ESCUELA TÉCNICA SUPERIOR DE INGENIERÍA (ICAI)
INSTITUTO DE INVESTIGACIÓN TECNOLÓGICA

Stochastic Dual Dynamic Programming Applied to Nonlinear Hydrothermal Models

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- Introduction
- Nonlinear constraints reformulation
  - McCormick envelope for bilinear terms
  - Disjunctive programming
- Stochastic Dual Dynamic Programming
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- Case Study and Numerical Results
- Conclusions

Introduction

- Linear hydrothermal model
  - Minimize total operating cost while satisfying demand for power
  - Constant hydro production function
- Advantages
  - Possibility of using LP-solvers
  - Monotonically decreasing and convex water value function
  - Solutions of large-scale stochastic models using decomposition techniques
- Disadvantage
  - Multiplicity of solutions for reserve profiles
  - Inadequate profiles

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Introduction

- A non linear hydrothermal model
  - Non constant hydro production function
  - Increases the production of the hydro plant with the head of the reservoir
- The hydro production function
  - Power (MW) = Water discharge (m^3/s) ·
    Head (m) ·
    Efficiency of hydro unit (head)

Introduction

- A non linear hydrothermal model
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    Head (m) ·
    Efficiency of hydro unit (head)

$$\boxed{P = q \cdot h \cdot \eta(h)}$$

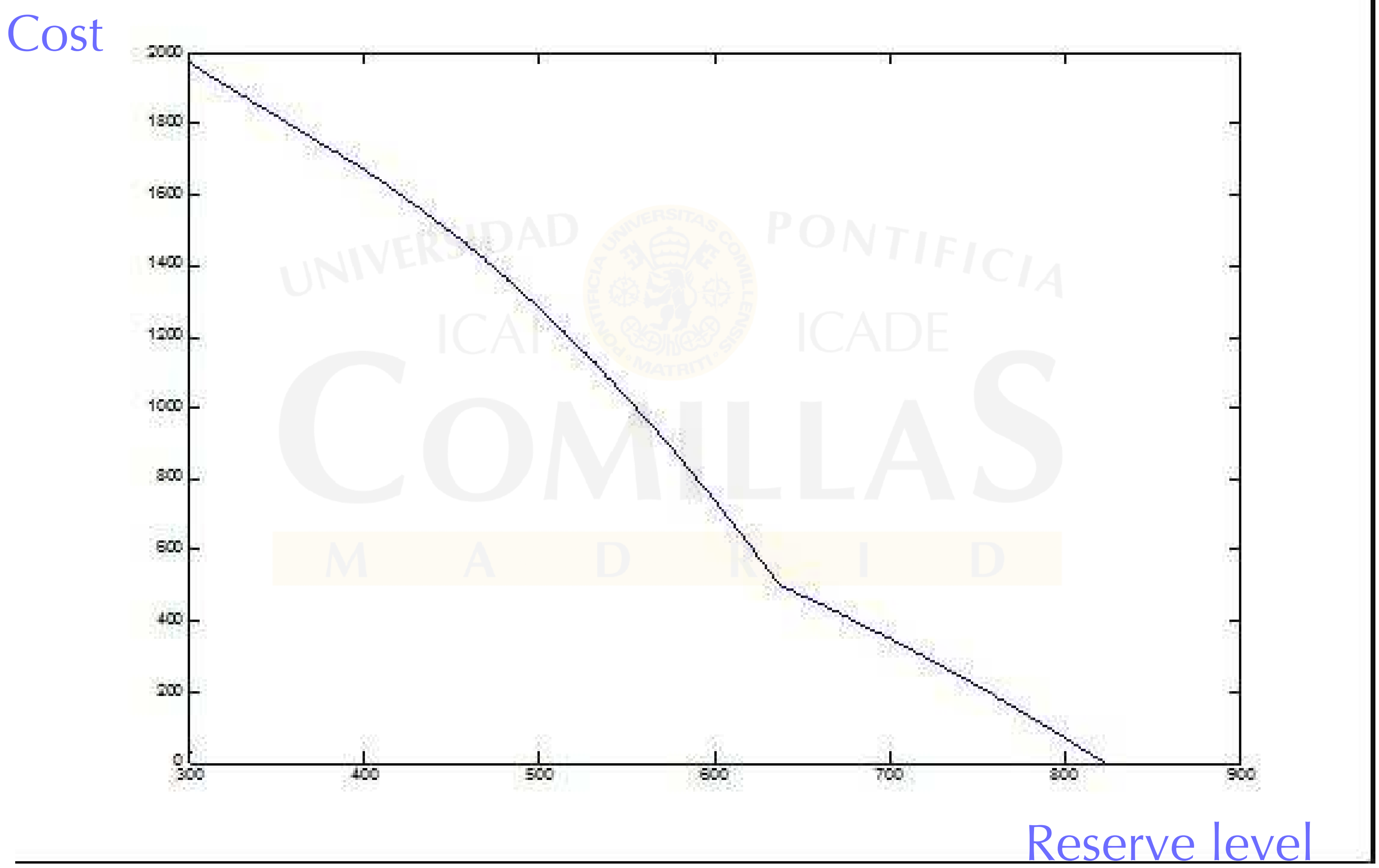
Simplification as an affine function

$$\boxed{P = q \cdot (\alpha + \beta h)}$$

Introduction

- Bilinear relations for modeling hydro production functions
  - Forces the use of nonlinear solvers
  - Possibility of stacking in a local minima
  - More computation time
  - Nonconvex recourse function
  - Difficulty of applying decompositions techniques
  - Difficulty of solving the stochastic problem
- Example of nonconvex recourse function

Introduction



Introduction

¿How to extend the Stochastic Dual Dynamic Programming decomposition technique to deal with this situation?

Nonlinear constraints reformulation

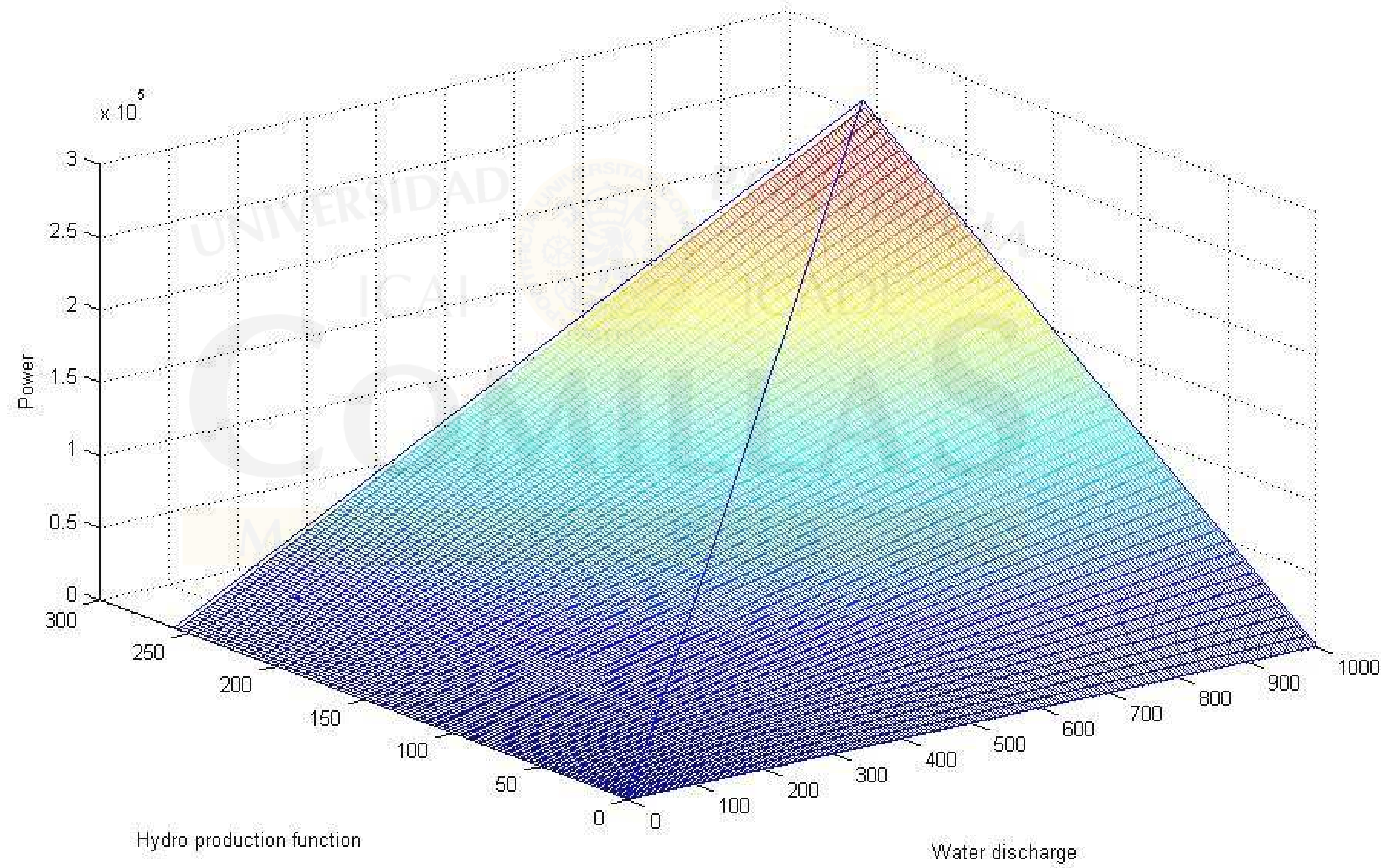
- Reformulate the bilinear terms using McCormick reformulation

