A Medium Term Bulk Production Cost Model Based on Decomposition Techniques

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Abstract- This model provides the minimum variable cost subject to operating constraints (generation, transmission and fuel constraints). Generation constraints include power reserve margin with respect to the system peak load, first Kirchhoff's law at each node, hydro energy scheduling, maintenance scheduling, and generation limitations. Transmission constraints cover the second Kirchhoff's law and transmission limitations. The generation and transmission economic dispatch is approximated by the linearized (also called DC) load flow. Network losses are included as a non linear approximation. Fuel constraints include minimum consumption quotas and fuel scheduling for domestic coal thermal plants.

This production costing problem is formulated as a large-scale non linear optimization problem solved by generalized Benders decomposition method. Master problem determines the interperiod decisions, i.e., maintenance, fuel and hydro scheduling, and each subproblem solves the intraperiod decisions, i.e., generation and transmission economic dispatch for one period.

The model has been implemented in GAMS, a mathematical programming language. An application to the large-scale Spanish electric power system is presented.

1 Introduction

Economic planning is a major concern of medium term operations planning studies devoted to predict the future operation of an electric utility. The remuneration of the Spanish electric utilities is determined, according to the Stable Legal Framework (SLF), as a function of some standard parameters and the results obtained from the operation of the system. The SLF establishes the methodology to determine the electric tariff by a combination of cost of service plus yardstick competition method and calculates the economic compensations among utilities.

This production cost model addresses all the operating decisions in an integrated framework and establishes a compromise and balance among different modeling decisions. Its prime purpose is to predict the future system operation and determine their parameters (costs, fuel consumption, productions, etc.) for the large-scale Spanish electric power system.

This model provides the minimum variable cost subject to operating constraints (generation, transmission and fuel constraints). Generation constraints include power reserve margin with respect to the system peak load, first Kirchhoff's law at each node, hydro energy scheduling, maintenance scheduling, and generation limitations. Transmission constraints cover the second Kirchhoff's law and transmission limitations. The generation and transmission economic dispatch is approximated by the linearized (also called DC) load flow equations. Network losses are included as a non linear approximation of the voltage angles at the nodes and resistance and reactance of the circuit. Fuel constraints include minimum consumption quotas and fuel scheduling for domestic coal thermal plants. The relevant decision variables and the real operation of the power system are adequately represented, two types of decisions are addressed:

- *Interperiod* decisions are those regarding resources planning for multiple periods. In particular, maintenance scheduling for thermal units, yearly hydro energy scheduling, seasonal operation of pumped-hydro[1] units, and fuel scheduling are represented. The model determines the optimal hydrothermal coordination (i.e., the use of hydro against thermal generation resources).
- *Intraperiod* decisions correspond to a generation and transmission economic dispatch. In particular, those related to weekly/dairy operation of pumped-storage units and commitment decisions of thermal units.

This production costing problem is formulated as a large-scale non linear optimization problem solved by generalized Benders decomposition method. Master problem determines the interperiod decisions and each subproblem solves the intraperiod decisions, i.e., generation and transmission economic dispatch for one period.

The model has been implemented in GAMS [1], a mathematical specification language specially suited for the solution of optimization problems, and solved by using OSL and MINOS, a well-known solvers. The model is being used as a medium term tool for economic planning by IBERDROLA Generation Area by representing the Spanish electric power system.

The medium term planning problem is stochastic by nature. Uncertainties arise in load, hydro inflows, generation and transmission availability, etc. However, the model described is deterministic. Stochasticity in units and lines availability and load can be naturally implemented within this methodology via scenarios. Uncertainty in hydro inflows is modeled deterministically because medium term economic planning is performed under the assumption of average hydrology. However, the model allows the automatic run of several sensitivities, for example, to different hydro inflows, demand increments or major overhaul of thermal units.

No model with this whole set of characteristics (i.e., fuel, maintenance and hydro scheduling on one hand and generation and transmission economic dispatch and commitment decisions on the other hand) has been found in the literature. On one hand, models deciding seasonal hydro scheduling, usually based on stochastic dynamic programming or decomposition methods [5], represent in detail the spatial hydro dependencies but usually ignore the fuel and maintenance scheduling problems. Medium term fuel scheduling is decided using a large-scale linear programming approach in [7, 10]. Maintenance scheduling has been solved by many different techniques [6], decomposition techniques [11] and integer programming [4] among others. Combined seasonal and weekly/dairy operation of pumped units has not been addressed so far. On the other hand, bulk production cost models usually ignore the interperiod decisions, such as hydro and maintenance scheduling, and treat each period independently [2, 3, 8, 9], .

