ESCUELA TÉCNICA SUPERIOR DE INGENIERÍA
INSTITUTO DE INVESTIGACIÓN TECNOLÓGICA

APMOD 2006

Applied Mathematical Programming and Modelling

20 June 2006, Madrid

STOCHASTIC BIDDING IN ELECTRICITY SPOT MARKETS. A MIP-ORIENTED BENDERS DECOMPOSITION APPROACH

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Introduction

- **Optimizing offer curves is still a challenge** for generation companies taking part in spot markets
  - A relevant part of their revenues stems directly from the spot market
  - Operation costs also depend on the results of the spot market
  - The spot market is a reference for longer-term transactions
- **Offer curves** derived with the optimization approaches proposed in the literature are in general **not valid** for real generation companies
  - They may not comply with the technical and strategic constraints required by the generation companies or with formal limitations imposed by the market operator

Introduction

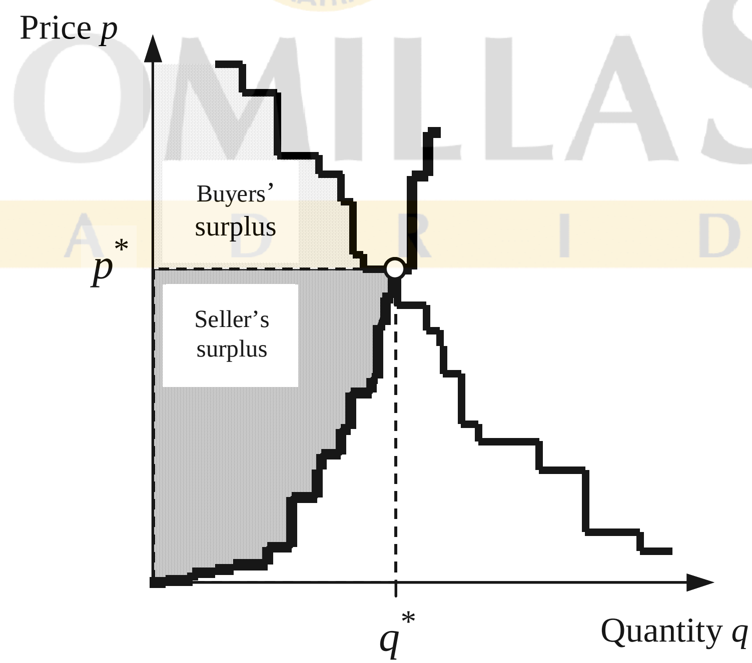
- We present a **methodology** to optimize offer curves considering a more practical approach
  - We take **valid offer curves** as an **initial point** for the optimization
  - We introduce modifications in these offer curves in order to maximize the expected profit of the generation company while complying with the constraints imposed by the user
- This assumption has evident **advantages**:
  - Solution existence is guaranteed
  - Modifications suggested by the model may provide valuable insight for the generation company strategy
  - The resulting offer curves are valid to be submitted to the market operator

Contents

- Introduction
- **Problem Description**
- Mathematical Programming Model
- Description of the Decomposition Algorithm
- Numerical Application To The Spanish Electricity Market
- Conclusion

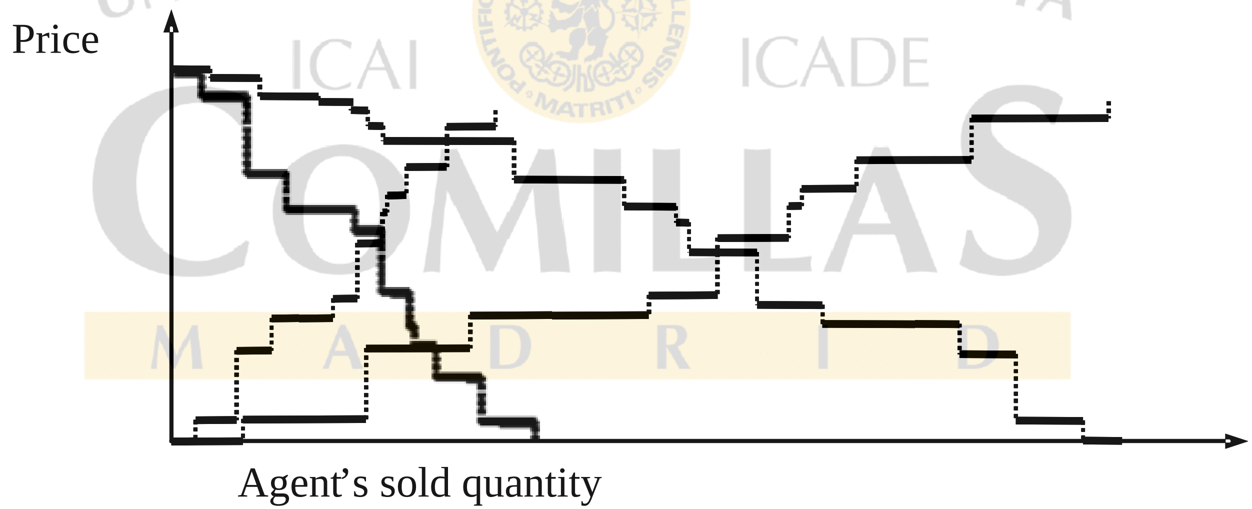
Problem Description

- We consider a **generation company** that owns a mixture of generation technologies
  - The aim of the company is to **maximize** its **long-term profit** through the operation of the generation units
- We focus our attention on the **Spanish Day-Ahead Market**



Problem Description

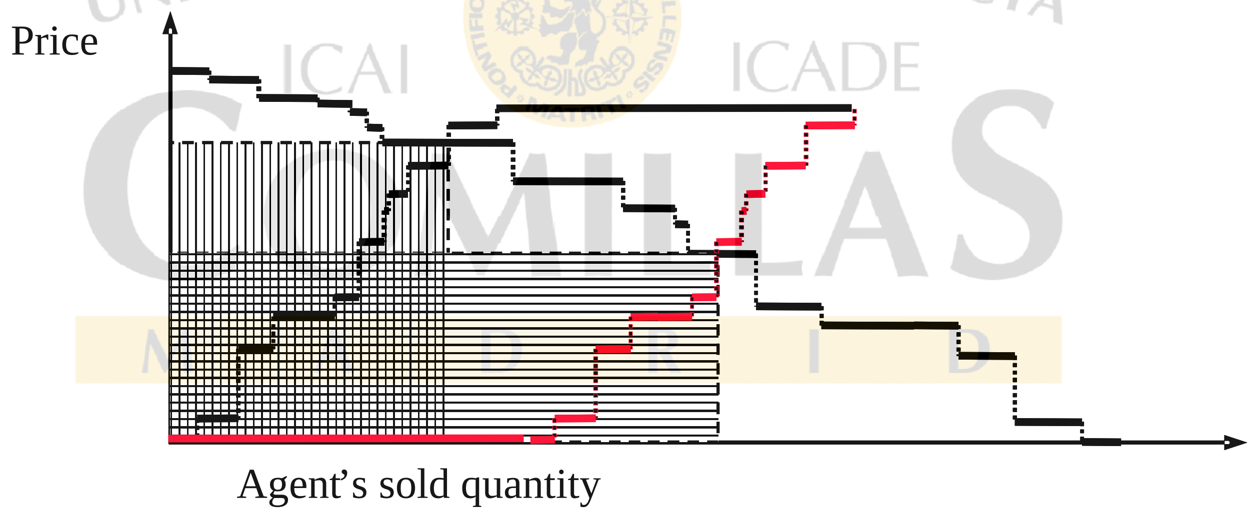
- **Revenue** depends not only on the company's offer curve but also on the offers submitted by **other agents**
  - We represent this effect by means of **residual demand** curves



  - Uncertainty will be considered by means of different **scenarios** of residual demand functions for each hourly auction

Problem Description

- Our method evaluates the impact of **increasing/reducing** the amount of energy offered in each block of the **initial offer curves**



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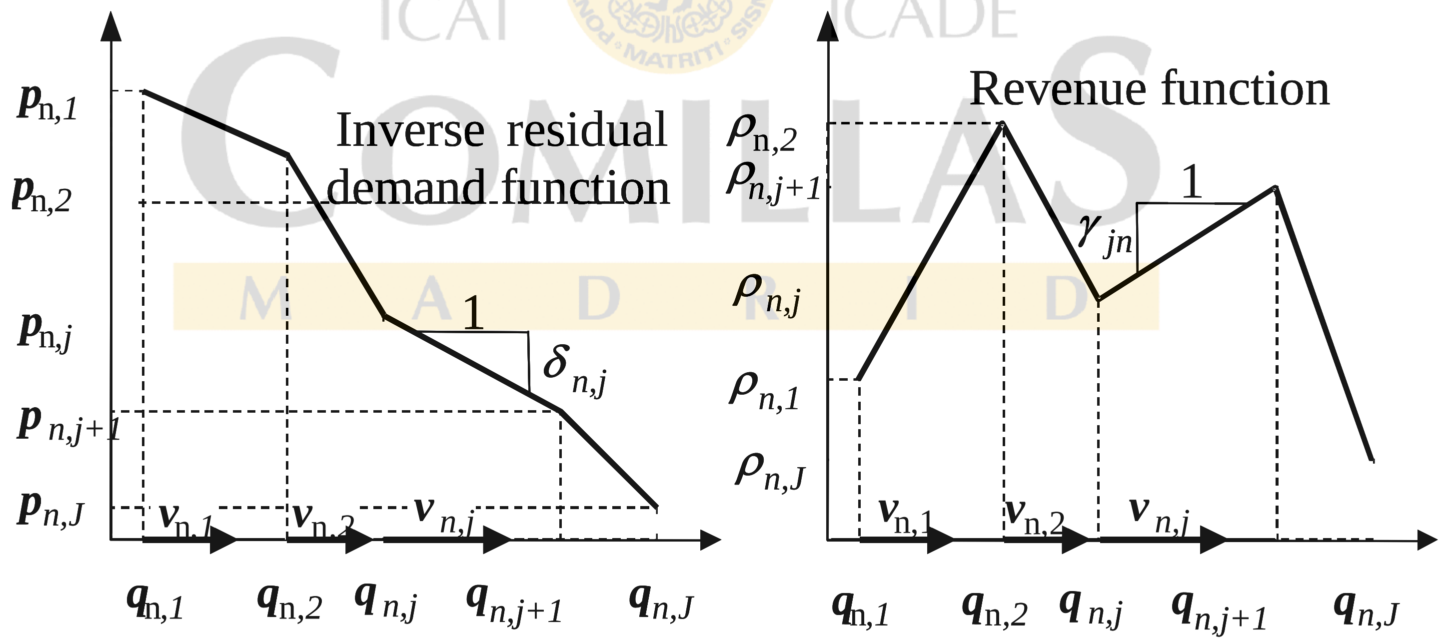
Mathematical Programming Model