$$\boxed{z = xy}$$

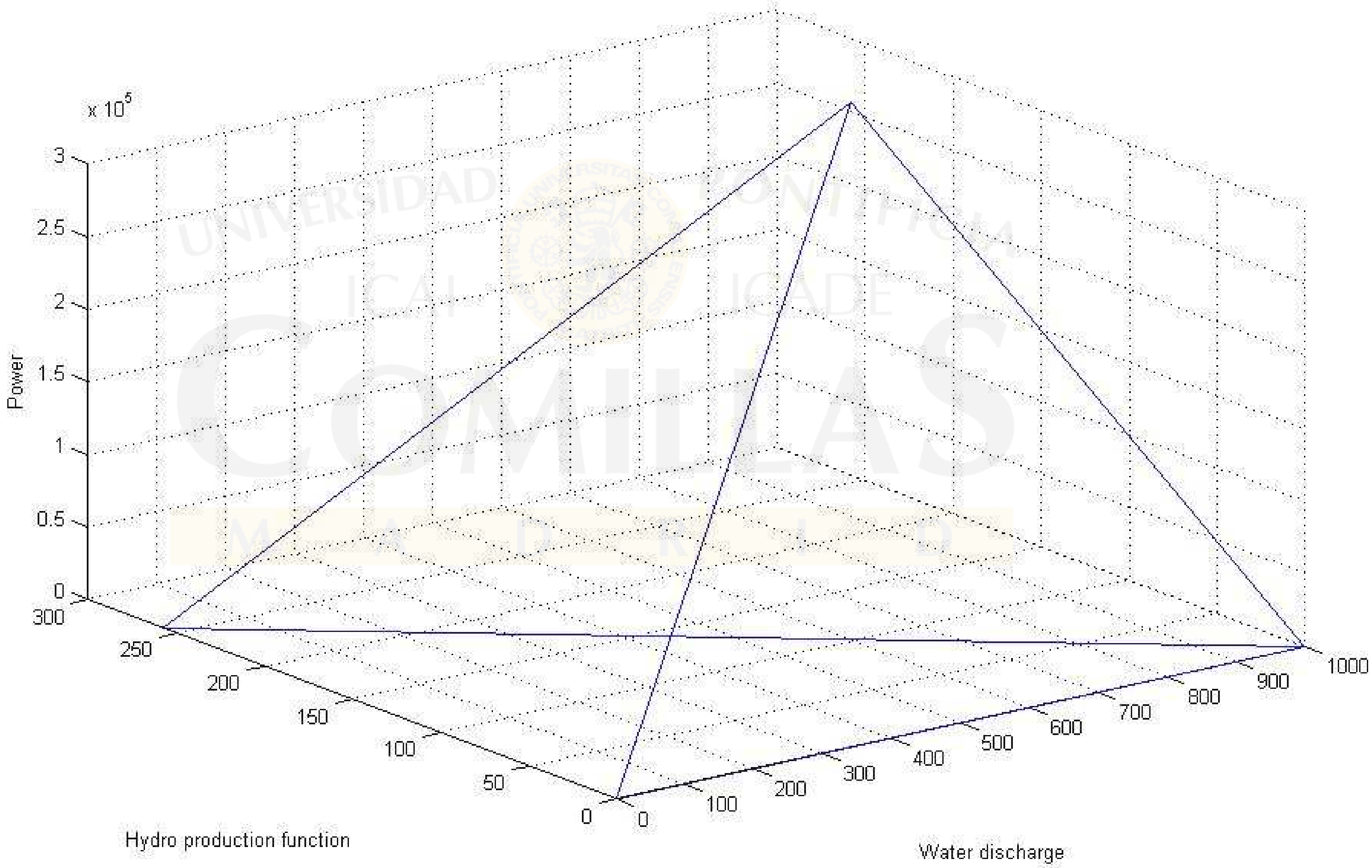
$$\boxed{\begin{aligned} z &\geq x\overline{y} + \overline{x}y - \overline{x}\,\overline{y} \\ z &\geq x\underline{y} + \underline{x}y - \underline{x}\,\underline{y} \\ z &\leq x\overline{y} + \underline{x}y - \underline{x}\,\overline{y} \\ z &\leq x\underline{y} + \overline{x}y - \overline{x}\,\underline{y} \end{aligned}}$$