The paper is organized as follows. Firstly, the notation used for the mathematical expressions is presented. In section 3 it is shown the system description and modeling of each type of generation and network. Then, the mathematical formulation of the model is stated. The solution procedure based on decomposition method is explained afterwards. The implementation of the model and some basic characteristics are discussed in section 6. Finally, it is described the application of the model to the Spanish electric power system where the convergence properties of the method are shown.

2 Notation

In this section all the symbols used along the paper are identified and classified according to their use into auxiliary, input and output variables.

A. Indexes and Auxiliary Variables

B number of pumped-storage units.
C number of thermal plants with fuel quotas.
C' number of thermal plants.
D number of nodes.
H number of hydro and/or pumped-hydro units.
K number of circuits per line.
L number of lines.
N number of load levels.
P number of periods.
S number of subperiods.
T number of thermal units.
θ_p recourse function of period p.

B. Input Variables

A_{hp} hydro inflows expressed in energy for hydro unit h in period p.
$\bar{b}_b, \underline{b}_b$ maximum and minimum capacity of pumped-storage unit b when pumping.
C_{cp} minimum quota of fuel consumption in thermal plant c at the beginning of period p.
c_c fuel storage cost in thermal plant c per unit of time.
d_{dnsp}, d'_{dnsp} customer and interruptible power demand in node d in load level n of subperiod s of period p.
D_{nsp} duration of load level n of subperiod s of period p.

[1] In the paper the following convention is used. A *pumped-hydro* unit is a pump-turbine having a large upper reservoir with seasonal storage capability that receives water from pumping and also from natural hydro inflows. By the contrary, a *pumped-storage* unit has a small upper reservoir filled only from pumped water allowing just a weekly or dairy cycle.

$\overline{e}_h, \underline{e}_h$ maximum and minimum capacity of pumped-hydro unit h when pumping.

$\overline{F}_{kl}, \underline{F}_{kl}$ maximum flow in one and the other way by circuit k in line l.

g maximum number of thermal units simultaneously in maintenance on the same plant.

$\overline{h}_{hp}, \underline{h}_{hp}$ maximum and minimum capacity of hydro unit h in period p.

k_t auxiliary services coefficient of thermal unit t.

m coefficient of the thermal installed capacity simultaneously on maintenance in any period.

M_t number of periods on scheduled maintenance for the thermal unit t.

o_t, o'_t heat rate (independent and linear terms) of thermal unit t.

$\overline{p}_t, \underline{p}_t$ maximum and minimum rated capacity of thermal unit t.

q_t EFOR of thermal unit t.

R power reserve margin.

$\overline{R}_h, \underline{R}_h$ maximum and minimum hydro energy reserve of hydro unit h.

r_{kl}, x_{kl} resistance and reactance of circuit k of line l.

r_t startup cost of thermal unit t.

s security coefficient for circuit flows.

S_B power base.

$\overline{S}_c, \underline{S}_c$ maximum and minimum fuel storage capacity of thermal plant c.

$\overline{t}_b, \underline{t}_b$ maximum and minimum capacity of pumped-storage unit b when generating.

u_t O&M variable cost of thermal unit t.

V_b upper reservoir limit of pumped-storage unit b.

v_t fuel cost of thermal unit t.

W penalty cost by power reserve defect.

w, w' unserved and interruptible energy cost.

η_b performance of pumped-storage unit b.

η_h performance of pumped-hydro unit h.

C. Output Variables

a_{tsp} commitment decision of thermal unit t in subperiod s of period p [0/1].

b_{bnsp}, t_{bnsp} consumption and generation by pumped-storage unit b in load level n of subperiod s of period p.

e_{hnsp} power consumption by pumped-hydro unit h in load level n of subperiod s of period p.

f_{klnsp} flow through the circuit k of line l in load level n of subperiod s of period p.

f_{sp} defect of power reserve in subperiod s of period p.

h_{hnsp} generation of hydro unit h in load level n of subperiod s of period p.

i_{tp} unavailability by scheduled maintenance for thermal unit t during period p [0/1].

n_{dnsp}, n'_{dnsp} non served and interruptible power in node d in load level n of subperiod s of period p.

p_{tnsp} generation of thermal unit t in load level n of subperiod s of period p.

