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An MIP Formulation for Joint Market-Clearing of Energy and Reserves Based on Ramp Scheduling

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***Abstract*—The day-ahead Unit-Commitment (UC)-based Market-Clearing (MC) is widely acknowledged to be the most economically efficient mechanism for scheduling resources in power systems. In conventional UC problems, power schedules are used to represent the staircase energy schedule. However, the realizability of this schedule cannot be guaranteed due to the violation of ramping limits, and hence conventional UC formulations do not manage the flexibility of generating units efficiently.**

This paper provides a UC-based MC formulation, drawing a clear distinction between power and energy. Demand and generation are modelled as hourly piecewise-linear functions representing their instantaneous power trajectories. The schedule of generating unit output is no longer a staircase function, but a smoother function that respects all ramp constraints. The formulation represents in detail the operating reserves (online and offline), their time deployment limits (e.g., 15 min), their potential substitution, and their limits according to the actual ramp schedule. Startup and shutdown power trajectories are also modelled, and thus a more efficient energy and reserves schedule is obtained. The model is formulated as a mixed-integer programming (MIP) problem, and was tested with a 10-unit and 100-unit system in which its computational performance was compared with a traditional UC formulation.

***Index Terms*—Mixed-integer programming, operating reserves, startup and shutdown ramps, UC-based market-clearing.**

NOMENCLATURE

Upper-case letters are used for denoting parameters and sets. Lower-case letters denote variables and indexes.

A. Indexes and Sets

$g \in \mathcal{G}$	Generating units, running from 1 to G.
$s \in \mathcal{S}_g$	Startup segments, running from 1 (the hottest) to S_g (the coldest).
$t \in \mathcal{T}$	Hourly periods, running from 1 to T hours.
$\kappa \in \mathcal{K}$	Index for reserve type: $2+$ and $2-$ for secondary up and down; $3+$ and $3-$ for tertiary up and down; 3N$+$ and 3N$-$ for offline tertiary up and down.
$\tau \in \Gamma$	Index for time interval: $15'$ for fifteen minutes, $30'$ for thirty minutes, and op for one hour.

B. Parameters

C_{gt}^{LV}	Linear variable production cost bid [$/MWh].
C_{gt}^{NL}	No-load cost bid [$/h].
C_{gt}^{SD}	Shutdown cost bid [$].
C_{gst}^{SU}	Startup cost bid for starting up at segment s [$].
C_{gt}^{κ}	Cost bid for reserve type κ [$/MW].
D_t	Instantaneous demand at the end of hour t [MW].
D_t^{κ}	System requirements for reserve type κ [MW].
E_{gt}	Energy capacity bid [MWh].
$\overline{P}_g$	Maximum power output [MW].
$\underline{P}_g$	Minimum power output [MW].
P_{gi}^{SD}	Power output at the beginning of the i^{th} interval of the shutdown ramp process [MW], see Fig. 2.
P_{gsi}^{SU}	Power output at the beginning of the i^{th} interval of the startup ramp process type s [MW], see Fig. 2.
$Q_g^{\mathrm{SD}\tau}$	Quick shutdown capability for $\tau \in \{30', op\}$ [MW].
$Q_g^{\mathrm{SU}\tau}$	Quick startup capability for $\tau \in \{30', op\}$ [MW].
R_{gt}^{κ}	Capacity bid for reserve type κ [MW].
RD_g^{τ}	Ramp-down capability for interval τ [MW/min].
RU_g^{τ}	Ramp-up capability for interval τ [MW/min].
SD_g^{D}	Duration of the shutdown process [h], see Fig. 2.
SU_{gs}^{D}	Duration of the startup process type s [h], see Fig. 2.
T_{gs}^{SU}	Time defining the interval limits of the startup segment s, $\left[T_{gs}^{\mathrm{SU}}, T_{g,s+1}^{\mathrm{SU}}\right)$ [h].
TD_g	Minimum down time [h].
TU_g	Minimum up time [h].

C. Decision Variables

e_{gt}	Energy schedule for hour t, excluding energy production during the startup and shutdown processes [MWh].
p_{gt}	Power output schedule at the end of hour t, production above the minimum output [MW].
$\widehat{p}_{gt}$	Total power output schedule at the end of hour t, including startup and shutdown trajectories [MW].
r_{gt}^{κ}	Reserve type κ schedule [MW].
u_{gt}^{κ}	Binary variable which is equal to 1 if the unit is providing up/down offline tertiary reserve ($\kappa \in \{3N+, 3N-\}$) and 0 otherwise.
u_{gt}	Binary variable which is equal to 1 if the unit is producing above $\underline{P}_g$ and 0 otherwise, see Fig. 2.
v_{gt}	Binary variable which takes the value of 1 if the unit starts up and 0 otherwise, see Fig. 2.
w_{gt}	Binary variable which takes the value of 1 if the unit shuts down and 0 otherwise, see Fig. 2.
δ_{gst}	Startup type s. Binary variable which takes the value of 1 if the unit starts up and has been previously down within $\left[T_{gs}^{\mathrm{SU}}, T_{g,s+1}^{\mathrm{SU}}\right)$ hours.

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I. Introduction

A. Motivation

DAY-AHEAD Market-Clearing (MC) is the central mechanism in electricity markets, despite the large variety in market designs across the world. Unit Commitment (UC)-based MC, in which energy and operating reserves are simultaneously cleared, is widely, if not universally, acknowledged to be the most economically efficient way to run day-ahead markets [1], [2]. The UC problem schedules the cheapest resources to supply the demand, while operating the system and units within secure technical limits [1], [3]. Moreover, simultaneous clearing avoids uneconomical out-of-merit operation and mitigates potential market power when hierarchical substitution of reserves is considered [2], [4], [5].

Current day-ahead scheduling practices do not exploit the real flexibility of power systems and could even endanger security of supply. This problem is faced by markets that are (physically) cleared on an hourly basis as well as on a sub-hourly one. An inherent problem of hourly-cleared markets is that they make an (staircase) hourly energy balance between supply and demand rather than matching the instantaneous generating power profiles with the power demand profile. In these kind of markets, generators are penalized if they deviate from their hourly energy schedule. Therefore, units operate by trying to match their power profile with the staircase energy blocks. This staircase behaviour creates large generation gradients at the beginning and at the end of every trading hour, causing large frequency deviations during these time intervals [6], [7]. As a consequence, even in the absence of uncertainty, power system security is being compromised and a significant quantity of operating reserves need to be deployed in real time to maintain the supply-demand balance. A report from The European Network of Transmission System Operators for Electricity (Entso-e) [8] summarizes the operational and economic impacts of this phenomenon on the power system and generating units.

Although sub-hour or real-time markets allow the mitigation of these problems, an inadequate day-ahead schedule may leave real-time markets unprepared to face real-time uncertainties. In fact, some power systems have experienced short-term scarcity events caused by resources with sufficient power capacity but insufficient ramp capability [9]. In response, independent system operators (ISOs) are developing market-based ramping products that will be acquired in day-ahead markets in order to increase real-time dispatch flexibility [9], [10].

In order to better prepare the power system to face real-time uncertainties, day-ahead scheduling approaches are required to efficiently manage power system flexibility by adequately utilizing ramping resources.

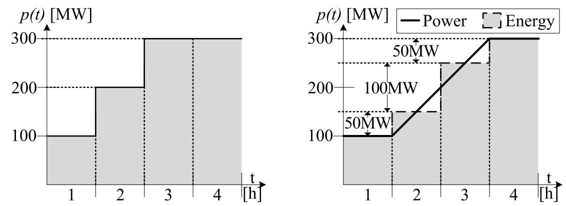
B. Literature Review

1) Inefficient Ramp Management —Energy vs. Power: Conventional day-ahead UC formulations fail to deal with ramp capabilities appropriately. Inefficient ramp management arises from applying ramp-constraints to energy levels or (hourly) averaged generation levels, which is standard practice in traditional UC models [1], [3], [5], [11]. As a result, energy schedules may not be feasible [12]. To illustrate this problem, consider the following scheduling example for one generating unit. This example assumes that the minimum and maximum generation outputs of the unit are 100 MW and 300 MW, respectively, and that the maximum ramp rate is 100 MW/h. As shown in Fig. 1a, if the unit ramps up at its maximum capability and has been producing 100 MW during the first hour, then the expected hourly energy levels for the second and third hours will be 200 MWh and 300 MWh, respectively. However, the unit cannot reach its maximum output before the end of the third hour due to its limited ramp rate, as shown in Fig. 1b. Consequently, the solution obtained in Fig. 1a is not feasible. In fact, the unit requires a ramping capability of 200 MW/h to be able to produce the energy presented in Fig. 1a.

Note that representing the generation in a staircase fashion (energy blocks) may lead to misleading estimations of a system's ramp availability. This in turn could leave the system unprepared to face real-time uncertainties. For example, if the unit in the previous example were actually scheduled to produce the energy profile presented in Fig. 1b then, since the first energy increase is 50 MW (half of the unit's ramp capability), the unit would be erroneously considered to have 50 MW of remaining upward ramp flexibility.

