

Applied Mathematics in the Electricity Industry Management

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Abstract

This paper shows that optimization models are a part of the curriculum of the School of Engineering at different levels from undergraduate to graduate students. Also it illustrates how optimization models can contribute to support decisions in the electricity industry. Mock-up models are used as learning tools for the students while high-end models are fully integrated in the real decision making of the electric companies.

Introduction

Operational Research (OR) can be defined as the application of advanced scientific analytical methods in improving the effectiveness in operations, decisions and management of a company. Other names also used for naming it are *Management Science*, *Business Analytics* or *Decision Science*. In short, it is also defined as the science of better (motto of the Operational Research Societies

(<http://www.scienceofbetter.org/>))

or as decision support models or advanced analytical methods. We can consider OR as a science in the frontier between primarily economics and engineering.

Life itself is a matter of OR is the slogan of the EURO (European Operational Research Societies). Effectively, there are many fields where OR methods can be applied to improve decision making. The electricity industry and the OR have always been good companions for many years, see Garver (1962). Decision support tools allow managing the operation and to evaluate long-term capital-intensive decisions in the electricity industry, see Delson (1992).

In this paper we present the practical experience of teaching applied mathematical methods at undergraduate, master and doctorate levels in a School of Engineering and some real decision support tools we have developed to help in taking operation and investment decisions in the electricity industry.

The paper is organised as follows. First, we introduce an overview of the different courses that constitute the curriculum of mathematical optimisation methods of the engineering students and the learning material they utilize. Then, we enumerate the competencies that the students attain within these courses. Later, we show the importance of developing mock-up models to consolidate their learning process. Finally, we establish the transition between the small models and the high-end ones used in the electricity industry and extract some conclusions.

Structure of the optimisation courses

In the paper, we are going to focus on optimization, as the most frequently used technique, although other classical OR techniques such as simulation are also adopted. Optimization selects the best decisions among the innumerable feasible options available. It is a prescriptive technique. Simulation evaluates the performance of a system under different conditions including mainly stochastic parameters or events. It is a descriptive approach.

At undergraduate level in the *Engineering Degree* we present just an introduction/overview of the many techniques that are under the name of *Mathematical Methods* (see the hourly content of the course in

<http://www.iit.upcomillas.es/aramos/MME.htm>)

see figure 1.

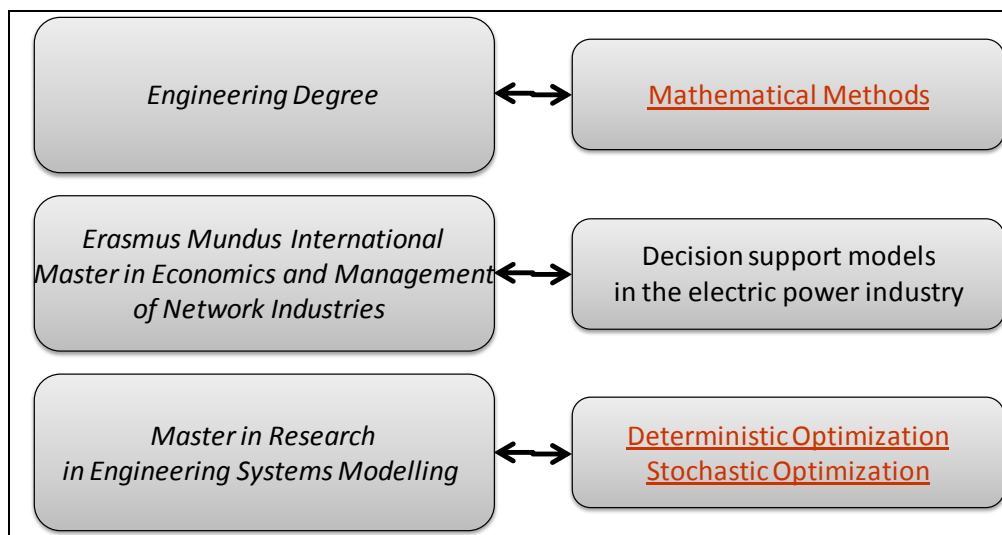


Figure 1. Summary of optimization methods in the engineering curriculum.