- Enables the use of LP solvers

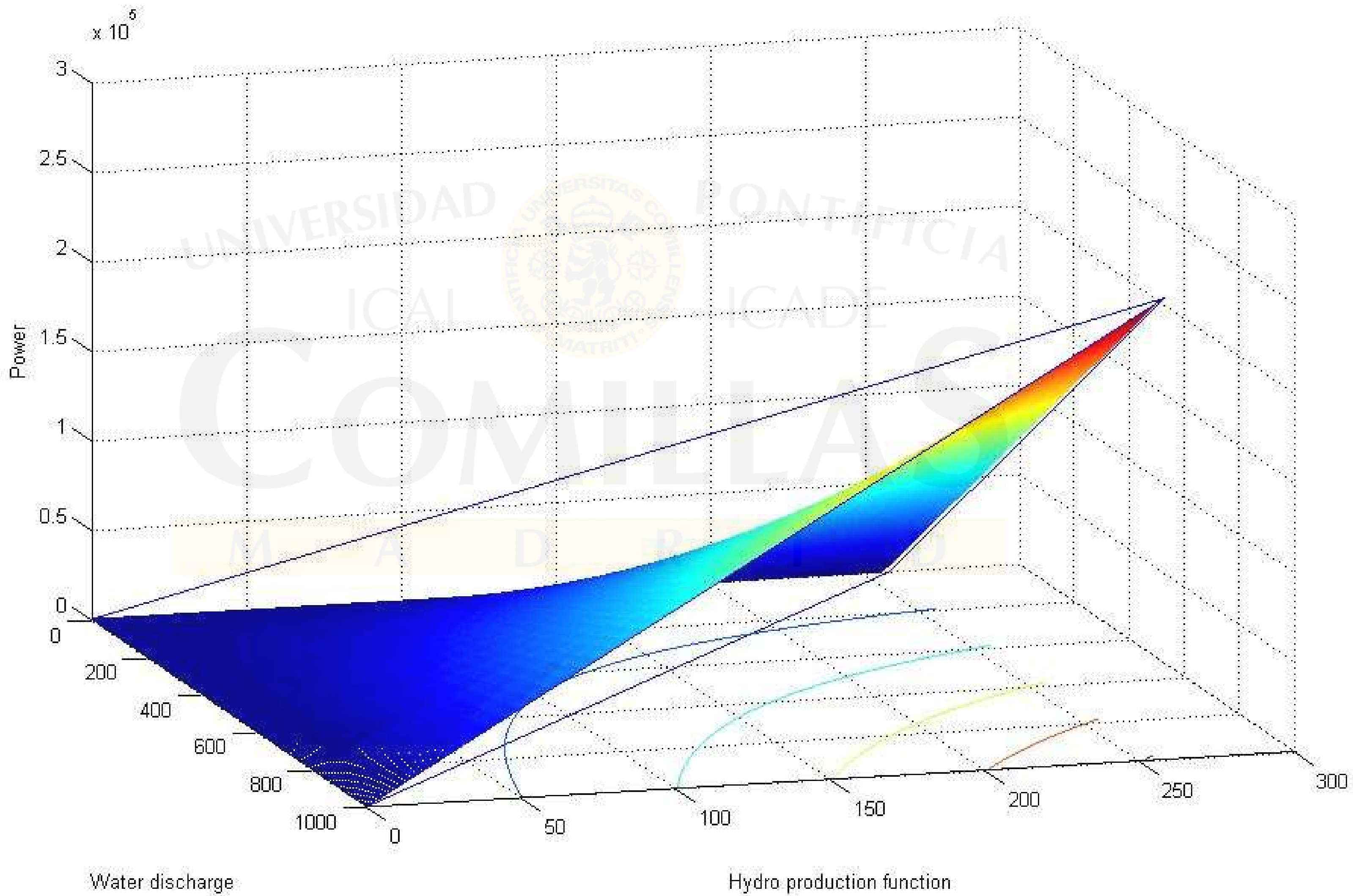
Nonlinear constraints reformulation



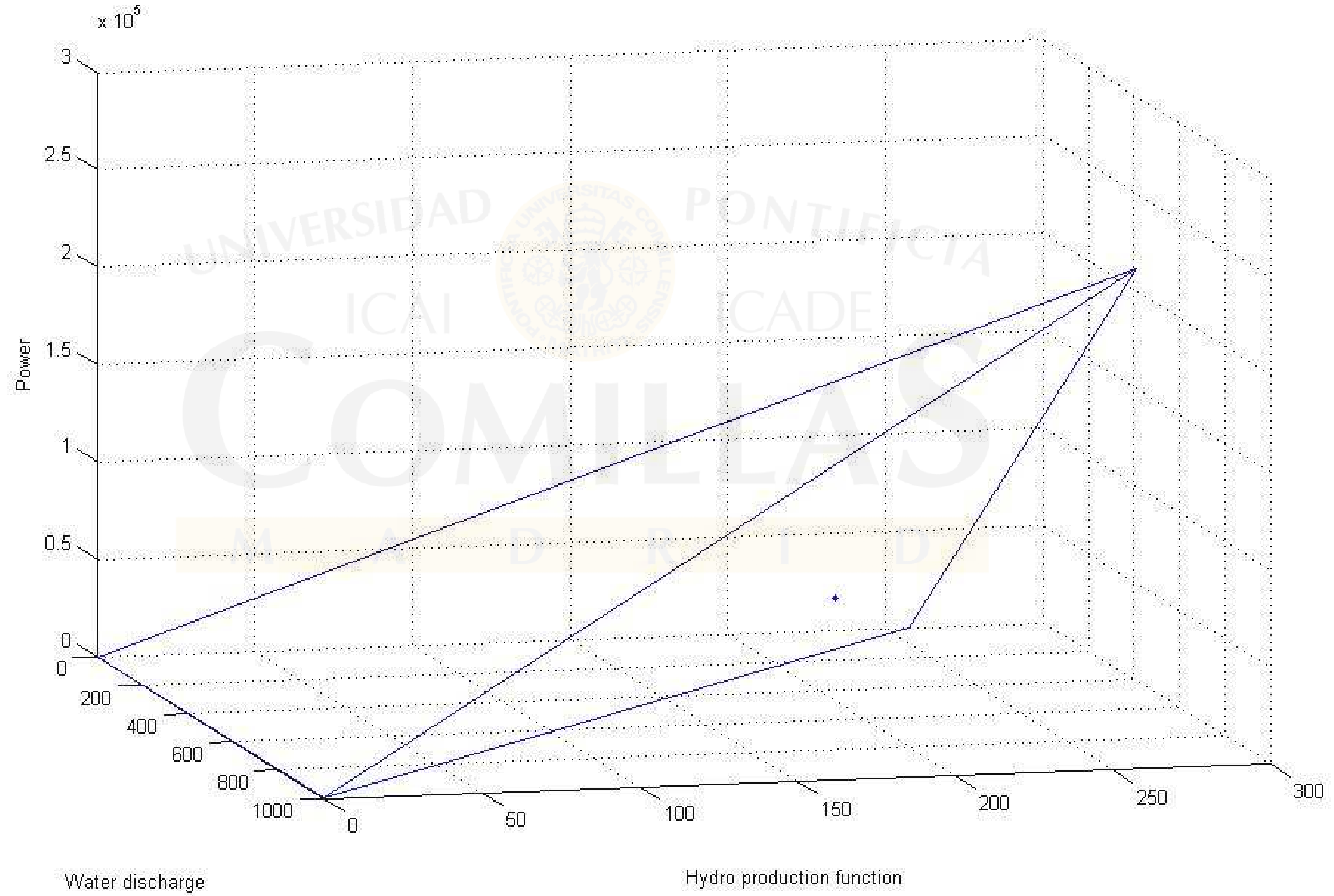
Nonlinear constraints reformulation



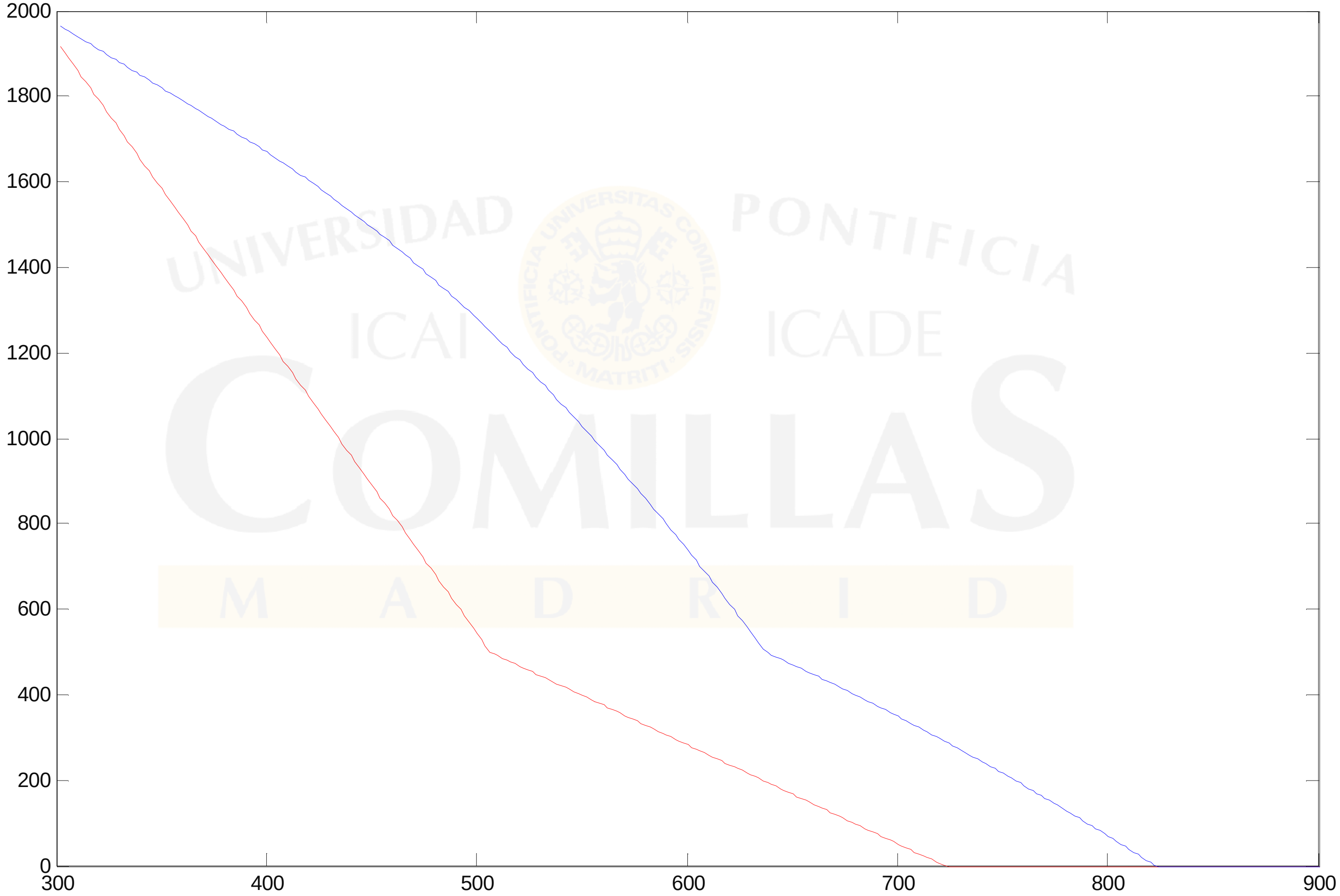
Nonlinear constraints reformulation



Nonlinear constraints reformulation



Nonlinear constraints reformulation



Nonlinear constraints reformulation

- A single McCormick envelope can be insufficient
- Construction of a grid for the variables of the bilinear relation
- Construction of the McCormick envelope for each rectangle of the grid
- Disjunctive programming forces the model to select just one tetrahedron out of the total
- Mathematical formulation using binary variables and a big-M approach

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Nonlinear constraints reformulation

$$z \geq x\overline{y}^{m} + \overline{x}^{n} y - u^{n,m}\overline{x}^{n}\overline{y}^{m} - \left(1 - u^{n,m}\right)K1^{n,m}$$

$$z \geq x\underline{y}^{m} + \underline{x}^{n} y - u^{n,m}\underline{x}^{n}\underline{y}^{m} - \left(1 - u^{n,m}\right)K2^{n,m}$$

$$z \leq x\overline{y}^{m} + \underline{x}^{n} y - u^{n,m}\underline{x}^{n}\overline{y}^{m} - \left(1 - u^{n,m}\right)K3^{n,m}$$

$$z \leq x\underline{y}^{m} + \overline{x}^{n} y - u^{n,m}\overline{x}^{n}\underline{y}^{m} - \left(1 - u^{n,m}\right)K4^{n,m}$$

