1388, August 2005
  - J. Cabero, Á. Baíllo, S. Cerisola, M. Ventosa, "Application of benders decomposition to an equilibrium problem," *Proceedings of the 15th PSCC, Power Systems Computing Conference. Liege, Belgium, 22-26 Agosto 2005*
  - M. Ventosa, A. Baíllo, A. Ramos, M. Rivier *Electricity Market Modeling Trends* Energy Policy Vol. 33 (7) pp. 897-913 May 2005
  - A. García-Alcalde, M. Ventosa, M. Rivier, A. Ramos, G. Relaño *Fitting Electricity Market Models. A Conjectural Variations Approach* 14th Power Systems Computation Conference (PSCC '02) Seville, Spain June 2002
  - M. Rivier, M. Ventosa, A. Ramos, F. Martínez-Córcoles and A. Chiarri *A Generation Operation Planning Model in Deregulated Electricity Markets based on the Complementarity Problem* in book *Complementarity: Applications, Algorithms and Extensions* Kluwer Academic Publishers. Dordrecht, The Netherlands. pp. 273-295. 2001

Valore

- **Purpose**
  - Oligolopolistic electricity markets simulation
- **Main characteristics**
  - Based on quadratic optimization (QP)
  - Medium-term
    - Allows detailed physical assets modeling
    - Extended for stochastic optimization (i.e. water inflows)
    - Network constraints (explicit and implicit transmission auctions)
- **References**

  - J. Barquín, M. Vázquez, Cournot Equilibrium Calculation in Power Networks: An Optimization Approach With Price Response Computation, *IEEE Trans. on Power Systems*, 23, no. 2, 317-326, May, 2008
  - J. Barquín, E. Centeno, J. Reneses , Stochastic Market Equilibrium Model For Generation Planning, *Probability in the Engineering and Informational Sciences*, 19, 533-546, August, 2005

Fuzzy Valore

- **Purpose**
  - Proposing an electricity market model based on the conjectural-price-response equilibrium when uncertainty of RDC is modeled using the possibility theory
- **Main characteristics**
  - Compute robust Cournot equilibrium by using possibilistic VAR for medium term analysis
  - Determine possibility distributions of main outputs (prices and incomes)
  - Novel variational inequalities (VI) algorithms with global and proved convergence that iteratively solve quadratic programming (QP) models
- **References**

  - F.A. Campos, J. Villar, J. Barquín, J. Reneses, "Variational inequalities for solving possibilistic risk-averse electricity market equilibrium," *IET Gener. Transm. Distrib.* vol. 2, no. 5, pp. 632-645, Sep 2008
  - F.A. Campos, J. Villar, J. Barquín, J. Ruipérez, "Robust mixed strategies in fuzzy non-cooperative Nash games," *Engineering Optimization*. vol. 40, no. 5, pp. 459-474, May 2008
  - F.A. Campos, J. Villar, J. Barquín, "Application of possibility theory to robust Cournot equilibriums in electricity market," *Probability in the Engineering and Informational Sciences*. vol. 19, no. 4, pp. 519-531, October 2005

Task assignment

- Take one model as the base case and run it modifying
  - The marginal cost a thermal units
  - The available water of a hydro reservoir
- Extend the model to consider an small electric system
- Introduce intermittent generation in the model
- Write a small report (1 page long)

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