OBJECTIVE FUNCTION:

$$\max \textit{Profit} = \sum_n E\left[\textit{Revenue}_n - \textit{Cost}_n\right]$$

- Hourly **piecewise-linear functions** as approximations of residual demand functions and revenue functions



- **Binary variables** are needed to model these functions

Mathematical Programming Model

OBJECTIVE FUNCTION:

$$\max \textbf{\textit{Profit}} = \sum_n E\left[\textbf{\textit{Revenue}}_n - \textbf{\textit{Cost}}_n\right]$$

– **Costs**:

- O&M and Fuel consumption for Thermal units

$$\boldsymbol{c}_n^t = o^t \boldsymbol{q}_n^t + f^t\left(b^t u_n^t + a^t\, \boldsymbol{q}_n^t / k^t\right)$$

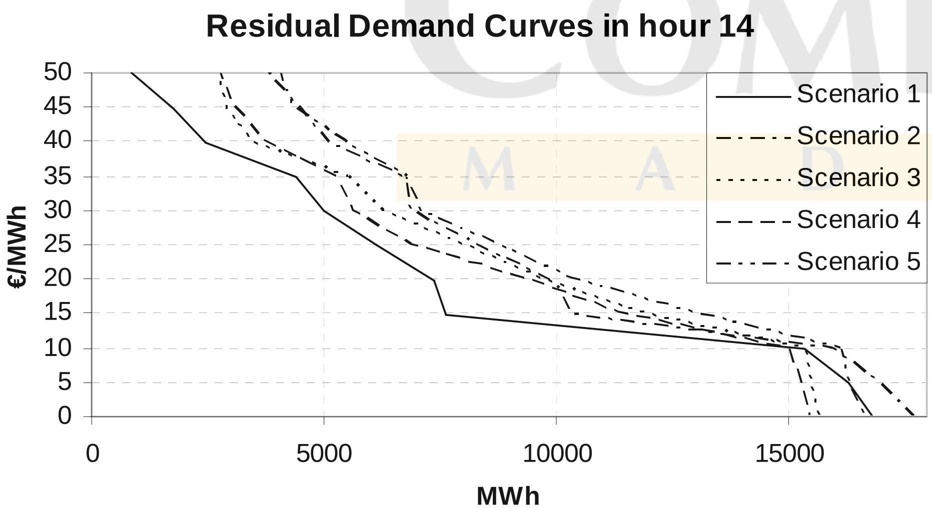
- Hydro and nuclear costs are neglected

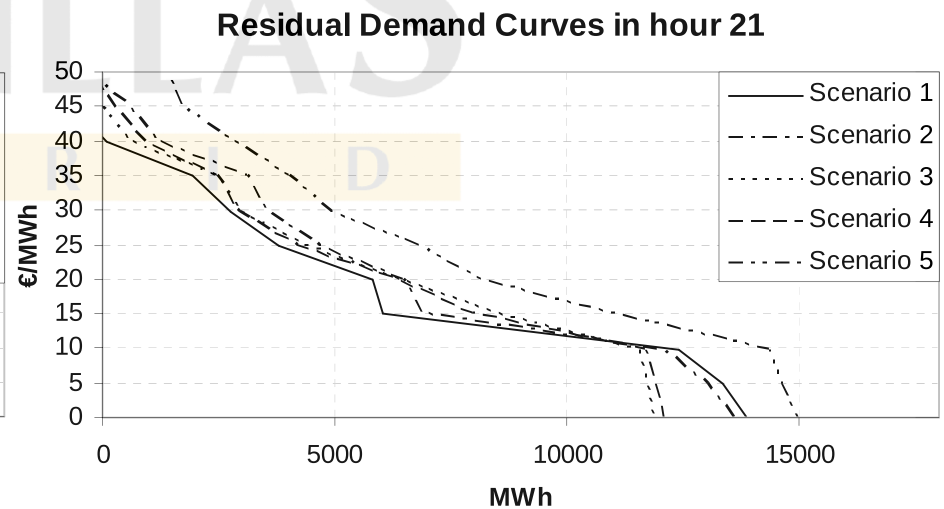
Mathematical Programming Model

OBJECTIVE FUNCTION:

$$\max \mathbf{Profit} = \sum_n E\left[\mathbf{Revenue}_n - \mathbf{Cost}_n\right]$$

– Modelling **uncertainty**:
  - Different scenarios of residual demand curves (and their corresponding revenue functions).

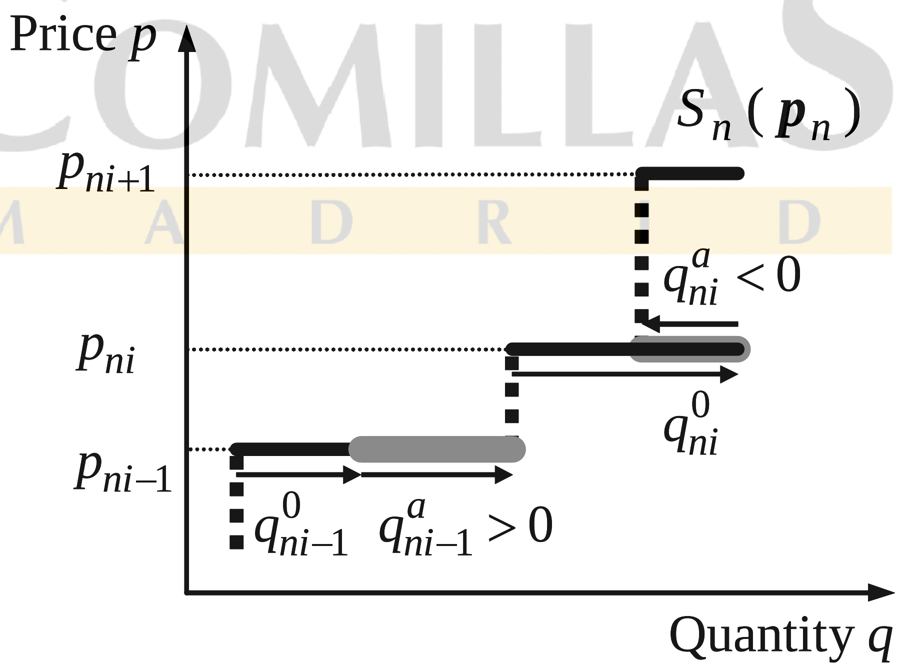




Mathematical Programming Model

CONSTRAINTS:

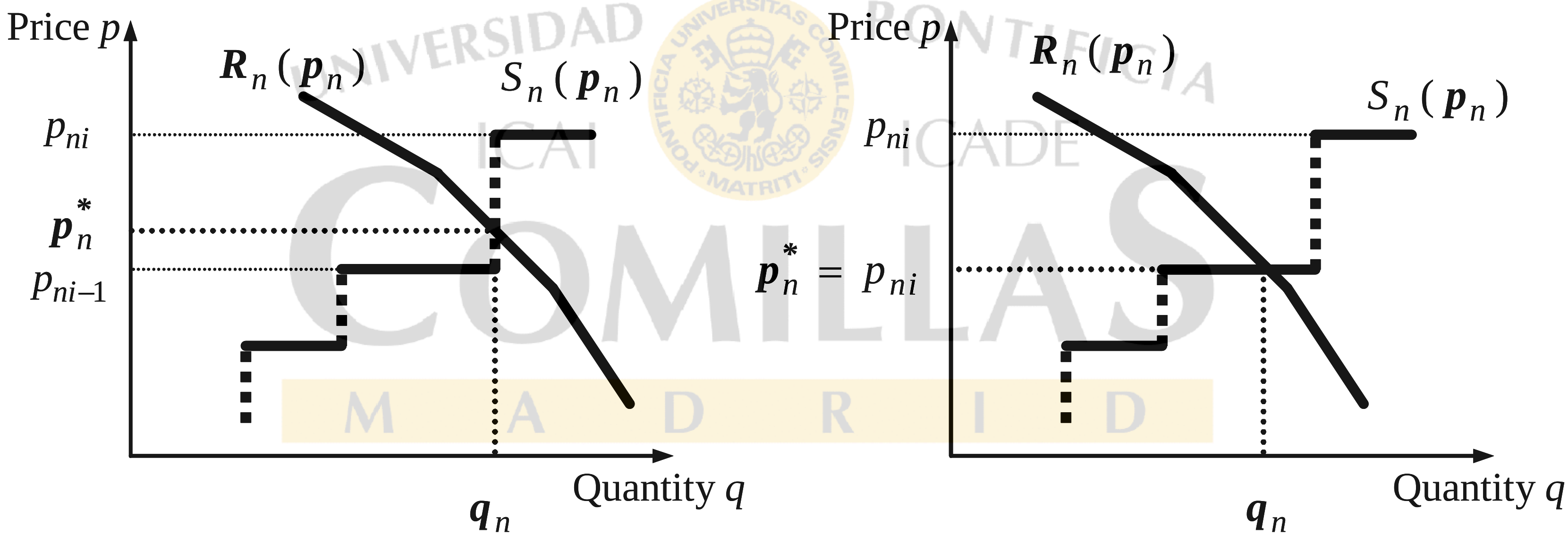
- Modeling the **offer curves** submitted by the company:
  - Hourly **stepwise curves** consisting of blocks defined in energy and price.
  - **Modifications** are introduced in **existing** blocks.