At graduate level we need to discriminate between professional-oriented master programs and research-oriented doctorate programs. As an example of the first one we can observe in the *Erasmus Mundus International Master in Economics and Management of Network Industries* whose general program is in this link (<http://www.upcomillas.es/emin/Program.aspx>).

In this master the spotlight is on decision support applications presented in the context of the electricity industry operation functions with a course entitled *Decision support models in the electric power industry*. As an example of the research-oriented doctorate program we can take the *Master in Research in Engineering Systems Modelling* following this link to the syllabus

(http://www.upcomillas.es/eng/estudios/estu_mast_inve_mode_cont.aspx?idIdioma=ENG).

In this research-oriented doctorate program the optimization theory is explained but still with an engineering point of view. Deterministic Optimization

(<http://www.iit.upcomillas.es/aramos/O.htm>)

and Stochastic Optimization

(<http://www.iit.upcomillas.es/aramos/OE.htm>)

are the two courses in the OR field.

Learning material

We follow an OpenCourseWare approach to provide all the material fully accessible via the web. For example, in the link

http://www.doi.icaei.upcomillas.es/intro_simio.htm

you can find the lecture notes and slides while in the link

http://www.iit.upcomillas.es/aramos/Ramos_CV.htm#ModelosAyudaDecision

are the more specific applications to the electricity industry and the mock-up models we provide as a basic step.

Competencies

In these courses we pay emphasis in two main objectives: learn how to build models for a certain decision problem and to understand the technique used to solve them. The student has to be able to develop optimization models using high-level languages. We encompass theory, numerical examples and computer examples along the course.

The specific contents of optimisation that are reviewed with different prominence are: linear programming (LP), mixed integer programming (MIP), nonlinear programming (NLP), mixed complementarity problem (MCP) and stochastic programming (SP). Also we review some specialised algorithms used to solve large-scale optimization problems such as Benders' decomposition or Lagrangian relaxation.

The general competencies of the courses can be summarised in the following list:

- Recognise the diverse fields where optimisation techniques can be applied
- Understand and apply the techniques used for decision making

- Model and solve prototype optimisation problems of diverse nature using an algebraic modelling language
- Analyse, synthesise and interpret the solutions obtained
- Present the model in a written report and orally
- Learn how to work in a group for doing the practice

Professional-oriented master students need to achieve a general understanding of the mathematical models. They must understand their input and output and their use in an industry context. A specific competency of this master is to understand how different functions of the company are done by means of mathematical models.

However, research-oriented master students must develop from scratch their own mathematical optimization model and, therefore. They must be able to develop a model following the different steps and to present orally and to write a research paper about it. Other specific competencies include:

- Understanding the mathematical principles that support the algorithms and their potential application
- Achieve mathematical rigorousness
- Learning how to model efficiently

Model development

To achieve these competencies the students learn by doing. They have to build mock-up optimization models by using an algebraic modelling language (in particular, we use General Algebraic Modelling System GAMS (<http://www.gams.de/>)) at both undergraduate and master levels. Relevant characteristics of these algebraic languages are:

- High-level computer programming languages for the formulation of complex mathematical optimization problems
- Notation similar to algebraic notation. They provide a concise and readable definition of problems in the domain of optimization
- Do not solve problems directly, but ready-for-use links to state-of-the-art algorithms. Therefore, allow the modeller to concentrate in the modelling process

These languages are used for rapid prototyping given that allow flexibility for continuous refinement of the model and therefore generate a huge decrease in maintenance time. Their main advantages are:

- Independency of the mathematical model and data, solution method (solver), operating system or user interface
- At the same time, models can benefit from advances in hardware, solution methods or interfaces to other systems

These advantages are also important for developing high-end models. Since more than ten years we have been using the GAMS language for teaching and for professional model development at the School of Engineering.

For practical purposes, Microsoft Excel is the preferred user interface (inputting the data and output of the results including their graphical representation).

