Modeling Transmission Ohmic Losses in a Stochastic Bulk Production Cost Model

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Abstract—Transmission ohmic losses affect the production cost of an electric power system in a non negligible percentage. Also, they have a relevant influence on spatial diversity of spot prices and on transmission revenues. So, transmission losses have to be considered in economic bulk pricing.

Five transmission losses approximations are analyzed in a stochastic bulk production cost model. Network is modeled using a DC load flow approach with losses. A comprehensive comparison among linear and nonlinear approximations has been done for the large-scale Spanish power system. Then, a trade-off between optimization time and accuracy is shown. Finally, a novel piecewise linear approximation is proposed with a good compromise between time and accuracy.

I. INTRODUCTION

Generation and transmission planning requires of computer models to study the economic and reliability impacts of additions of new plants and circuits, different operation policies and random system parameters [1]. Specifically, this production cost model addresses the medium term transmission planning.

Transmission ohmic losses are included in this model because of two main reasons:

- they affect system production costs in a non negligible percentage.
- they modify the spatial diversity of spot prices and line revenues according to marginalist theory [2, 3, 4].

Direct current load flow (DCLF) approximation without losses is applied to include the network in many composite reliability models [5, 6] or in some production cost models [7]. Other models add total losses effect after solving the optimization problem using a polynomial expression [8, 9]. Quadratic and piecewise linear losses approximations per circuit have also been previously used in different models, see references [2, 10]. Using an alternate current load flow (ACLF), losses are implicitly considered [11], but its optimization time can be cumbersome for a stochastic model.

This production cost model uses a DCLF with several losses approximations: two nonlinear, one mixed binary and two relaxed piecewise linear. Comparing results are obtained for the large-scale Spanish system (around 450 buses, 750 circuits and 200 generators).

For each approximation, a trade-off exists between accuracy in losses estimation and computer requirements. These requirements include optimization & execution times and memory. Usually, each approximation method implies its own requirements and accuracy.

This paper is divided into seven sections. In Section II, the stochastic bulk production cost model is described. In Section III, different approximations of transmission losses are shown. Section IV contains aspects of relaxed approximations, as the impact of number of segments per circuit, computation of intersection points and fictitious losses. Implementation details and required optimization algorithms are explained in Section V. Section VI compares results with different approximations referred to a classical nonlinear approximation. Conclusions, acknowledgments and references are situated in Sections VII, VIII & IX respectively. At the back, appendix contains the overall mathematical model formulation per level.

II. PRODUCTION COST MODEL

Production cost models are common tools for medium term economic planning studies. They allow the evaluation of alternative expansion plans and foresee economic results, i.e., determine the values of the operation variables obtaining the minimum variable cost for the model scope. Security aspects are also considered in this production model. Below, more general aspects are explained.

A. Scope

Scope is the time horizon analyzed by the model (usually, one year), and is divided into P periods. A different scheduled maintenance status in generation units and transmission circuits is associated to each period. Hydrology and forced outages at units and circuits are stochastically sampled per period. To model the impact of load variation, each period is subdivided into M levels (Fig.1) (e.g., peak, plateau and off-peak hours). Maintenance, duration and levels of a period are defined by the user. All levels of a period have the same elements available and the same hydro trajectory, (although hydro productions per unit can be different per level).

SCOPE

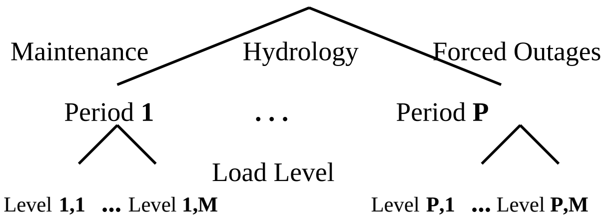

Fig.1. The Model Scope

B. Thermal Generation

Each thermal unit has a minimum (i.e., minimum output when it is on line) and maximum power limits and also, a constant and independent failure rate. Its production cost is a linear function of its power output.

C. Hydroelectric Generation

Hydro scheduling requires the use of a higher level hydro-thermal coordination model. Depending on the system Hydrology, this coordination tool must provide the optimum amount of hydro energy to be produced per unit and level and also, a marginal value of that hydro energy.