Mathematical Programming Model

CONSTRAINTS:

- Modeling the **Market clearing process**:
  - **Two** different **situations** are considered



  - We explicitly consider the case of **partially accepted blocks**
  - Some **binary variables** are needed to model the Market clearing process

Mathematical Programming Model

CONSTRAINTS:

- Modelling **generation units**:
  - Modelling **thermal** units:
    - Operation **Limits**:

$$\underline{q}^t k^t u_n^t \le \boldsymbol{q}_n^t \le \overline{q}^t k^t u_n^t$$

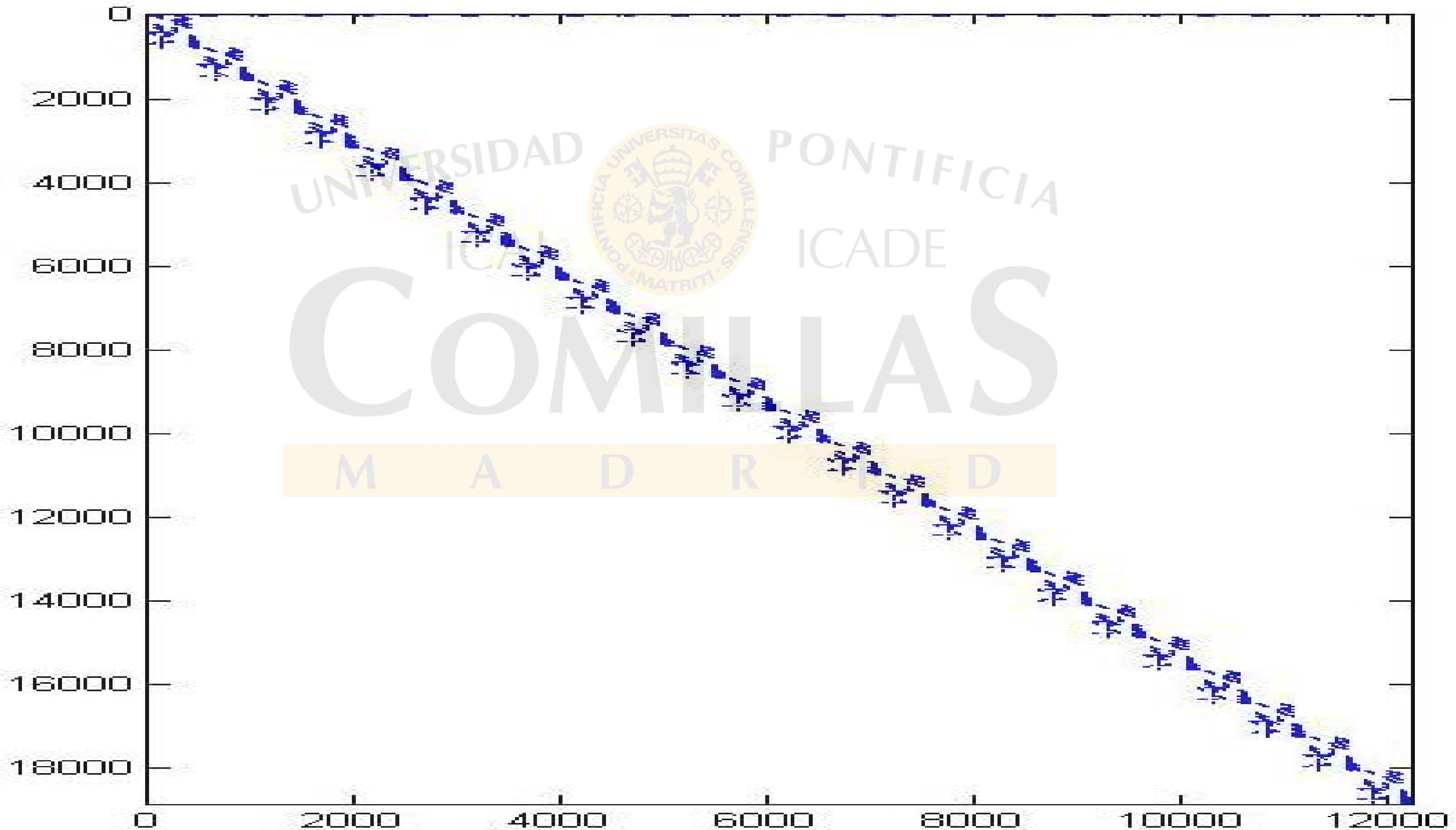
  - Modelling **hydro** units:
    - The **total amount of energy** offered at each price is forced to remain **constant**

$$\sum_n q_{ni}^a = 0$$

    - **This constraint links the 24 hourly auctions**

Mathematical Programming Model

MATRIX OF CONSTRAINTS:



Mathematical Programming Model

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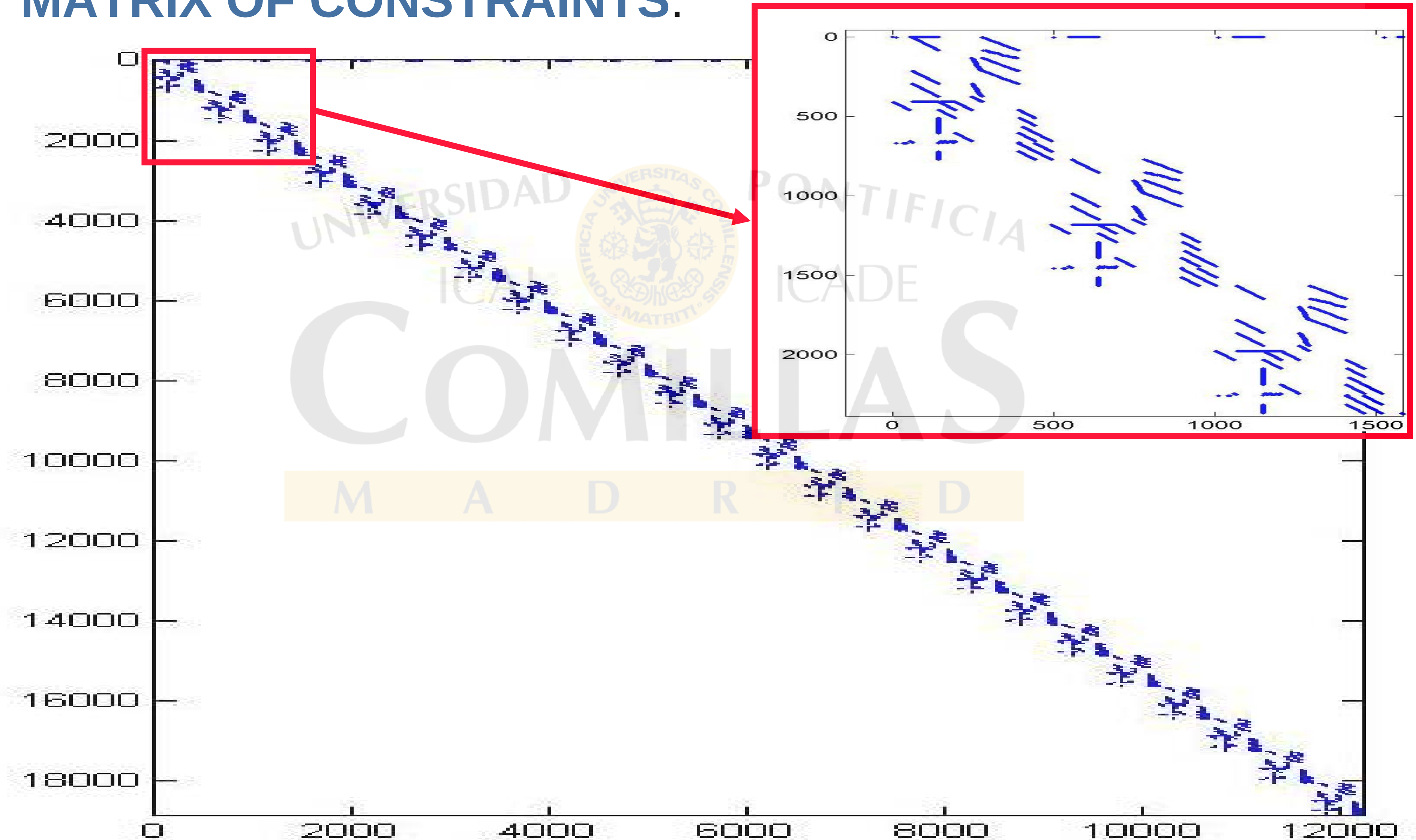