High-end models

Students develop mock-up optimization models to learn their use and to attain the competencies. However, commercial-grade models are needed to support decisions for large-scale electric systems. Scaling-up models is a major task, not only from a computational point of view but also from a mathematical point of view. A very careful computer implementation has to be followed and probably a specific optimization algorithm must be used (e.g., Benders' decomposition). People from the electric companies are aware of the usefulness of mathematical models. However, in these models it is crucial to balance the mathematics and algorithms involved and the practical solutions provided by them.

In the next section we present some of the paradigmatic models used in the electricity industry and how we have stated and solved it.

Short-term daily unit commitment model

ROM model (<http://www.iit.upcomillas.es/aramos/ROM.htm>) objective is to determine the technical and economic impact of intermittent generation (IG) and other types of emerging technologies (active demand response, electric vehicles, concentrated solar power, solar photovoltaic) into the medium-term system operation including reliability assessment, see Dietrich (2012). Results consist of generation output including IG surplus, pumped storage hydro and storage hydro usage, and adequacy reliability measures. The benefits of improving IG predictions can also be determined by changing forecast error distributions and re-running the model.

Next there is a list of the main characteristics of the model:

- A daily stochastic optimization model formulated as a mixed integer programming (MIP) problem followed by a sequential hourly simulation

This system modelling in two phases reproduces the usual decision mechanism of the system operator. Detailed operation constraints such as minimum load, ramp-rate, minimum up-time and downtime of thermal units and power reserve provision

are included into the daily stochastic unit commitment model. The hourly simulation is run for the same day to account for IG production errors, demand forecast errors and unit failure and therefore revising the previous schedule.

- A chronological approach to sequentially evaluate every day of a year

Decisions above this scope as the weekly scheduling of pumped storage hydro plants are done internally in the model by heuristic criteria. Yearly hydro scheduling of storage hydro plants is done by higher hierarchy models, as for example, a hydrothermal scheduling model (see next section).

- Monte Carlo simulation of many yearly scenarios that deal with IG or hydro inflows stochasticity

The model scheme based on a daily sequence of planning and simulation is similar to an open-loop feedback control used in control theory.

A mock-up Stochastic Daily Unit Commitment Model for the master students can be found at (http://www.iit.upcomillas.es/aramos/StarNetLite_SDUC.zip).

Hydrothermal scheduling model

Hydrothermal scheduling models (HTCM) manage the integrated operation planning of both hydro and thermal power plants, see Ramos (2011).

By nature, these models are high-dimensional, dynamic, nonlinear, stochastic and multiobjective. Solving these models is still a challenging task for large-scale systems. One key question for them is to obtain a feasible operation for each hydro plant, which is very difficult because the models require a huge amount of data, by the complexity of hydro subsystems, and by the need to evaluate multiple hydrological scenarios. For these models no aggregation or disaggregation process for hydro power input and output is established. Besides, thermal power units are considered individually.

A HTCM determines the optimal yearly operation of all the thermal and hydro power plants taking into account multiple cascaded reservoirs in multiple basins. The objective function is based on cost minimization because the main goal is the medium term hydro operation.

This model is connected with other models within a hierarchical structure. At an upper level, a stochastic market equilibrium model (see next section) with monthly periods is run to determine the hydro basin production. At a lower level, a stochastic simulation model with daily periods details hydro plant power output, see Latorre (2007). This later model analyzes for several scenarios the optimal operational policies proposed by the HTCM. Adjustment feedbacks are allowed to assure the coherence among the output results.

This model has the following main characteristics:

- Specially suited for large-scale hydroelectric systems
- Deals with multireservoir, multiple cascaded hydro plants
- Consider nonlinear water head effects
- Takes into account stochastic hydro inflows
- Formulated as a multi-stage stochastic optimization solved by a state-of-the-art solution method, stochastic dual dynamic programming, see Cerisola (2012)

A mock-up Medium Term Stochastic Hydrothermal Coordination Model can be found at (http://www.iit.upcomillas.es/aramos/StarGenLite_SHTCM.zip).

