A Two-Stage Stochastic Model for Energy Contracting Decisions of an Industrial Consumer

Emilio Gómez-Villalva* Andrés Ramos†

Abstract

To encourage industrial consumers to participate more actively in deregulated energy markets, it is necessary to provide them with optimization tools to manage the risk derived from energy price uncertainty. With the risk measures selected, safety-first and value-at-risk, two bi-objective mixed-integer linear stochastic problems are implemented. These models obtain, through a risk-aversion parameter, a tradeoff between the risk measure and the expected cost of the total energy supply cost of industrial consumers. The efficient frontiers obtained with the safety-first and value-at-risk models are compared in a realistic case example. The model presented here extends the use of stochastic programming as an integrated decision support tool for industrial consumers to participate in energy markets.

Keywords: risk management, stochastic optimization, liberalized energy markets, industrial plants, cogeneration.

1 Introduction

The price uncertainty and the new contracting possibilities that have arisen from the recent liberalization of energy markets show the necessity of new optimization tools for decision-making processes [12]. Specifically, industrial consumers of electricity and heat who have their own energy supply system need to decide which energy contracts to sign and how to operate their system [7].

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Energy supply systems, mainly composed of cogeneration plants and boilers, have been broadly modeled as deterministic optimization problems [4, 7, 11, 21]. However, very few models optimize contracting and system operation decisions simultaneously [7, 11, 21]. These two concepts have to be considered in the optimization problem since contracting decisions depend on the quantity of energy traded, which is determined by the optimal operation of the plant.

Although deterministic optimization represents a powerful technique to model the complexity of these types of problems, it is very limited for treating the uncertainty of the parameters. To overcome this obstacle, stochastic programming plays a key role. In this field, Paravan *et al.* [21] proposed a risk-neutral stochastic model for the decision-making process concerning contracts and energy supply system operation of cogeneration plants. In this paper, we go one step beyond risk-neutral approaches proposing two multi-objective stochastic optimization models for the energy risk management of industrial consumers.

The problem consists on determining energy contracts and supply system operation while keeping total annual energy costs at a minimum. It is a medium term decision support model, formulated as a two-stage stochastic optimization problem derived from the detailed deterministic approach presented in [7]. The first stage decisions concern the different energy contracts while the second stage decisions represent the system (boiler and cogeneration plant) operation for each time period of the year. Binary variables appear in both stages, related to contract selection in the first stage and to boiler and cogeneration plant commitment in the second stage. Uncertainty in fuel oil, natural gas and electricity prices is considered in the second stage parameters. This uncertainty is represented by means of a scenario tree in which all of the scenarios come from a single root node with no additional branching. The model has these main characteristics:

- It is an integrated tool because it includes contract and operation optimization and price generation modules. Complexity and richness of contract modeling is specially considered.
- The different modules are easy to parameterize and use.
- Input data used by the model are easy and, when applicable, publicly available.

The paper is organized as follows. Firstly, we present a general description of the problem in Section 2. In Section 3 we formulate a risk-neutral model with the expected energy supply cost as the objective function and show its drawbacks. The formulation of other risk-averse measures and their corresponding models are described in Section 4. These models obtain a compromise, through a risk-aversion parameter, between the risk measure and the expected energy supply cost. The two risk measures considered are: Value-at-Risk (VaR) and Safety-first or maximum cost. These risk measures are easy to interpret and penalize only high costs, which reflect the risk aversion of industrial consumers. The procedure carried out to determine efficient frontiers with the chosen models is presented in Section 5. To illustrate the working of the models, we offer a realistic numerical application in Section 6. Finally, conclusions are presented in Section 7.

2 Mathematical problem description

We consider a system composed of a steam boiler and a cogeneration plant fed by fuel oil and natural gas, respectively. With this configuration, the thermal demand is satisfied by the boiler or the cogeneration plant, whereas the electric demand is covered by the electric network or the cogeneration plant (Fig. 1). This configuration is quite flexible, since it is also valid for consumers without a cogeneration system or thermal demand.

An industrial consumer with an energy supply system of these characteristics, negotiates with retailers the following types of contracts (Fig. 1):

1. Purchase of electricity for those periods in which the cogeneration plant is shut down.

2. Purchase of fuel oil for the boiler.

3. Purchase of natural gas for the cogeneration plant.

4. Sale of surplus electricity produced by the cogeneration plant.

Different examples of representative contracts of each type of asset needing negotiation are modeled according to the alternatives and current situation of energy markets. These contracts range from spot to fixed prices as presented in Fig. 2 and are discussed in Section 6.

