Combining economic and fluid dynamic models to determine the optimal spacing in very large wind-farms

Richard J. A. M. Stevens[1,2], Benjamin F. Hobbs[3,*], Andrés Ramos[4] and Charles Meneveau[1]

[1]Department of Mechanical Engineering & Center for Environmental and Applied Fluid Mechanics, Johns Hopkins University, Baltimore, Maryland 21218, USA

[2]Department of Physics, Mesa+ Institute, and J. M. Burgers Centre for Fluid Dynamics, University of Twente, 7500 AE Enschede, The Netherlands

[3]Department of Geography & Environmental Engineering and E^{2}SHI, Johns Hopkins University, Baltimore, Maryland 21218, USA

[4] Institute for Research in Technology (IIT), ICAI School of Engineering, Comillas Pontifical University, C/Sta. Cruz de Marcenado 26, 28015 Madrid, Spain

[*] Presenter

Now that wind-farms are becoming increasingly larger, the economics and physics of wind farms become intrinsically coupled and important when designing large wind farms. It is important to develop wind farm models in which economic considerations can be combined with physical considerations in a transparent and intuitive way. For smaller wind-farms the majority of the turbines can be placed such that physical wake effects are relatively limited and thus physical effects may be less important. However, for large wind farms (e.g. with hundreds or thousands of turbines) it is important to consider the influence of wake effects on the optimal turbine spacing. For the design of wind-farms the industry uses site-specific, detailed optimization calculations for wind-turbine placement based on wake models (1; 2; 3; 4; 5; 6). Such calculations aim to place the turbines such that wake effects are limited with respect to the prevailing incoming wind-directions at the site under consideration. There are also academic studies that use wake models to optimize turbine placement using Monte Carlo simulations (7), genetic algorithms (8), or evolutionary algorithms (9; 10). In this work we will combine economic and fluid dynamic models to determine the main parameters that are important for the design of very large wind-farms.

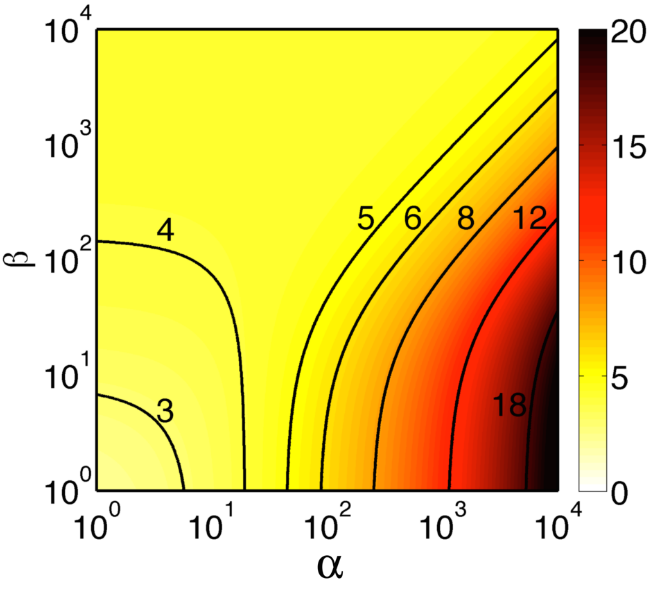
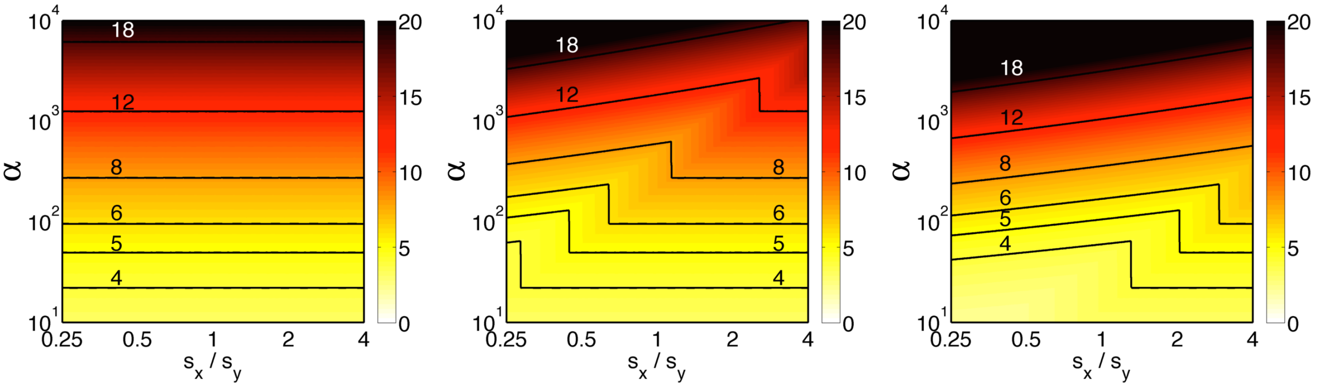
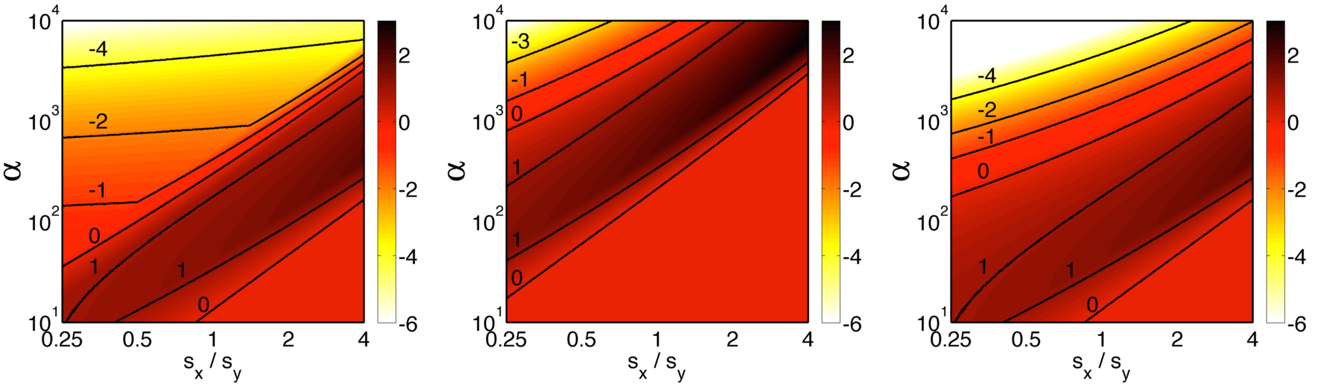