Each hydro unit can vary its output between a minimum (e.g., ecological flow) and maximum limits (depending on its technical characteristics and its reservoir level). Hydro production of each unit that exceeds the scheduled amount by the coordination model is valued at its marginal value. Hydro productions between minimum and reference limits have associated no costs in order to reach those reference values. Maximum and reference hydro productions vary per unit and level. Each hydro unit has associated a constant and independent failure rate.

D. Network and Demand

Network is composed of buses linked by circuits. Generators and loads are located at buses. Network flows are modeled using a DCLF approximation. Standard DC approximation has been improved to include transmission losses. This approximation constitutes a reasonable balance between a full AC load flow and a DC approximation without losses. Other aspects, as reactive power and bus voltages, are not considered in this model. Transformers and transmission lines are modeled as circuits. Each circuit have a constant failure rate, a resistance and reactance values.

E. Uncertainty

System Hydrology and forced outages at generators and circuits are modeled as random parameters. Hidrology is approximated using a triangular distribution function. Forced outages are discrete parameters, whose possible states are *available* and *unavailable.* Each element (units & circuits) has associated a Bernoulli probability distribution. Sampling is based on inverse transform method [12], i.e., extracting uniform random numbers, U[0,1], and using them with the corresponding cumulative distribution inverse function. In case of forced outages, the inverse function is the following:

$$CDF_i^{-1}\begin{cases} 0 \le U[0,1] < q_i; & unavailable \\ q_i \le U[0,1] < 1; & available \end{cases} \tag{1}$$

$$p_i + q_i = 1 \tag{2}$$

where, q_i is the forced outage rate of element *i*.

Random circuit failures can create several independent electric systems, called *islands*. To avoid a likely unbalance between load and generation capacity in these islands, two types of variables are used: *unserved power* per load bus and *excess of generation* per generator. Both variables are highly penalized in the objective function (A.1).

F. Statistical Results

Means and confidence intervals for output random variables are calculated using recursive and mathematically stable expressions as:

$$\overline{X}_n = \frac{(n-1)}{n}\,\overline{X}_{n-1} + \frac{x_n}{n} \tag{3}$$

$$\overline{X^2_n} = \frac{(n-1)}{n}\,\overline{X^2_{n-1}} + \frac{x_n^2}{n} \tag{4}$$

$$IC\,\overline{X}_n = 2 \cdot Z_{1-\alpha/2} \cdot \sqrt{\frac{\overline{X^2_n} - (\overline{X}_n)^2}{n-1}} \tag{5}$$

$\overline{X}_n$, $\overline{X^2_n}$: Means of x and x^2 at sample n.

x_n, x_n^2: Sample n of x and x square.

$IC\,\overline{X}_n$: Confidence interval of mean of x at sample n.

$Z_{1-\alpha/2}$: Upper 1-α/2 critical point for normal distribution.

Stopping criteria are based on confidence interval length. Confidence interval length depends on the root square of the ratio between variance of $\overline{X}_n$ and sample size minus one (5). Some spatial output variables may require a large sample size and then, an excessive computation time.

To obtain confidence intervals of aggregated means, it is necessary to evaluate the existence of sample correlations. Main correlations are situated among levels of same period because of identical random inputs. The remaining correlations have a negligible effect.

III. TRANSMISSION OHMIC LOSSES APPROXIMATIONS

This production cost model, based on a DCLF approximation, considers circuit losses as fictitious loads connected at their end buses, as shown in Figure 2. Each fictitious load represents half of the circuit losses. Losses computation can be done using different approximations.

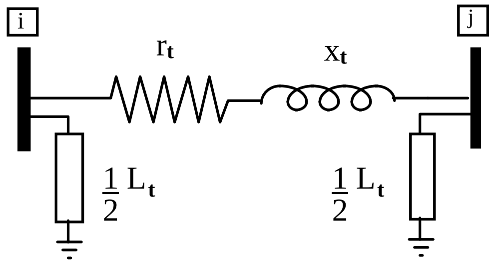

Fig. 2. Circuit model characteristics.

L_t: Losses at circuit t [W].

r_t : Circuit resistance [Ω].

x_t: Circuit reactance [Ω].

In this section two classes of transmission losses approximations are considered:

- nonlinear approximations (Subheadings A & B).
- piecewise linear approximations of a nonlinear function (Subheadings C, D & E).