Although it has been proven that delivering the energy schedule obtained from these energy-block formulations may not be feasible [12], insufficient attention has been paid to this issue. Formulations drawing a clear distinction between power and energy have been proposed, guaranteeing that staircase energy schedules can be realized [13]–[15]. In [13] a smooth nonlinear programming problem which does not take into account discrete decisions is proposed (e.g. commitment). The work in [14] presents a formulation with feasible energy delivery constraints, which is further extended in [15], where a sub-hourly UC is formulated. These formulations are focused on feasible energy schedules rather than on matching generating and demand power profiles. In fact, these formulations supply hourly energy demand with power profiles that vary from staircase [15] to oscillating power trajectories [16], which are far from matching the instantaneous power demand forecast. This indiscriminate use of ramping resources from the scheduling stage does not permit the effective management of the system ramp capabilities to face real-time uncertainties. In addition, the formulations do not model operating reserves.

2) Startup and Shutdown Power Trajectories: Conventional day-ahead UC models assume that units start/end their production at their minimum output. That is, UC models ignore the intrinsic startup (SU) and shutdown (SD) power trajectories of thermal units. Consequently, there is an increasing amount of energy that is not being allocated by day-ahead scheduling approaches because, first, units provide energy (and ramp) during the SU and SD processes, affecting the total load balance; and second, thermal units are being shut down and started up more often due to the increasing penetration of variable generation [17]. As a result, there is an inefficient deployment in real time of resources that are required to accommodate the power trajectories that were ignored in the day-ahead schedule, so that the balance between supply and demand is maintained



(a) Traditional Energy Schedule (b) Actual Deployment

Fig. 1: Scheduling vs. Deployment

[18]. Furthermore, as discussed in [19], ignoring these power trajectories can significantly change commitment decisions, which in turn increases operating costs. Recent papers indicate an awareness of the importance of the SU and SD processes [20]–[22]. However, SU and SD power trajectories continue being ignored because the resulting model will supposedly be considerably more complex, and thus lead to prohibitive solving times.

An adequate day-ahead schedule not only must take into account these SU and SD power trajectories, but also must optimally schedule them to avoid the aforementioned drawbacks.

3) Reserve Modelling: Another drawback of conventional UC-based MC formulations is related to the accuracy of reserve modelling. Reserves must be scheduled on the basis of their required time deployment (e.g. 15 min) and not as an hourly requirement, as has been commonly modelled [1], [5], [11]. The formulations presented in [4], [23] and [24] guarantee possible reserve deployments in a few minutes, although these models are on an hourly-basis. However, they do not consider the real reserve availability of a unit which depends on its actual ramp schedule.

A correct modelling of ramp constraints, which must be applied to power trajectories, is then required to guarantee the execution of the power schedules and correctly represent the real availability of operating reserves at any moment within the hour.

For further details of the drawbacks of conventional UC-based scheduling approaches, the reader is referred to [18].

C. Ramp-Based Scheduling Approach: An Overview

This paper proposes a day-ahead UC-based MC formulation in which the operating ramping of generators is optimally scheduled to supply an instantaneous power demand forecast. In addition, the formulation guarantees that operating reserves can be deployed in a given (required) time. The formulation is represented as a mixed-integer programming (MIP) problem. MIP is becoming widely used in the electricity sector due to significant improvements on MIP solvers [25].

The proposed formulation draws a clear distinction between power and energy. Ramp constraints are thus applied on power trajectories rather than on energy blocks, which is a common drawback of conventional UC formulations [1], [3], [5], [11]. Power production and demand are modelled as an hourly piecewise-linear function representing their instantaneous power trajectories. This overcomes the disadvantages of power-based scheduling models [13]–[15] by having the clear objective of matching the instantaneous power demand with the total power generation profile, thus avoiding an indiscriminate use of ramping resources in the scheduling stage.

Unlike previous works that have modelled reserves [1], [3], [5], [11], [23], [24], the proposed formulation provides the actual ramp schedules,and thus defines the available ramp capability that can be used to provide reserves. Although the formulation is based on time periods of one hour, it also guarantees that reserves can be deployed within the time requirements (a few minutes) imposed by the reliability authorities (for each type of reserve) [26].

In addition, the formulation considers SU and SD power trajectories, thus avoiding power discontinuities in the scheduling stage which result in an inefficient deployment of resources in the real-time operation.

The proposed formulation would help ISOs to draw up an efficient day-ahead ramp resources schedule in order to better prepare the system to face real-time uncertainties. For the case of an hourly-cleared market (such as those in Europe), if the proposed approach were followed, generating units would be penalized if they deviated within the hour from the scheduled power trajectory. As a result, in comparison with the staircase approach, the aggregated generation would better fit the power demand. This strategy would avoid large frequency deviations at the hour limits and the unnecessary reserve use caused by the mismatch between supply and demand. In addition, power systems with real-time markets would be better prepared to face real-time if their day-ahead schedules followed piecewise power profiles rather than staircase energy blocks. This is because, in comparison with the conventional staircase scheduling approach, the scheduled power profiles would be a better approximation of the units' real production and the optimal ramp scheduling would correctly estimate the ramp availability of power systems.

This paper is focused on scheduling quantities, and the problem of determining the prices that will allow generators to recover their non-convex costs, is beyond the scope of this work. However, a pricing mechanism for a multi-part bidding with different commodities [27], such as startup and shutdown costs, can be applied directly. It is important to highlight that the proposed ramp-based approach presents great challenges in terms of market design. Both the definition of a proper pricing mechanism that copes with continuous power profiles and the consideration of demand bids expressed as continuous functions are some examples that require further research. Nevertheless, the ideas presented in this paper have potential for broad applications, such as for reliability UC, which guarantee the feasibility of the scheduling obtained after forward markets have been cleared [9], [10]. Finally, for the sake of simplicity and without loss of generality, transmission constraints are not considered in this paper.

D. Contributions

The principal contributions of this paper are as follows:

1) A day-ahead UC-based MC formulation is proposed in which the total power generation follows the instantaneous power profile of the demand forecast. This is

achieved by taking into account piecewise-linear power-trajectories instead of staircase energy-blocks, and also scheduling the SU and SD power trajectories of thermal units.

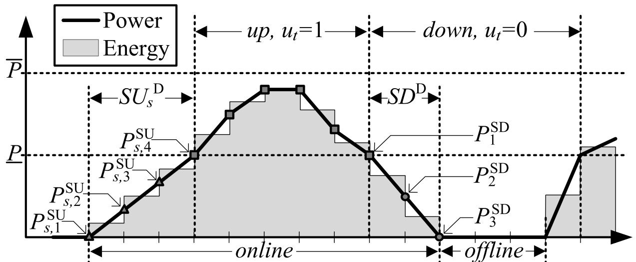
2) The actual reserve availability is accurately defined, based on the units' ramp schedules. The formulation takes into account different ramp-rate limits, and it guarantees that reserves can be deployed within their different time requirements. Consequently, the reserve capabilities of a system are optimally scheduled, taking a better advantage of units' flexibility.
3) The core of the proposed MIP formulation is built upon the tight and compact formulations presented in [19] and [28], thus taking advantage of their mathematical properties. These formulations reinforce the convergence speed by reducing the search space (tightness) and at the same time by increasing the searching speed with which solvers explore that reduced space (compactness). That is, the formulations are simultaneously tight and compact. If compared with a traditional UC formulation, with no SU an SD ramps and representing a single reserve type, the proposed formulation involves a low computational burden and solving times were even decreased when a large study case was carried out.

E. Paper Organization

The remainder of this paper is organized as follows. Section II details the mathematical formulation of different operating reserves (secondary, tertiary online and tertiary offline) and their links with the ramp schedules. Section III presents some numerical examples as well as a comparison with a conventional UC. Finally, concluding remarks are made in Section IV.

II. Proposed Approach

This section details the mathematical formulation of the proposed unit-commitment (UC)-based market-clearing approach. This paper models secondary and tertiary reserves using European standards as a benchmark [26]. The up/down reserve provided by a generating unit is defined as the amount of power that the unit can increase/decrease over its scheduled power output within a time limit. Secondary up (r_{gt}^{2+}) and down (r_{gt}^{2-}) reserves are provided by online units that respond to a continuous automatic generation control (AGC). The secondary reserve must be fully available within 15 min. Tertiary reserve is composed of online up (r_{gt}^{3+}) and down (r_{gt}^{3-}) reserves, as well as offline up ($r_{gt}^{3\mathrm{N}+}$) and down ($r_{gt}^{3\mathrm{N}-}$) reserves. The tertiary reserve is manually activated by ISOs and it is used to release the secondary reserve or prevent its activation. After being called, the tertiary reserve must be fully available within 30 min. Although the formulation follows these time deployments, the adaptation to US standards [29] is straightforward. For example, the 10-min spinning reserve can be modelled in the same way as the (15-min) secondary reserves by simply modifying the parameters established for the time deployments.



t [h]	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
u_t	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	1
v_t	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1
w_t	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0

Fig. 2: Unit operation states, including SU and SD power trajectories

The formulation takes into account different ramp limits to model different reserve time deployments. These limits change depending on the duration of the ramping process, i.e. the shorter a sustained ramping process, the larger the ramp limits without shortening the rotor life [30]. For the sake of simplicity and without loss of generality, ramp-rate limits are considered to be constant during the unit's *up* state; however, the formulation can be further extended to deal with dynamic ramps [31].