Figure 1: Optimal spacing as function of dimensionless turbine cost (α) and dimensionless cable cost (β), see equation (2). The optimal spacing increases with increasing α or decreasing β.

It is known that wake models do not capture the effect of the interaction between the atmosphere and very large wind-farms correctly (11). These effects are described by "top-down" single-column type models which are based on momentum analysis and horizontal averaging. These models give a vertical profile of the average velocity profile by assuming the existence of two logarithmic regions, one above the turbine hub-height and the other below (12; 13; 14; 15; 16; 17; 18; 19). The mean velocity at hub-height is used to predict the wind-farm performance as function of the main design parameters. Meyers & Meneveau (20) showed, by accounting for the turbine and land costs, that the optimal spacing in the limit of very large wind-farms can be found analytically using "top-down" models. For typical land costs (e.g. Texas) they found an optimal spacing of 12 − 15 turbine diameters, which is



(a) "Best" (b) Staggered (c) Aligned



(d) Staggered-aligned (e) "Best"-staggered (f) "Best"-aligned

Figure 2: Optimal turbine spacing based on the normalized power per unit cost for different wind-farm layouts as function of the ratio s_x/s_y and α, for $\beta = 0$. (a) "best" layout in which the power production is limited by the vertical kinetic energy flux that can be obtained for a given turbine density, and a (b) staggered and an (c) aligned layout in which the area is not necessarily used optimally. Panels (d-f) give the differences between the results shown in panels (a-c) in different ways. Positive values indicate that turbines in an aligned (or staggered) wind-farm should be placed closer together than using the "best" (or staggered) configuration.

significantly larger than the value found in actual wind-farms ($6 - 10$ turbine diameters). However, their approach did not take into account cable costs, specific turbine placements, profit considerations and maintenance costs. In this paper, we generalize the framework to include these considerations.