R_{hp} hydro energy reserve of hydro unit h at the beginning of period p.

S_{cp} fuel storage capacity of thermal plant c at the beginning of period p.

θ_{dnsp} voltage angle in node d in load level n of subperiod s of period p.

π^j_{hnsp} dual variable of the first Kirchhoff's law constraint at iteration j.

ρ^j_{tsp} dual variable of the constraint commitment decision lower than the availability at iteration j.

σ^j_{cp} dual variable of the constraint associated to heat consumption of thermal plant at iteration j.

ξ_{tp} auxiliary variable in the equation of contiguity among maintenance periods.

3 System Description

A production cost model determines the variables defining the system operation at minimum variable cost for the scope of the model. Let us define *horizon* as the point in time for which the system operation is to be modeled and *scope* as the duration of the time interval to be studied. In this medium term model, the horizon is two or three years ahead and the scope is usually one year. The scope is divided into *periods*, *subperiods* and *load levels*. Typically, periods will correspond to months, subperiods to weekdays and weekends of a month, and load levels to peak, plateau and off-peak hours.

The load for each period is modeled as a staircase load duration curve, where an step is a load level. Hence, generation will be constant for each load level. Load is individually specified for each node.

Each thermal unit is divided into two blocks, being the minimum load block the first. Heat rate is specified by a straight line with independent and linear terms. Random outages are deterministically modeled by derating the unit's full capacity by its equivalent forced outage rate. A thermal plant consists of units in a physical plant. Fuel constraints affect the fuel consumption of domestic coal thermal plants.

Very small hydro units are aggregated. Spatial dependencies among hydro plants are considered irrelevant to the medium term thermal generation scheduling problem and ignored. Therefore, the variation in the hydro energy reserve of a reservoir due to the generation in a hydroelectric plant located upstream is not taken into account.

Only the economic utilization of pumped units is considered. This economic function includes both the transference of energy from off-peak hours to peak hours and the alleviation of minimum load conditions in off-peak hours or maximum load conditions in peak hours. Additionally, these units may be operated for reliability purposes keeping their upper reservoirs full at the beginning of each week but this operation is not represented in the model.

Each line of the high voltage transmission network is modeled individually by its resistance and reactance and a thermal capacity limit.

4 Model Formulation

As mentioned previously this medium term production cost model performs hydro, maintenance and fuel scheduling, seasonal operation of pumped-hydro units, weekly/dairy operation of pumped-storage units, generation/transmission economic dispatch and thermal unit commitment for a generation system. The model is formulated as a large-scale non linear *global* optimization problem. The objective function to be minimized is the total variable cost for the scope of the model subject to operating constraints. These can be classified into inter and intraperiod, according to the spanning periods, as previously defined.

It follows the mathematical formulation of the objective function, the constraints and the variables involved in the problem.

A. Objective Function

The objective function represents the fuel costs (including independent and linear terms of the heat rate and O&M variable costs) plus startup costs plus storage costs of fuel stocks plus some penalties (due to non served power, interruptibility, and reserve margin defect) for all the load levels, subperiods and periods of the time scope.

$$\begin{aligned}
\text{Min} \quad & \sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{n=1}^{N}\sum_{t=1}^{T} D_{nsp} v_t \left(o_t a_{tsp} + o'_t \frac{p_{tnsp}}{k_t} \right) \\
& \sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{n=1}^{N}\sum_{t=1}^{T} D_{nsp} u_t p_{tnsp} \\
+ & \sum_{p=1}^{P}\sum_{s=2}^{S}\sum_{t=1}^{T} r_t (a_{ts-1p} - a_{tsp}) \\
+ & \sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{n=1}^{N}\sum_{c=1}^{C} c_c D_{nsp} S_{cp} \\
+ & \sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{n=1}^{N}\sum_{d=1}^{D} D_{nsp} (w n_{dnsp} + w' n'_{dnsp}) \\
+ & \; W \sum_{p=1}^{P}\sum_{s=1}^{S} f_{sp}
\end{aligned} \tag{1}$$

In order to allow separability among periods the startup costs from weekend to workdays have been assumed to be equal to the shutdown costs from workdays to weekend and these are the costs considered into the equation.