Depending on the implemented approximation, number of optimization variables and constraints changes

A. Approximation by Cosine.

Exact active power dissipated in a circuit, L_t, without taking into account shunt losses, is calculated as:

$$L_t = G_t(V_i^2 + V_j^2) - 2\, G_t\, V_i\, V_j\, \cos(\Delta\theta_t) \tag{6}$$

where,

V_i , V_j: Voltages at end buses *i* and j[V].

G_t : Conductance of Circuit *t* [mho].

$\Delta\theta_t$: Voltage angular difference at circuit *t* [rad].

Assumpting that voltages magnitudes are constant and equal to 1 p.u., the former equation of L_t becomes:

$$L_t = 2G_t\left[1 - \cos(\Delta\theta_t)\right] \tag{7}$$

Equation (7) has been drawn with a thick line in Figure 3:

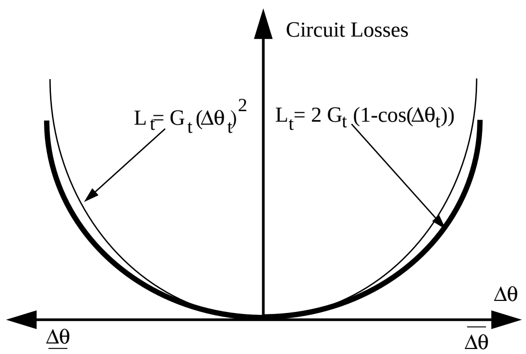

Fig. 3. Quadratic and by Cosine approximations.

Circuit losses expression (7) is included into the bus power balance equation (A.2). This approximation does not modify the number of constraints or variables of the model, but becomes the original linear problem into a nonlinear one.

This approximation is considered as a benchmark with reference to the following approximations.

B. Quadratic Approximation.

This approximation consists of modeling circuit losses in a circuit as its conductance multiplied by the square angular difference. Then, its mathematical formulation becomes:

$$L_t = G_t\, (\Delta\theta_t)^2 \tag{8}$$

In Figure 3, A & B approximations are depicted. The quadratic one is drawn with a thin line. This approximation increases its deviation from approximation by cosine as angular difference grows

C. Mixed-Binary Approximation.

This approximation is based on a piecewise linear fitting to the cosine's curve. $\Delta\theta$-axes are divided into segments, and each segment has assigned a slope, m_i, and an optimization variable, $\Delta\theta_i$. Figure 4 shows deviations between the approximation by cosine (thin line) and a piecewise linear approximation with three segments per axis (thick line).

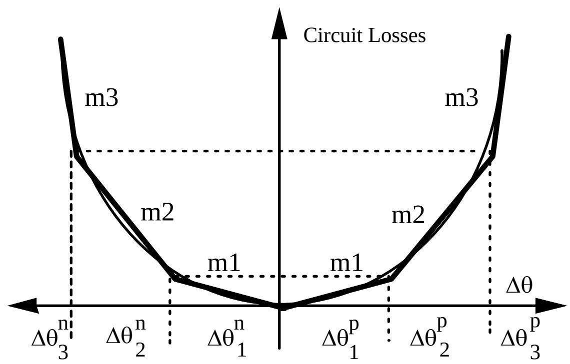

Fig.4. Mixed-Binary Approximation.

The mathematical formulation of this piecewise linear approximation for circuit *t* is composed by:

$$L_t = \sum_{k=1}^{K_t} m_{k,t}\, (\Delta\theta_{k,t}^p + \Delta\theta_{k,t}^n) \tag{9}$$

$$\Delta\theta_t = \sum_{k=1}^{K_t} \left(\Delta\theta_{k,t}^p - \Delta\theta_{k,t}^n\right) \tag{10}$$

$$\overline{\Delta\theta}_{k,t}\; W_{k+1,t}^p \le \Delta\theta_{k,t}^p \le \overline{\Delta\theta}_{k,t}\; W_{k,t}^p \qquad k = 1,\ldots, K_t \tag{11}$$

$$\overline{\Delta\theta}_{k,t}\; W_{k+1,t}^n \le \Delta\theta_{k,t}^n \le \overline{\Delta\theta}_{k,t}\; W_{k,t}^n \qquad k = 1,\ldots, K_t \tag{12}$$

$$W_{1,t}^p + W_{1,t}^n = 1 \tag{13}$$

where,

Constant Parameters:

K_t: Number of segments per axis of circuit *t*.
$m_{k,t}$: Slope of segment *k* [W/rad].
$\overline{\Delta\theta}_{k,t}$: Maximum angular increment of segment k [rad].
Optimization Variables:
$\Delta\theta^{p}_{k,t}$, $\Delta\theta^{n}_{k,t}$: Positive and negative angular increments of segment *k* [rad].
$w^{p}_{k,t}$, $w^{n}_{k,t}$: Binary variables of positive and negative axis segments.