Retailers will bid contracts of the four above-mentioned products to the industrial consumer, who will annually choose one contract of each product among the proposed ones. The time scope of the problem is one year, since this is the most frequent duration of contracts between consumers and retailers. Therefore, the industrial consumer decides which contracts to sign

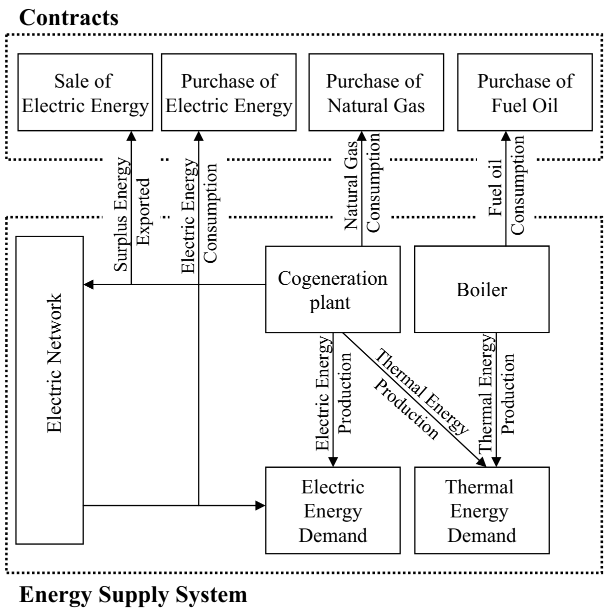

Figure 1: Configuration of the energy supply system and types of contracts to sign.

before the beginning of the planning year. For this purpose, in each period of the time scope, the optimal operation of the energy supply system of the consumer is taken into account.

On the one hand, the main decision to be made with this model is the contract choice for the whole scope and, therefore, contracts are modeled in detail. On the other hand, some simplifications in the annual operation modeling can be allowed as they significantly reduce the problem size without losing relevant information. The simplifications considered are mentioned below:

- Linear relations among variables are used for equipment modeling. Gas engines have a quite linear behavior along their operating range, as do the boilers. Thus, this representation can be acceptable, furthermore, taking into account that operation variables represent average values for each period with duration of several hours.

- Temperature and pressure variations are not significant for average values of thermal energy as considered in the presented mid-term problem and, therefore, these variations have been neglected. Consequently,

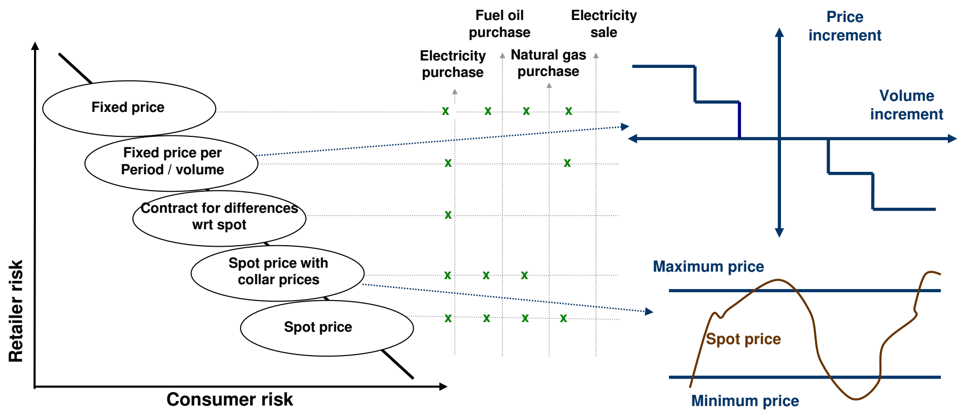

Figure 2: Different types of contracts to sign.

variables that represent thermal energy are exclusively functions of mass flow.

- Start-up and shut-down costs can be neglected in gas engines for mid-term problems.

Other characteristics of the model are:

- Only surplus electricity produced by the cogeneration facility can be sold.
- Oil tanks have no storage capacity.
- Unused thermal energy is lost, it can not be sold.

This problem was formulated as a MIP model in [7]. The objective of this deterministic model is to minimize the total energy supply cost. This cost comprises the ones related to the energy contracts signed as well as those related to the operation and maintenance of the cogeneration plant and the boiler. In this model, three sets of constraints were basically formulated:

- *Boiler and cogeneration plant operation*: To determine the economic dispatch and the unit commitment of the energy supply system.
- *Energy Balance*: To satisfy the electric and thermal demands of the factory.

- *Contracts*: To evaluate the contracts to choose and the quantity of energy or fuel associated with each one.

In this formulation, binary variables are used for modeling the unit commitment of the boiler and cogeneration plant, the contracting decisions and some types of contracts.

3 Risk-neutral stochastic formulation

In this section we extend the deterministic problem stated in the previous one to a risk-neutral stochastic model in order to consider the uncertainty of the parameters of the problem.