B. Interperiod Constraints

These constraints span all the periods considered in the model and correspond to maintenance, fuel and hydro scheduling.

1. Maintenance Scheduling

   The units will be shutdown an integer number of periods for scheduled maintenance according to the specified requirement.

$$\sum_{p=1}^{P} i_{tp} = M_t \tag{2}$$

   Also limits on the maximum number of thermal units simultaneously on maintenance on the same plant and on the maximum thermal capacity simultaneously on maintenance in any period with respect to the total installed thermal capacity are imposed.

$$\sum_{t \in c} i_{tp} \leq g \tag{3}$$

$$\sum_{t}^{T} i_{tp} \overline{p}_t \leq m \sum_{t}^{T} \overline{p}_t \tag{4}$$

   Contiguity among the maintenance periods is required too, if more than one is specified.

$$i_{tp} \leq \sum_{p' \leq M_t} (\xi_{tp+p'-1} + \ldots + \xi_{tp+M_t-1}) \tag{5}$$

   being the sum of the variables ξ_{tp} equal to 1.

2. Fuel Scheduling

   For each thermal plant, the stock level at the beginning of each period is a function of the previous stock and the purchase and consumption done during the period. The initial and final storage levels are specified by the user. It represents the must-buy fuel purchase mandated by socioeconomic and political considerations for domestic coal plants, although their cost can be more expensive than other available fuels.

$$\sum_{s=1}^{S}\sum_{n=1}^{N}\sum_{t\in c} D_{nsp}(o_t a_{tsp}+o'_t\frac{p_{tnsp}}{k_t}) \geq C_{cp}+S_{cp}-S_{cp+1} \tag{6}$$

3. Hydro Scheduling

   For each hydro unit, the hydro reserve level at the beginning of each period is a function of the previous level, the hydro inflow, pumping and generation on that period. The initial and final hydro reserves are specified by the user.

$$\sum_{s=1}^{S}\sum_{n=1}^{N} D_{nsp}(h_{hnsp}-\eta_h e_{hnsp}) \leq A_{hp}+R_{hp}-R_{hp+1} \tag{7}$$

C. Intraperiod Constraints

These constraints are internal to each period and represent the security constraint based on the reserve margin, first and second Kirchhoff's laws and the weekly/dairy operation of pumped-storage units, and the thermal generation constraints.

1. Reserve Margin

   A power reserve margin for the peak load level of each subperiod must be met. This constraint represents the condition imposed to provide some amount of power available to account for increments in demand or failures of committed generation units.

$$\sum_{t=1}^{T}\overline{p}_t k_t a_{tsp}+\sum_{h=1}^{H}\overline{h}_{hp}+\sum_{b=1}^{B}\overline{t}_b+f_{sp} \geq d_{1sp}(1+R) \tag{8}$$

2. First Kirchhoff's Law

   Balance between generation and demand at any node for any load level including non served power and interruptibility. Thermal losses of each circuit are modeled as fictitious loads at each end node of the line. Each load represents half of the total losses of the line that are calculated as a non linear approximation of the voltage angles at their extreme nodes and resistance and reactance of the circuit. It is the only non linear equation of the model.

$$\begin{aligned}
&\sum_{t\in d} p_{tnsp} \\
+\ &\sum_{h\in d}(h_{hnsp}-e_{hnsp})+\sum_{b\in d}(t_{bnsp}-b_{bnsp}) \\
+\ &\sum_{l\to d}\sum_{k=1}^{K} f_{klnsp}-\sum_{l\leftarrow d}\sum_{k=1}^{K} f_{klnsp} \\
+\ &n_{dnsp}+n'_{dnsp} \\
\geq\ &d_{dnsp} \\
+\ &\sum_{l\leftrightarrow d}\frac{r_{kl}S_B}{r_{kl}^2+x_{kl}^2}\left[1-\cos(\theta_{insp}-\theta_{dnsp})\right] \quad : \pi^J_{hnsp}
\end{aligned} \tag{9}$$

3. Second Kirchhoff's Law

   Relation between the flow through a circuit of a line and the voltage angles in the extreme nodes.