The first part of this section presents the general formulation. The second part describes how to obtain the ramp-capability and power-capacity constraints using the proposed ramp-based scheduling approach. The following two parts are devoted to modelling the reserve constraints for slow- and quick-start units, respectively. Finally, the last subsection lists some specific characteristics that make the formulation computationally efficient.

A. General Formulation

In order to obtain computational advantage, the generation output above and bellow $\underline{P}$ is managed independently [28]. This also facilitates the inclusion of SU and SD power trajectories in the model [19]. Therefore, the *up* and *down* states are distinguished from the *online* and *offline* states, as shown in Fig. 2. The unit is *online* when providing energy to the system and *offline* otherwise. During the *up* period, the unit has the flexibility to follow any trajectory being limited by its power-capacity and ramp-capability limits. Consequently, the unit can only provide reserves when it is *up*. On the other hand, the unit's power output follows a predefined power trajectory when it is starting up or shutting down. The SU power trajectory depends on the unit's previous down time, unlike the SD process.

1) Objective Function: The objective of the MC is to procure energy and reserves at the minimum cost:

$$\min \sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}}\Bigg[\sum_{\kappa\in\mathcal{K}} C_{gt}^{\kappa} r_{gt}^{\kappa} + \sum_{s\in\mathcal{S}_g} C_{gst}^{\mathrm{SU}}\delta_{gst} + C_{gt}^{\mathrm{SD}} w_{gt} + C_{gt}^{\mathrm{NL}} u_{gt} + C_{gt}^{\mathrm{LV}} e_{gt}\Bigg]. \quad (1)$$

Note that the startup cost C_{gst}^{SU} includes the energy spent by two different actions: first, the energy required to bring the thermal unit online, which does not result in any MW generation [32]; second, the cost of the energy that is provided to the system during the SU process, i.e., the energy which is produced until the unit achieves its minimum output, *up* state. Both the cost of bring the unit online and the duration of the SU ramp, depend on how long the unit has been down [19]. Similarly, the C_{gt}^{SD} includes the cost of the energy provided to the system during the SD ramp process.

2) Power System Requirements: The power system requirements for demand and reserves are presented as follows:

$$\sum_{g\in\mathcal{G}} \widehat{p}_{gt} = D_t \qquad \forall t \quad (2)$$

$$\sum_{g\in\mathcal{G}} r_{gt}^{2+} \geq D_t^{2+} \qquad \forall t \quad (3)$$

$$\sum_{g\in\mathcal{G}} r_{gt}^{2-} \geq D_t^{2-} \qquad \forall t \quad (4)$$

$$\sum_{g\in\mathcal{G}} \left[r_{gt}^{2+} + r_{gt}^{3+} + r_{gt}^{3\mathrm{N}+}\right] \geq D_t^{3+} + D_t^{2+} \qquad \forall t \quad (5)$$

$$\sum_{g\in\mathcal{G}} \left[r_{gt}^{2-} + r_{gt}^{3-} + r_{gt}^{3\mathrm{N}-}\right] \geq D_t^{3-} + D_t^{2-} \qquad \forall t. \quad (6)$$

The demand balance in (2) is calculated at the end of hour t. Note that the energy balance for the whole hour is automatically achieved by satisfying the power demand at the beginning and end of each hour, and by considering a piecewise-linear power profile for demand and generation. Constraints (3) and (4) represent the supply of up and down secondary reserves. The constraints satisfying the tertiary reserve requirements, (5) and (6), also consider the substitution of a higher quality reserve for a lower quality reserve [2], [4], [22], [24]. In other words, the secondary reserves can technically substitute tertiary reserves as long as this reduces the total procurement costs.

3) Commitment Logic and Minimum Up/Down Times: The relation between the commitment, startup and shutdown variables is presented in (7). Constraints (8) and (9) ensure the minimum up and down times respectively [33].

$$u_{gt} - u_{g,t-1} = v_{gt} - w_{gt} \qquad \forall g, t \quad (7)$$

$$\sum_{i=t-TU_g+1}^{t} v_{gi} \leq u_{gt} \qquad \forall g, t \in [TU_g, T] \quad (8)$$

$$\sum_{i=t-TD_g+1}^{t} w_{gi} \leq 1 - u_{gt} \qquad \forall g, t \in [TD_g, T] \quad (9)$$

where the minimum up/down constraints ensure that a unit cannot start up and shut down simultaneously. Note that (8) and (9) guarantee (dominate over) the inequalities $v_{gt} \leq u_{gt}$ and $u_{gt} \leq 1 - w_{gt}$ respectively which, combined, become $v_{gt} + w_{gt} \leq 1$. In addition, given that u_{gt} is defined as a binary variable, (7) forces v_{gt} and w_{gt} to take binary values, even if they are defined as continuous.

4) Selection of SU type: The SU type and the SU and SD power trajectories are obtained using the tight and compact formulation proposed in [19], which considerably reduces the computational burden in comparison with analogous formulations commonly found in the literature. The SU type is selected with

$$\delta_{gst} \leq \sum_{i=T_{gs}^{\mathrm{SU}}}^{T_{g,s+1}^{\mathrm{SU}}-1} w_{g,t-i} \qquad \forall g, s\in[1, S_g), t\in\left[T_{g,s+1}^{\mathrm{SU}}, T\right] \quad (10)$$

$$\sum_{s\in\mathcal{S}_g} \delta_{gst} = v_{gt} \qquad \forall g, t \quad (11)$$

where (10) allows that the startup segment s can be selected ($\delta_{gst} \leq 1$) if the unit has been previously down within $\left[T_{gs}^{\mathrm{SU}}, T_{g,s+1}^{\mathrm{SU}}\right)$ hours. Constraint (11) forces the selection of a unique SU type if the unit actually starts up.

As discussed in [19] and [28], the variables δ_{gst} take binary values even if they are defined as continuous. This is due to the tightness characteristic of the startup-cost formulation. Note that (10) is not defined for the first hours. See [19] for details of how the initial conditions define δ_{gst} for these first hours.

5) Total Power Output: Although all units' technical constraints are applied to the output variable p_{gt}, which is production above $\underline{P}_g$, the total power production $\widehat{p}_{gt}$ is needed to satisfy the power demand (2).

As presented in [19], the total power output including the SU and SD power trajectories for slow-start units is obtained with:

$$\begin{aligned}\widehat{p}_{gt} =& \underbrace{\sum_{s=1}^{S_g}\sum_{i=1}^{SU_{gs}^{\mathrm{D}}} P_{gsi}^{\mathrm{SU}} \delta_{gs,\left(t-i+SU_{gs}^{\mathrm{D}}+2\right)}}_{\text{(iii) SU trajectory}} + \underbrace{\sum_{i=2}^{SD_g^{\mathrm{D}}+1} P_{gi}^{\mathrm{SD}} w_{g,(t-i+2)}}_{\text{(ii) SD trajectory}} \\ &+ \underbrace{\underline{P}_g\left(u_{gt} + v_{g,t+1}\right) + p_{gt}}_{\text{(i) Output when being } up} \qquad \forall g, t. \quad (12)\end{aligned}$$

For a better understanding of this constraint, we can analyse how the power trajectory example in Fig. 2 is obtained from the three different parts in (12):

1) Output when the unit is *up*: Although the unit is *up* for five consecutive hours, there are six total power values, from $\widehat{p}_{g,4}$ to $\widehat{p}_{g,9}$, greater than or equal to $\underline{P}_g$ (see the squares in Fig. 2). When $t=4$, the term $v_{g,t+1}$ in (i) becomes $v_{g,5}$ ensuring (the first) $\underline{P}_g$ at the beginning of the up period, and the term u_{gt} adds (the remaining five) $\underline{P}_g$ for $t = 5 \ldots 9$. In addition, p_{gt} adds the power production above $\underline{P}_g$.
2) SD power trajectory: This process lasts for two hours, $SD_g^{\mathrm{D}} = 2$; then, the summation term (ii) becomes $P_{g,2}^{\mathrm{SD}} w_{gt} + P_{g,3}^{\mathrm{SD}} w_{g,t-1}$, which is equal to $P_{g,2}^{\mathrm{SD}}$ for $t=10$ and $P_{g,3}^{\mathrm{SD}}$ for $t=11$, being zero otherwise. This provides the SD power trajectory (see the circles in Fig. 2).
3) SU power trajectory: the SU power trajectory can be obtained using a procedure similar to that used in 2) (see the triangles in Fig. 2). The possible SU trajectory is given by the chosen segment s (see Section II-A4), which depends on how long the unit has been down.