Here we expand the method used by Meyers and Meneveau (20) to determine the optimal turbine spacing in a very large wind-farm. The influence of several additional aspects, such as cable costs, maintenance costs, the wind-farm layout, and the effect of optimizing net revenue instead of normalized power per unit cost, are addressed. This abstract summarizes a selection of these results, addressing the effect of cable costs and wind-farm layout. Here we consider the difference between several wind-farm layouts. For very large wind-farms the turbine power production for a given turbine density is limited by the vertical kinetic energy flux that can be obtained on that land area when the distance between turbines placed directly in each others wakes is maximized (11). This limit we call the "best" layout, which we will compare with the aligned and staggered layouts.

Following Meyers and Meneveau (20) a simple cost function that includes the turbine, land, and cable cost, is formulated as follows:

$$\mathrm{Cost} = \mathrm{Cost}_{\mathrm{turb}} + (\mathrm{sD})\mathrm{Cost}_{\mathrm{cable}} + (\mathrm{sD})^2\mathrm{Cost}_{\mathrm{land}}. \tag{1}$$

The objective is to minimize the levelized cost per unit output (\$/MWh) or to maximize its reciprocal (power per unit cost). Here α and β are two non-dimensional parameters, representing turbine and cable costs, compared to land costs, respectively. They are defined as:

$$\alpha = \frac{\mathrm{Cost}_{\mathrm{turb}}/(\frac{1}{4}\pi D^2)}{\mathrm{Cost}_{\mathrm{land}}}, \qquad \beta = \frac{\mathrm{Cost}_{\mathrm{cable}}/D}{\mathrm{Cost}_{\mathrm{land}}}. \tag{2}$$

These dimensionless cost ratios are used to define the normalized power per unit cost as

$$P^* = \frac{P_\infty(s_\mathrm{x}, s_\mathrm{y}, \mathrm{layout}, ...)}{s} \frac{\mathrm{Cost}_\mathrm{land}}{\mathrm{Cost}_\mathrm{turb}/(\mathrm{sD})^2 + \mathrm{Cost}_\mathrm{cable}/(\mathrm{sD}) + \mathrm{Cost}_\mathrm{land}} = \frac{P_\infty(s_\mathrm{x}, s_\mathrm{y}, \mathrm{layout}, ...)}{s} \frac{4s^2/\pi}{\alpha + 4\beta s/\pi + 4s^2/\pi}, \tag{3}$$

where $P_\infty(s_\mathrm{x}, s_\mathrm{y}, \mathrm{layout}, ...)$ indicates the normalized predicted power output for the turbines in the wind-farm. The coupled wake boundary layer (CWBL) model is used to evaluate the wind-farm performance as a function of the spacing between the turbines in the streamwise s_x and spanwise s_y direction, the wind-farm layout, the size of the wind-farm (19), etc.. It has been shown that the CWBL model agrees well with results from large eddy simulations (LES) and field measurements in Horns Rev and Nysted (11; 21).

In agreement with the results of Meyers and Meneveau (20), figure 1 shows that without cable costs ($\beta = 0$) the optimal turbine spacing strongly depends on the turbine to land cost ratio α. For $\alpha \approx 3000$, which is typical for Texas (20), the obtained turbine spacing is around 15 turbine diameters. However, when cable costs are included the obtained optimal spacing is significantly smaller and much closer to the value observed in Texas's wind-farms, which is in the range of $8 - 12$ turbine diameters (19). This result is based on an order-of-magnitude estimate of $\beta = 100$, resulting from a cable cost of 200\$/m, a turbine diameter of 100 meters, and a land cost 0.4\$/m^2.

To assess the influence of the wind-farm layout we evaluated the optimal spacing for aligned, staggered, and "best" layout in figure 2 as function of α and the ratio between the streamwise and spanwise turbine spacing $s_\mathrm{x}/s_\mathrm{y}$. The differences in the optimal spacing for the staggered, aligned and "best" layout are shown in panel (d-f) and are an effect of the stronger wake effects for the aligned or staggered configuration compared to the "best" layout. Note that for sufficiently large enough streamwise spacing (large enough s_x/s_y) the optimal spacing for the aligned and staggered configuration becomes equal to the "best" layout. Here it is important to note that the figure shows the optimal spacing and not the profitability, which together with effects of maintenance costs has also been considered in our work. In addition we note that determining the profitability per turbine gives us insight in the required (nondimensionalized) revenue that is necessary to operate a wind-farm for a given α and β while the profitability can also be maximized per land area, which results in different optimal spacings.