Then, equation (9) is substituted in the bus balance equations (A.2). Constraints (11) & (12) force an ordered filling of segments in both axes. Moreover, constraint (13) fixes the sign of angular increments using binary variables of the first segment.

Using this formulation two equality constraints, (10) & (13), are added per circuit into the model. Also, the number of inequalities per circuit is increased in four times the number of segments per axis, (13) & (14). The binary and real optimization variables added per circuit, is equal to the double of number of segments per axis.

The model becomes a too much complex problem using this approximation for all circuit losses. Therefore, it has not been implemented in the production cost model. Nevertheless, two relaxations of this piecewise linear approximation that have been implemented, are shown in next subheadings.

D. Linear Relaxed Approximation by Variables.

This relaxation consists of suppressing binary variables of the previous approximation. Its drawing is the same as the mixed-binary one (Fig.4). Then, the optimization model becomes a linear problem (see Appendix).

Based on the previous formulation, this approximation contains equalities (9) & (10), and substitutes the remaining constraints by:

$$0 \le \Delta\theta^{p}_{k,t},\ \Delta\theta^{n}_{k,t} \le \overline{\Delta\theta}_{k,t}\ ;\ k = 1,\ldots,K_t. \qquad (14)$$

This relaxation by variables adds only one equality (10) per circuit. The added number of real variables is equal to the double of number of segments per axis, K_t.

E. Linear Relaxed Approximation by Inequalities.

This relaxation is analogous to the previous one. Both approximations obtain the same results, although their mathematical formulations are different.

In Figure 5, a three segment example has been drawn (the negative side has been omitted).

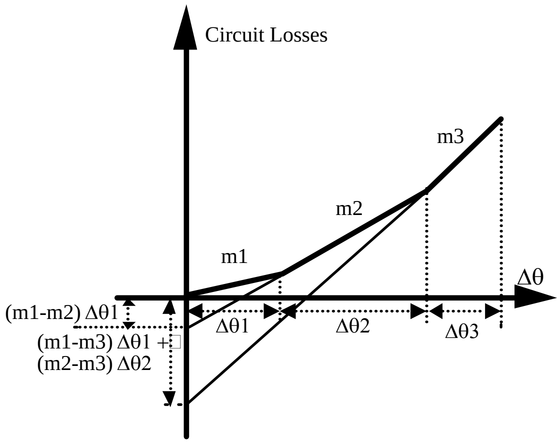

Fig. 5. Relaxed Approximation by Inequalities.

The mathematical formulation of this approximation for circuit *t* is composed by:

$$L_t \ge m_{k,t}\,(\Delta\theta^{p}_{t} + \Delta\theta^{n}_{t}) + \sum_{r=1}^{k-1} \Delta\theta_{r,t}\,(m_{r,t} - m_{k,t});\ k = 1,\ldots,K_t \qquad (15)$$

$$\Delta\theta_t = \Delta\theta^{p}_{t} - \Delta\theta^{n}_{t} \qquad (16)$$

$$0 \le \Delta\theta^{p}_{t},\Delta\theta^{n}_{t} \le \overline{\Delta\theta}_t \qquad (17)$$

where,
Constant Parameters:
$\overline{\Delta\theta}_t$: Maximum angular difference of circuit *t* [rad].
$\Delta\theta_{r,t}$: Size of segment *r* of circuit *t* [rad].
Optimization Variables:
$\Delta\theta^{p}_{t}$, $\Delta\theta^{n}_{t}$: Positive and negative angular differences of circuit *t* [rad].
L_t: Losses at circuit t [W].

This relaxation adds per circuit one equality (16) and as many inequalities (15) as the number of segments per axis, K_t. Moreover, three optimization variables per circuit ,$\Delta\theta^{p}_{t}$,$\Delta\theta^{n}_{t}$ and L_t, are added. Finally, the formulation of bus balance equations (A.2) is not modified.