6) Energy Schedule: The energy produced by a unit during the *up* state, following an hourly piecewise-linear power profile, is obtained with:

$$e_{gt} = \underline{P}_g u_{gt} + \frac{p_{g,t-1} + p_{gt}}{2} \qquad \forall g, t. \quad (13)$$

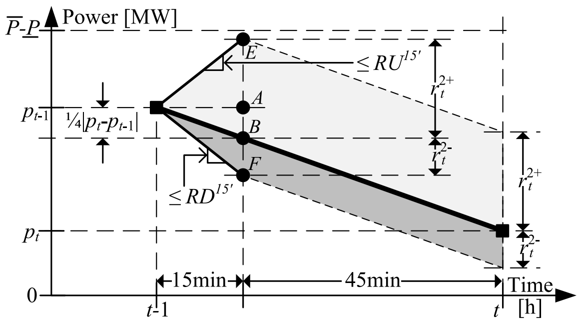


Fig. 3: Relation between secondary reserves, power trajectory and ramps

This energy is used to represent the unit's production cost during the *up* state in (1). The energy produced during the SU and SD processes is internalized in the SU and SD costs, as discussed in Section II-A1. The total energy schedule can easily be calculated using $\widehat{p}_{gt}$ after the optimization problem is solved.

7) Operating ramps: The traditional ramp constraints for the unit operation are presented as follows:

$$-60RD_g^{op} \leq p_{gt} - p_{g,t-1} \leq 60RU_g^{op} \qquad \forall g,t \quad (14)$$

B. Obtaining the Reserve Constraints

This subsection is made for illustrative purposes in order to aid understanding of how the ramping and capacity constraints are derived. For the sake of simplicity only secondary reserves are considered here. The complete formulation also taking into account the tertiary reserves is presented in Sections II-C and II-D for slow- and quick-start units, respectively. In other words, the equations (1)-(14) together with (21)-(45) provide the complete formulation that is proposed in this paper. A formulation that only models secondary reserve, ignoring online and offline tertiary reserves, is described by (1)-(14) together with (15)-(20).

1) Ramping Limits: The up (down) secondary reserve provided by a generating unit is the amount of power that the unit can increase (decrease) over its scheduled power output within 15 minutes. Therefore, as observed in Fig. 3, the segment *EB* (*BF*) defines the up (down) secondary reserve, which is the power above (below) the scheduled power output level *B*. The following constraints ensure that the unit has the ramp capability to provide up (*EB*) and down (*BF*) secondary reserves:

$$\underbrace{\underbrace{\tfrac{1}{4}\left(p_{gt} - p_{g,t-1}\right)}_{BA} + \underbrace{r_{gt}^{2+}}_{EB}}_{EA} \leq 15RU_g^{15'} \qquad \forall g,t \quad (15)$$

$$\underbrace{\underbrace{-\tfrac{1}{4}\left(p_{gt} - p_{g,t-1}\right)}_{AB} + \underbrace{r_{gt}^{2-}}_{BF}}_{AF} \leq 15RD_g^{15'} \qquad \forall g,t. \quad (16)$$

As shown in Fig. 3, when the unit is ramping up, the 15-min ramp excursion resulting from the scheduled power trajectory (*BA*) and the down up secondary reserve (*EB*) cannot exceed the 15-min ramp capability (15). Similarly, when the unit is ramping down, the 15-min ramp excursion due to the scheduled power trajectory (*AB*) plus the down secondary reserve (*BF*) cannot exceed the 15-min ramp capability (16)

As shown in Fig. 3, due to the hourly piecewise-linear power profile, the ramp excursion of the power trajectory during a 15 min period is a quarter of that obtained during an hour.

The reserve that is available within one hour depends directly on the unit power trajectory during that hour. For example, the up (down) secondary reserve availability increases (decreases) if the scheduled power is ramping down. This is the case in Fig. 3, where the up secondary reserve (*EB*) can even be greater than the 15-min ramp rate limit.

2) Capacity limits: The reserve interval (grey areas in Fig. 3) must not exceed the unit's capacity limits at the end of the hour:

$$p_{gt} + r_{gt}^{2+} \leq \left(\overline{P}_g - \underline{P}_g\right)\left(u_{gt} - w_{g,t+1}\right) \qquad \forall g,t \quad (17)$$

$$p_{gt} - r_{gt}^{2-} \geq 0 \qquad \forall g,t. \quad (18)$$

Constraint (17) also guarantees that the unit is at the minimum output $\underline{P}_g$ at the instant when the SU (SD) power trajectory finishes (starts), thus connecting the production above $\underline{P}_g$ with the SU (SD) power trajectory, as discussed in Section II-A5. This can be observed in the example presented in Fig. 2, where (17) makes p_{gt} equal to zero at the end of hours 4 ($p_{g,4} = 0$) and 9 ($p_{g,9} = 0$), which are the beginning and end of the *up* state period respectively.

It is important to note that (17) and (18) do not ensure that the unit operates within its capacity limits during the whole hour. When the unit is ramping down (up), the unit can violate its maximum (minimum) power limit at minute 15, as indicated with point *E* (*F*) in Fig. 3. This problem is avoided by ensuring that point *E* is below the maximum power limit (19) and *F* is above the minimum (20).

$$\underbrace{\underbrace{\tfrac{1}{4}p_{gt} + \tfrac{3}{4}p_{g,t-1}}_{B} + \underbrace{r_{gt}^{2+}}_{EB}}_{E} \leq \overline{P}_g - \underline{P}_g \qquad \forall g,t \quad (19)$$

$$\underbrace{\underbrace{\tfrac{1}{4}p_{gt} + \tfrac{3}{4}p_{g,t-1}}_{B} - \underbrace{r_{gt}^{2-}}_{BF}}_{F} \geq 0 \qquad \forall g,t. \quad (20)$$

In short, secondary reserves can be provided at any time within the hour by guaranteeing that the reserve interval (grey areas in Fig. 3) does not exceed the ramp-capability and power-capacity limits at the end of the hour and at minute 15.

C. Secondary and Tertiary Reserves for Slow-Start Units

The complete formulation for secondary and tertiary reserves is presented in this subsection. The formulation guarantees a simultaneous or independent (either secondary or tertiary) reserve deployment. All equations are derived in a similar fashion to the constraints presented in Section II-B.

1) Ramping Limits: The simultaneous deployment of secondary and tertiary reserves cannot exceed the unit ramping limits. The following constraints ensure that the unit operates

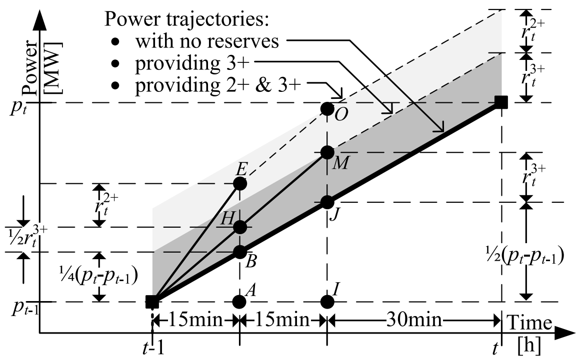


Fig. 4: Relation between upward reserves, power trajectory and ramps

within its 30-min ramp limits

$$\underbrace{\underbrace{\tfrac{1}{2}p_{gt}-\tfrac{1}{2}p_{g,t-1}}_{JI}+\underbrace{r^{3+}_{gt}}_{MJ}}_{MI} \leq 30RU^{30'}_{g} \qquad \forall g,t \quad (21)$$

$$-\tfrac{1}{2}p_{gt}+\tfrac{1}{2}p_{g,t-1}+r^{3-}_{gt} \leq 30RD^{30'}_{g} \qquad \forall g,t \quad (22)$$

and the operation within the unit's 15-min ramp limits are ensured with

$$\underbrace{\underbrace{\tfrac{1}{4}p_{gt}-\tfrac{1}{4}p_{g,t-1}}_{BA}+\underbrace{\tfrac{1}{2}r^{3+}_{gt}}_{HB}+\underbrace{r^{2+}_{gt}}_{EH}}_{EA} \leq 15RU^{15'}_{g} \qquad \forall g,t \quad (23)$$

$$-\tfrac{1}{4}p_{gt}+\tfrac{1}{4}p_{g,t-1}+\tfrac{1}{2}r^{3-}_{gt}+r^{2-}_{gt} \leq 15RD^{15'}_{g} \qquad \forall g,t. \quad (24)$$

As shown in Fig. 4, the 30-min ramp excursion due to the scheduled power trajectory (*JI*) plus the up tertiary reserve (*MJ*) cannot exceed the 30-min ramp rate limit (21). Similarly, the 15-min ramp excursion due to the scheduled power trajectory (*BA*), plus the possible 15-min ramp excursion due to up tertiary reserve (*HB*), plus the up secondary reserve (*EH*) cannot exceed the 15-min ramp rate limit (23). Analogously to these constraints, down reserve limits, (22) and (24), can be easily obtained.