In short, we have extended the work of Meyers and Meneveau (20) on the determination of the optimal spacing in very large wind-farms using analytical models. We now included the effect of the cable cost, the maintenance costs, the wind-farm layout, and the influence of maximizing the profitability instead of the normalized power per unit cost, in the analysis. However, it is emphasized that for the design of an actual wind-farm local effects and restrictions should always been taken into account. An analytical analysis as shown here is useful to provide insight in the main trends and to develop intuition of the factors that are important for the actual design of wind-farms.

References

[1] P. B. S. Lissaman, *Energy Effectiveness of Arbitrary Arrays of Wind Turbines*, J. of Energy **3**, 323 (1979).

[2] N. O. Jensen, *A note on wind generator interaction*, Risø-M-2411, Risø National Laboratory, Roskilde (1983).

[3] I. Katić, J. Højstrup, and N. O. Jensen, *A simple model for cluster efficiency*, European Wind Energy Association Conference and Exhibition, 7-9 October 1986, Rome, Italy 407 (1986).

[4] J. Choi and M. Shan, *Advancement of Jensen (PARK) wake model*, EWEA Conference, Vienna, February 2013 (2013).

[5] A. Peña and O. Rathmann, *Atmospheric stability-dependent infinite wind-farm models and the wake-decay coefficient*, Wind Energy (2013).

[6] M. Bastankhah and F. Porté-Agel, *A new analytical model for wind-turbine wakes*, Renewable Energy **70**, 116 (2014).

[7] G. Marmidis, S. Lazarou, and E. Pyrgioti, *Optimal placement of wind turbines in a wind park using Monte Carlo simulation*, Renewable Energy **33**, 1455 (2008).

[8] A. Emami and P. Noghreh, *New approach on optimization in placement of wind turbines within wind farm by genetic algorithms*, Renewable Energy **35**, 1559 (2010).

[9] A. Kusiak and Z. Song, *Design of wind farm layout for maximum wind energy capture*, Renewable Energy **35**, 685 (2010).

[10] B. Saavedra-Moreno, S. Salcedo-Sanz, A. Paniagua-Tineo, L. Prieto, and A. Portilla-Figueras, *Seeding evolutionary algorithms with heuristics for optimal wind turbines positioning in wind farms*, Renewable Energy **36**, 2838 (2011).

[11] R. J. A. M. Stevens, D. F. Gayme, and C. Meneveau, *Coupled wake boundary layer model of wind-farms*, Submitted to J. Renewable and Sustainable Energy (2014).

[12] B. G. Newman, *The spacing of wind turbines in large arrays*, Energy conversion **16**, 169 (1977).

[13] N. O. Jensen, *Change of surface roughness and the planetary boundary layer*, Quart. J. R. Met. Soc. **104**, 351 (1978).

[14] S. Frandsen, *On the wind speed reduction in the center of large clusters of wind turbines*, J. Wind Eng. and Ind. Aerodyn. **39**, 251 (1992).

[15] S. Emeis and S. Frandsen, *Reduction of horizontal wind speed in a boundary layer with obstacles*, Bound-Lay. Meteorol. **64**, 297 (1993).

[16] S. Frandsen, R. Barthelmie, S. Pryor, O. Rathmann, S. Larsen, J. Højstrup, and M. Thøgersen, *Analytical modelling of wind speed deficit in large offshore wind farms*, Wind Energy **9**, 39 (2006).

[17] M. Calaf, C. Meneveau, and J. Meyers, *Large eddy simulations of fully developed wind-turbine array boundary layers*, Phys. Fluids **22**, 015110 (2010).

[18] C. Meneveau, *The top-down model of wind farm boundary layers and its applications*, J. of Turbulence **13**, 1 (2012).

[19] R. J. A. M. Stevens, *Dependence of optimal wind-turbine spacing on wind-farm length*, Submitted to Wind Energy (2014).

[20] J. Meyers and C. Meneveau, *Optimal turbine spacing in fully developed wind farm boundary layers*, Wind Energy **15**, 305 (2012).

[21] R. J. A. M. Stevens, D. F. Gayme, and C. Meneveau, *Generalized coupled wake boundary layer model: applications and comparisons with field and LES data for two real wind farms*, Submitted to Wind Energy (2015).