- We determine the most accurate big-M values that enter in above constraints

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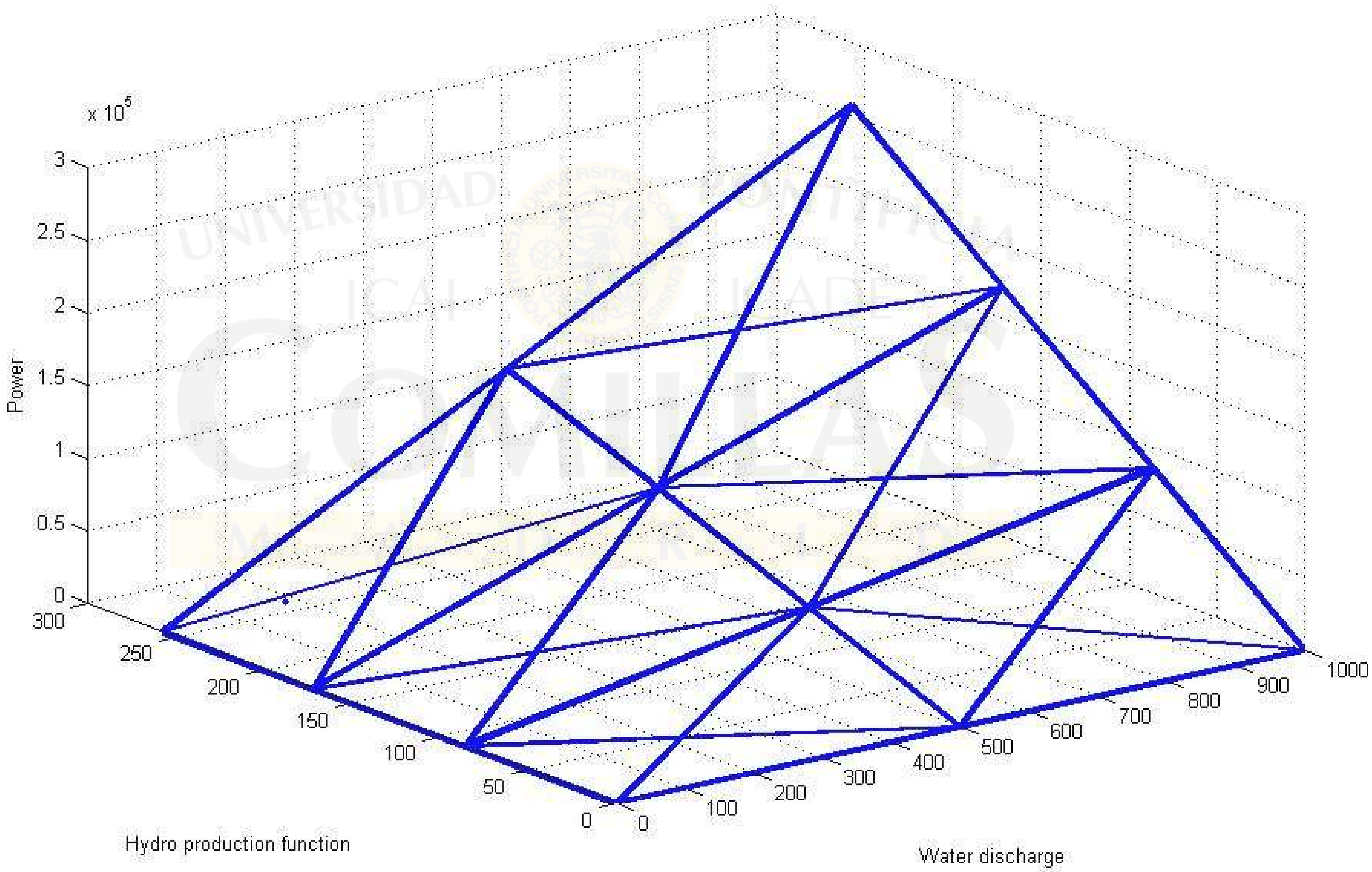
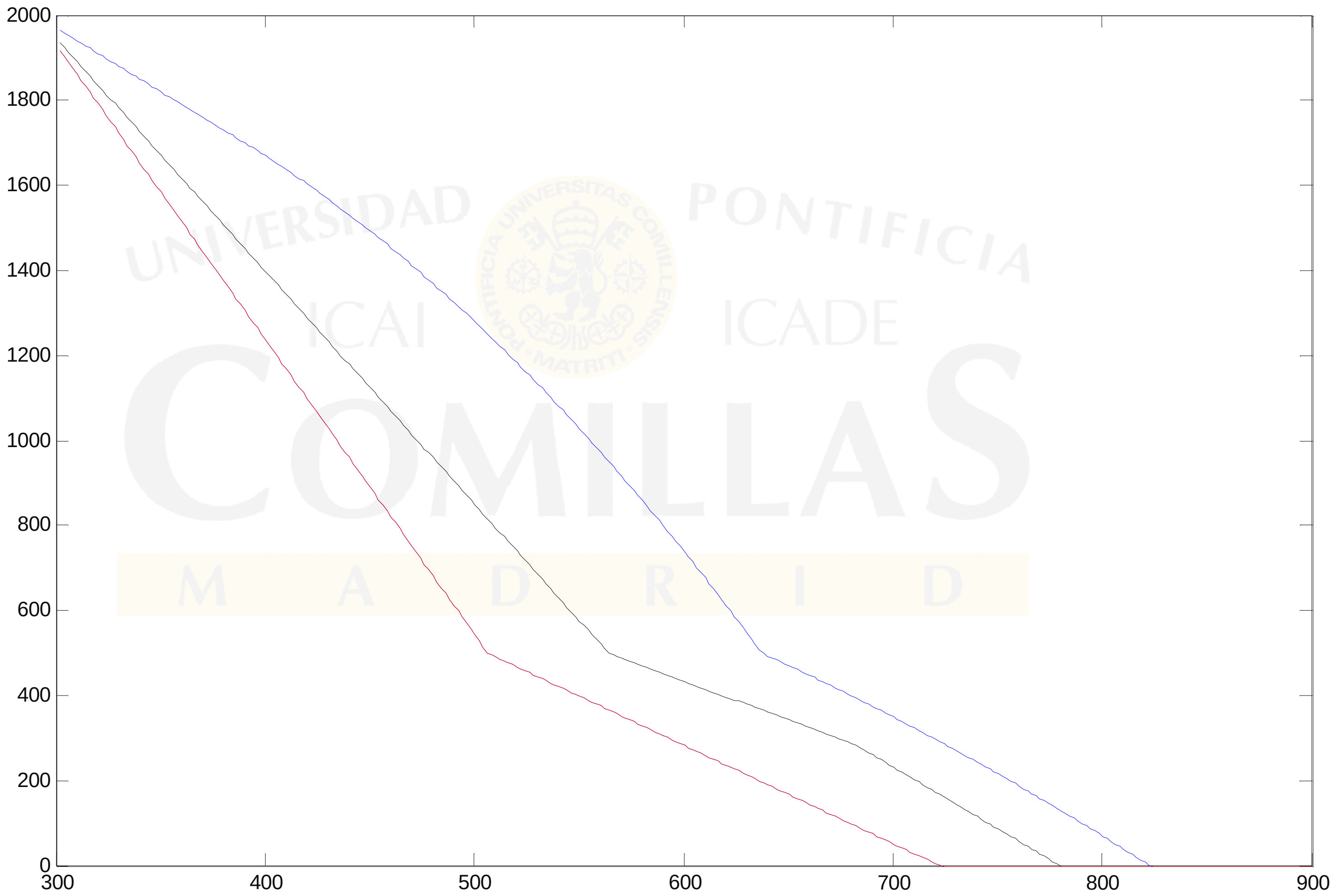
Nonlinear constraints reformulation



Nonlinear constraints reformulation



Stochastic Dual Dynamic Programming

- Multiperiod
- Stochasticity given by means of a recombining tree

$$
\begin{aligned}
&\min z = c^1 x^1 + E_{\xi^2}\left[\min c^2 x^2 + E_{\xi^3}\left[\min c^3 x^3 + \cdots\right]\right] \\
&A x^t \leq b^t && t : 1,\ldots,T \\
&B^t\left(x^t\right) = d^t && t : 1,\ldots,T \\
&T x^t + W^{t+1} x^{t+1} = h^{t+1}\left(\xi^{t+1}\right) && t : 1,\ldots,T-1
\end{aligned}
$$

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Stochastic Dual Dynamic Programming

- Traditional decomposition in master problem and subproblem

Master Problem

$$\min z = c^1 x^1 + E_{\xi^2}\left[Q^1\left(x^1, \xi^2\right)\right]$$
$$s.t: Ax^1 \leq b^1, B^1\left(x^1\right) = d^1$$

Primal Proposals

Subproblem

$$Q^{t-1}\left(x^{t-1}, \xi^t\right) = \min c^t x^t + E_{\xi^t}\left[Q^t\left(x^t, \xi^{t+1}\right)\right]$$
$$Ax^t \leq b^t$$
$$B^t\left(x^t\right) = d^t$$
$$Wx^t \leq h\left(\xi^t\right) - Tx^{t-1}$$

Outer approximations

Stochastic Dual Dynamic Programming

$$
\begin{aligned}
&Q^{t-1}\left(x^{t-1},\xi^{t}\right)=\min c^{t}x^{t}+E_{\xi^{t}}\left[Q^{t}\left(x^{t},\xi^{t+1}\right)\right]\\
&Ax^{t}\leq b^{t}\\
&B^{t}\left(x^{t}\right)=d^{t}\\
&Wx^{t}\leq h\left(\xi^{t}\right)-Tx^{t-1}
\end{aligned}
$$

Bilinear relations

Nonlinear subproblem

Non convex recourse function

$$
\begin{aligned}
&Q^{t-1}\left(x^{t-1},\xi^{t}\right)=\min c^{t}x^{t}+E_{\xi^{t}}\left[Q^{t}\left(x^{t},\xi^{t+1}\right)\right]\\
&Ax^{t}\leq b^{t}\\
&Mx^{t}=d^{t}\\
&Wx^{t}\leq h\left(\xi^{t}\right)-Tx^{t-1}
\end{aligned}
$$

MCormick refomulation

Linear subproblem

Convex recourse function

Slack approximation

$$
\begin{aligned}
&Q^{t-1}\left(x^{t-1},\xi^{t}\right)=\min c^{t}x^{t}+E_{\xi^{t}}\left[Q^{t}\left(x^{t},\xi^{t+1}\right)\right]\\
&Ax^{t}\leq b^{t}\\
&Mx^{t}+Nu^{t}=d^{t}\\
&Wx^{t}\leq h\left(\xi^{t}\right)-Tx^{t-1}
\end{aligned}
$$

