The Worst-case Wind Scenario for Adaptive Robust Unit Commitment Problems

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***Abstract*—In this article, we present a simple deterministic formulation for the unit commitment (UC) problem under the adaptive robust optimization (ARO) approach for the case of wind production uncertainty. We show that the worst-case wind power scenario can be obtained before solving the UC then the ARO-UC problem becomes a simple single-scenario deterministic UC. This avoids the bilinear optimization problem associated with the second-stage dispatch actions in traditional ARO formulations.**

***Index Terms*—Adaptive robust optimization (ARO), unit commitment (UC), wind power production, worst-case scenario.**

I. Introduction

In recent years, higher penetration of variable generation (e.g., wind and solar power) has motivated developments in unit commitment (UC) formulations in order to manage these uncertainties. The adaptive robust optimization (ARO) approach has been recently applied to UC formulations as an alternative to the conventional stochastic optimization (SO) approach, see [1], [2] and references therein for further details.

Traditional ARO-UC formulations require the solution of a mixed-linear integer (MIP) problem together with a bilinear program, which is nonconcave and NP-hard, to obtain the worst-case scenario. In contrast, we show that by taking into account the wind curtailment, the worst-case wind power scenario can be obtained before solving the UC. Therefore, the ARO-UC problem becomes a deterministic MIP. We present two simple examples to illustrate that the commitment solutions (first-stage) obtained with this deterministic UC guarantee a feasible operation (second-stage) of the power system for any realization of wind uncertainty within the uncertainty set.

II. Obtaining the Worst-case Wind Scenario

The two-stage adaptive robust UC seeks to minimize the worst-case dispatch cost considering any possible realization of wind nodal injection w within the uncertainty set $\mathcal{W}$. Here, we present a compact matrix formulation:

$$\begin{aligned} \min_{x} \quad & \left(\mathbf{c}^\top x + \max_{w\in\mathcal{W}} \min_{y(\cdot)} \mathbf{b}^\top y\left(w\right)\right) \\ \text{s.t.} \quad & \mathbf{F}x \le \mathbf{f},\ x \text{ is binary} && (1) \\ & \mathbf{H}y\left(w\right) \le \mathbf{h},\ \forall w \in \mathcal{W} && (2) \\ & \mathbf{A}x + \mathbf{B}y\left(w\right) \le \mathbf{g},\ \forall w \in \mathcal{W} && (3) \end{aligned}$$

where x, y and w are variables and bold letters are parameters. The binary variable x is a vector of commitment related decisions of each generation unit for each time interval over the planning horizon. The continuous variable y is a vector of each unit dispatch decision for each time interval. The continuous variable w is a vector of each uncertain wind nodal injection for each time interval, and the set of uncertainty $\mathcal{W}$ is defined by the continuous interval $[\underline{\mathbf{w}}, \overline{\mathbf{w}}]$.

The objective function is to minimize the sum of commitment cost and worst-case dispatch cost (max-min expression) over the planning horizon. Constraint (1) involves only commitment-related constraints, e.g., minimum up and down. Constraint (2) contains dispatch-related constraints, e.g., energy balance, transmission limit constraints. Constraint (3) couples the commitment and dispatch decisions. The reader is refereed to [3] for a detailed UC formulation.

The above formulation can be rewritten in the following stochastic equivalent form, which is suitable to obtain the worst-case scenario:

$$\begin{aligned} \min_{x,z(\cdot)} \quad & \mathbf{c}^\top x + z\left(y_s\right) \\ \text{s.t.} \quad & z\left(y_s\right) \ge \mathbf{b}^\top y_s,\ \forall s \in \mathcal{S} && (4) \\ & \mathbf{F}x \le \mathbf{f},\ x \text{ is binary} && (5) \\ & \mathbf{H}y \le \mathbf{h},\ \forall s \in \mathcal{S} && (6) \\ & \mathbf{A}x + \mathbf{B}y_s \le \mathbf{g},\ \forall s \in \mathcal{S} && (7) \\ & w_s = \underline{\mathbf{w}} + \boldsymbol{\delta}_s^\top\left(\overline{\mathbf{w}} - \underline{\mathbf{w}}\right),\ \forall s \in \mathcal{S} && (8) \end{aligned}$$

where (4) guarantees that the continuous variable z takes the worst-case dispatch cost from any possible scenario s realization. Although there are infinite possible wind scenario realizations within the interval $[\underline{\mathbf{w}}, \overline{\mathbf{w}}]$, the worst-case wind scenario lies on extreme values $\{\underline{\mathbf{w}}, \overline{\mathbf{w}}\}$, which are the possible vertices of the uncertainty set, as also stated in [1], [2]. Constraint (8) takes into account all these possible vertices by introducing the parameter $\boldsymbol{\delta}_s$, which is a vector containing binary values of each node (where there is wind injection) for each time interval. Therefore, the set $\mathcal{S}$ contains all different binary combinations of $\boldsymbol{\delta}_s$, which is equal to $2^{T\cdot N}$, where T is the number of periods over the planning horizon and N is the number of nodes where there is wind injection.