IV. Relaxation Aspects

A. Selection of Number of Segments.

This selection can only be implemented into relaxed approximations. Number of segments per circuit affects directly to the losses calculation. There are two ways to determine this number:

- Fixing that number as an option for all circuits.
- Fixing a maximum deviation in MW per circuit and then, automatically the model determines the number of segments for each circuit, depending on its electric parameters.

This selection modifies the optimization problem size, affecting the computer memory requirements. The problem sizes per level of the Spanish case are shown in Table I using different approximations and number of segments.

TABLE I
LEVEL PROBLEM SIZES

Approximation	r	c	nz

	r	c	nz
Without losses	1250	3780	11945
Approximation by Cosine	1250	3780	13690
Quadratic Approximation	1250	3780	13650
Relaxation by variables (1 segment)	2025	5300	18100
Relaxation by variables (2 segments)	2025	6860	22700
Relaxation by variables (3 segments)	2025	8430	27430
Relaxation by variables (4 segments)	2025	9740	29436
Relaxation by inequalities (3 segments)	4330	6100	23510

* r: rows c: columns nz: nonzero elements

Using three segments, the relaxation by variables requires approximately the double memory requirements of nonlinear approximations. Comparing the relaxation by variables and inequalities, the last one duplicates the number of rows although reduces its number of columns and nonzero elements.

B. Intersection Point Calculation.

The intersection points between each pair of segments (given a number of segments) are calculated by minimizing the quadratic deviation between the approximation by cosine and the linear piecewise approximation (Fig.4). Once fixed the number of segments, this calculation is repeated for several maximum angular differences and then, interpolation is used. Intersection points are not forced to be on the cosine's curve.

C. Drawback of Relaxed Approximation.

The main drawback for using relaxations is the very unlikely but possible existence of *fictitious* transmission ohmic losses. In the Spanish case, the impact of fictitious losses is negligible. The appearance ratio is one circuit per thirty five thousand without fictitious losses.

Usually, production cost minimization also minimizes transmission losses. However, there are some specific situations where a circuit losses increment stands or reduces this cost. There are two types of fictitious losses:

- The first one appears in zones where an increment of generation maintains or reduces the system production cost. This situations may happen when there is hydro energy in excess at no cost.
- The second one appears when there are overloaded circuits in the system. Then, additional losses at circuits adjacent to overloaded ones can relax some active constraints and therefore, diminish the objective function value.

These fictitious losses can be suppressed using mixed binary or nonlinear losses approximations exclusively on circuits with fictitious losses.

Next example shows a situation of the second type. Fig.6 shows the topology of a three bus system. Tables II, III & IV show the characteristics of this small system. Tables V & VI contain generation outputs and circuit flows using the approximation by cosine. Tables VII & VIII contain the same results using relaxed approximation by variables with three segments per axis. Characteristics of circuit segments are detailed in table IX.

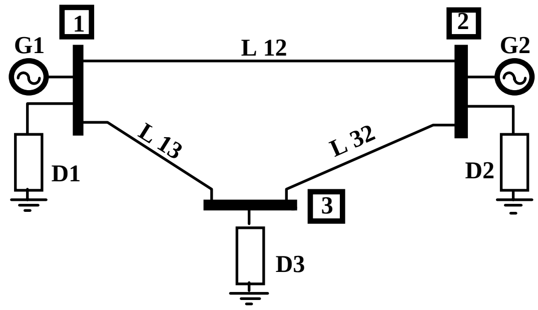

Fig.6. Example Case

TABLE II
GENERATION DATA

Unit	Minimum (MW)	Maximum (MW)	Cost ($/MWh)
G1	0	1000	1
G2	0	400	60

TABLE III
DEMAND DATA

	D1	D2	D3
Demand (MW)	100	700	200

TABLE IV
CIRCUIT DATA

Circuit	Resistance (p.u.)	Reactance (p.u.)	Capacity (MW)
L12	0.00062	0.02631	1000
L13	0.00159	0.00653	500
L32	0.00090	0.01600	200

TABLE V
GENERATION OUTPUTS USING APPROXIMATION BY COSINE

Unit	Output (MW)	Cost ($/h)
G1	724	724
G2	279.08	16744.8
Sum	1003.08	17468.8

TABLE VI
CIRCUIT FLOWS USING APPROXIMATION BY COSINE

Circuit	Flow (MW)	Loss (MW)	Angular difference (rad)
L12	221.25	0.30	0.05821
L13	401.39	2.42	0.02621
L32	200 *	0.36	0.03200

* : Overloaded circuit.