MCormick surface

MIP subproblem

Non convex recourse function

Tight approximation

Stochastic Dual Dynamic Programming

- Convexification of the recourse function using Lagrangean Relaxation

$$\max w\left(x^{t-1}, \xi^t, \lambda^t\right)$$

$$w\left(x^{t-1}, \xi^t, \lambda^t\right) = \min c^t x^t + E_{\xi^t}\left[Q^t\left(x^t, \xi^{t+1}\right)\right] + \lambda^t\left(Tx^{t-1} + Wx^t - h\left(\xi^t\right)\right)$$

$$Ax^t \le b^t$$

$$B^t\left(x^t\right) = d^t$$

Nonlinear Subproblem

Local Minima

$$\max w\left(x^{t-1}, \xi^t, \lambda^t\right)$$

$$w\left(x^{t-1}, \xi^t, \lambda^t\right) = \min c^t x^t + E_{\xi^t}\left[Q^t\left(x^t, \xi^{t+1}\right)\right] + \lambda^t\left(Tx^{t-1} + Wx^t - h\left(\xi^t\right)\right)$$

$$Ax^t \le b^t$$

$$\mathrm{M}^t x^t + N u^t = d^t$$

MIP Subproblem

Use the Best Bound

Stochastic Dual Dynamic Programming

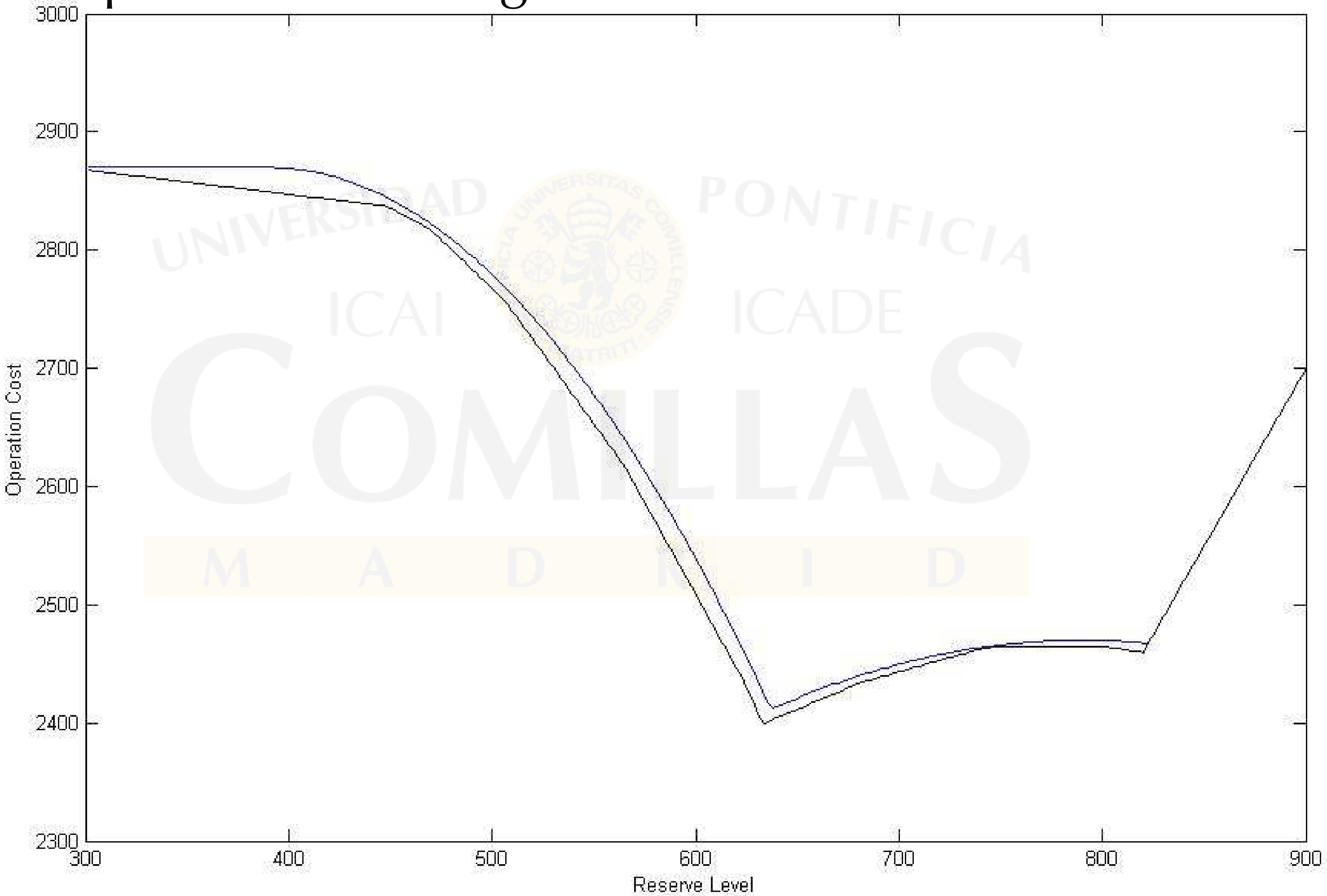
- We adopt the reformulation given by the McCormick Surface for the convexification routine
- We avoid the large number of Lagrangean Relaxation iterations for the optimization of the dual function
- We chose a proper multiplier and perform just one evaluation of the Lagrangean subproblem
  - Heuristic 1. Solution of the McCormick envelope subproblem and obtain the dual variable of the coupling constraints. Set the optimal multiplier

$$\lambda^t = -\pi^t$$

  - Heuristic 2. Combine the coefficients of previously computed Benders cuts to create the proper multiplier

Stochastic Dual Dynamic Programming

- An example for a two stage situation



Stochastic Dual Dynamic Programming

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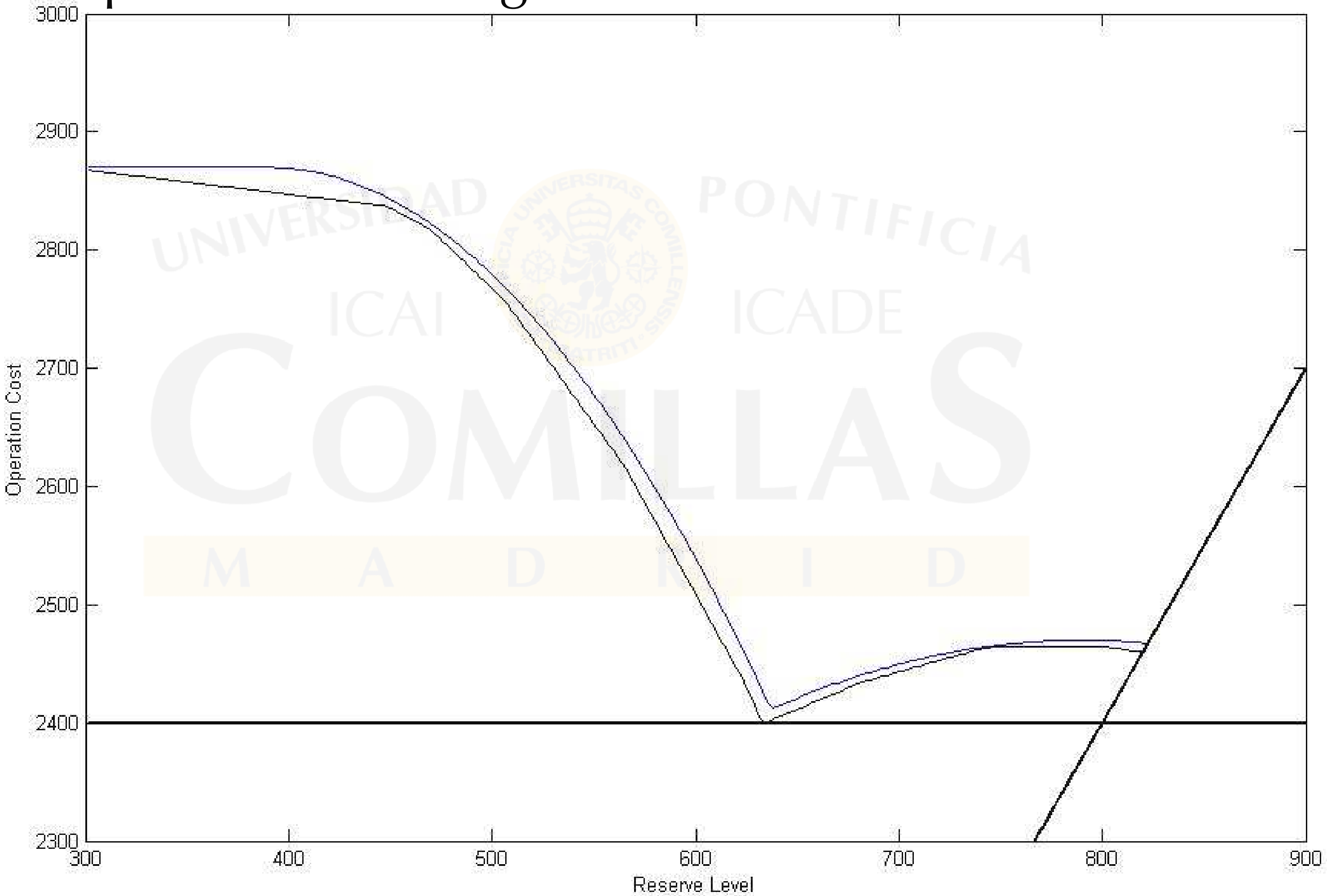