To cope with contracting and energy system operation decisions under uncertainty, we propose a two-stage stochastic model. The contracts to sign are chosen in the first stage. These are the so-called *here-and-now decisions*, since they are made under uncertainty and before the first period of the time scope of the problem. In the second stage, the boiler and cogeneration plant operation are determined in each time period taking into account the known stochastic parameters and the contracts that were chosen in the first stage. These are the so-called *wait-and-see decisions*, since they are made once the uncertainty has been revealed. Given the two-stage structure of the problem, scenarios are represented as independent time series with only the root node in common (Fig. 3).

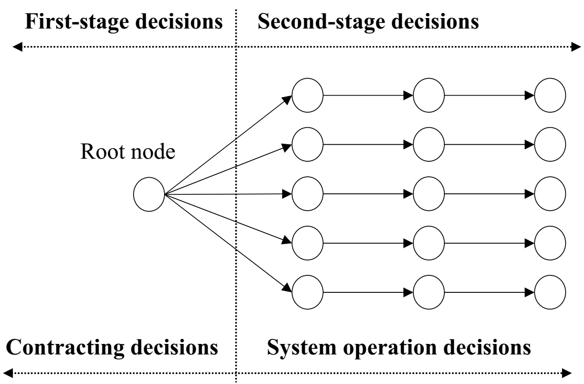

Figure 3: Structure of the scenario tree.

3.1 Scenario tree generation

The stochastic parameters of the problem are the electricity, natural gas and fuel oil prices, whereas electric and thermal demands are considered deterministic since demand volatility is insignificant compared to that of prices. Price uncertainty is represented through a scenario tree (Fig. 4), in which electricity prices are estimated for each load level (peak, plateau and off-peak) and fuel prices are calculated monthly.

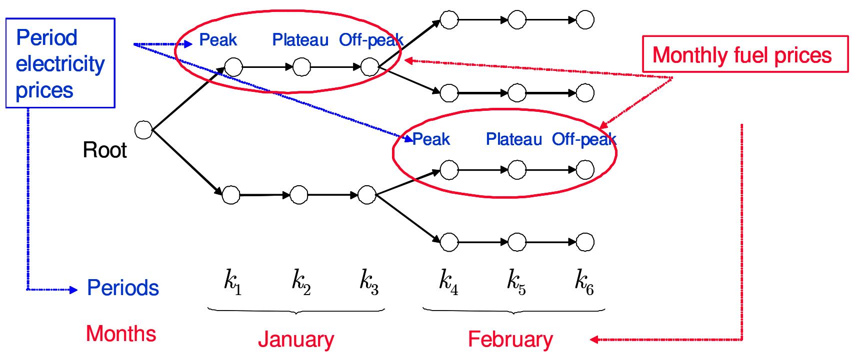

Figure 4: Scenario tree of electricity and fuel prices.

Due to the lack of any significant correlation between electricity and fuel prices in the Spanish energy markets, price scenarios are generated independently. On the one hand, three electricity price scenarios were obtained by sampling from historical data distribution (Fig. 5). This method is reasonable given the difficulties in forecasting electricity prices in Spain with an annual scope [16], although we are conscious that further research is needed in this field. On the other hand, five fuel oil and natural gas price scenarios were generated with the algorithm proposed in [8] (Fig. 5). Basically, this algorithm generates fuel prices through Brent spot prices, which are calculated from historical distributions of Brent spot and futures prices.

3.2 Problem formulation

The discrete probability function of the total annual energy cost $c_T \in R^{\overline{G}}$, where $\overline{G}$ is the number of scenarios, is defined as:

$$c_T = f(\beta, e_r, g_o, f_a, e_{oe}) \tag{1}$$

where f is a function of the following vectors of state variables:

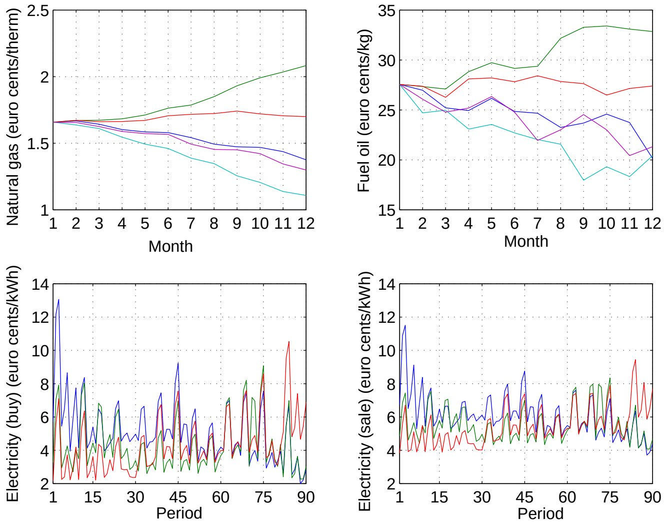

Figure 5: Natural gas, fuel oil and electricity (buy and sale) price scenarios for the case example.