Mathematical Programming Model

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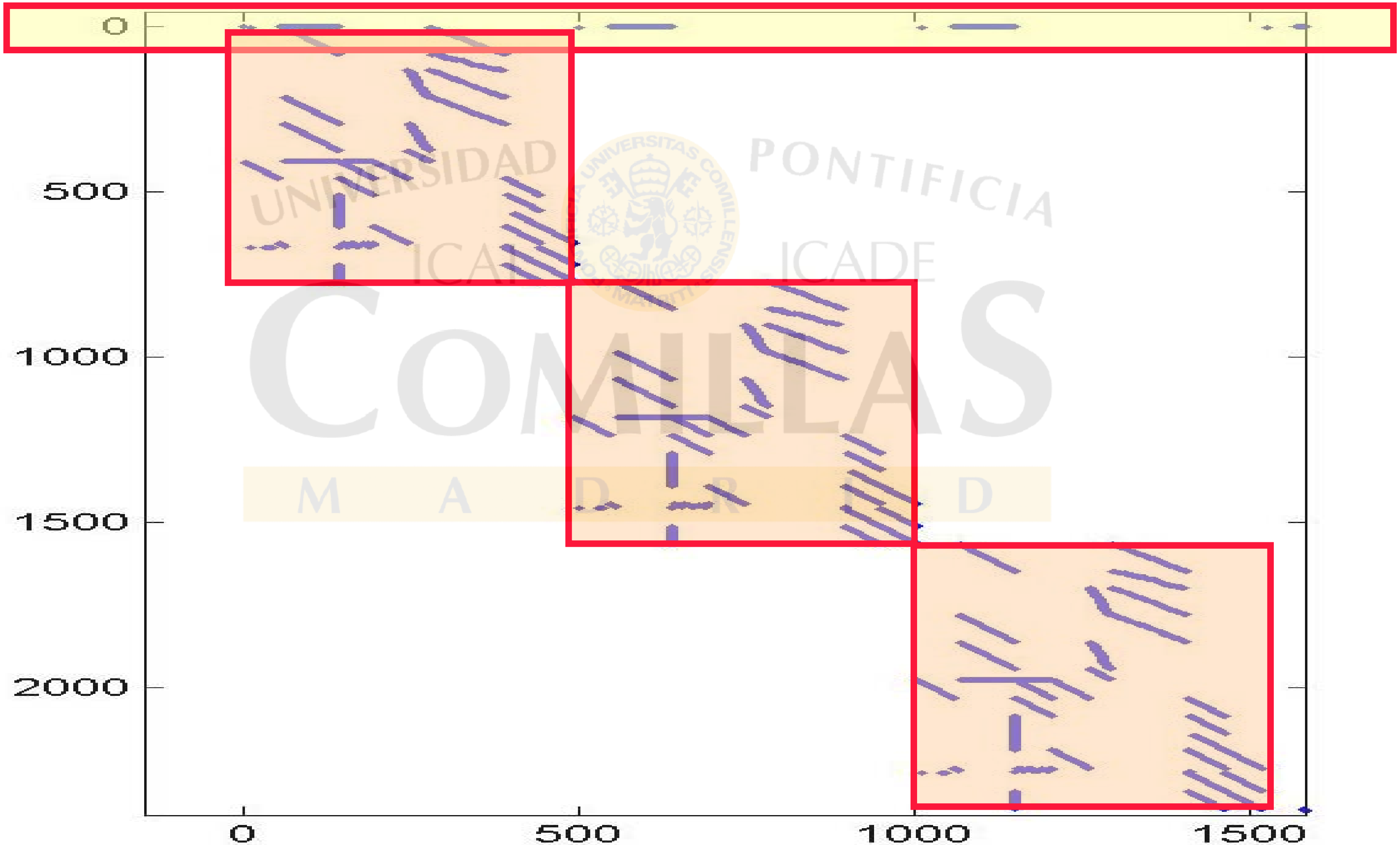


Mathematical Programming Model

CONSTRAINTS:

- We **re-formulate** this complicating constraint in order to create a **stair-case** matrix structure:
  - We introduce a new **variable** that represents the accumulated modifications in energy offered

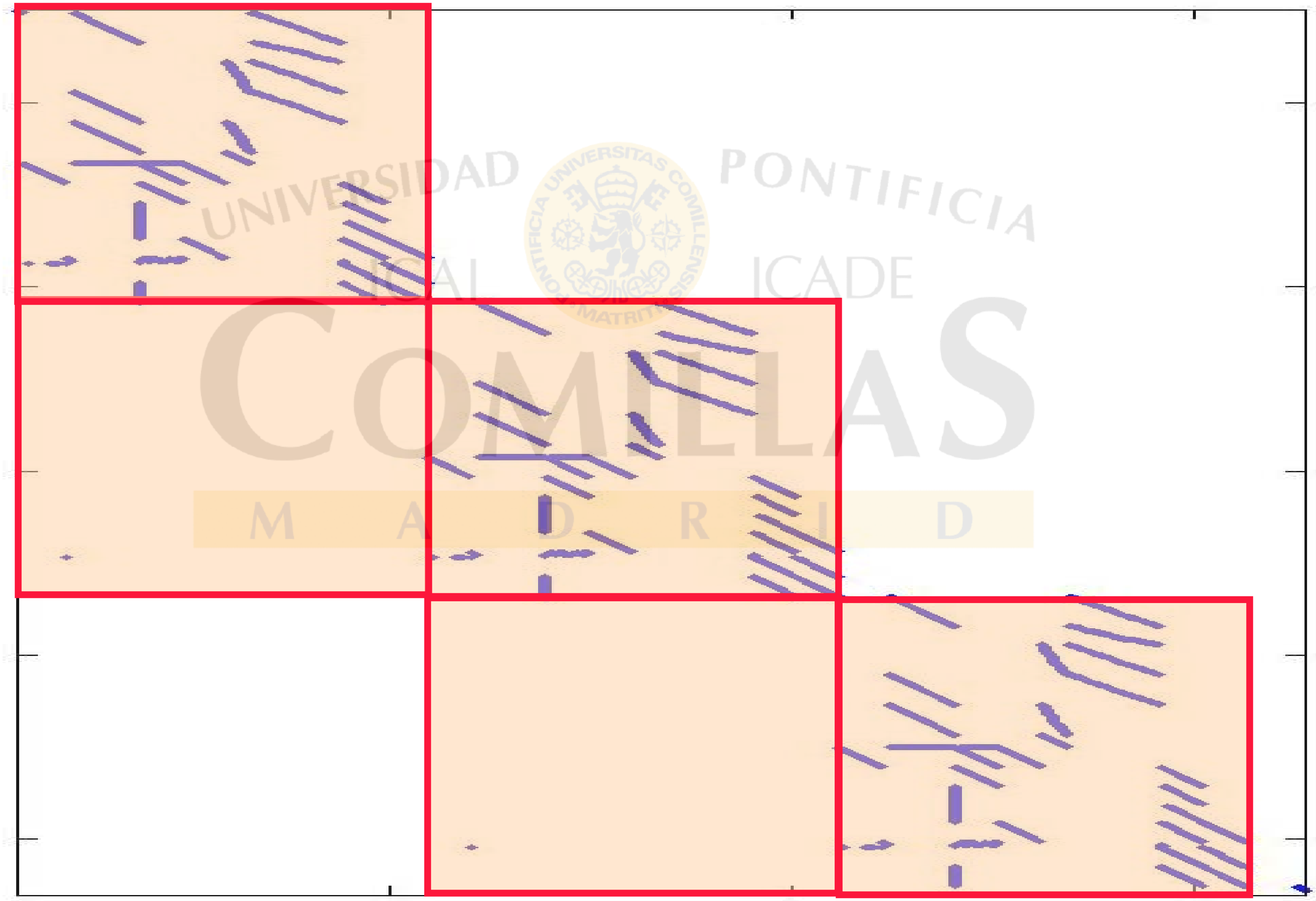
$$q_{n,i}^{acum} = q_{n-1,i}^{acum} + q_{n,i}^{a} \qquad n = 1, 2, \ldots, 24$$

  - Now the previous **constraint** is formulated as:

$$q_{24,i}^{acum} = 0$$

Mathematical Programming Model

MATRIX OF CONSTRAINTS:



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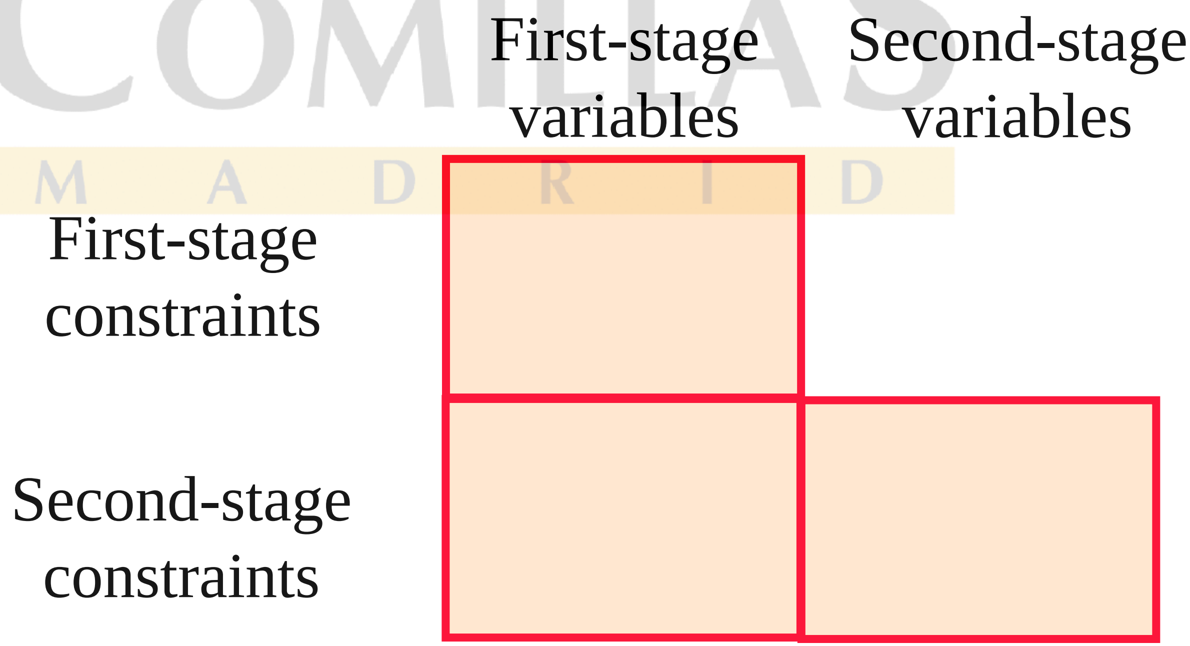
Description of the Decomposition Algorithm

- **Benders** method is oriented to solve mathematical programming problems with a **L-Shape** structure
  - This structure permits identifying **two stages** in the problem that are known as first and second stage
  - **Variables** are usually identified as first-stage variables and second-stage variables



Description of the Decomposition Algorithm

- **Benders algorithm** iterates between the resolution of both stages.
  - First-stage is denoted the **master problem** and incorporates the part of the objective function corresponding to first stage variables and a partial approximation of the recourse function.

$$\max cx + qy \qquad Tx + Wy = h$$

Master Problem

$$\max cx + \theta(x)$$

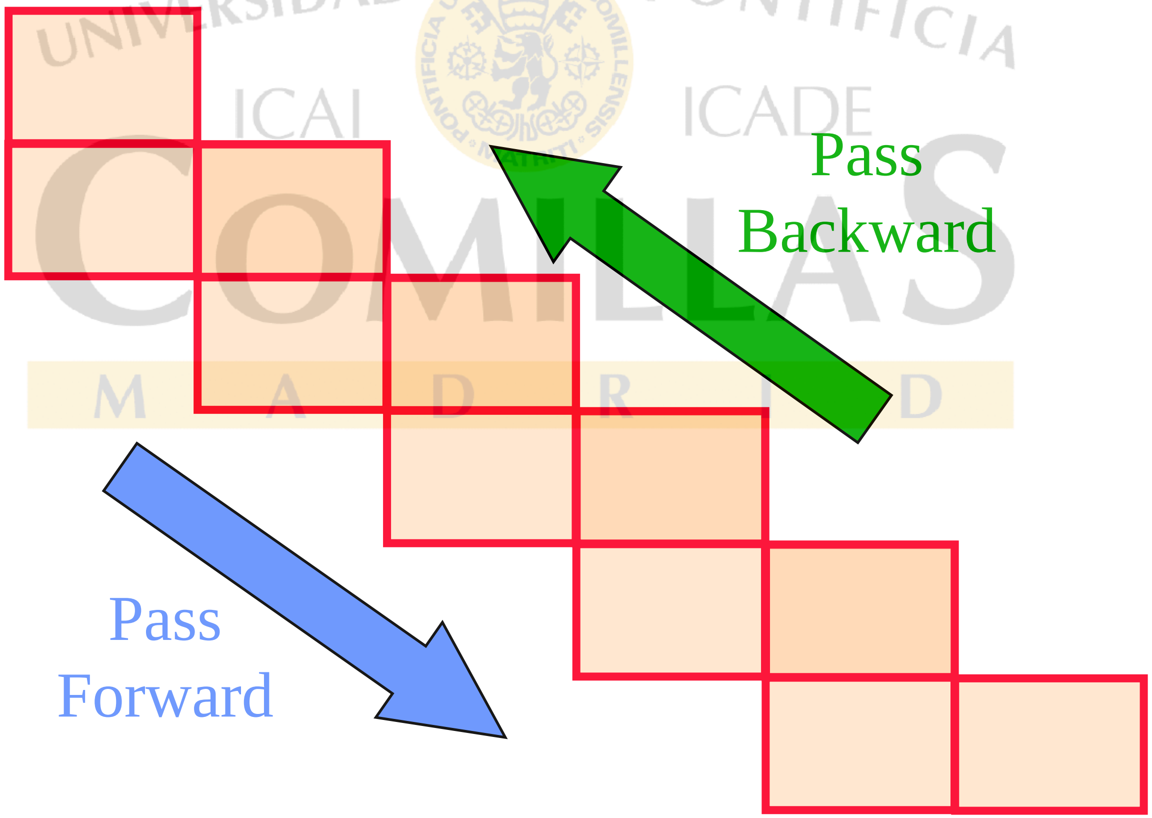
Subproblem

$$\theta(x) = \max qy \qquad Wy = h - Tx$$

  - The **recourse function** represents second-stage objective function value as a function of first-stage decisions

Description of the Decomposition Algorithm

- In case of **multiple-stages problems**, the decomposition method is extended in a natural manner for problems with a stair case structure



Description of the Decomposition Algorithm

- **Binary variables** complicates the construction of the recourse function approximation.
  - This recourse function is nor **convex** neither **continuous**.
  - Traditional Benders approximation needs to be revisited

- We follow the **Generalized Benders Decomposition** and approximates the recourse function by solving the **Lagrangean Relaxation** of the **subproblem**
  - The relaxed constraints are those that connect different stages

Contents

Numerical Application To The Spanish Electricity Market

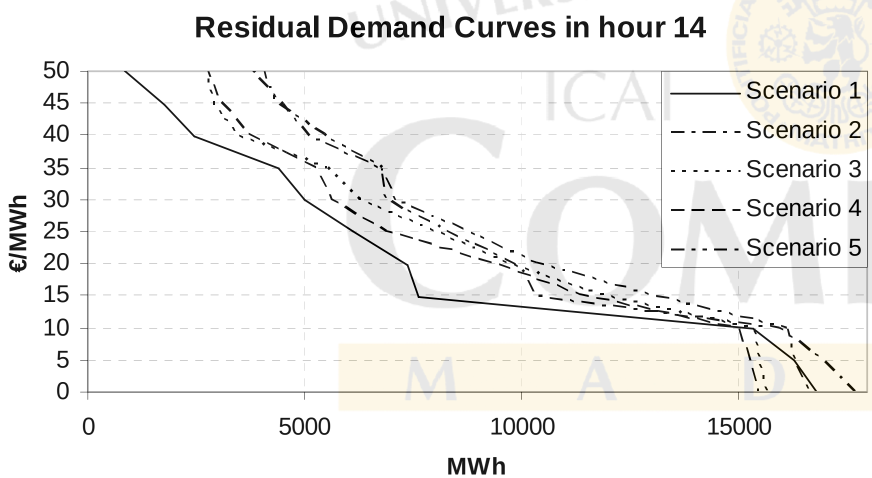
- **Fictitious** power generation company owning a number of randomly chosen generation units present in the **Spanish** power system

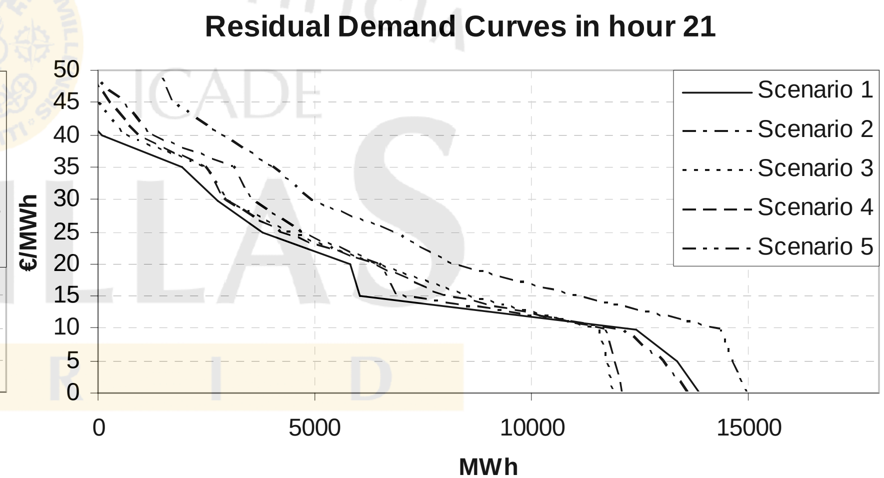
TOTAL	Nuclear	Hydro	Pumping	Fuel	Gas	Coal	CCGT	
100	12.73	32.20	7.07	4.70	7.10	25.07	11.11	[%]

- **Initial offer curves** are constructed by aggregating the offers corresponding to the selected generation units in a certain **day of the past** (July 29th 2005)