Note that if all ramp limits are the same $RU^{op}_{g}=RU^{30'}_{g}=RU^{15'}_{g}$ and $RD^{op}_{g}=RD^{30'}_{g}=RD^{15'}_{g}$, then the 15-min ramp constraints (23)-(24) dominate over the 30-min (21)-(22) and one-hour (14) constraints. Consequently, although (21)-(22) and (14) would not be necessary, these constraints take advantage of the different units' ramp limits. To illustrate how this formulation works with different ramping limits, $RU^{15'}_{g}>RU^{30'}_{g}>RU^{op}_{g}$, we can analyse the upwards reserve deployment for the following example. Consider that unit g presents a zero ramping excursion during a given hour t, then $p_{gt}-p_{g,t-1}=0$ and thus (14) is automatically satisfied. Constraint (23) now ensures $\frac{1}{2}r^{3+}_{gt}+r^{2+}_{gt}\leq 15RU^{15'}_{g}$, then we have the two extreme feasible solutions $r^{2+}_{gt}=15RU^{15'}_{g}, r^{3+}_{gt}=0$ and $r^{2+}_{gt}=0, r^{3+}_{gt}=30RU^{15'}_{g}$. The former solution does not violate the ramp limits, but the latter implies that the unit may operate 30 minutes at 15-min ramp rate which clearly violates the 30-min ramp limit. Therefore, (21) is necessary to ensure that deploying r^{3+}_{gt} does not violate the unit's 30-min ramp limit $30RU^{30'}_{g}$.

2) Capacity Limits: The following constraints ensure that the reserve intervals remain within the power capacity limits at the end of the hour:

$$p_{gt}+r^{2+}_{gt}+r^{3+}_{gt} \leq \left(\overline{P}_{g}-\underline{P}_{g}\right)\left(u_{gt}-w_{g,t+1}\right) \qquad \forall g,t \quad (25)$$

$$p_{gt}-r^{2-}_{gt}-r^{3-}_{gt} \geq 0 \qquad \forall g,t \quad (26)$$

As discussed in Section II-B, these capacity limits at the end of the hour (25)-(26) do not guarantee that the unit operates within its capacity limits during the whole hour. Note in Fig. 4 that either the point O or the point E may exceed the maximum power limit when the unit is ramping down. Therefore, (27) and (28) are needed to keep the points O and E below the maximum power limit:

$$\underbrace{\underbrace{\tfrac{1}{2}p_{gt}+\tfrac{1}{2}p_{g,t-1}}_{J}+\underbrace{r^{2+}_{gt}}_{OM}+\underbrace{r^{3+}_{gt}}_{MJ}}_{O} \leq \overline{P}_{g}-\underline{P}_{g} \qquad \forall g,t \quad (27)$$

$$\underbrace{\underbrace{\tfrac{1}{4}p_{gt}+\tfrac{3}{4}p_{g,t-1}}_{B}+\underbrace{r^{2+}_{gt}}_{EH}+\underbrace{\tfrac{1}{2}r^{3+}_{gt}}_{HB}}_{E} \leq \overline{P}_{g}-\underline{P}_{g} \qquad \forall g,t. \quad (28)$$

Analogously, (29) and (30) ensure that the unit is always producing above its minimum:

$$\tfrac{1}{2}p_{gt}+\tfrac{1}{2}p_{g,t-1}-r^{2-}_{gt}-r^{3-}_{gt} \geq 0 \qquad \forall g,t \quad (29)$$

$$\tfrac{1}{4}p_{gt}+\tfrac{3}{4}p_{g,t-1}-r^{2-}_{gt}-\tfrac{1}{2}r^{3-}_{gt} \geq 0 \qquad \forall g,t. \quad (30)$$

Finally, apart from keeping the units' energy and reserve within their technical limits, the formulation must also constrain energy and reserves by the bidding limits:

$$0 \leq r^{\kappa}_{gt} \leq R^{\kappa}_{gt} \qquad \forall \kappa,g,t \quad (31)$$

$$0 \leq e_{gt} \leq E_{gt} \qquad \forall \kappa,g,t \quad (32)$$

where the energy bid E_{gt} should be greater than or equal to $\underline{P}_{g}$, so that the unit can be committed.

In conclusion, constraints (14) and (21)-(32) guarantee that the unit can provide simultaneously (or independently) secondary and tertiary reserves at any time within the hour without violating its technical and bidding limits (ramp capability and power capacity).

D. Secondary and Tertiary Reserves for Quick-Start Units

Unlike the slow-start units, the quick-start units can ramp up (down) from 0 (more than $\underline{P}_{g}$) to more than $\underline{P}_{g}$ (0) within one hour. This makes them technically capable of providing offline tertiary reserves. Similarly to (12), which includes the SU and SD trajectories for slow-start units, (33) presents the total power output for quick-start units.

$$\widehat{p}_{gt} = \underline{P}_{g}u_{gt}+p_{gt} \qquad \forall g,t. \quad (33)$$

1) Up and Down Offline Tertiary Reserves: Due to the minimum power output $\underline{P}_{g}$, the offline up reserve that is scheduled must be above $\underline{P}_{g}$ and below the 30-min quick-SU power capability of the unit, as presented in (34). Models commonly found in the literature fail to capture this technical characteristic ($\underline{P}_{g}$) when modelling offline (or non-spinning) reserves. Similarly, the offline down reserve must be between

$\underline{P}_g$ and the 30-min quick-SD capability, as is shown in (35). We consider the tertiary offline down reserve as the down reserve that involves the shut down of the unit.

$$\underline{P}_g u_{gt}^{3N+} \leq r_{gt}^{3+} \leq Q_g^{SU30'} u_{gt}^{3N+} \qquad \forall g,t \quad (34)$$

$$\underline{P}_g u_{gt}^{3N-} \leq r_{gt}^{3-} \leq Q_g^{SD30'} u_{gt}^{3N-} \qquad \forall g,t \quad (35)$$

Constraint (36) ensures that the unit can provide offline up reserves if the unit is down but not shutting down, and (37) ensures that the unit must be up but not starting up to provide the offline down reserve.

$$u_{gt}^{3N+} + u_{gt} + w_{gt} \leq 1 \qquad \forall g,t \quad (36)$$

$$u_{gt}^{3N-} - u_{gt} + v_{gt} \leq 0 \qquad \forall g,t \quad (37)$$

Although two binary variables are needed to deal with offline tertiary reserves, one of them is always fixed by u_{gt}. If $u_{gt}=0$, then (37) implies $u_{gt}^{3N-}=0$, and when $u_{gt}=1$, then (36) makes $u_{gt}^{3N+}=0$.

2) Capacity Limits: To provide the offline down reserve for a given hour, the unit must be operating below the 30-min SD capability during that hour. This is ensured by the upper limit constraints of the unit at the beginning of the hour (38), at the end (39):

$$p_{gt}+r_{gt}^{2+}+r_{gt}^{3+} \leq \left(\overline{P}_g-\underline{P}_g\right) u_{gt}-\left(\overline{P}_g-Q_g^{SD30'}\right)u_{g,t+1}^{3N-} -\left(\overline{P}_g-Q_g^{SU}\right)v_{gt}-\left(\overline{P}_g-Q_g^{SD}\right)w_{g,t+1} \qquad \forall g,t \quad (38)$$

$$p_{gt}+r_{gt}^{2+}+r_{gt}^{3+} \leq \overline{P}_g-\underline{P}_g-\left(\overline{P}_g-Q_g^{SD30'}\right)u_{g,t}^{3N-} \qquad \forall g,t \quad (39)$$

and at minute 30 (40) and 15 (41):

$$\tfrac{1}{2}p_{gt}+\tfrac{1}{2}p_{g,t-1}+r_{gt}^{2+}+r_{gt}^{3+} \leq \overline{P}_g-\underline{P}_g-\left(\overline{P}_g-Q_g^{SD30'}\right)u_{gt}^{3N-} \qquad \forall g,t \quad (40)$$

$$\tfrac{1}{4}p_{gt}+\tfrac{3}{4}p_{g,t-1}+r_{gt}^{2+}+\tfrac{1}{2}r_{gt}^{3+} \leq \overline{P}_g-\underline{P}_g-\left(\overline{P}_g-Q_g^{SD30'}\right)u_{gt}^{3N-} \qquad \forall g,t \quad (41)$$