β contracts to sign (binary variables); e_r electricity imported from the electric network; g_o natural gas consumed by the cogeneration plant; f_a fuel oil consumed by the boiler; and e_{oe} surplus electricity exported.

The first vector (β) corresponds to the first-stage variables, whereas the remaining are the second-stage random variables. These latter variables are the energy or fuel associated with the chosen contract of acquisition of electricity, natural gas, fuel oil and of sale of electricity. Natural gas (g_o) and fuel oil (f_a) consumption is also responsible for determining the maintenance costs of the cogeneration plant and the boiler, respectively.

The random variable c_T is composed of the cost of each scenario c_T^g, with $g = \{1, .., \overline{G}\} \in G$. Then, if p^g is the probability of each scenario, the expected cost of c_T can be written as:

$$E[c_T] = \sum_{g \in G} p^g c_T^g \tag{2}$$

The problem constraints X are the same as in the deterministic problem (system operation, energy balance and contract formulation) but in their stochastic versions. These are not shown in order to focus the analysis on the risk management modeling and its interpretation.

Therefore, the risk-neutral stochastic model, which minimizes the expected cost, can be formulated as:

$$
\begin{aligned}
&\min_{c_T \in R^{\overline{G}}} \quad E[c_T] \\
&x \in X
\end{aligned}
\tag{3}
$$

where x is the set of variables of the problem.

This model takes into account the uncertainty of the parameters explicitly, although it does not perform risk management. With this formulation, the model will select, for example, a spot price contract instead of a fixed price one if the former is slightly cheaper. This is not realistic. In this case a consumer will prefer a fixed price contract so as to hedge himself against the possibility of high costs that can appear once the price uncertainty is revealed. As shown in the next section, this limitation of the risk-neutral model is resolved with the risk-averse formulation.

4 Risk-averse stochastic formulation

Contract selection is greatly influenced by the price-risk attitude of consumers. In general, an industrial consumer is very risk averse. Usually, the core of its business is not energy management and thus, he is reluctant to have surprises in his energy costs.

Taking this into account, in this section we propose bi-objective stochastic models. The industrial consumer will obtain, through a risk-aversion parameter, a tradeoff between the expected cost and a risk measure of the total energy supply cost function.

Among the most commonly used risk measures in financial and energy markets it is worth to mention: Variance [1, 17], Total absolute deviation [9, 20], Reference cost [10], Utility function formulated as an exponential [13] or piecewise linear [3] function, Fleten's approach [5, 19], Regret with linear [24] and nonlinear [18] approaches, Safety-first or maximum cost [23], Value-at-Risk (VaR) [6, 14, 15], and Conditional Value-at-Risk (CVaR) [22, 25].

To decide among the above-mentioned measures, two items are considered: the mathematical formulation of the measures and the definition of risk for consumers. On the one hand, we have a MIP model and therefore this formulation does not admit nonlinear measures to be practically solvable. On the other hand, according to our point of view, an industrial consumer perceives the risk as the potential of high costs and, thus, measures which penalize low costs are inappropriate. As a consequence, the measures variance, total absolute deviation, regret and utility function are not suitable for industrial consumers.

The measure that we note as the reference cost penalizes values above a target (reference cost), although it does not use a penalty function. The stochastic model with this measure can be formulated as:

$$\min_{c_T, c_T^+ \in R^{\overline{G}}} \quad \sum_{g \in G} p^g c_T^{g+} \tag{4}$$

$$x \in X$$

$$E[c_T] \leq S_{cr} \tag{4a}$$

$$c_T^{g+} \geq c_T^g - R \qquad \forall\, g \in G \tag{4b}$$

$$c_T^{g+} \geq 0 \qquad \forall\, g \in G \tag{4c}$$

The objective of this model is to minimize the risk measure while maintaining the expected cost below the risk-aversion parameter S_{cr} (constraint (4a)). This model only penalizes costs above the reference R, being risk-neutral for costs below R (Fig. 6). The penalization is done through c_T^{g+}, which computes the excess of the cost c_T^g of each scenario g over the reference cost R (constraints (4b) and (4c)).