Market equilibrium model

The market equilibrium model is stated as the profit maximization problem of each generation company (GENCO) subject to the constraint that determines the electricity price as a function of the demand, which is the sum of all the power produced by the companies. Each company profit maximization problem includes all the operational constraints that the generating units must satisfy.

When considering the Cournot's approach the decision variable for each company is its total output while the output from competitors is considered constant. In the conjectural variation approach the reaction from competitors is included into the model by a function that defines the sensitivity of the electricity price with respect to the output of the company. This function may be different for each company.

Operating constraints include fuel scheduling of the power plants, hydro reservoir management for storage and pumped-storage hydro plants, run-of-the-river hydro plants and operation limits of all the generating units.

The model incorporates several sources of uncertainty that are relevant in the long term, such as water inflows, fuel prices, demand, electricity prices and output of each company sold to the market. This is done by classifying historical data into a multivariate scenario tree. The introduction of uncertainty extends the model to a stochastic equilibrium problem and gives the company the possibility of finding a hedging strategy to manage its market risk. With this intention, we force currently future prices to coincide with the expected value of future spot prices that the equilibrium returns for each node of the scenario tree. Future's revenues are calculated as gain and losses of future contracts that are cancelled at the difference between future and spot price at maturity. Transition costs are associated to contracts and computed when signed.

The risk measure used is the *Conditional Value at Risk* (CVaR), which computes the expected value of losses for all the scenarios in which the loss exceeds the *Value at Risk* (VaR) with a certain probability.

All these components set up the mathematical programming problem for each company, which maximizes the expected revenues from the spot and the futures market minus the expected thermal variable costs and minus the expected contract transaction costs. The operating constraints deal with fuel scheduling, hydro reservoir management, operating limits of the units for each scenario, while the financial constraints compute the CVaR for the company for the set of scenarios. Linking constraints for the optimization problems of the companies are the spot price equation and the relation of future price as the expectation of future spot prices.

The KKT optimality conditions of the profit maximization problem of each company together with the linear function for the price define a *mixed linear complementarity problem*. Thus the market equilibrium problem is created with the set of KKT conditions of each GENCO plus the price equation of the system, see Rivier (2001). The problem is linear if the terms of the original profit maximization problem are quadratic and, therefore, the derivatives of the KKT conditions become linear.

The results of this model are the output of each production technology for each period and each scenario, the market share of each company and the resulting electricity spot price for each load level in each period and each scenario. Monthly hydro system and thermal plant production are the magnitudes passed to the medium-term hydrothermal coordination model, explained below.

A mock-up Cournot Model can be found at

(http://www.iit.upcomillas.es/aramos/StarMrkLite_CournotEn.gms)

Transmission expansion planning model

Transmission expansion planning determines the investment plans of new facilities (lines and other network equipment) for supplying the forecasted demand at minimum cost. Tactical planning is concerned with time horizons of 15-30 years. Its objective is to evaluate the future network needs. The main results are the guidelines for future structure of the transmission network.

Here we present a decision support system for defining the transmission expansion plan of a large-scale electric system at a tactical level. A transmission expansion plan will be defined as a set of network investment decisions for future years. The candidate lines are pre-defined by the user so the model determines the optimal decisions among those specified by the user.

TEPES transmission expansion planning model

(<http://www.iit.upcomillas.es/aramos/TEPES.htm>)

will determine automatically optimal expansion plans that satisfy simultaneously several main attributes. Their main characteristics are:

- **Dynamic:** the scope of the model will be several years at a long-term horizon, 2020 or 2030 for example.
- **Stochastic:** several stochastic parameters that can influence the optimal transmission expansion decisions will be considered. Besides, the model must consider stochasticity scenarios associated to: renewable energy sources, electricity demand, hydro inflows, and fuel costs. These yearly scenarios are grouped in: operation scenarios (hydroelectricity, etc.) and reliability scenarios (N-1 generation and transmission contingencies)
- **Multicriteria:** some of the main quantifiable objectives will be incorporated in the objective function, the model considers: transmission investment and variable operation costs (including generation emission cost), reliability cost associated to N-1 generation and transmission contingencies.