$$f_{klnsp}=\frac{S_B}{x_{kl}}(\theta_{insp}-\theta_{jnsp}) \tag{10}$$

4. Pumped-Storage Units

   Balance between pumped and generated energy by pumped-storage units in a period and a reservoir limit imposed to the pumped energy.

$$\sum_{s=1}^{S}\sum_{n=1}^{N} D_{nsp}(\eta_b b_{bnsp}-t_{bnsp})=0 \tag{11}$$

$$\sum_{s=1}^{S}\sum_{n=1}^{N} D_{nsp} b_{bnsp} \leq V_b \tag{12}$$

5. Thermal Generation Constraints

   For each thermal unit the maximum generation is less than the maximum available capacity and the minimum generation is greater than the minimum load. Thermal unit commitment related constraints state that the unit's output during higher load levels must be larger than its generation in lower load levels and that the commitment decision in a higher load subperiod (weekdays) must be greater than those in a lower load subperiod (weekends).

$$\underline{p}_t k_t(1-q_t)a_{tsp} \leq p_{tNsp} \tag{13}$$

$$p_{t1sp} \leq \overline{p}_t k_t(1-q_t)a_{tsp} \tag{14}$$

$$p_{tn+1sp} \leq p_{tnsp} \tag{15}$$

$$a_{tsp} \leq 1 - i_{tp} \qquad : \rho^J_{tsp} \tag{16}$$

$$a_{ts+1p} \leq a_{tsp} \tag{17}$$

The above constraints enforce a minimum generation for each thermal unit committed at peak load level. Note that since the heat rate curves are represented as linear curves, during any load level all the committed units will be at their maximum output except one marginal unit if the network does not introduce further constraints.

D. Variables

All the variables involved in the previous formulation are subject to the following bounds:

- maintenance decisions for thermal units, $i_{tp} = \{0,1\}$, $0 \leq \xi_{tp} \leq 1$
- fuel stock levels, $\underline{S}_c \leq S_{cp} \leq \overline{S}_c$
- hydro productions and consumption of seasonal pumped-hydro units, $\underline{h}_{hp} \leq h_{hnsp} \leq \overline{h}_{hp}$ and $\underline{e}_h \leq e_{hnsp} \leq \overline{e}_h$
- hydro energy reserves, $\underline{R}_h \leq R_{hp} \leq \overline{R}_h$
- commitment decisions of thermal units, $a_{tsp} = \{0,1\}$
- thermal generation bound explicitly considers the deterministic criterion, $p_{tnsp} \leq \overline{p}_t k_t (1 - q_t)$
- generation and consumption of weekly/dairy pumped-storage units, $\underline{t}_b \leq t_{bnsp} \leq \overline{t}_b$ and $\underline{b}_b \leq b_{bnsp} \leq \overline{b}_b$
- non served power and interruptible power at each node, $n_{dnsp} \leq d_{dnsp}$ and $n'_{dnsp} \leq d'_{dnsp}$
- reserve margin defect, $f_{sp} \leq d_{1sp}(1 + R)$
- load flows bounds consider a coefficient s that takes into account in a simple way preventive dispatch considerations, $-s\underline{F}_{kl} \leq f_{klnsp} \leq s\overline{F}_{kl}$
- voltage angle in nodes bounds included from an algorithmic point of view, $-0.75 \leq \theta_{dnsp} \leq 0.75$

The variables regarding operation of the pumped-hydro and pumped-storage units are defined only for the periods, subperiods and load levels where they are meaningful according to the system operation.

The previous formulation has been presented in a general framework of a bulk production cost model. Then the commitment and maintenance decisions for thermal units cause the problem to be mixed integer with the associated difficulty to be solved. However, the following operating criterion has been used that simplifies this bulk production cost model. The discrete maintenance decisions are previously determined with the transmission network reduced to a single node with a much smaller optimization problem. These values are taken as input for the bulk production cost model. Unit commitment variables are relaxed to be in the interval [0,1].

5 Solution Procedure

This optimization problem has an special staircase structure that can be exploited. Decomposition techniques are useful in solving this problem by two reasons. First, the global problem is divided into smaller and easier to solve problems. Second, it allows the use of different solution algorithms for master and subproblem. In particular, Benders method is applied when some variables "complicate" the solution of the problem. In this model the complicating variables are those regarding maintenance, fuel and hydro scheduling. Benders method is specially indicated in this context of hydrothermal coordination, where hydro generation is decided in the master problem and thermal operation is determined in the subproblem. Generalized Benders procedure is required due to the non linearity appearing in balance constraints.