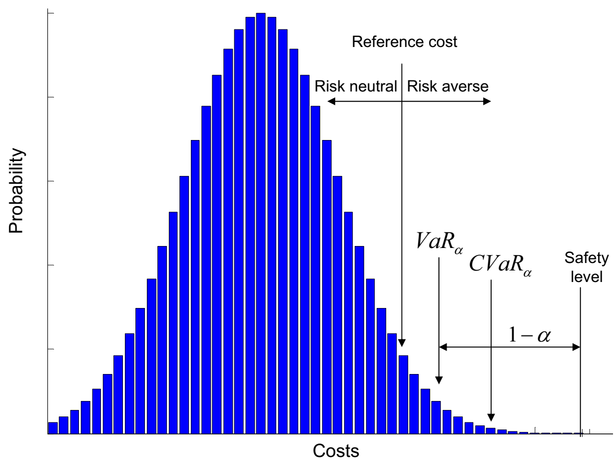

Figure 6: Discrete density function of energy supply costs.

Whether or not to use this model depends on the consumer's preferences. Particularly, we think that the reference cost R can be difficult to select for some consumers and, thus, this model was not implemented. In addition, a confidence level (provided by the safety-first, VaR and CVaR models) seems to be a more intuitive risk measure than the linear penalization used in the reference cost model. Specifically, the safety-first model is formulated as:

$$\min_{c_T \in R^{\overline{G}}} \quad E[c_T] \tag{5}$$
$$x \in X$$
$$c_T^g \leq S_{sl} \quad \forall g \in G \tag{5a}$$

This model minimizes the expected cost while keeping the cost of all the scenarios below a safety level or maximum allowed cost S_{sl}, which is the risk-aversion parameter (constraint (5a)). Therefore, the risk measure, which is the maximum cost, corresponds to the value of the cost distribution with a confidence level equal to 1.

Among the other measures, VaR and CVaR, the former was selected. As previously stated, the CVaR has the main advantage in that it can be modeled as a linear problem. However, a reduced number of scenarios (i.e., 15) was considered due to the large size of the deterministic problem[1] and, therefore, it does not make sense to analyze values above the VaR. On the other hand, the deterministic model is formulated as a MIP problem, so it is feasible to use a risk measure with binary variables. The VaR model with binary variables is formulated as:

$$\min_{c_T \in R^{\overline{G}}, \delta \in B^{\overline{G}}, \zeta \in R} \quad \zeta \tag{6}$$
$$x \in X$$
$$E[c_T] \leq S_{VaR} \tag{6a}$$
$$\sum_{g \in G} p^g \delta^g \leq 1 - \alpha \tag{6b}$$
$$c_T^g \leq \zeta + M\delta^g \quad \forall g \in G \tag{6c}$$

where $B = \{0, 1\}$, ζ is the VaR for the confidence level α, M is a constant value above the highest cost among all the scenarios c_T^g and δ^g are auxiliary binary variables for each scenario g.

The VaR (ζ) is minimized in the objective function while the expected cost, the other objective variable, is limited to the risk-aversion parameter

[1] 5,883 constraints, 7,590 continuous variables, 1,087 binary variables and 32,887 non-zero coefficients of the constraint matrix.

(S_{VaR}) (constraint ($6a$)). To determine which scenario the VaR is, two equations are needed: ($6b$), which limits the number of binary variables (δ^g) that can have the value of 1 to the number of scenarios with cost above the VaR; and ($6c$), which forces the binary variables (δ^g) of the cost scenarios above the VaR to have the value of 1 and establishes the VaR in the scenario of the highest cost with auxiliary variable δ^g equal to 0.

Comparing both chosen models, VaR and safety-first, VaR is more flexible since it allows the user to analyze solutions obtained with different confidence levels, whereas the safety-first model only limits the highest cost (confidence level 1). As a negative aspect of the VaR model, including binary variables in the formulation increases the computation time considerably, despite only one binary variable is needed per scenario.

5 Determination of efficient frontiers

An efficient frontier refers to the set of optimal contract portfolios obtained by varying the risk-aversion parameter [24]. These portfolios represent a tradeoff between the two objectives: expected cost and risk measure.

The efficient frontier with the safety-first model is calculated as follows. First, the risk-neutral model (equations (3)) is solved. The maximum value of the cost distribution obtained is used as a cap value for the safety level. Next, while the problem remains feasible, the safety-first model (equations (5)) is solved and the safety level is decreased iteratively. In this process, optimal solutions of the two stages of the stochastic problem and different contract portfolios are obtained in each iteration.

The same type of procedure cannot be applied when determining efficient frontiers with the VaR model (equations (6)). This model has as objective function the cost of the scenario which corresponds to that of the VaR for a given confidence level. Thus, for this scenario, the VaR model obtains optimal solutions of the first-stage (contracts) and second-stage (energy supply system operation) variables. However, for the other scenarios, only first-stage variables (common for all scenarios) are optimal. The reason for this is that the cost of the scenarios different from the VaR is not penalized in the objective function and, as a consequence, the model does not obtain their optimal values.