Finally, the total power output must be greater than the summation of all downward reserves. This is guaranteed in the lower limit constraints of the unit at the beginning of the hour (42), at the end (43):

$$p_{g,t-1}-r_{g,t-1}^{2-}-r_{g,t-1}^{3-}-\left(r_{gt}^{3N-}-\underline{P}_g u_{gt}^{3N-}\right)\geq 0 \qquad \forall g,t \quad (42)$$

$$p_{gt}-r_{gt}^{2-}-r_{gt}^{3-}-\left(r_{gt}^{3N-}-\underline{P}_g u_{gt}^{3N-}\right)\geq 0 \qquad \forall g,t \quad (43)$$

and at minutes 30 (44) and 15 (45):

$$\tfrac{1}{2}p_{gt}+\tfrac{1}{2}p_{g,t-1}-r_{gt}^{2-}-r_{gt}^{3-}-\left(r_{gt}^{3N-}-\underline{P}_g u_{gt}^{3N-}\right)\geq 0 \qquad \forall g,t \quad (44)$$

$$\tfrac{1}{4}p_{gt}+\tfrac{3}{4}p_{g,t-1}-r_{gt}^{2-}-\tfrac{1}{2}r_{gt}^{3-}-\left(r_{gp}^{3N-}-\underline{P}_g u_{gt}^{3N-}\right)\geq 0 \qquad \forall g,t \quad (45)$$

E. Computational Efficiency

The computational performance of an MIP formulation depends mainly on its tightness (distance between relaxed and integer solutions) and compactness (quantity of data to process), as stated in the literature of integer programming [34], [35]. The full exploitation of these two characteristics has meant a breakthrough in off-the-shelf MIP solvers (through cutting planes and root presolve) [36], [37].

The core of the proposed MIP formulation is built upon the tight and compact formulations presented in [19] and [28], and thus takes advantage of these mathematical properties. Although detailing the mathematical properties of the proposed formulation is beyond the scope of this paper, some specific aspects are worth mentioning to aid the understanding of its computational efficiency:

1) The number of binary variables is a very poor indicator of the difficulty of an MIP model [34], [35]. Increasing the number of binary variables, as in the case of the proposed formulation, is actually used as a tightening strategy [35]. See [19] and [28] for further details. In addition, the variables v_{gt}, w_{gt} and δ_{gst} can be defined as continuous because the formulation (tightness of the model) forces them to take binary values. Therefore, declaring these variables as binary does not increase the combinatorial complexity and allow MIP solvers to use powerful strategies that exploit their integrality characteristic [28], [35], [36].
2) The only binary variable that is actually needed for slow-start units is u_{gt}. On the other hand, the quick-start units require two extra binary variables u_{gt}^{3N+} and u_{gt}^{3N-}. However, one of them is always fixed by u_{gt} (see Section II-D1). In the worst case, they only add the complexity of one-single binary variable. In any case, and fortunately, quick-start units are usually a minority in the power system mixes.
3) The modelling of variable SU costs with δ_{gst} and (10)-(11) make a formulation significantly more tight and compact in comparison with common SU-cost models (e.g., [11]), as reported in [28]. Apart from taking this computational advantage, this paper fully exploits the inclusion of δ_{gst} to model the SU power trajectories in (12) [19].
4) Including r_{gt}^{κ} together with the equations in Section II-C (Section II-D) further constrains the operation of slow (quick)-start units. This means that the formulation is actually being further tightened. A similar conclusion was drawn in [38], where including ramping constraints actually improved the MIP formulation.
5) Finally, the variables $\widehat{p}_{gt}$ and e_{gt} are used in this work for the sake of clarity. However, they are not strictly needed, as the former could be directly included in (2) and the latter in (1). Their values can be obtained after solving the problem, without changing the results.

III. Numerical Results

The following case studies were conducted to illustrate the proposed market-clearing formulation, given by (1)-(14) together with (21)-(45). The power system data was based on that in [11]. This power system was adapted to consider SU and SD power trajectories. Table I presents the technical and economic data of the thermal units, including different SU ramps. Units 8 to 10 are quick-start units with hourly SU and

Table I: Generator Data

Unit	Technical Data							Cost Coefficients		StartUp Ramping Information								
	$\overline{P}$ [MW]	$\underline{P}$ [MW]	TU/TD [h]	RU/RD [MW/min]	p_0 [MW]	IniState [h]	SD^{D} [h]	C^{NL} [$/h]	C^{LV} [$/MWh]	SU_1^{D} [h]	T_1^{SU} [h]	C_1^{SU} [$]	SU_2^{D} [h]	T_2^{SU} [h]	C_2^{SU} [$]	SU_3^{D} [h]	T_3^{SU} [h]	C_3^{SU} [$]
1	455	150	8	3.75	455	8	3	1000	16.19	3	8	3000	5	11	7500	6	14	9000
2	455	150	8	3.75	245	8	3	970	17.26	3	8	4000	5	11	8000	7	14	10000
3	130	20	5	0.83	0	-5	2	700	16.60	2	5	300	3	7	800	5	10	1100
4	130	20	5	0.83	0	-5	2	680	16.50	2	5	560	3	7	950	5	10	1120
5	162	25	6	1.00	0	-6	2	450	19.70	2	6	600	3	8	1400	5	11	1800
6	80	20	3	1.00	0	-3	1	370	22.26	1	3	170	3	8	340	—	—	—
7	85	25	3	1.00	0	-3	1	480	27.74	1	3	260	3	6	520	—	—	—
8*	55	10	1	2.25	0	-1	—	660	25.92	—	1	30	—	2	60	—	—	—
9*	55	10	1	2.25	0	-1	—	665	27.27	—	1	30	—	2	60	—	—	—
10*	55	10	1	2.25	0	-1	—	670	27.79	—	1	30	—	2	60	—	—	—

*This is a quick-start unit

SD capabilities of 55 MW, and 50 MW for the 30-min SU and SD capabilities. For slow-start units, the power outputs P_{gsi}^{SU} (P_{gi}^{SD}) for the SU (SD) power trajectories are obtained as an hourly linear change from 0 ($\underline{P}_g$) to $\underline{P}_g$ (0) for a duration of SU_{gs}^{D} (SD_g^{D}) hours. The energy costs due to SU and SD processes are added to the SU and SD costs shown in Table I.

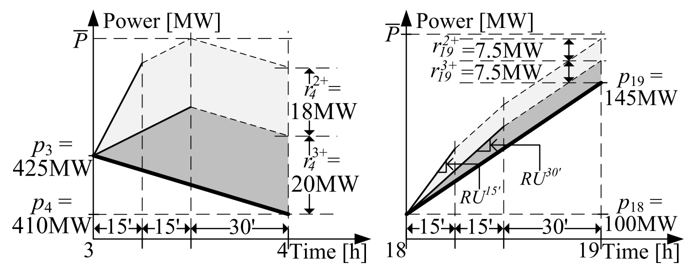
All tests were carried out using CPLEX 12.4 under GAMS [39] on an Intel-i7 2.4 GHz with 4 GB of RAM memory. Problems were solved until they hit a CPU time limit of 1000 seconds or until they reached optimality (more precisely to 10^{-6} of relative optimality tolerance). Apart from this, CPLEX default values were used for all the experiments.

This section is divided into three parts. The first part illustrates how the formulation deals with the reserves. The second part compares the difference in commitment schedules between the proposed formulation and the conventional energy-block scheduling. The last part compares the computational performance of the proposed formulation with a UC formulation commonly found in the literature [11].

A. Ramp and Reserve Schedules

For this case study, the previously described power system must meet the power demand D1, shown in Table II, at the end of each hour. The up/down secondary and tertiary reserve requirements of 2.5% and 5% of the power demand have to be met for each hour. The 15- and 30-minute ramp capabilities of the units are assumed to be equal to 150% and 100% of their operation ramp rates respectively. For simplicity, we assume that all units offer secondary, tertiary and offline-tertiary reserves at 20%, 10% and 40% of their energy variable cost C_g^{LV}, respectively. Each unit is considered to have the same bids for upward and downward reserves. The maximum reserve offered by each unit is set to the maximum available reserve.

Fig. 6 shows the generation and reserve schedules for each generation unit. Note the piecewise-linear profiles of power schedules which follow the instantaneous demand forecast profile. Reserves are scheduled as constant power availability for each hour. Note in Fig. 6 (bottom section) that all the scheduled offline tertiary reserves are above the units' minimum output (10 MW for quick-start units 7 to 10). As mentioned in Section II-D, the units providing offline tertiary reserves cannot be called to produce below their minimum output.