TABLE VII
GENERATION OUTPUTS USING RELAXED APPROXIMATION BY VARIABLES

Unit	Output (MW)	Cost ($/h)
G1	726.75	726.75
G2	278.77	16726.20
Sum	1005.52	17452.95

TABLE VIII
CIRCUIT FLOWS USING RELAXED APPROXIMATION BY VARIABLES

Circuit	Flow (MW)	Loss (MW)	Angular difference (rad)
L12	221.55	0.30	0.05829
L13	402.61	4.88	0.02629
L32	200 *	0.35	0.03200

* : Overloaded circuit.

TABLE IX
SIZES AND SLOPES OF SEGMENTS

Circuit	m_1	m_2	m_3	$\overline{\Delta\theta_1}$	$\overline{\Delta\theta_2}$	$\overline{\Delta\theta_3}$
L12	0.0510	0.2188	0.3840	0.0759	0.0933	0.0939

L13	0.2498	1.0744	1.8902	0.0095	0.0116	0.0116
L32	0.0244	0.1048	0.1844	0.0093	0.0114	0.0114

The overloaded circuit L32 is constraining the actual transmission capacity of circuit L12 (tables VI & VIII). An hypothetical increment in the flow of circuit L12 would allow to increase output of G1 and therefore, the system generation cost will be reduced.

Using the relaxed approximation by variables, the circuit L12 flow raises 0.3 MW referred to the approximation by cosine. This increase is because of fictitious losses at circuit L13 (2.46 MW, comparing tables VI & VIII). Finally, system generation cost has diminished from \$17468.8 to \$17452.95 (tables V & VII).

Table X shows circuit flows using mixed-binary approximation at circuit L13 to force its proper segment filling.

TABLE X
CIRCUIT FLOWS USING
MIXED-BINARY APPROXIMATION

Circuit	Flow (MW)	Loss (MW)	Angular difference (rad)
L12	221.25	0.30	0.05821
L13	401.40	$\boxed{2.46}$	0.02621
L32	200 *	0.35	0.03200

* : Overloaded circuit.

V. IMPLEMENTATION

The model has been programmed in GAMS language (General Algebraic Modeling System), version 2.25 [13]. This code gives clarity and compactness to the source code structure. GAMS avoids the use of interfaces to call different solvers. Previous losses approximations becomes the production cost model into linear (LP), nonlinear (NLP) and mixed integer 0/1 problems (MIP). CPLEX 3.0 optimization package [14] has been used to solve LP and MIP problems, and MINOS 5.3 package [15] has been used to solve NLP problems.

CPLEX implements simplex and interior point algorithms to solve LP problems, and a branch & bound (B&B) algorithm for MIP problems. A relative stopping criterion for B&B (difference between the best integer found solution and the best possible solution) has been set to 0.1%.

MINOS uses a projected augmented Lagrangian algorithm to solve the production model with nonlinear losses approximations.

When circuits with fictitious losses are detected inside optimization results, the problem is formulated and optimized again using mixed binary approximation only on these specific circuits until fictitious losses are suppressed. This suppression of fictitious losses is an option of the model.

VI. COMPUTATIONAL RESULTS

Table XI contains details of the Spanish study case. Random outages at units and circuits have been modeled using a FOR of 10% for all units and of 0.2% for circuits. The sample size to obtain statistical results has been 100 scope samples.

TABLE XI
SPANISH CASE CHARACTERISTICS

Buses	458
Circuits	742
Hydro Units	129
Thermal Units	75
Periods	12
Levels per period	2

Table XII shows optimization time reductions and accuracy deviations using different losses approximations (referred to the approximation by cosine).

TABLE XII
TIME REDUCTIONS & DEVIATIONS REFERRED
TO APPROXIMATION BY COSINE

Losses Approximations	ΔT %	ΔC %	ΔL %	ΔG %	ΔI %
α. Without losses	-89	-5.26	-100	-2.28	-53
β. 1 segment (Variables)	-78	+5.77	+176.8	+ 2.53	-63
γ. 2 segments (Variables)	-64	+0.87	+33.9	+ 0.4	-38
δ. 3 segments (Variables)*	-50	+0.22	+7.6	+ 0.1	-11
ε. 4 segments (Variables)	-25	+0.06	+2.8	+ 0.02	-2
ζ. 3 segments (Inequalities)	-21	+0.22	+7.6	+ 0.1	-11
η. Quadratic approximation	-10	+0.07	+0.2	+ 0.01	-1.6

* Suppressed fictitious losses.
ΔT: Optimization time reduction. ΔC: Generation cost deviation.
ΔL: Ohmic losses deviation. ΔG: Energy production deviation.
ΔI: Transmission revenue deviation.