Notice that w_s does not have any flexibility because (8) imposes that w_s takes a fixed wind realization. However, wind has some flexibility because it can be curtailed. Therefore, what is uncertain is not the wind production range but rather the upper bound of the possible wind dispatch. Consequently, to correctly model this level of wind flexibility, Constraint (8) must constrain the upper bound of w_s instead of imposing a fixed wind realization:

$$w_s \le \underline{\mathbf{w}} + \boldsymbol{\delta}_s\left(\overline{\mathbf{w}} - \underline{\mathbf{w}}\right),\ \forall s \in \mathcal{S} \qquad (9)$$

Notice that in the formulation given by (4)-(7) and (9) there is a unique scenario that is active for all the constraints. By finding this worst-case scenario, we can remove all the others and their associated constraints. Note in (9) that the constraint with the lowest upper bound, $\boldsymbol{\delta}_s = \mathbf{0}$, dominates all the others, hence this is the unique scenario that will be active. Consequently, the formulation (4)-(7) and (9) can be rewritten in function of

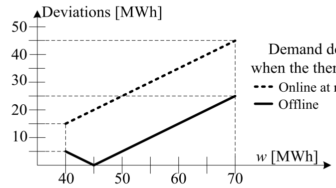
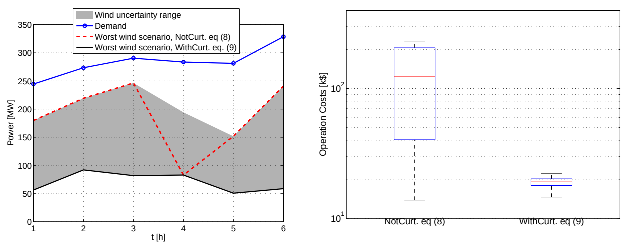


Fig. 1: Demand balance deviations in function of expected wind production.



(a) Demand and wind production

(b) Total operation costs

Fig. 2: Demand and simulation results

the dominant scenario without changing its optimal solution:

$$\min_{x,y} \quad \mathbf{c}^\top x + \mathbf{b}^\top y$$

$$\text{s.t.} \quad \mathbf{F}x \leq \mathbf{f},\ x \text{ is binary} \tag{10}$$

$$\mathbf{H}y \leq \mathbf{h} \tag{11}$$

$$\mathbf{A}x + \mathbf{B}y \leq \mathbf{g} \tag{12}$$

$$w \leq \underline{\mathbf{w}} \tag{13}$$

It is important to highlight that this is a deterministic formulation where the only scenario that is considered is the lowest expected wind $\underline{\mathbf{w}}$ within the uncertainty set. If this formulation has a feasible optimal solution w^* then it guarantees that all other possible wind realizations within the uncertainty set are feasible. That is, all scenarios can become w^* by curtailment. Consequently, all scenarios can be dispatched and, in the worst case, the maximum quantity of wind that can be dispatched for any scenario would be w^*.

In short, $\underline{\mathbf{w}}$ is the worst-case scenario, which ensures feasibility, but it is too conservative because it does not guarantee that wind scenarios above $\underline{\mathbf{w}}$ can be dispatched.

III. Numerical Results and Conclusions

Not allowing wind curtailment in the ARO formulation leads to misleading solutions. To illustrate this, consider the two following examples. The ARO formulations considered here are: 1) the traditional ARO-UC, presented in [1], [2], that does not allows curtailment (4)-(8), and 2) the formulation that allows curtailment (4)-(7) and (9), which are labelled as NotCurt. and WithCurt., respectively.

1) One-period scheduling example: A fixed demand of 45 MWh needs to be supplied by one thermal and one wind generating units. The thermal unit has 20 and 40 MW as min and max generation capacity, respectively. The wind production is within the uncertainty range $[40, 70]$ MWh. To provide the demand, it is necessary to make an energy balance. Fig. 1 shows the demand deviations (shortage/surplus) for every value of wind within its uncertainty range. Notice that for a given value of wind, the deviations are always lower if the thermal unit is offline. Following the ARO approach, where these unbalances are highly penalized [1], [2], the objective is to minimize the maximum penalization (the worst-case scenario). Consequently, the thermal unit will always be offline for this example. Therefore, following the NotCurt. formulation, there will be non-suply energy when the wind production is bellow 45 MWh, which is not acceptable in the electricity sector.

However, it is easy to see that the optimal and satisfactory solution for this example is that the thermal unit produces at its minimum output 20 MWh and the wind production provides 25 MWh thus supplying the demand, the remaining possible wind production would be spilled, thus always guaranteeing the energy supply. In fact, this is the solution that would be achieved by the WithCurt. formulation.

2) Six-bus system: The system in [4] is used for a time span of six periods. Fig. 2a shows the demand and the total wind uncertainty range. All possible vertices of the uncertainty range are considered with 64 wind scenarios (2^6). Similarly to [1], [2], we introduce a penalty cost of 5000 \$/MWh for any power unbalance or transmission capacity violation.

Fig. 2a shows the worst-case wind scenarios found by the two ARO-UC formulations. Notice that, as expected, the worst-case scenario found by WithCurt. was $\underline{\mathbf{w}}$. The commitment solutions of these two formulations are tested by simulating the possible dispatch of the 64 wind scenarios. Fig. 2b shows the total operation costs of the formulations in a box-plot fashion for all scenarios. Similarly to the previous example, the operation costs of NotCurt. are significantly higher because it committed insufficient units in order to dispatch the total wind production of the worst-case scenario (dashed line in Fig. 2a); therefore, this solution incurs in power unbalance and transmission capacity violations in most of the scenarios.

In conclusion, to adequately model wind flexibility, the ARO-UC formulations must take into account the wind curtailment. Otherwise, the solutions may not commit sufficient generating units to supply the demand. By modelling wind curtailment, the worst-case wind power scenario can be obtained before solving the UC. Therefore, the ARO-UC problem becomes a deterministic MIP, thus avoiding the bilinear problem needed by common ARO-UC formulations.

Although the worst-case scenario ensures feasibility, it is too conservative because it does not guarantee that other wind scenarios can be dispatched. However, any UC with different objective function can become more robust by adding it this single-scenario set of constraints. Further research is needed to obtain deterministic formulations, avoiding the bilinear problem, that allows to control the level of conservatism of the ARO solution (similarly to the “budget of uncertainty” [1]).

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