(a) Unit 2 hour 4 (b) Unit 5 hour 19

Fig. 5: Examples of units power and upwards reserve schedules

We will now examine some cases in which the available reserve of units were bound by the capacity and ramping limits:

1) *Reserves bound by capacity limits*: Interestingly, unit 2 is scheduled to ramp down in hour 4 while the demand is increasing during that hour, as shown in Fig. 6. Unit 2 reduces its production during hour 4 in order to provide upward reserves to the system. Fig. 5a shows the power production and upward-reserve schedules for unit 2 during hour 4. In the event that unit 2 provides all the upward scheduled reserves, the resulting power trajectory (see the uppermost solid line in Fig. 5) will ramp up and achieve the maximum unit capacity output (455 MW) after 30 minutes. Note that the capacity limit is only reached if the unit starts providing all the upward reserves at the beginning of the hour.
2) *Reserves bound by ramping limits*: Fig. 5b shows the power production and upward-reserve schedules for unit 5 during hour 19. The unit is scheduled to ramp up at 0.75 MW/min during normal operation. The unit 30-min ramping limit $RU^{30'}$ is 1 MW/min, which means that the unit has an extra ramp capability of 0.25 MW/min, which results in 7.5 MW in the reserve that the unit can provide within 30 minutes r^{3+}. In addition, the unit has a 15-min ramping limit $RU^{15'}$ of 1.5 MW/min. This means that for 15-min reserve deployment r^{2+}, the unit has an available ramp capability of 0.5 MW/min, which results in a power reserve capacity of 7.5 MW that can be provided in 15 minutes.

In conclusion, as discussed in Section II, even though the formulation is on an hourly basis, it guarantees that that the unit (capacity and ramping) limits are not violated within the

Table II: Power and Energy Demand Profiles (MW)

Hour	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
$D1^*$	750	850	950	1000	1100	1150	1200	1300	1400	1450	1500	1400
$D2^*$	725	875	925	1025	1075	1175	1175	1325	1375	1475	1475	1425
$D^{\mathrm{E}\dagger}$	725	800	900	975	1050	1125	1175	1250	1350	1425	1475	1450

Hour	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
$D1^*$	1300	1200	1050	1000	1100	1200	1400	1300	1100	900	800	700
$D2^*$	1475	1425	1275	1225	1025	1025	1075	1225	1375	1325	1075	925
$D^{\mathrm{E}\dagger}$	1350	1250	1125	1025	1050	1150	1300	1350	1200	1000	850	750

*Power [MW] at the end of the hour †Total Energy [MWh] for the hour

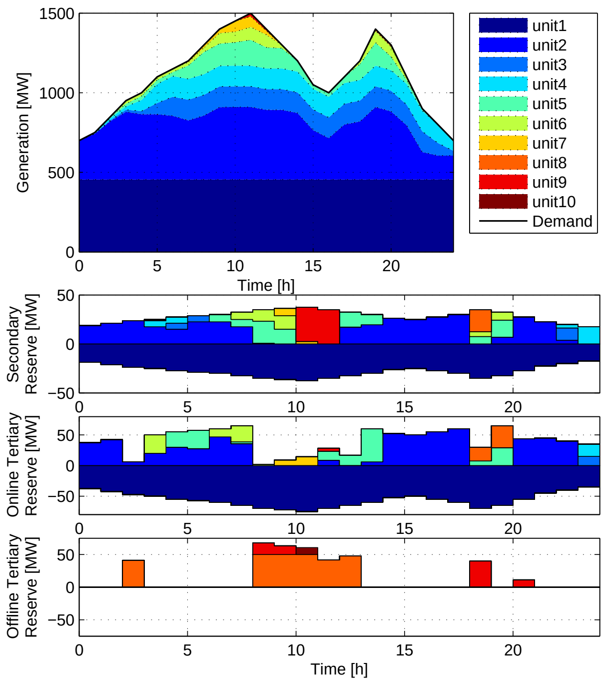


Fig. 6: Generation and reserves schedules. For the reserve schedules, positive and negative values refer to up and down reserves respectively.

hour when providing reserves with shorter deployment times.

B. Conventional vs. Ramp-Based Scheduling Approaches: Commitment and Economic Impact

To illustrate the difference in schedules between the conventional and the proposed scheduling approaches, the lowest production cost is obtained for two different demand profiles assuming full knowledge of system conditions. That is, it is assumed that the power demand profile is known perfectly and that no uncertain events will happen. Therefore, there should be no need for operating reserves and hence they are not considered (i.e. $D_t^\kappa, r_{gt}^\kappa = 0\,\forall\kappa$). Although this situation is hypothetical, it helps to evaluate [40] and compare the two scheduling approaches.

The proposed ramp-based scheduling formulation, labelled as *PropRmpSch*, and the conventional staircase energy approach, labelled as *ConvlEnSch*, are used to optimally schedule the 10-unit system, in Table I, to supply the power demand profiles $D1$ and $D2$ presented in Table II. Note that $D1$ and $D2$ present the same energy profile (D^{E} in Table II) but different ramp requirements. Table III shows the optimal energy schedules found by *PropRmpSch* and *ConvlEnSch* to supply $D1$ and $D2$. While *TradEnSch* directly provides the energy schedules, *PropRmpSch* provides the piecewise-linear power schedules (see Table IV), and obtaining the resulting energy schedule is straightforward ($\widehat{p}_{gt}/2 + \widehat{p}_{g,t-1}/2$). Note that in Table III, *ConvlEnSch* provides the same optimal scheduling solution for $D1$ and $D2$ because they present the same energy profile. On the other hand, *PropRmpSch* provides different optimal scheduling for $D1$ and $D2$, although both scheduling solutions satisfy the same total energy demand.

One power profile has a unique energy profile and hence satisfying a power profile automatically satisfies the energy profile. However, one energy profile has infinite possible power profiles [12], [14], [16]; therefore, even though *ConvlEnSch* could provide a given energy profile, it cannot guarantee that all possible resulting power profiles can be supplied [12]. Moreover, *ConvlEnSch* suffer from the following shortcomings in comparison with *PropRmpSch*, due to the inability of *ConvlEnSch* to perceive a given power profile:

1) *Ramp Scarcity*: The power demand $D2$ is ramping at 100 MW/h during hour 4 (see Table II) and the optimal schedule of *ConvlEnSch* only provides 60 MW/h of ramp capability. Note that in Table III only three units are up during hour four, where units 1 and 2 are producing at their maximum capacity. Consequently, unit 5 is the only unit that can ramp up and its ramping capability is 1 MW/min (see Table I).
2) *Capacity Scarcity*: The demand peak of $D1$ is 1500MW and occurs at the end of hour 11. Note that *ConvlEnSch* scheduled seven units for this hour having a total production capacity of 1497 MW. This is in contrast to *PropRmpSch*, which committed seven units at hour 11 to satisfy the peak demand of $D1$.
3) *Infeasible Energy Delivery*: There are many hours where units cannot comply with their scheduled energy profile provided by *ConvlEnSch*. For example, unit 5 must produce at its minimum output (25 MW) during the whole hour 3 to deliver its scheduled 25 MWh. If the unit ramps up at its maximum capability (60 MW/h), then the production at the end of hour 4 will be 85 MW, providing a maximum of 55 MWh for hour 4, and thus failing to deliver its scheduled energy level of 65 MWh. Similarly, unit 6 must produce 80 MW at the end of hour 12 to provide its schedule energy for that hour. If the unit ramps down at its maximum capability, it can provide a minimum of 50 MWh for hour 13, thus failing to deliver its scheduled energy level of 20 MWh.

Table V shows the comparison of the optimal scheduling costs where *ConvlEnSch* presents the highest scheduling costs. This can be explained as follows: Although both *PropRmpSch* and *ConvlEnSch* consider the cost of the intrinsic energy produced during the SU and SD processes, *ConvlEnSch* does not include this energy in the scheduling stage. As a consequence, *ConvlEnSch* cannot accommodate the SU and SD power trajectories, which contribute to satisfying the demand (energy and ramp). This also causes an inefficient deployment of resources in real time to accommodate these trajectories that were ignored in the scheduling stage [18], [19].

In short, the conventional energy scheduling approach does not guarantee that enough resources will be available to satisfy an expected power profile. Furthermore, *ConvlEnSch* cannot even guarantee a feasible energy delivery of its resulting

Table III: Optimal Energy Schedules

	Unit	Hour 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
PropRmpSch - D1	**1**	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455
	2	266.67	327.5	405.83	437.5	430	421.67	417.5	438.33	455	455	455	455	455	435	362.5	285	302.5	355	410	455	422.5	335	241.25	176.25
	3	.	3.33	10	16.67	45	95	125	130	130	130	130	130	130	130	130	130	130	130	125	95	45	15	5	.
	4	3.33	10	16.67	45	95	125	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	112.5
	5	.	4.17	12.5	20.83	25	25	37.5	80	136	162	162	162	146	100	47.5	25	32.5	70	130	152.5	115	55	18.75	6.25
	6	.	.	.	.	.	3.33	10	16.67	44	74	80	50	10	.	.	.	.	10	50	62.5	32.5	10	.	.
	8	.	.	.	.	.	.	.	.	.	.	27.5	51.5	24	.	.	.	.	.	.	.	.	.	.	.
	9	.	.	.	.	.	.	.	.	.	19	35.5	16.5	.	.	.	.	.	.	.	.	.	.	.	.
PropRmpSch - D2	**1**	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455
	2	266.67	327.5	405.83	437.5	430	425	412.5	417.5	455	455	455	455	446.5	446.5	370	285	310	370	430	455	385	256.25	173.75	150
	3	.	3.33	10	16.67	45	95	125	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	105	55
	4	3.33	10	16.67	45	95	125	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	110	90
	5	.	4.17	12.5	20.83	25	25	45	95	142.5	161	162	162	132	78.5	40	25	25	55	115	115	55	18.75	6.25	.
	6	.	.	.	.	.	.	3.33	10	16.67	50	80	74	44	10	.	.	.	10	40	65	45	10	.	.
	7	.	.	.	.	.	.	4.17	12.5	20.83	44	63	44	12.5	.	.	.	.	.	.	.	.	.	.	.
ConvlEnSch - D1&D2	**1**	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455
	2	270	320	420	455	455	455	440	435	455	455	455	455	455	455	385	285	310	385	455	455	380	260	150	150
	3	.	.	.	.	20	70	120	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	90	40
	4	.	.	.	.	20	70	120	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	80
	5	.	25	25	65	100	75	40	100	160	162	162	162	135	80	25	25	25	50	110	145	85	25	25	25
	6	.	.	.	.	.	.	.	.	20	68	80	80	20	.	.	.	.	.	20	35	20	.	.	.
	7	.	.	.	.	.	.	.	.	.	25	63	38	25	.	.	.	.	.	.	.	.	.	.	.