Approximations α to ε show the effect of increasing the number of segments on the evolution of result accuracies and optimization times. Transmission revenue is the most sensitive global result to number of segments. Figure 7 shows that evolution over optimization time and transmission revenue accuracy changing from zero to four segments per axis.

Without modeling transmission losses, α approximation, optimization time is reduced an almost 90%. However, its transmission revenue deviation is high. On the other hand, the relaxation by 1 variable (β approximation) is worse than α approximation. This situation is due to the low usage rate of transmission Spanish system.

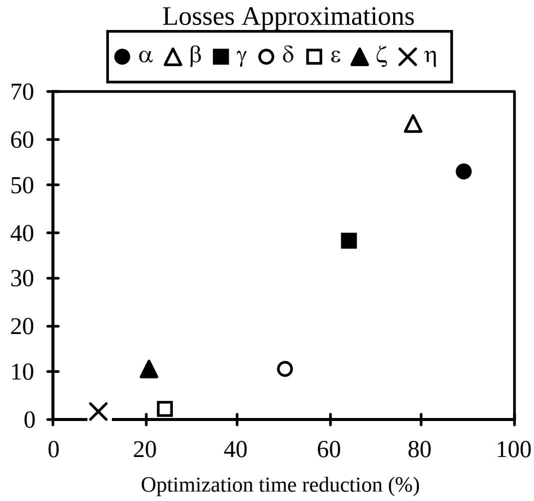

Fig. 7. Trade-off between Time Reduction- Accuracy.

By comparing relaxations by variables and inequalities (δ versus ζ), three consequences can be extracted. First one is a confirmation that both relaxations reach the same model results. Second one is that relaxation by variables is more than twice of fast than by inequalities. Third one is that the effect of fictitious losses on these global results is negligible, because of accuracy percentages are the same, suppressing fictitious losses (δ) and without suppressing them (ζ).

δ approximation (three segments per axis) in this implementation is considered a good trade-off between optimization time and result accuracies. Adding more segments to the approximations with three segments improves a little the result accuracies and rises drastically the optimization times. Besides, the memory requirements increases dramatically with the number of segments.

Quadratic approximation (η) obtains an excellent accuracy. However, its time reduction is the smallest, a 10%. In addition, this η approximation also becomes the model problem into a nonlinear one.

The optimization time using approximation by cosine has been 25275 seconds (in a 125 SPECint92 and 121 SPECfp92 workstation).

VII. CONCLUSIONS

In this study, five transmission ohmic losses approximations have been analyzed and compared among them using the actual Spanish case. This group of approximations is composed by two novel 'relaxed' piecewise linear ones, two classical nonlinear ones and one mixed binary.

Aspects related with piecewise linear approximations, as the impact of number of segments, the intersection point calculation and the fictitious losses drawback are studied.

From the comparison among these approximations, a relaxation by variables obtains a good trade-off between optimization time reduction and result accuracies to evaluate the impact of transmission losses. Specifically, this relaxed approximation reduces the optimization time to a half the approximation by cosine although its memory requirement is duplicated.

VIII. ACKNOWLEDGEMENTS

This work was sponsored by RED ELECTRICA DE ESPAÑA. (REE) through a Ph. D. grant for the first author. We thank the comments done by Michel Rivier (I.I.T.).

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APPENDIX

MATHEMATICAL FORMULATION OF PRODUCTION COST MODEL PER LEVEL.

A. Glossary of Terms.

Definition of indices and sets:

tr: Thermal unit bus.
TR_p: Set of buses with available thermal generation during period p.
u: Thermal generation unit number.
U_{tr}: Set of thermal units at bus tr.
hr: Hydro unit bus.
HR_p: Set of buses with available hydro generation during period p.
n: Bus number.
N: Set of System's buses.
t: Circuit number.
i, j: End nodes of circuit t.
L_p: Set of available circuits during period p.
$\Omega^{+}_{p,n}$: Set of available circuits during period p whose flows enters at bus n.
$\Omega^{-}_{p,n}$: Set of available circuits during period p whose flows leaves bus n.