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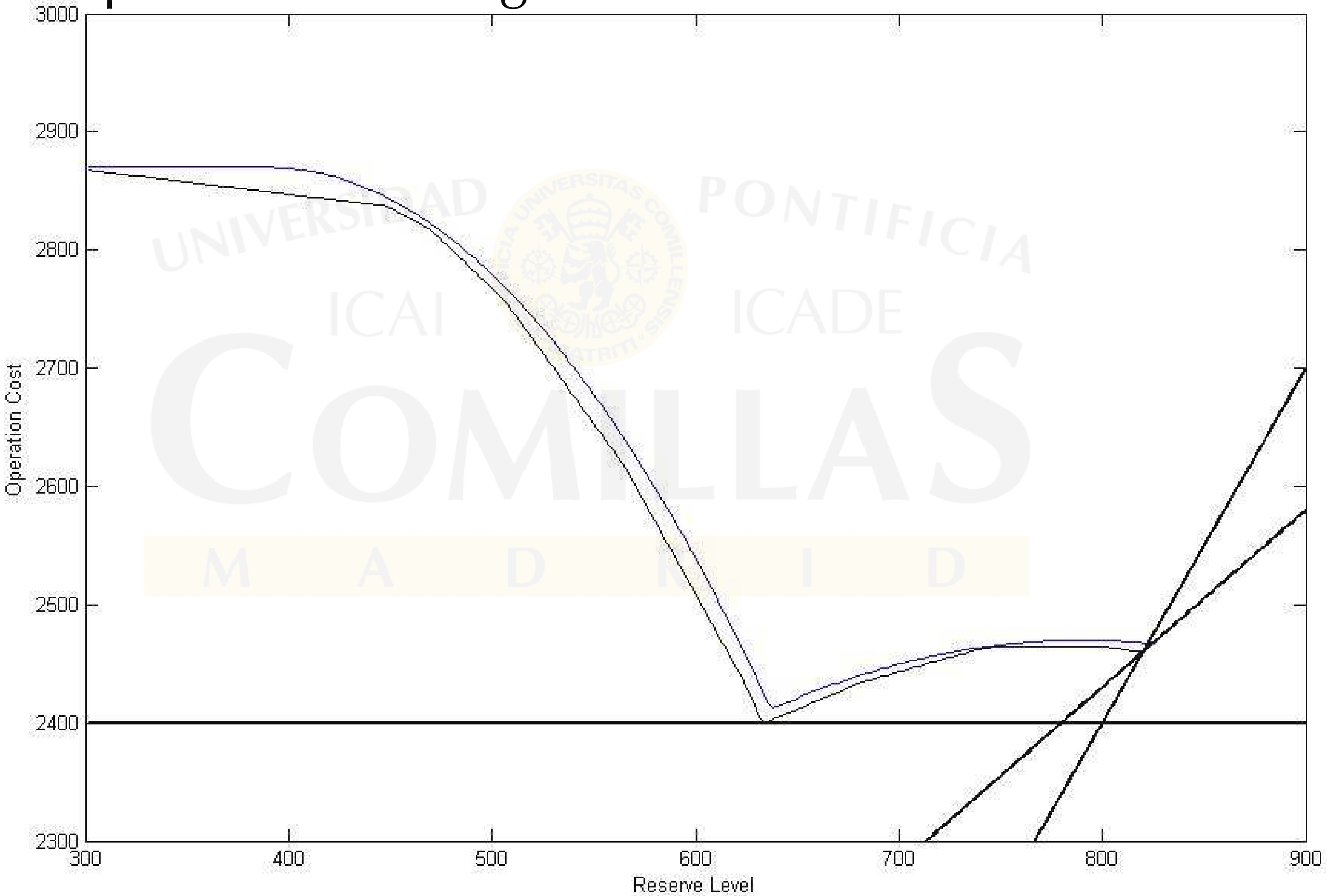
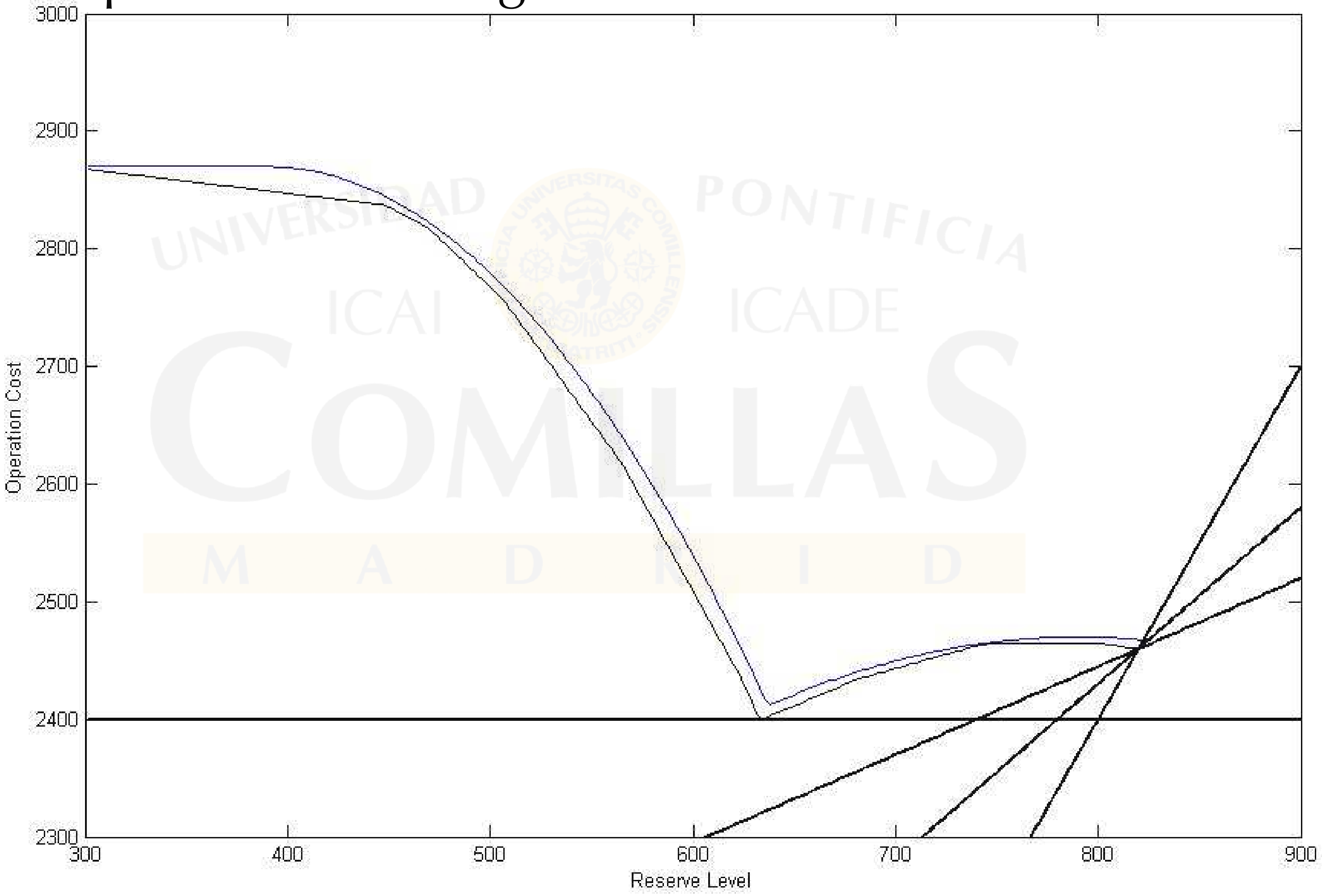


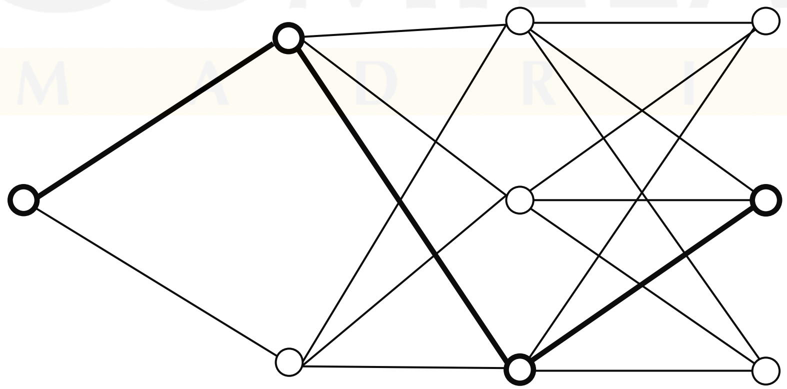
Stochastic Dual Dynamic Programming

- An example for a two stage situation



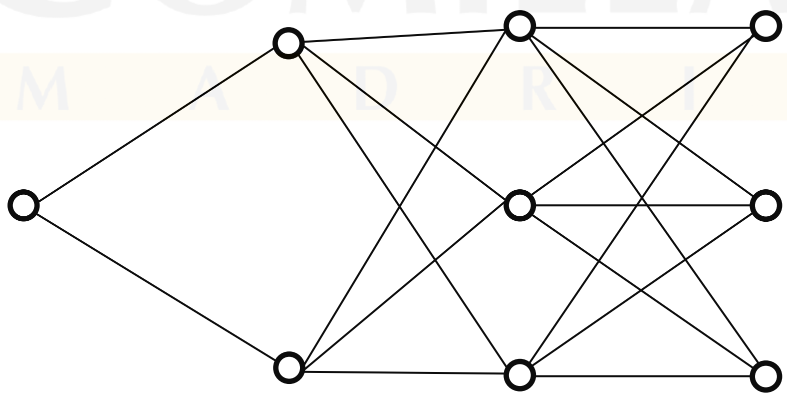
Stochastic Dual Dynamic Programming

- Description of the multistage situation
- Forward pass
  - Sample a scenario (path from the root through the tree)
  - Solve each node of the scenario (MIP subproblem)
  - Store the primal solution and the coefficients of the active Benders cuts



Stochastic Dual Dynamic Programming

- Description of the multistage situation
- Backward pass
  - Solve each node of each period
  - Create the proposed multiplier
  - Evaluate the Lagrangean subproblem (MIP)
  - Store the objective function and create a new Benders cut