Numerical Application To The Spanish Electricity Market

- A number of sets of **day-ahead market scenarios** is constructed selecting **different previous days** similar to the day of study





Numerical Application To The Spanish Electricity Market

- Resulting **optimization problems**:

Number of Scenarios	1	5	10	20
Number of Equations	18873	63082	125872	251692
Number of Variables	12215	37615	73961	146485
Number of Binary Variables	6211	18910	37874	75586

- **Direct** Resolution **Vs. Decomposed** Resolution (subproblems comprising 2 hourly auctions)

Number of Scenarios	1	5	10	20
Direct Solution Time	10 secs	8 h 30 min	> 1 day	???
Decomposed Solution Time	2 min	20 min	40 min	2 h 30 min

Numerical Application To The Spanish Electricity Market

- Results submitting the **Original** Offer-curves:

Total Profit [€]	4782377
Total Accepted Quantity [GWh]	90.233
Weighted Average Price [€/MWh]	75.12

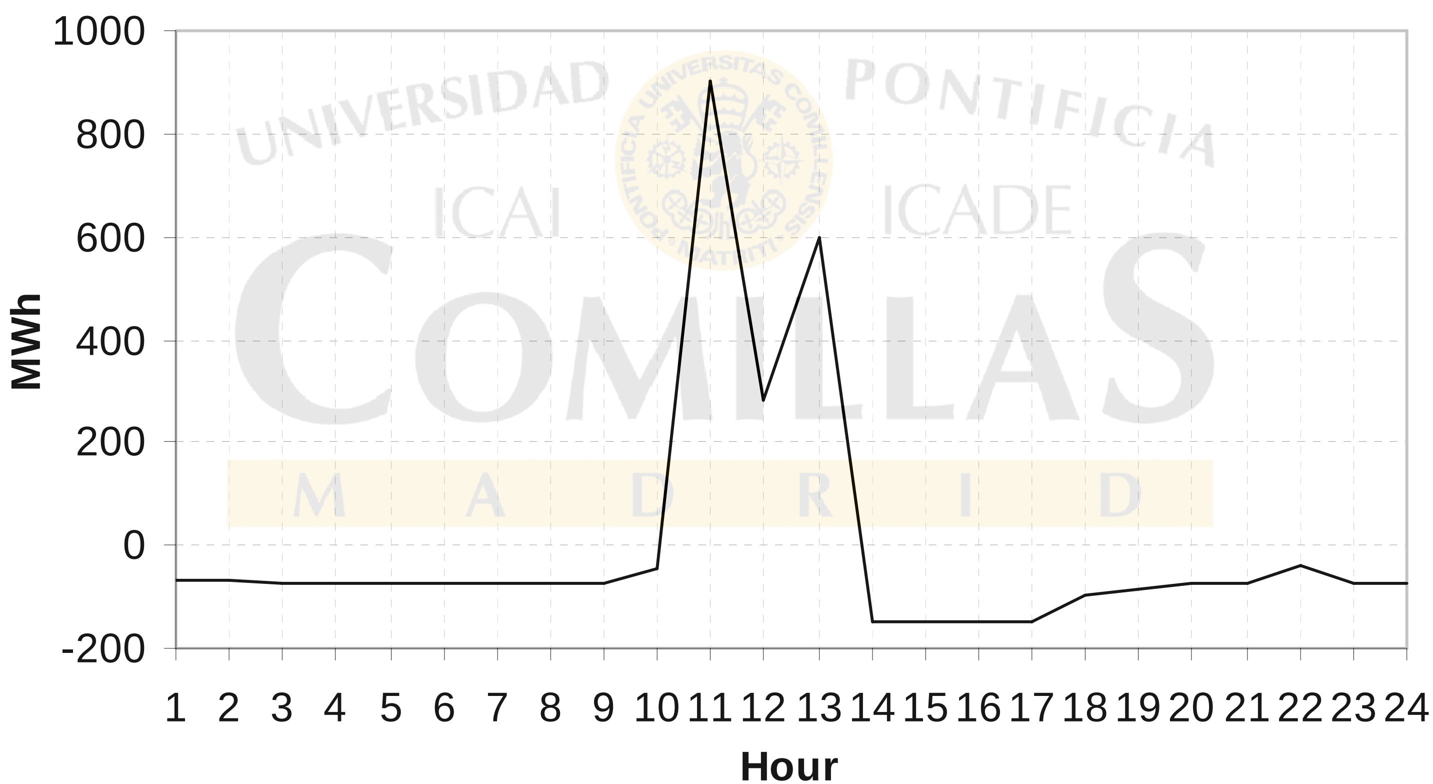
- **Results Optimizing** a **hydro unit** must-run energy:

Total Profit [€]	4854244
Total Accepted Quantity [GWh]	90.366
Weighted Average Price [€/MWh]	75.53

  – Results after optimization show a **1.5% increase in profits** while not modifying significantly market clearing results

Numerical Application To The Spanish Electricity Market

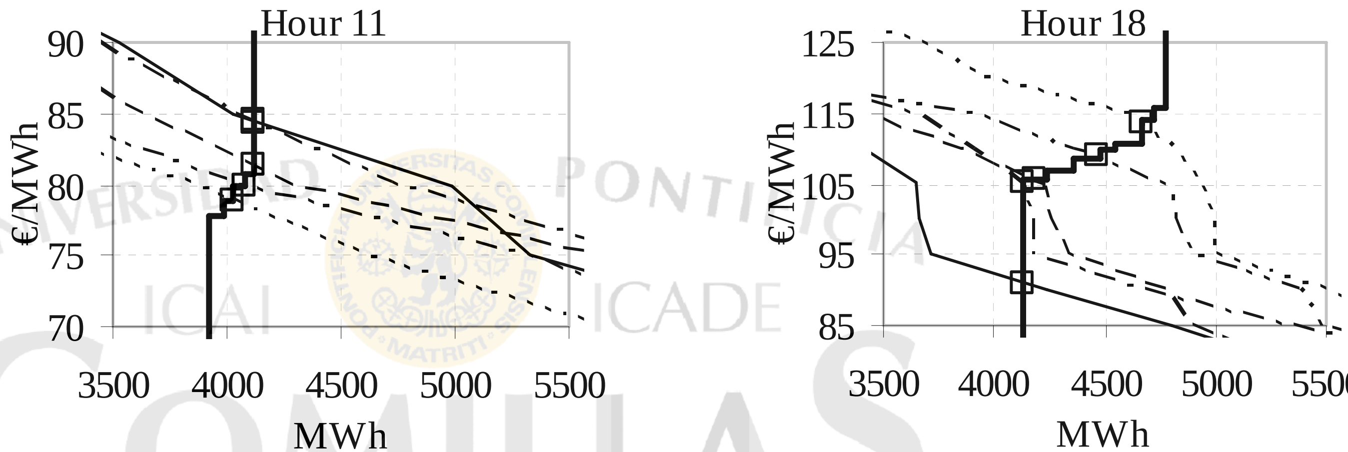
- **Modifications** in Hydro Production:



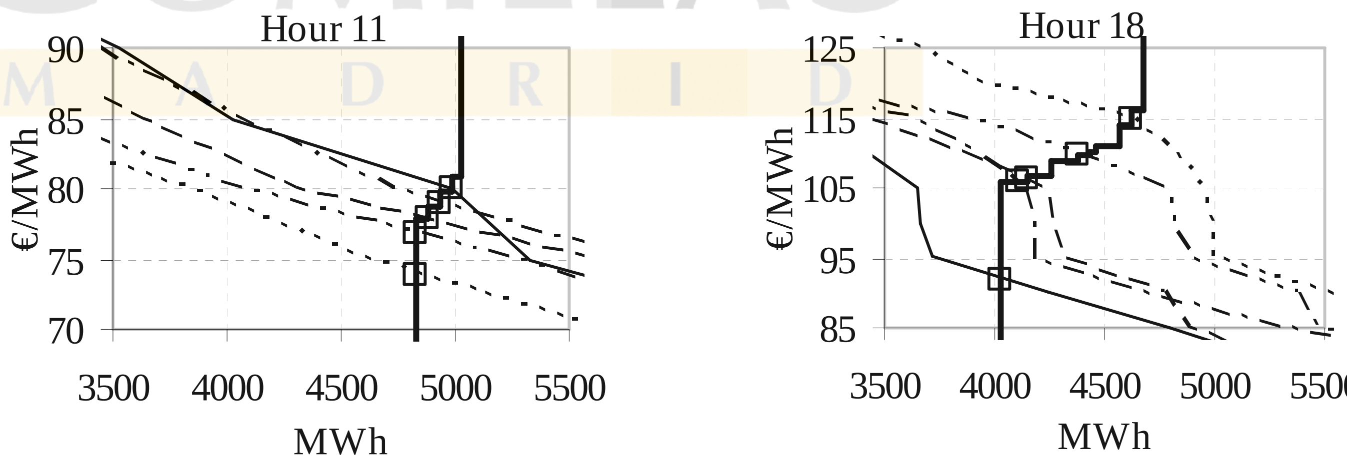
Numerical Application To The Spanish Electricity Market

- **Market Clearing Results** comparison:

Original Offer-curve



Modified Offer-curve



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Conclusion

- **Slight modifications** introduced in the company's original offer curves turn into an **increment** of company's **expected profit**
  - This seems to **confirm** the **validity** of our approach.