Generation related terms:

PT_{tr}^{u} :	Thermal power output of unit *u* at bus *tr*.
CT_{tr}^{u} :	Linear cost of power output of thermal unit *u* at bus *tr*.
$\underline{PT}_{tr}^{u}, \overline{PT}_{tr}^{u}$:	Minimum and maximum thermal limits.
$PH1_{hr}$:	Scheduled component of hydro power output at bus *hr*.
$PH2_{hr}, CH_{hr}$:	Excess above scheduled hydro power at bus *hr* and its penalty cost.
$PET_{tr}^{u}, CET_{tr}^{u}$:	Excess of thermal power output and its penalty cost.
PEH_{hr}, CEH_{hr} :	Excess of hydro power output and its penalty cost.

Load related terms:

D_n:	Load demand at bus *n*.
NSP_n, CNS_n :	Unserved power at bus *n* and its penalty cost.

Network related terms:

$FL1_t$:	Power flow in circuit *t* between safe margins.
$FL2_t$:	Excess above positive safe margin of flow in circuit *t* and its penalty cost.
$FL3_t$:	Excess above negative safe margin of flow in circuit *t* and its penalty cost.
T_i:	Voltage angle at bus *i*.
L_t:	Ohmic losses at circuit *t*.
X_t:	Circuit *t* reactance.
$\overline{FL}_t$:	Maximum capacity limit of circuit *t*.
$\underline{FL1}_t, \overline{FL1}_t$:	Safe margins of circuit *t*.

B. Formulation of Level Economic Dispatch.

Objective Function:
Minimize

$$\sum_{tr \in TR_p} \left(\sum_{u}^{U_{tr}} CT_{tr}^{u} PT_{tr}^{u}\right) + \sum_{hr \in HR_p} CH_{hr} PH2_{hr} + \sum_{n}^{N} CNS_n NSP_n + \sum_{t \in L_p} CL2_t FL2_t + \sum_{t \in L_p} CL3_t FL3_t + \sum_{tr \in TR_p} \left(\sum_{u}^{U_{tr}} CET_{tr}^{u} PET_{tr}^{u}\right) + \sum_{hr \in HR_p} CEH_{hr} PEH_{hr} \tag{A.1}$$

Bus Power Balance Equations (Kirchhoff Current Law):

$$\sum_{t \in \Omega^+_{p,n}} (FL1_t + FL2_t + FL3_t) - \sum_{t \in \Omega^-_{p,n}} (FL1_t + FL2_t + FL3_t) + \sum_{u}^{U_n} PT_n^u + PH1_n + PH2_n - \sum_{u}^{U_n} PET_n^u - PEH_n = D_n - NSP_n + \frac{1}{2} \sum_{t \in \Omega^+_{p,n} \cup \Omega^-_{p,n}} L_t \quad ; \; n = 1 \dots N. \tag{A.2}$$

Circuit Flow (Kirchhoff Voltage Law) and capacity limits:

$$FL1_t + FL2_t - FL3_t = \frac{(T_i - T_j)}{X_t} \quad ; \; t \in L_p \tag{A.3}$$

$$\underline{FL1}_t \le FL1_t \le \overline{FL1}_t \tag{A.4}$$

$$0 \le FL2_t \le \overline{FL}_t - \overline{FL1}_t \tag{A.5}$$

$$0 \le FL3_t \le \left| -\overline{FL}_t - \underline{FL1}_t \right| \tag{A.6}$$

Output limits of thermal and hydro generation:

$$\underline{PT}_{tr}^{u} \le PT_{tr}^{u} \le \overline{PT}_{tr}^{u} \tag{A.7}$$

$$\underline{PH1}_{tr} \le PH1_{tr} \le \overline{PH1}_{tr} \tag{A.8}$$

$$0 \le PH2_{tr} \le \overline{PH2}_{tr} \tag{A.9}$$

Limit of unserved power and excess of generation:

$$0 \le NSP_n \le D_n \tag{A.10}$$

$$0 \le PET_{tr}^{u} \le \underline{PT}_{tr}^{u} \tag{A.11}$$

$$0 \le PEH_{hr} \le \underline{PH}_{hr} \tag{A.12}$$

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