The optimization method used is based on a functional decomposition between an automatic plan generator (based on optimization) and an evaluator of the transmission plans from different points of view (operation costs for several operating conditions, reliability assessment for N-1 generation and transmission contingencies, etc.). The model is formulated as a two-stage stochastic optimization solved by *Benders' decomposition* where the master problem proposes line investment decisions and the operation subproblem determines the operation cost for this investment decisions and the reliability subproblems determine the not served power for the generation contingencies given that investment decisions.

The operation model (evaluator) is based on a DC load flow. By nature the transmission investment decisions will be binary. The current network topology will be considered as starting point.

A mock-up Long Term Transmission Expansion Model can be found at

(http://www.iit.upcomillas.es/aramos/StarNetLite_TEPM.zip)

Conclusions

Mathematical model development and the use of models for taking decisions is a part of the curriculum of the School of Engineering at undergraduate and graduate level. The main competencies associated to the courses are focused on understanding the process to develop models and their potential application. There is a natural continuation between mock-up models that are explained to the students and high-end models that are developed as part of funded research.

Mathematical formulation of models allows the students to advance in their logical thinking, writing them in an algebraic modelling language, familiarize them with reality and how the models can be employed for decision support.

References

- Cerisola, S., Latorre, J.M., Ramos, A. (2012) “Stochastic Dual Dynamic Programming Applied to Nonconvex Hydrothermal Models” *European Journal of Operational Research*, 218, 687–697 [10.1016/j.ejor.2011.11.040](https://doi.org/10.1016/j.ejor.2011.11.040)
- Delson, J.K. and Shahidehpour, S.M. (1992) “Linear Programming Applications to Power System Economics, Planning and Operations” *IEEE Transactions on Power Systems* (7) 3: 1155-1163
- Dietrich, K., Latorre, J.M., Olmos, L., Ramos, A. (2012) “Demand Response in an Isolated System with high Wind Integration” *IEEE Transactions on Power Systems* 27 (1): 20-29 [10.1109/TPWRS.2011.2159252](https://doi.org/10.1109/TPWRS.2011.2159252)
- Garver, L.L. (1962) “Power generation scheduling by integer Programming-Development of theory,” *Power Apparatus and Systems, Part III. Transactions of the American Institute of Electrical Engineers*, 81 (3), 730–734.
- Latorre, J.M. Cerisola, S. Ramos, A. Bellido, R. Perea A. (2007) “Creation of Hydroelectric System Scheduling by Simulation” in the book H. Qudrat-Ullah, J.M. Spector and P. Davidsen (eds.) *Complex Decision Making: Theory and Practice* pp. 83-96 Springer ISBN 9783540736646 [10.1007/978-3-540-73665-3_5](https://doi.org/10.1007/978-3-540-73665-3_5)
- Ramos, A., Cerisola, S., Latorre, J.M. (2010) “A Decision Support System for Generation Planning and Operation in Electricity Markets” in the book P.M. Pardalos, S. Rebennack, M.V.F. Pereira and N.A. Iliadis (eds.) *Handbook of Power Systems II* pp. 797-817 Springer ISBN 9783642126857
- Ramos, A. Cerisola, S. Latorre, J.M. Bellido, R. Perea, A. and Lopez E. (2011) “A Decision Support Model for Weekly Operation of Hydrothermal Systems by Stochastic Nonlinear Optimization” in the book G. Consigli, M. Bertocchi and M.A.H. Dempster (eds.) *Stochastic Optimization Methods in Finance and Energy*. Springer ISBN 9781441995858 [10.1007/978-1-4419-9586-5_7](https://doi.org/10.1007/978-1-4419-9586-5_7)
- Rivier, M., Ventosa, M., Ramos, A., Martínez-Córcoles, F., Chiarri, A. (2001) “A Generation Operation Planning Model in Deregulated Electricity Markets based on the Complementarity Problem” in the book M.C. Ferris, O.L. Mangasarian and J-S. Pang (eds.) *Complementarity: Applications, Algorithms and Extensions* pp. 273-295 Kluwer Academic Publishers ISBN 0792368169