Stochastic Dual Dynamic Programming

- Stopping criteria
  - Lower Bound: solution of the root node
  - Upper Bound: random variable. Estimation after n scenarios together with a confidence interval

$$\overline{z} = \frac{1}{N}\sum_{t=1}^{T} c_t \left(x_t^{\xi_t}\right)^n \qquad \sigma_n = \sqrt{\frac{1}{N}\sum_{n=1}^{N}\left(\sum_{t=1}^{T} c_t \left(x_t^{\xi_t}\right)^n - \overline{z}\right)^2}$$

$$I = \left(\overline{z} - \frac{1.96}{\sqrt{N}}\sigma_n, \overline{z} + \frac{1.96}{\sqrt{N}}\sigma_n\right)$$

  - Stopping rule:
    - Lower bound within the confidence interval
    - Confidence interval with a given tolerance

Case Study

- Real size hydrothermal coordination problem
- One year planning horizon
- Weekly period representation
- 84 thermal units
- 24 hydro plants
- 3 basins and multiple cascade reservoirs
- Recombining scenario tree created with clustering techniques
- Approximation of the bilinear relation with the McCormick surface with different pieces for the hydro production variable and the water discharge variable

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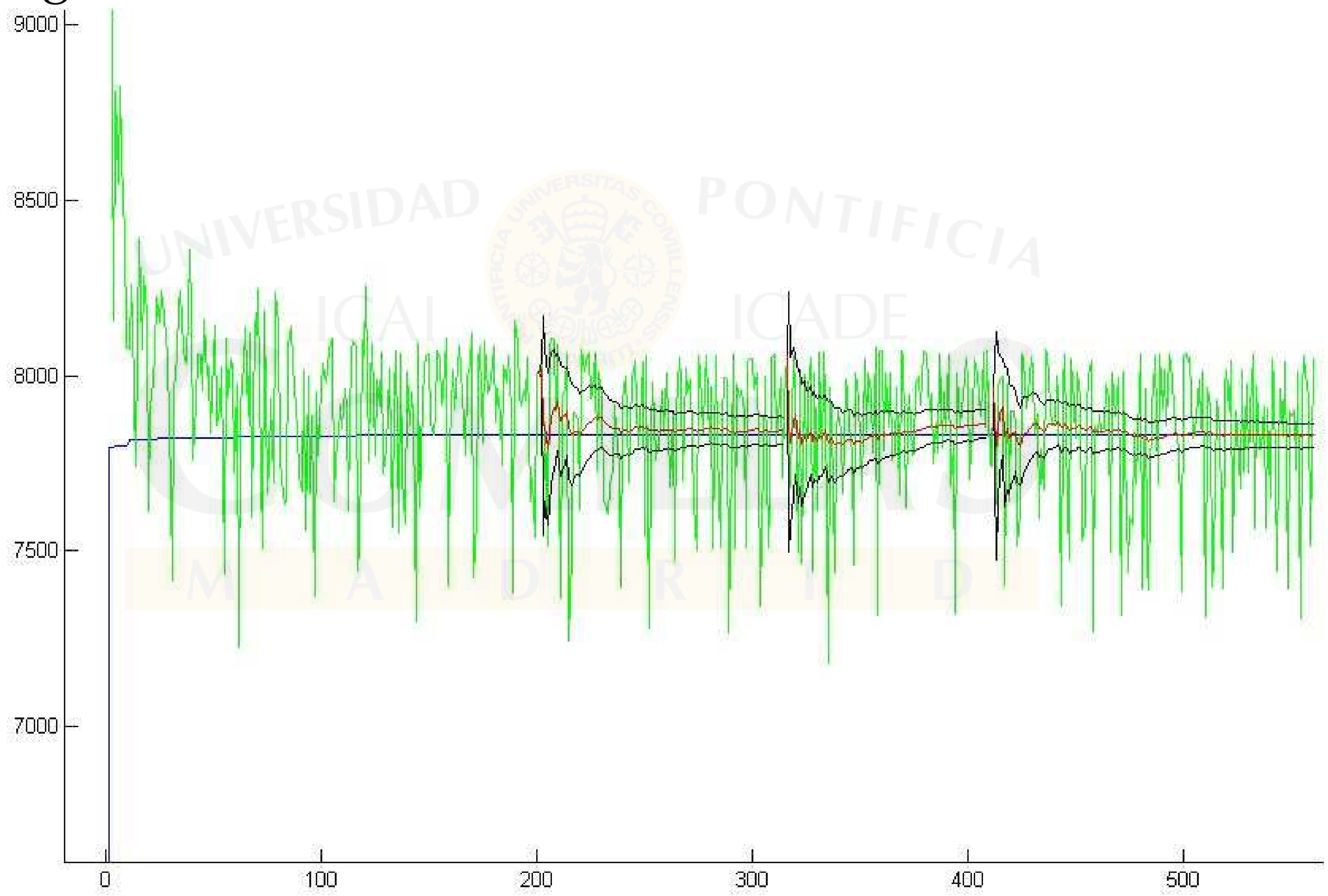
Case Study

- Practical implementation of the decomposition method
- Phase1
  - Forward and backward solution of the linear relaxation of the node subproblems
- Phase 2
  - Forward solution of the linear relaxation of the node subproblems.
  - Backward solution of Lagrangean subproblem evaluations. Multiplier proposed combining the coefficients of the active cuts in the forward pass (heuristic 2)
- Phase 3
  - Forward solution of the MIP subproblems.
  - Backward solution of the Lagrangean subproblem evaluations using heuristic 1

Case Study

- Convergence evolution



Iterations

Case Study

Branching	Sc	Lower	Upper	Interval	Tolerance
Every 4 weeks	2^{12}	7821.0	7817.7	[7803.5,7831.9]	0.0036
Every 2 weeks	2^{25}	7828.0	7839.3	[7826.4,7852.2]	0.0033
Every 1 week	2^{51}	7839.0	7850.3	[7831.6,7868.9]	0.0047
Every 4 weeks	3^{12}	7828.6	7830.8	[7791.7,7869.9]	0.0099

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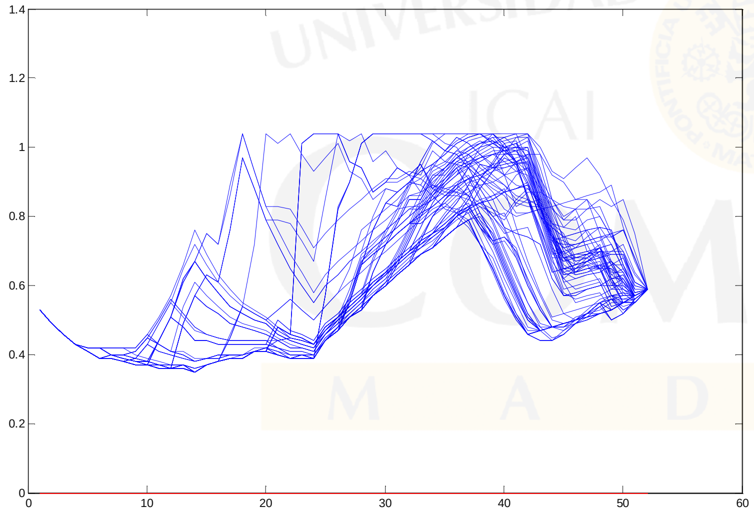
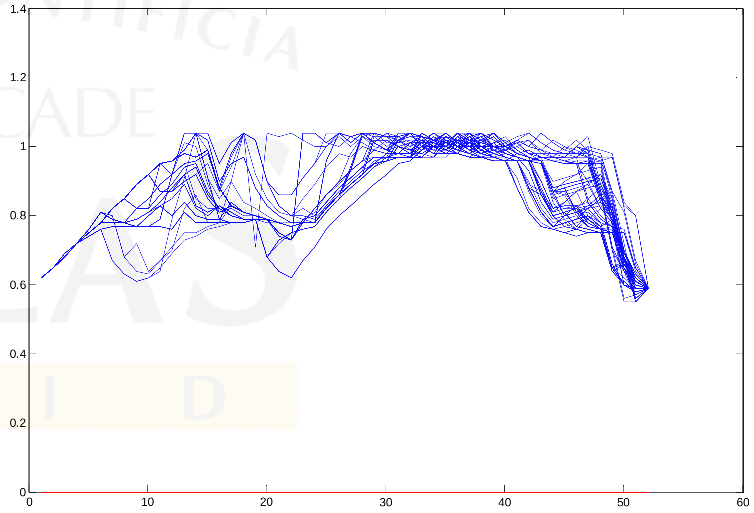
Case Study: Evolution of the reserve profiles





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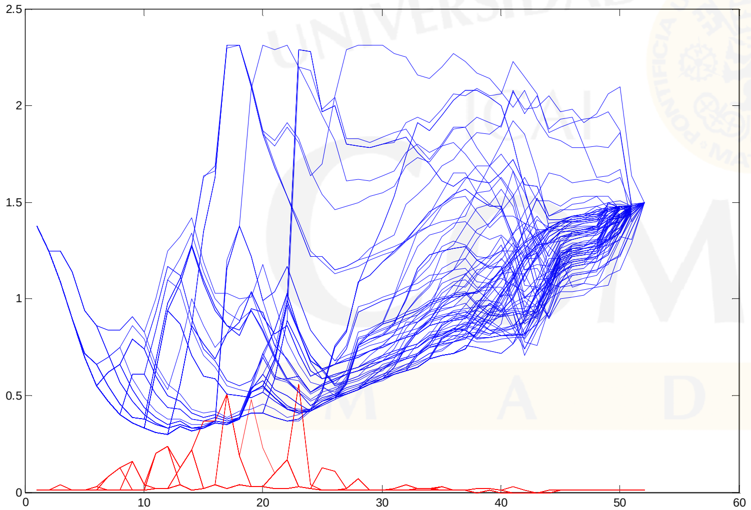