- The use of **decomposition techniques** allow us to apply our model in a realistic manner, obtaining optimal offer curves that can be directly submitted to a **real electricity market**

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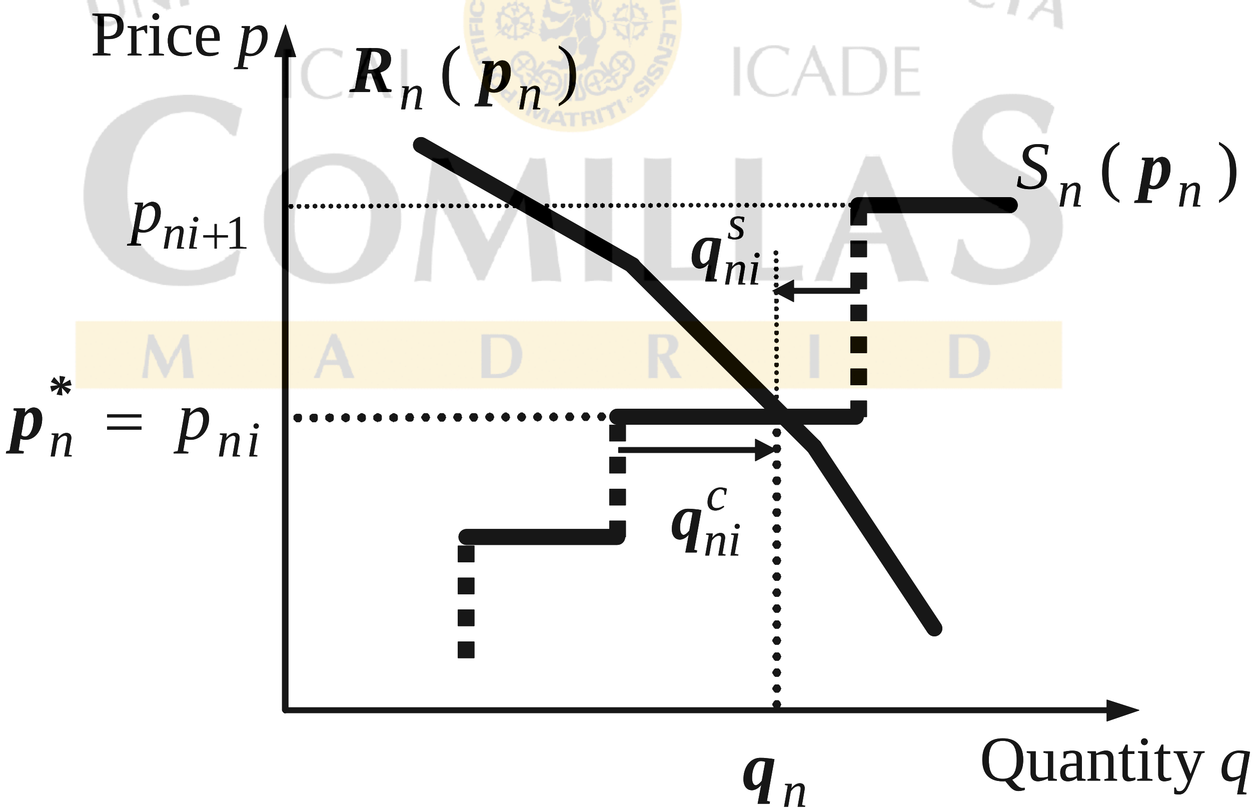

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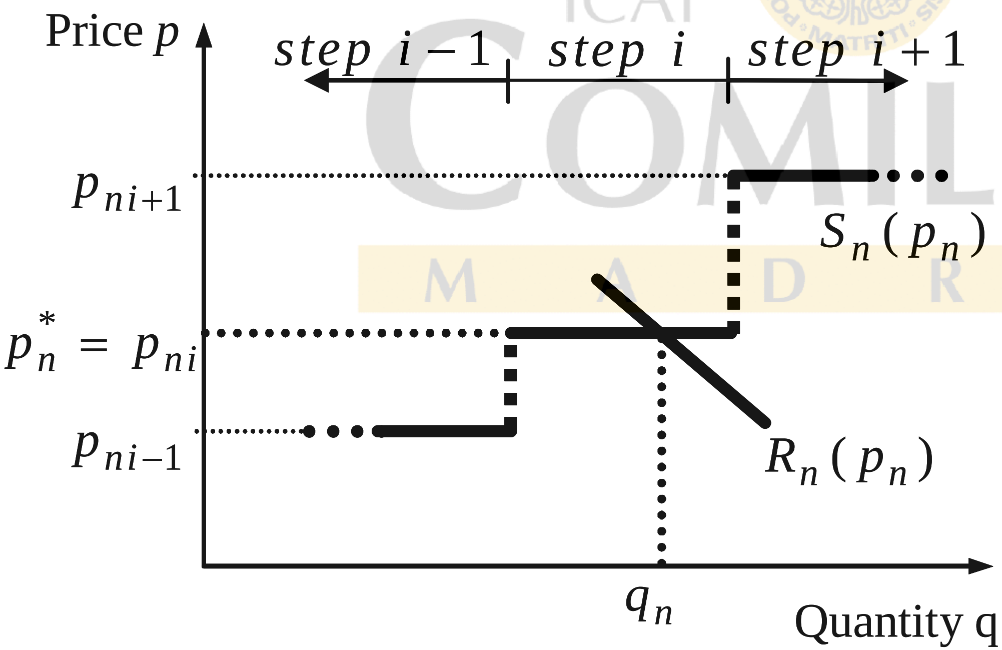


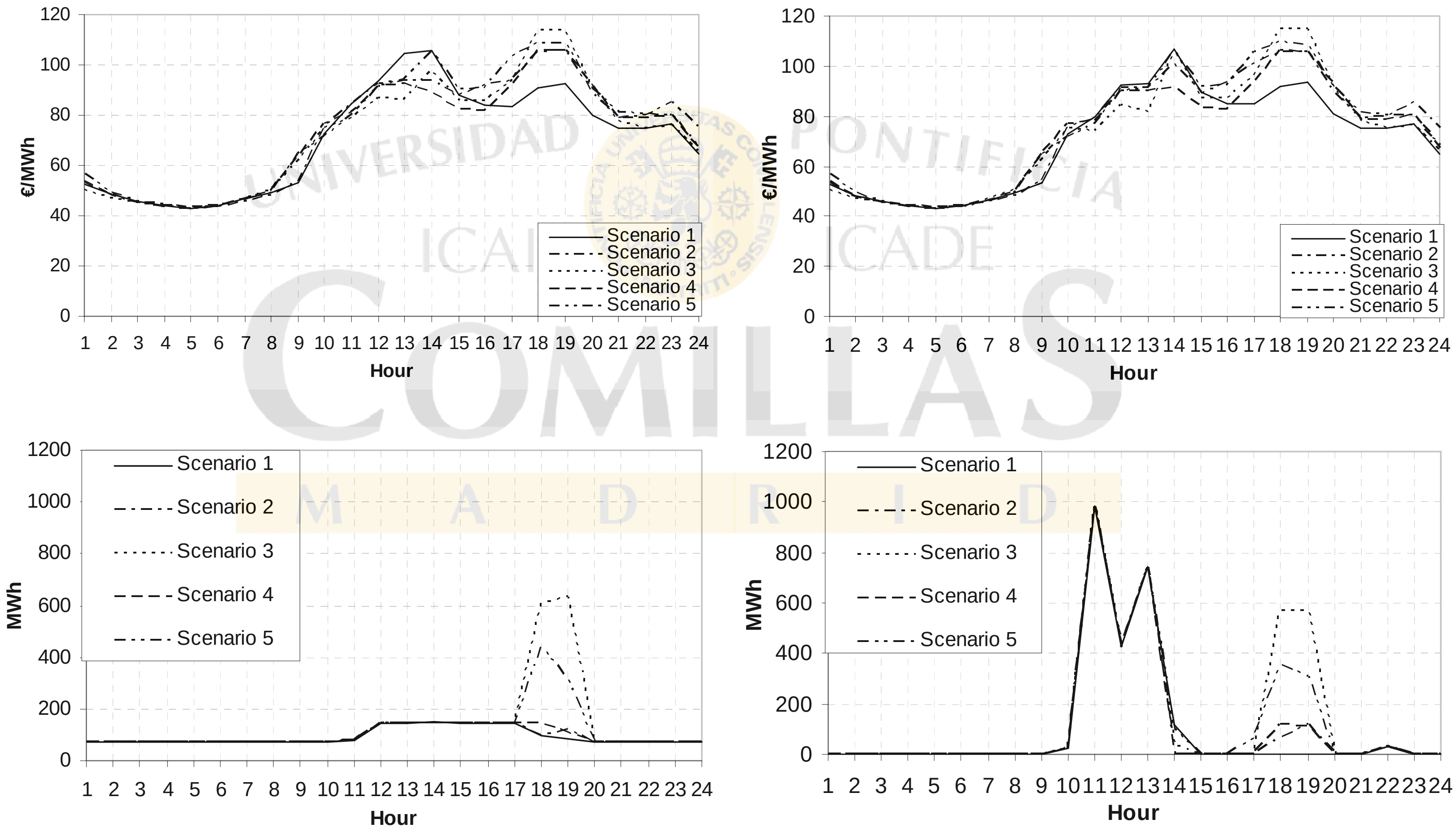
$step\ i-1$	$step\ i$	$step\ i+1$
$q^c_{ni-1} > 0$	$q^c_{ni} > 0$	$q^c_{ni+1} = 0$
$q^s_{ni-1} = 0$	$q^s_{ni} > 0$	$q^s_{ni+1} > 0$
$y_{ni-1} = 1$	$y_{ni} = 1$	$y_{ni+1} = 0$
$z_{ni-1} = 0$	$z_{ni} = 1$	$z_{ni+1} = 0$

Description of the Decomposition Algorithm

- **Infeasibilities** management
  - The implemented decomposition algorithm takes two phases
  - An **initial phase** in which the integrality requirements are removed and a solution of the **LP relaxation** is obtained by the use of the linear nested decomposition algorithm.
    - Infeasible decisions are avoided with the construction of a feasibility cut.
  - A **second phase** in which
    - Each stage subproblem is solved with **MIP techniques** in the forward pass
    - Each subproblem is solved with the **RL method** in a backward pass. In case of infeasibility it is solve the RL of the minimization of infeasibilities subproblem.