To obtain the efficient frontier, the optimal VaR and expected cost are necessary, which cannot be achieved solely with the VaR model. To overcome this problem we propose to obtain each value of the efficient frontier in two phases:

1. In the first phase, the first-stage variables (contract portfolio) are obtained from the resolution of the VaR problem.

2. Next, a risk-neutral problem, in which the contracts obtained in the previous phase are fixed, is solved.

The second problem determines the same VaR as the first problem as well as the optimal second-stage variables. The expected cost obtained with the risk-neutral model is used as the threshold of the risk-aversion parameter of the VaR model, below which contracting decisions change.

The results of one iteration of the method are depicted in Fig. 7, which shows the distribution functions obtained when solving the two phases with a stochastic problem of 15 scenarios and a confidence level of 0.9. The VaR model obtains optimal VaR and contracts as well as an expected cost, far from its optimal value, of 646 k€. Fixing the contracts obtained and solving the risk-neutral model, the optimal expected cost, which equals 564 k€, is determined (solid line in Fig. 7). These numbers show how important the boiler and cogeneration plant operation is for risk management. While contracts mainly hedge consumers against price risk, energy supply system operation manages energy and fuel volume uncertainty.

This proposed method is used for determining the efficient frontier of the VaR model shown in the next section.

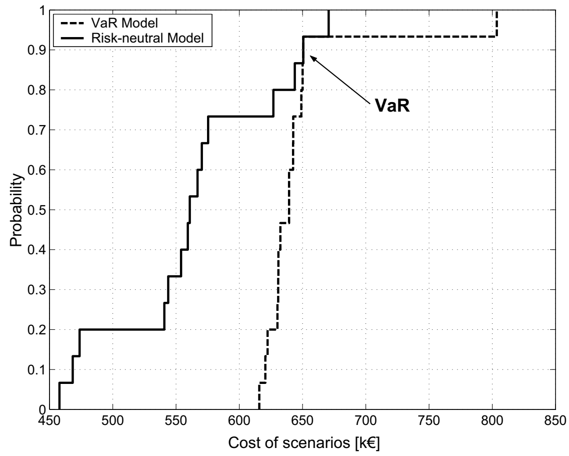

Figure 7: Distribution functions in one iteration of the two-phase method proposed for obtaining the efficient frontier for the VaR model.

6 Case study

The models described in this paper were implemented using data from a cellulose paper factory in Spain. Both the cogeneration plant and the steam boiler, which constitute the energy supply system, have enough capacity to supply 2 MW of peak thermal demand. The surplus thermal energy produced by the supply system is dispelled into the atmosphere. The cogeneration plant, with an electrical production capacity of 2.76 MW, can satisfy the peak electricity demand (1.22 MW) and sell the surplus.

The industrial consumer will annually sign one contract among the proposed by retailers of each of the following products: electricity acquisition, fuel oil acquisition for the boiler, natural gas acquisition for the cogeneration plant, and surplus electricity sale. The types of contracts considered in the model, which in general can be used for any product, are:

Type 1: Fixed annual price.

Type 2: Fixed annual price plus bonus or penalty by consumption. The price of this contract varies according to a stepwise linear function of the energy or fuel annual consumption (see Fig. 2).

Type 3: Fixed annual price indexed monthly to a variable of interest for the consumer, such as raw material costs or product sale prices.

Type 4: Three-section time-of-use (TOU) rate. Typically these sections are: peak, plateau and off-peak. This type of contract is only used for negotiating electricity.

Type 5: Contract for differences. The price of this contract varies in each time period according to the following expression: $[\lambda \cdot \mathit{Spot\ price} + (1-\lambda) \cdot \mathit{Contract\ fixed\, price}]$ where the parameter $\lambda \in [0, 1]$ typically has the value of 0.5.

Type 6: Spot price plus cap and floor (*collar*) prices (see Fig. 2). The energy under negotiation is paid at a cap price if this price is below the spot price, at a floor price if this price is above the spot price, or at the spot price if this price is between the cap and the floor ones.

Type 7: Spot price plus bonus or penalty by consumption. It is analogous to type 2 but referenced to the spot price instead of to a fixed annual price.

Type 8: Spot price.

Specifically, the number of contracts of each type and product considered in this example is stated in Table 1.

Table 1: Number of contracts of each type and product included in the case example

	Acquisition			Sale
Type of Contract	Electricity	Fuel Oil	Natural Gas	Electricity
1	1	1	1	1
2	2			
3	2			
4	2			1
5	2			
6	2	1	1	
7		1	1	
8	1	1	1	1
Total	12	4	4	3

The time periods of the problem are grouped into 4 representative days per month. These are the combination of working and non-working days according to the Spanish electricity tariffs and on and off production status of the factory. Each representative day is composed of 3 periods corresponding to peak, plateau and off-peak hours in working days and to 8 consecutive hours in non-working days. The number of periods considered in the planning year is 90, since not all the months have 4 representative days.