Highlighted cells indicate that the unit is either starting up or shutting down

Table IV: Optimal Power Schedules

	Unit	Hour 0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
PropRampSch - D1	**1**	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455
	2	245	288.33	366.67	445	430	430	413.33	421.67	455	455	455	455	455	455	415	310	260	345	365	455	455	390	280	202.5	150
	3	.	.	6.67	13.33	20	70	120	130	130	130	130	130	130	130	130	130	130	130	130	120	70	20	10	.	.
	4	.	6.67	13.33	20	70	120	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	95
	5	.	.	8.33	16.67	25	25	25	50	110	162	162	162	162	130	70	25	25	40	100	160	145	85	25	12.5	.
	6	.	.	.	.	.	.	6.67	13.33	20	68	80	80	20	.	.	.	.	.	20	80	45	20	.	.	.
	8	.	.	.	.	.	.	.	.	.	.	.	55	48	.	.	.	.	.	.	.	.	.	.	.	.
	9	.	.	.	.	.	.	.	.	.	.	38	33	.	.	.	.	.	.	.	.	.	.	.	.	.
PropRampSch– D2	**1**	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455	455
	2	270	263.33	391.67	420	455	405	445	380	455	455	455	455	455	438	455	285	285	335	405	455	455	315	197.5	150	150
	3	.	.	6.67	13.33	20	70	120	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	80	30
	4	.	6.67	13.33	20	70	120	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	130	90	90
	5	.	.	8.33	16.67	25	25	25	65	125	160	162	162	162	102	55	25	25	25	85	145	85	25	12.5	.	.
	6	.	.	.	.	.	.	.	6.67	13.33	20	80	80	68	20	.	.	.	.	.	60	70	20	.	.	.
	7	.	.	.	.	.	.	.	8.33	16.67	25	63	63	25	.	.	.	.	.	.	.	.	.	.	.	.

Highlighted cells indicate that the unit is either starting up or shutting down

Table V: Comparison of Total Optimal Scheduling Costs

Approach	Demand	Scheduling Cost ($)
PropRmpSch	$D1$	562738.61
	$D2$	562573.80
ConvlEnSch	$D1$ and $D2$	567392.22

energy profile, as also previously reported in [12], [14], [21]. Consequently, *ConvlEnSch* would require *ad-hoc* operations in real time in order to deal with these problems and keep the balance between supply and demand. However, *PropRmpSch* overcomes these problems by an adequate resource scheduling.

C. Computational Performance

In order to assess the computational burden of the proposed formulation, its computational performance was compared with the UC model proposed in [11]. The work in [11] presents a basic formulation that only considers one-single upward reserve and ignores the SU and SD power trajectories. The model in [11] is implemented using the case study detailed in Section III-A and the hourly spinning reserve is assumed to be 10% of the hourly demand (which is similar to the 5% of the hourly demand assumed for the half-hour tertiary reserve in the proposed formulation).

Two different problem sizes were simulated: 10-unit (presented in Table I) and 100-unit power systems, the latter being the 10-unit power system replicated ten times. This replication introduces symmetry in the MIP problem which makes it harder to solve than usual [35]. The load demand was accordingly multiplied by 10 for the latter power system case.

Table VI shows the model size and computational performance of [11] and the proposed formulation, which is labelled as “Prop”. The proposed model presents around 8% and 5% more constraints and non-zeros in the constraint matrix than [11]. This is an insignificant increase taking into account the fact that the proposed formulation includes SU and SD power trajectories and five types of reserves more than [11]. However, although the proposed formulation needs around 1.6 and 5.7 times as many real and binary variables as [11] respectively,

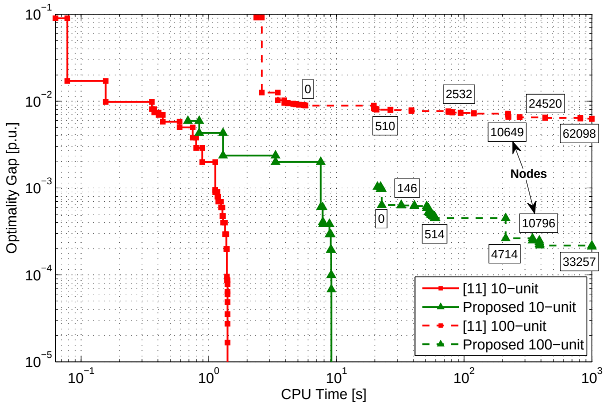


Fig. 7: Comparison of convergence evolution to optimal solutions. Values inside squares indicate the number of explored nodes by the solver.

this does not necessarily mean an increase in computational burden. In fact, increasing the number of binary variables may lower the complexity of an MIP formulation [35] as in the case of the tight and compact formulations presented in [19] and [28], which are the core of the formulation proposed in this paper.

As stated in Section II-E, the computational burden of an MIP formulation mainly depends on the strength of its linear program (LP) relaxation, where the LP relaxation of an MIP problem is obtained by relaxing its integrality requirements. In other words, the nearer the LP relaxed solution is to its MIP integer solution, the faster the search for optimality. The strength (or tightness) of a MIP formulation can be measured with the integrality gap [28], [35] which is defined as $(Z_{\text{MIP}} - Z_{\text{LP}})/Z_{\text{MIP}}$, where Z_{MIP} and Z_{LP} are the optimal values of the MIP and the relaxed LP respectively. The integrality gap of the two formulations which are not modelling exactly the same problem should not be directly compared; however, these gaps provide an indication of the strength of each formulation. Note that in Table VI, the proposed formulation presents a smaller integrality gap (around 5 times lower) in comparison with [11], which indicates that the proposed formulation is significantly tighter.

Finally, Fig. 7 shows the convergence evolution, for both formulations, for the two different system sizes. The proposed formulation took longer to find an initial feasible solution mainly due to the greater number of binary variables (In general, a large number of integer variables complicates the process of finding initial feasible solutions). For the 10-unit case, the impact is significant due to the short solving times (less than 10s). However, for the 100-unit case, even though the proposed formulation took longer to find an initial feasible solution, the optimality gap achieved by this solution (with zero nodes explored) is better than all the solutions found by [11] within the time limit. This evolution of convergence, as well as the quality of the initial solutions, is mainly due to the tightness of the proposed formulation.

IV. Conclusions

A unit-commitment (UC)-based market clearing formulation was proposed using continuous power trajectories for both generating units and demand instead of the commonly established staircase profile for energy blocks. The use of an instantaneous power profile allows the model to efficiently schedule reserve and ramping resources. In comparison with conventional UC models, the proposed formulation guarantees that, first, energy schedules can be delivered and, second, that operating reserves (secondary, tertiary online and tertiary offline) can be deployed within their given time requirements while respecting the ramping and capacity limits of generating units. In addition, the model takes into account the normally neglected power trajectories that occur during the startup and shutdown processes, thus optimally scheduling them to provide energy (and ramp), which help to satisfy the power demand. The formulation was tested on a 10-unit and 100-unit system, where the computational burden was lowered in comparison with common UC formulations.

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Table VI: Comparison of Different Formulations

Case	Model Size								Computational Performance							
	# of Constraints		# of Real Variables		# of Binary Variables		Non-zero elements		Time (s)		OptTol (p.u.)		Nodes		Int. Gap ($\times 10^{-3}$)	
(# Gens)	Prop	[11]	Prop	[11]	Prop	[11]	Prop	[11]	Prop	[11]	Prop	[11]	Prop	[11]	Prop	[11]
10	4436	4089	1584	984	1365	240	21767	20867	9.11	1.42	4e-15	2e-15	1123	1058	6.41	21.92
100	43271	40449	15840	9624	13650	2400	217661	208445	1001	1000	2e-4	6.3e-3	33257	62098	3.33	17.6

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