Numerical Application To The Spanish Electricity Market

- **Results** comparison:



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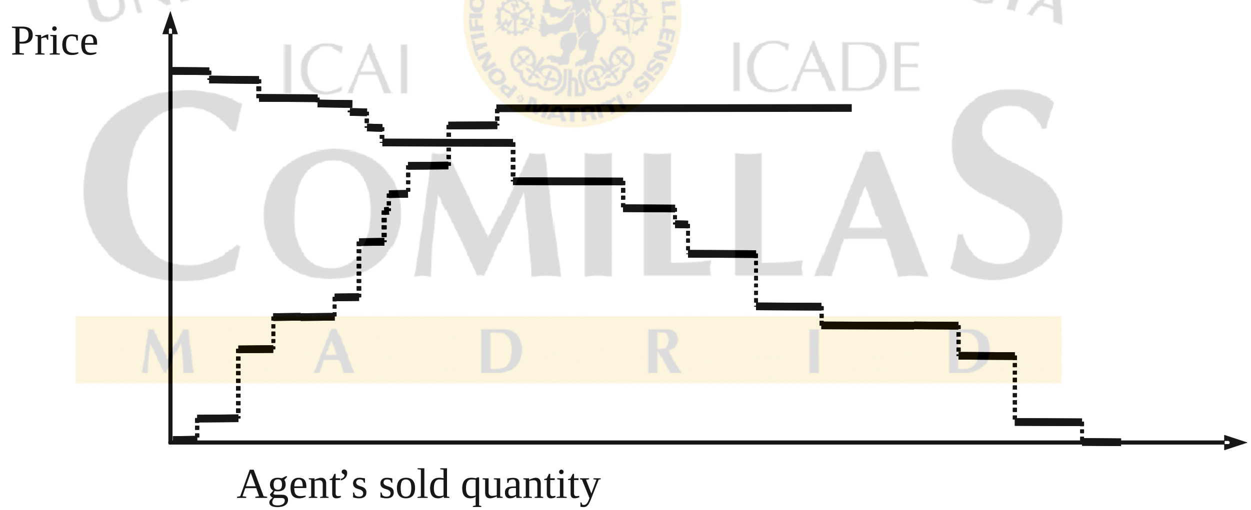

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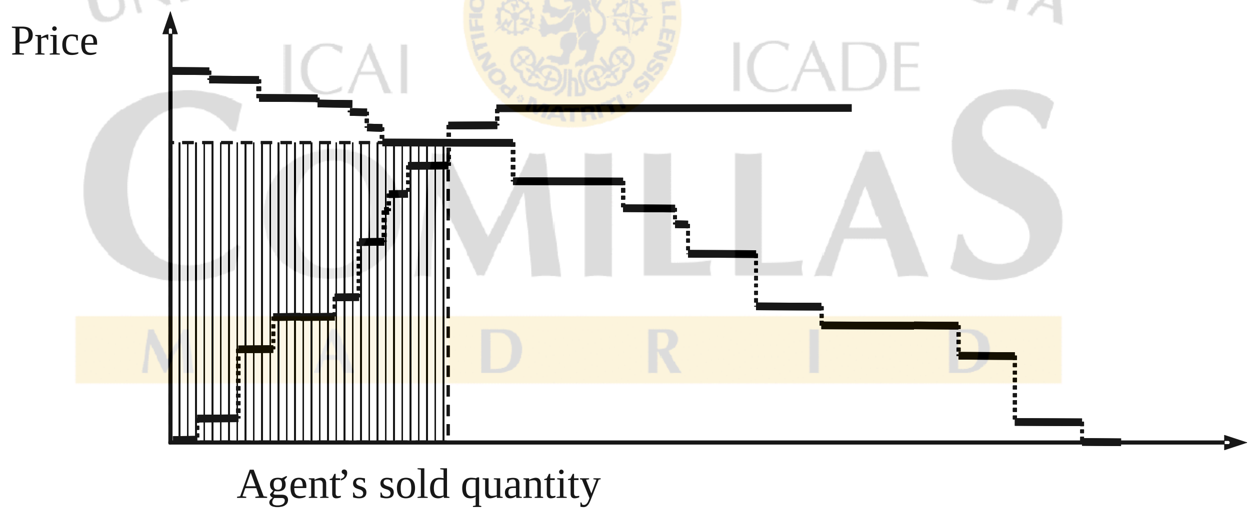

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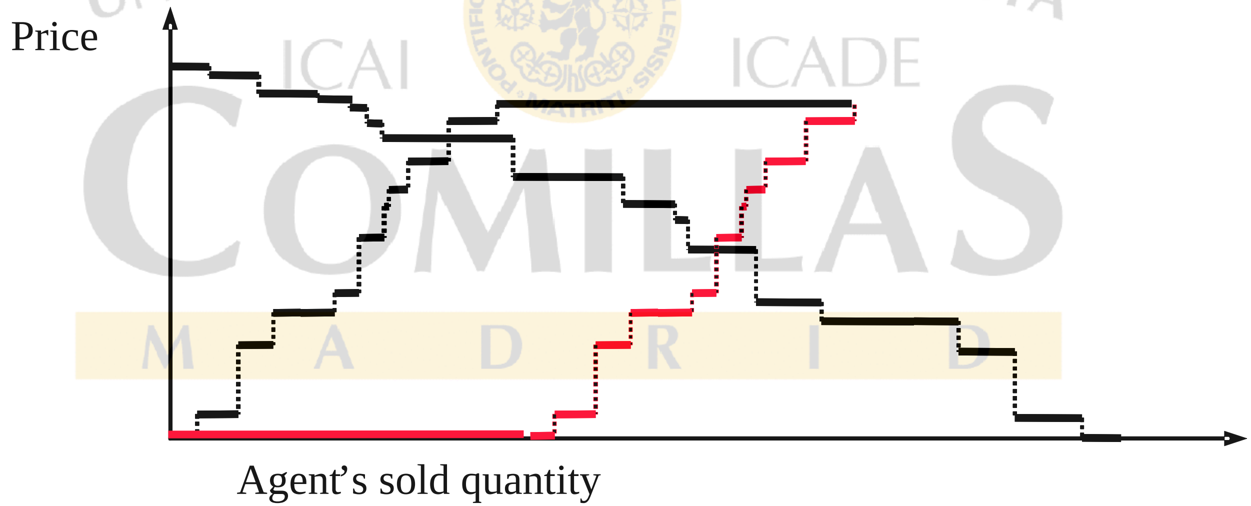

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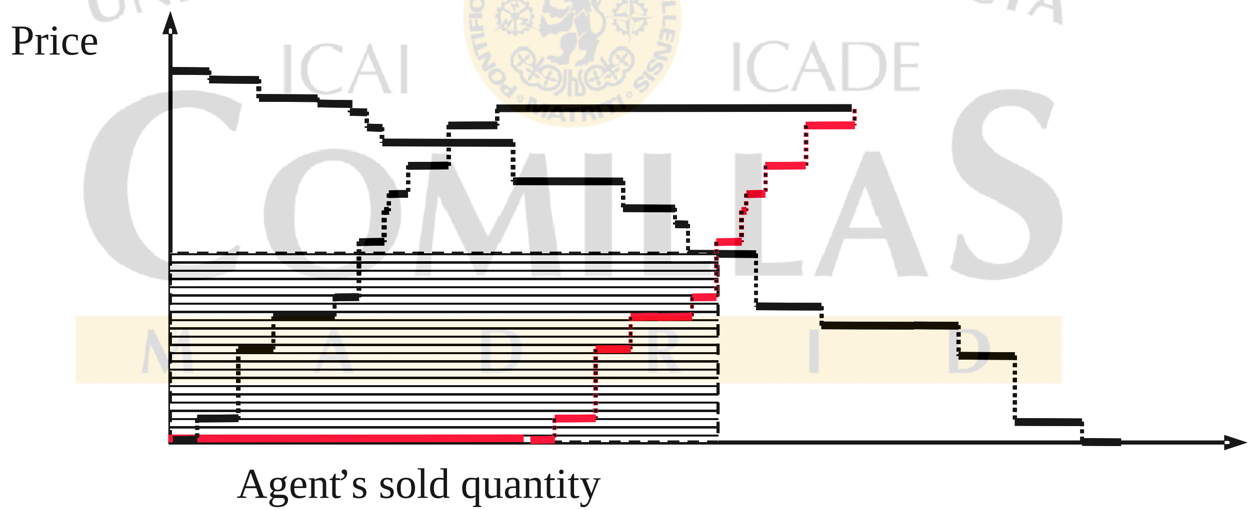

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