Besides these considerations, the size of the non linear optimization problem resulting for the Spanish power system prevents the direct use of a non linear code.

Before the formulation of the master and subproblem the constraints associated to fuel scheduling are restated for the problem to be modeled using the Benders method.

$$\sum_{s=1}^{S} \sum_{n=1}^{N} \sum_{t \in c} D_{nsp} \left[o_t a_{tsp} + o'_t \frac{p_{tnsp}}{k_t} \right] \geq P_{cp} \qquad : \sigma^J_{cp} \tag{18}$$

$$0 \leq P_{cp} \leq \sum_{s=1}^{S} \sum_{n=1}^{N} \sum_{t \in c} D_{nsp}(1 - i_{tp}) \left[o_t + o'_t \overline{p}_t (1 - q_t) \right]$$

$$P_{cp} = C_{cp} + S_{cp} - S_{cp+1} \tag{19}$$

A. Master Problem

The master problem has the following objective function:

$$\text{Min} \sum_{p=1}^{P} \theta_p + \sum_{p=1}^{P} \sum_{s=1}^{S} \sum_{n=1}^{N} \sum_{c=1}^{C} c_c D_{nsp} S_{cp} \tag{20}$$

and all the intraperiod constraints excepting the fuel scheduling equation (6) replaced by (19) and the bounds of the corresponding variables.

Moreover, for each iteration j a Benders cuts, one per period, are added

$$\begin{aligned}\theta_p - \theta_p^{j-1} \;\geq\; & -\sum_{s=1}^{S}\sum_{t=1}^{T} \rho_{tsp}^{j-1}(i_{tp} - i_{tp}^{j-1}) \\ & - \sum_{s=1}^{S}\sum_{n=1}^{N}\sum_{h=1}^{H} \pi_{hnsp}^{j-1}\Big(h_{hnsp} - h_{hnsp}^{j-1} \\ & \qquad -e_{hnsp} + e_{hnsp}^{j-1}\Big) \\ & + \sum_{c=1}^{C} \sigma_{cp}^{j-1}(P_{cp} - P_{cp}^{j-1})\end{aligned} \tag{21}$$

where the superscript indicates the value of the variable for that iteration.

B. Subproblem

The subproblem is separable in as many as periods. Each subproblem represents the operation of the system for each period and has the same objective function than the global problem excepting the storage costs term now included into the master problem and the remaining constraints (18) and (8) to (17) and the limits of the corresponding variables.

C. Benders Decomposition Method

Benders decomposition method achieves the solution of the global problem by iteratively solving master and subproblem and establishing an information link between both problems. For each iteration j the master problem determines a trial maintenance, fuel and hydro scheduling defined by the variables i_{tp}^j, h_{hnsp}^j, e_{hnsp}^j, and P_{cp}^j that are passed to the subproblem. Given these proposals, the subproblems are solved and return the sensitivity information ρ_{tsp}^J, π_{hnsp}^J and σ_{cp}^J to the master. The master problem is then modified, adding new Benders cuts based on the solution from the subproblems and solved again. This iterative procedure is solved until no further improvement can be achieved. Convergence is guaranteed if problems are convex.

A careful study of the number of constraints and variables involved in each formulation -global, master and subproblem- has been conducted and the sizes of the different problems are presented in the table 5.

6 Implementation

The model has been implemented in GAMS version 2.25 [1]. It allows the creation of large and complex problems in a concise and reliable manner. This language lets the user to concentrate on the modeling problem by eliminating the writing details of special code in the preliminary stages of algorithmic development. GAMS is flexible and powerful. This flexibility is crucial in the development and test of new algorithms. Several LP and NLP solvers can be used in conjunction with the GAMS language, for example, CPLEX and OSL as linear and MINOS as non linear solvers. The implementation of this model and its resolution using this compact and elegant algebraic language takes only 1400 lines of code. The model can be used in any hardware platform where GAMS and the solvers were available. Currently, a workstation is being used.

Careful attention when solving a large-scale optimization problem should be paid to the scalation of constraints and variables. So GW is taken as the natural unit for power, TWh for energy, Tpta for monetary unit and kTcal for heat consumption.