The stochastic MIP problem has 15 price scenarios, 90 periods and, as a result, a probability tree with 1350 nodes. This problem contains 88,035 constraints, 129,879 variables, 16,043 of which are binary and 492,818 non-zero coefficients of the constraint matrix. The model was programmed in GAMS [2] and solved with the solver CPLEX 9.0.

The efficient frontier obtained with the safety-first model is depicted by the solid line in Fig. 8. The solutions are labeled in capital letter, whereas the crosses ($\times$) are the VaR values with a confidence level of 0.9 for each optimal safety-first alternative. Contract portfolios above the efficient frontier have higher values in at least one of the two objectives: expected cost and risk measure, whereas there are no feasible solutions below the efficient frontier.

The types of contracts obtained for each product and their cost or income are shown in Table 2. The difference between the extreme solutions is significant. Option E increases the expected cost with respect to A in 9.3%,

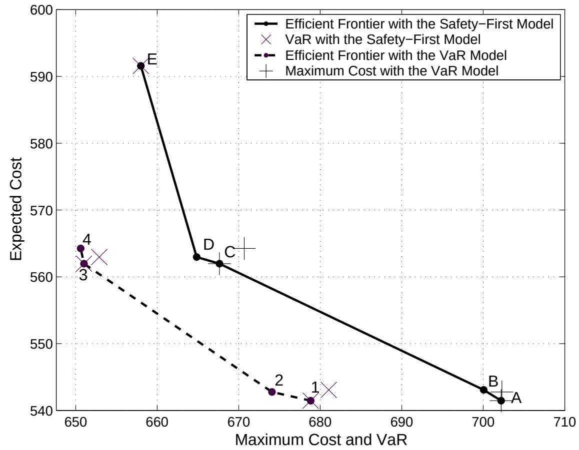

Figure 8: Efficient frontier with the safety first and VaR models [k€].

although the former decreases the maximum cost in 6.3%. The consumer will choose among these alternatives depending on his risk aversion.

Table 2: Solutions of the efficient frontier with the safety-first model [k€]

	Alternative				
Contract	A	B	C	D	E
Acq. of Elec. Type 2	16.7		11.6		0.4
Acq. of Elec. Type 6				21.9	
Acq. of Elec. Type 8		19.9			
Acq. of F. Oil Type 7	26.8	29.0	17.3	31.3	
Acq. of N. Gas Type 6			914.1	881.3	958.0
Acq. of N. Gas Type 8	872.6	868.0			
Sale of Elec. Type 4					827.3
Sale of Elec. Type 8	812.5	809.4	825.2	804.8	
Expected Cost	541.5	543.1	562.0	563.0	591.6
Maximum Cost	702.2	700.1	667.6	664.8	658.0
$VaR_{0.9}$	678.8	681.0	651.0	652.8	658.0

Three groups of solutions can be appreciated (A-B, C-D and E), each one having the same contracts of acquisition of natural gas and sale of surplus electricity. Solutions within the same group have similar costs, since fuel oil and electricity acquisition contracts are much cheaper than the others. The reason for this is that the cogeneration plant produces most of the periods because of the profitability of selling surplus electricity.

The efficient frontier illustrates how contracts are chosen for risk hedging. Thus, the contract portfolio with the highest risk (alternative A) corresponds to spot price contracts of the most expensive products (natural gas and surplus electricity). On the opposite side is E, the most risk-averse alternative, for which the model selects a three-section TOU rate contract for the sale of surplus electricity and a spot price contract with cap and floor prices for the acquisition of natural gas.

The distribution functions of the five alternatives obtained are depicted in Fig. 9. The spreading of the distributions is higher for solutions of higher risk and lower expected cost. The difference of low cost scenarios is higher than that of high cost scenarios; however, low cost scenarios are not taken into account since the consumer perceives the risk as the potential of high costs.

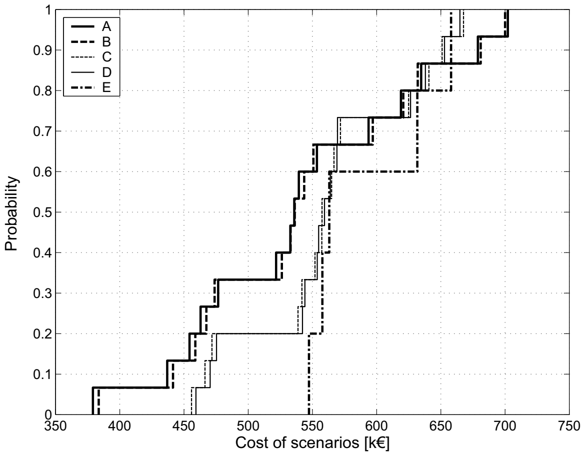

Figure 9: Distribution functions of the solutions of the efficient frontier calculated with the safety-first model.