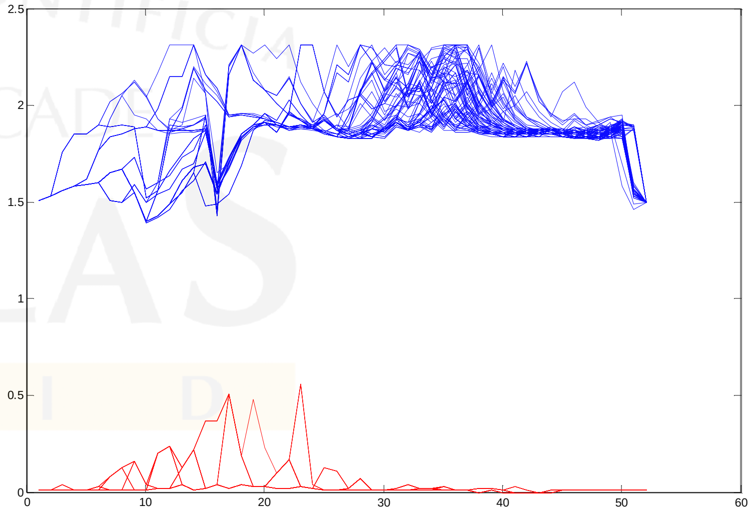

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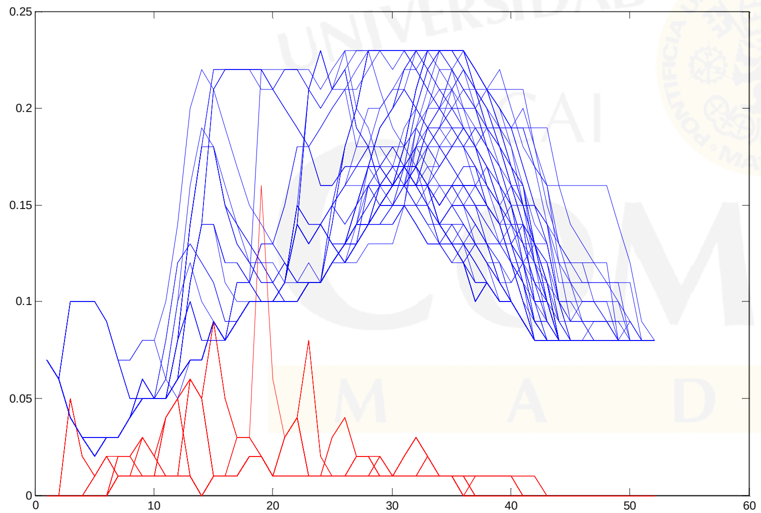

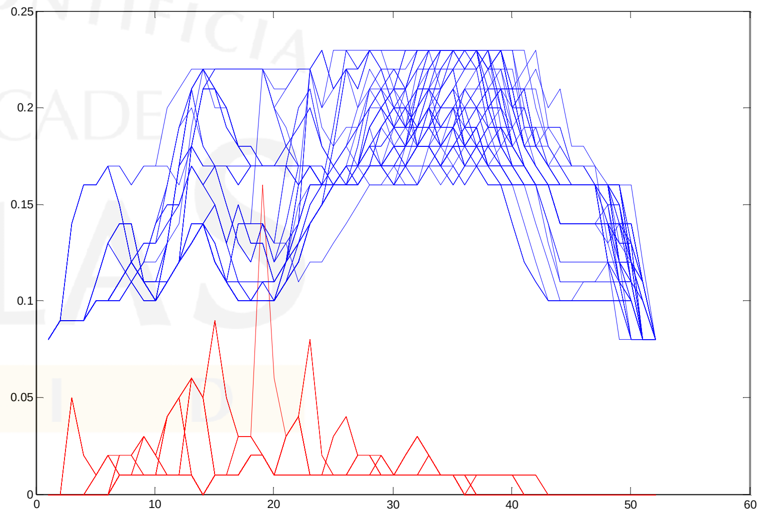

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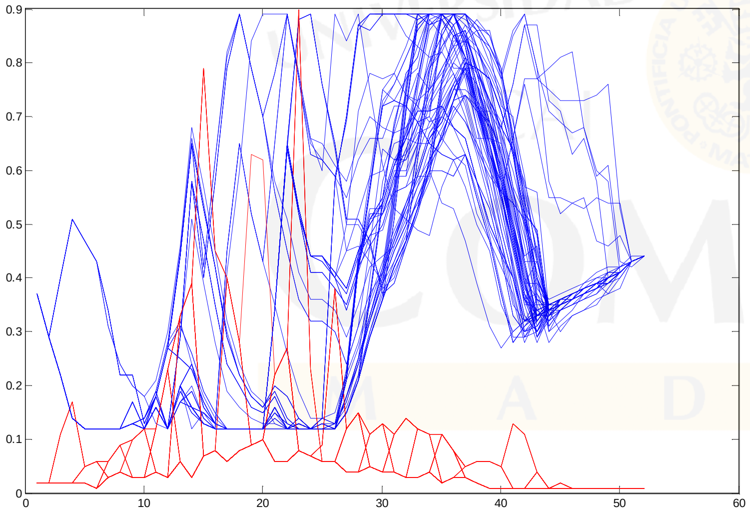

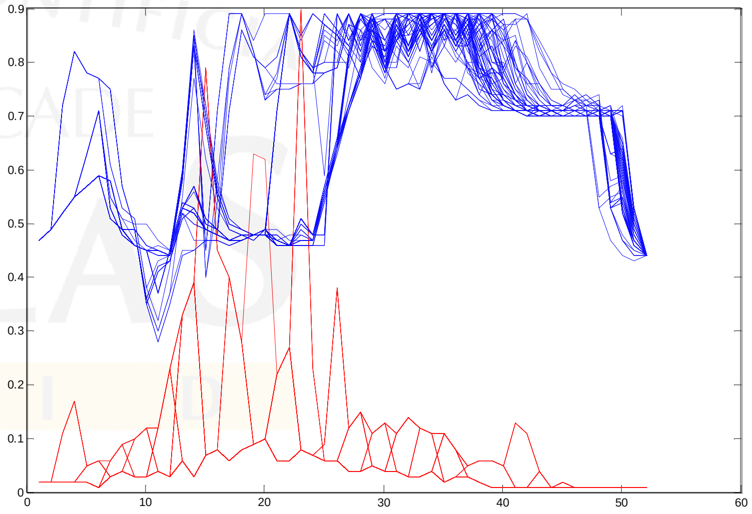

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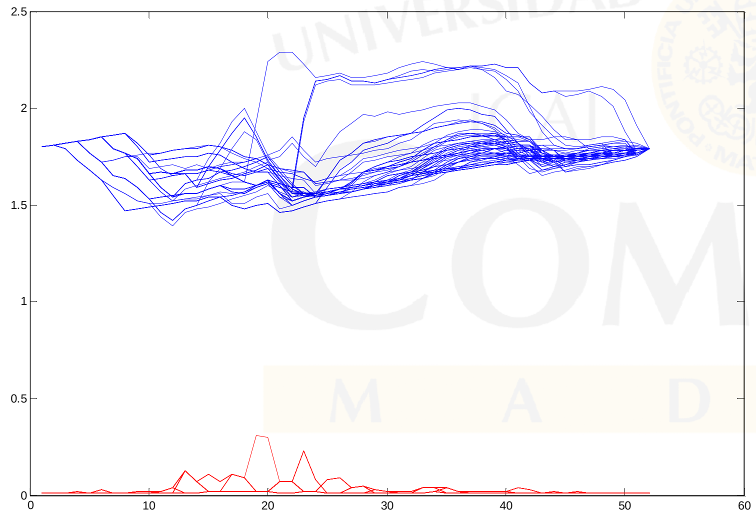

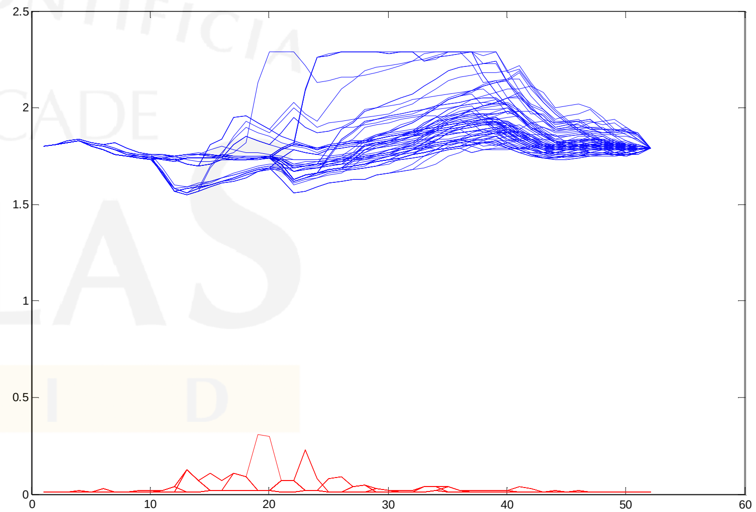

Conclusions

- Extension of the Stochastic Dual Dynamic Programming algorithm for nonlinear subproblems reformulated as MIP subproblems
- Remarkable results for the hydrothermal coordination problem. Acceptable reserve profiles
- Future developments
  - Sensitivity analysis for the uncertainty representation and the grid precision for the McCormick Surface
  - Adjust the stopping rule criteria with the theory developments in literature
  - Incorporate variance reduction techniques to reduce the computation time
  - Explore the possibility of performing the algorithm by solving small recombining subtrees during the iterations
  - Risk constraints for risk control of spillages

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