For solving this large-scale non linear problem (due to losses) it is provided as initial point the optimal solution of the global linear problem (with no losses). Its size for the Spanish electric power system is about 38000 rows, 52000 variables and 153000 non zero elements in the constraints matrix. In particular, recent developments in interior point methods are specially suited for the solution of these large problems. In this model it is solved using an interior point method of OSL. The non linear problem is solved by Benders decomposition, using MINOS for master and subproblem.

The model is a very powerful and flexible tool that can easily be adapted to any electric power system. Several options have been provided to customize their use to different needs, such as the following:

- production cost model with a single node with aggregation or deaggregation of the hydro units, generation/transmission production cost model with or without losses
- specification of the periods, subperiods and load levels when the pumped-hydro and pumped-storage units can be used for pumping or generation
- maintenance based exclusively on input data, discrete optimization with some prespecified maintenance and discrete optimization with no prespecified data
- demand increment with respect to a base case whose data are provided
- multiyear use for concatenated evaluation of an scope composed by several years
- sensitivities to a demand increment, different hydrologies, release of the minimum consumption quotas, and major overhaul of thermal units

The main results obtained from the model are: net and gross energy production for each generation unit; utilization, commitment and shutdown hours for each unit; fuel consumption for thermal plant separated in guaranteed

	CONSTRAINTS
Global Problem	$2T + P(2B + C + C' + H + 1) + SP(3T + 1) + NSP(D + KL + T)$
Master Problem	$2T + P(2C + C' + H + T + 1) + J - 1$
Subproblems	$(2B + C - T) + S(3T + 1) + NS(D + KL + T)$
	VARIABLES
Global Problem	$-C - H + P(C + H + 2T) + SP(T + 1) + NSP(2B + 3D + 2H + KL + T - 1)$
Master Problem	$-C - H + P(2C + H + 2T + 1) + 2NSPH$
Subproblems	$J + S(T + 1) + NS(2B + 3D + KL + T - 1)$

and spot market; fuel, startup, storage costs; maintenance schedule; flows and spot prices at the nodes. These results can be presented for load level, for each period or grouped by electric utility.

7 Case Study

The model has been designed to represent yearly operation of the Spanish electric power system. The scope of the model has been split into 12 periods (months) with 2 subperiods each (weekdays and weekends) and 3 and 2 load levels per each subperiod.

According to data extracted from 1994 statistical records, the Spanish power system met a maximum peak load of 25336 MW and a yearly energy demand of 145670 GWh. The installed generation capacity is 42096 MW (16110 MW are hydro, 10675 MW coal, 7910 MW oil/gas and 7401 MW nuclear). The share among energy produced by different utilities corresponds to IBERDROLA in a 40 %, ENDESA 40 %, UEFSA 15 % and other the remaining.

There are about 71 thermal generators (8 nuclear, 36 coal and the remaining oil/gas) grouped in 16 thermal plants. Their production is about 80 % of the total generation.

There are 70 hydro units with capacity greater than 5 MW and annual energy production greater than 100 GWh, that can be grouped into about 10 basins. Other 52 smaller hydro units have been considered. The maximum capacity at the same location is 915 MW. They produce as an average about 20 % of the total generation, ranging in between 13 % and 28 %, depending on the hydrology.

There are 8 pumped storage units, but their impact on annual energy production is minimum (about 1 %).

The high voltage transmission network has been simplified to 138 nodes and 263 lines.

For this case study, size (defined by rows, variables and elements), time and number of iterations required for solving the different problems are presented in the following table.

Time is expressed in seconds for a workstation Axil 311 Model 5.1 with performance of 83 SPECfp92 and 65 SPECint92.

Taking the optimal solution of the linear problem as initial point for the decomposition algorithm it takes 13 iterations to achieve converge under a tolerance of 1e-3 with one cut per period added in the master problem for each iteration. The difference in variable costs due to losses are about 0.2 %.

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	Solver time	Number of iterations
GLOBAL problem 36625 r, 51335 v, 149217 e		
OSL Interior3	1185	39
MASTER 1477+12J r, 8675 v, ≈ 13917+300J e		
MINOS Primal simplex	35	2000
SUBPROBLEM 2932 r, 3487 v, 10887 e		
MINOS Primal simplex	19	450

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