The other efficient frontier, determined with the risk-neutral and VaR models as mentioned in the previous section, is depicted by the doted line in Fig. 8. The points, labeled with numbers, are the optimal VaR values with a confidence level of 0.9, whereas the plus signs (+) represent the maximum cost of the distributions for the optimal VaR. The types of contracts chosen for the VaR model and their costs or income can be found in Table 3.

The solutions calculated when optimizing VaR are similar to those generated with the safety-first model. Specifically, solutions 1 and A on the one hand, and 3 and C on the other hand, correspond to the same contract portfolio. The other solutions of the VaR efficient frontier, 2 and 4, are similar to B and D, respectively. In fact, although 2 and 4 are not optimal solutions from a safety-first perspective, they are very close to the efficient frontier obtained with the safety-first model.

The main difference between both efficient frontiers is option E, whose

Table 3: Solutions of the efficient frontier with the VaR model [k€]

	Alternative			
Contract	1	2	3	4
Acq. of Elec. Type 2	16.7	17.4	11.6	12.5
Acq. of F. Oil Type 6		30.0		21.5
Acq. of F. Oil Type 7	26.8		17.3	
Acq. of N. Gas Type 6			914.1	910.8
Acq. of N. Gas Type 8	872.6	867.9		
Sale of Elec. Type 8	812.5	809.4	825.2	823.8
Expected Cost	541.5	542.8	562.0	564.3
$\text{VaR}_{0.9}$	678.8	674.1	651.0	650.6
Maximum Cost	702.2	702.3	667.6	670.7

maximum cost and VaR have the same value (see Fig. 8). This alternative does not appear when solving the VaR model since both the expected cost and VaR of this option are higher than those of alternative 4 and, therefore, option E is not an efficient VaR solution.

Although the efficient frontiers obtained for both risk measures are similar, the computation time is very different. The VaR approach requires much more time because of the implicit scenario selection involved in VaR evaluation. Specifically, the VaR model is solved in around 22 h, whereas the safety-first model takes 6 h and the deterministic model only takes 20 seconds[2]. In order to decrease these times, decomposition techniques for stochastic MIP models should be used.

Lastly, it is worth noting that all of the portfolios obtained contain contracts linked to spot prices. In this example, portfolios without price uncertainty are not efficient because the premium paid by the consumer for limiting the price risk is too high. These types of portfolios have a null variance, however, the risk associated with them, measured as VaR or maximum cost, is high. Although the parameters of the contracts of this example are realistic, it is possible that other parameters provided by retailers could lead to efficient fixed price contracts. Nevertheless, this example shows the usefulness of the models developed for contract evaluation and selection. It is worth to know that the number of industrial consumers that can use this stochastic model as decision support tool for their contracting decisions is quite large. For example, in Spain there are approximately seven hundred cogenerators and one hundred corresponds to the paper industry, as the cellulose paper factory used in this case study.

[2]Models were executed on a Pentium IV 3 GHz.

7 Conclusions

In this paper we have presented multi-objective stochastic optimization models for the energy management of industrial consumers working under liberalized energy markets. These original models optimize contracting and energy supply system operation decisions simultaneously taking into account the consumer's risk attitude. The integrated tool developed allows the consumers to decrement the energy bill and control the assumed risk. Other application of this tool is the analysis of new contracting possibilities for the retailers.

Starting from the deterministic problem stated in [7], we have extended this problem to two-stage stochastic models. In the first stage, before the first time period of the problem, contracting decisions are made. Simultaneously, in each period of the problem, the energy supply system operation is determined once the uncertainty is revealed.

The first model presented, the risk-neutral approach, does consider the price uncertainty explicitly, however, it is unable of performing risk management. To overcome this drawback, the stochastic models formulated obtain a tradeoff between a risk measure and the expected cost through a risk-aversion parameter. To formulate the stochastic problem, safety-first and VaR risk measures have been implemented. Both reflect the potential of high costs and measure confidence levels and, therefore, represent the risk attitude of consumers and are easy to interpret.

When determining efficient frontiers with VaR as the risk measure, the problem encountered is that not all the second-stage variables calculated are optimal, since they are not penalized in the objective function. This problem can be solved with the proposed two-phase method for obtaining each value of the efficient frontier. In the first phase, first-stage variables (contracts) and VaR are calculated with the VaR model. Next, in a second phase, these contracts are fixed in a risk-neutral model, which obtains second-stage variables (boiler and cogeneration operation) and the optimal expected cost.

Finally, we have illustrated the working of the models with a realistic case example. The results show that the models proposed can be valuable for reducing consumers' energy costs while keeping control of price risk.

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