FITTING ELECTRICITY MARKET MODELS. A CONJECTURAL VARIATIONS APPROACH

Antonio García-Alcalde Mariano Ventosa Michel Rivier Andrés Ramos Gregorio Relaño

INSTITUTO DE INVESTIGACIÓN TECNOLÓGICA
Universidad Pontificia Comillas
Alberto Aguilera 23
28015 Madrid, SPAIN
antonio.garcia@iit.upco.es

Dirección de Mercados de Electricidad
Iberdrola
Hermosilla 3,
28001 Madrid, SPAIN
grelano@iberdrola.es

Abstract – The worldwide electricity industry has been embedded in a significant restructuring process toward deregulation and competition during the last decade. Simultaneously, an important research effort has been made to properly incorporate market features within generation operation models. Cournot equilibrium has been one of the theoretical frameworks most widely used to model market behavior. However, it presents relevant weakness related to its high sensitivity to the demand elasticity. This paper proposes the use of firms' conjectural variations to overcome this difficulty and shows a procedure to determine them. The method, designed for long-term operation models, infers the implicit values associated to the firms' residual demand employing only public market information.

Keywords: Competitive electricity market, generation scheduling, equilibrium models, Complementarity Problem.

1 INTRODUCTION

In the new deregulated power markets, electric firms assume much more risk and become highly responsible for their own economic decisions. Therefore, firms need decision-support models that fulfill these new requirements. An important developmental effort has been made to design original models and tools that explicitly represent the electricity market behavior. In this context, the market mechanisms such as individual profit maximization and risk management are the driving forces that explain the generation units operation and the energy expected price.

A great number of market models that try to represent GENCOs' long-term behavior are based on Cournot market equilibrium. Although advances in market equilibrium[1] models have been notable, limitations persist in estimating several relevant market issues.

This paper proposes a fitting procedure designed for long-term operational models based on conjectural variations. This approach allows a more flexible representation of firms' behavior and a more accurate price generation process than Cournot-based models. These improvements are achieved by means of modeling firms' residual demand functions, which provide information about the energy that firms are able to sell at each price. These residual demand functions are difficult to estimate because they are related to the other firms' supply functions.

The proposed methodology characterizes each residual demand function by its elasticity and estimates the implicit values of this parameter. Thus, this procedure computes the long-term residual demand elasticity that each firm infers when bidding. The inference about firms' bidding perceptions is based on clearing market information provided by the Market Operator, instead of the real supply functions submitted by the firms. An estimation process based on short-term supply functions has been dismissed because they do not reflect properly firms' long-term strategies but just firms' short-term tactics.

This paper is organized as follows. Section 2 provides an overview of several electricity market models based on Cournot equilibrium that are available in the literature. Section 3 details the conjectural variations approach while section 4 shows how this approach can be considered in a MCP-based scheme. Section 5 states the estimation procedure of the implicit elasticities that allows to fit market models based on conjectural variations. Section 6 describes an application of the proposed approach to the Spanish Electricity Market, and finally, section 7 provides the conclusions drawn from the study.

2 COURNOT-BASED MARKET MODELS

The Cournot equilibrium suitability to model competition among few firms has been widely discussed in economics' literature [1]. The vast majority of the models used in an imperfect competitive context somehow consider firms' Cournot behavior. In particular, the Cournot equilibrium approach applied to electricity markets allows a more realistic modeling of electrical generation features than any other complex methodologies, such as Supply Function Equilibria (SFE)[2].

An interesting work devoted to a conjectured supply function approach can be found in [2]. The aim of this

[1] Market equilibrium provides a set of decisions such that no firm, taking its competitors' decisions as given, wishes to change its own unilaterally.

[2] Supply Function Equilibria (SFE) considers firms compete through out their bidding supply curves. Although this approach has been proved to be solid, its application to real markets is avoided due to the multiplicity of solutions and the poor representation of generation operation.

paper is to present a more realistic method to model imperfect competition in power networks than the traditional ones. This approach combines the computational tractability of Cournot models with a solid way of competition such as SFE.

Interest in the research community regarding the development of Cournot-based equilibrium models as behavior-pattern of electricity markets has grown and has been demonstrated as such in numerous publications. First developments [3] have been extended later by developing an iterative procedure to obtain the market equilibrium in the Californian Market [4]. An alternative procedure to compute the Cournot market equilibrium by an equivalent optimization problem can be seen in [5].

Hydrothermal coordination in an imperfect competition framework was first considered in [6]. The proposed model uses dual dynamic programming in which a duopoly Cournot model is solved in each stage. An analytic statement of this problem's equilibrium conditions is introduced in [7]. This approach provides interesting conclusions about the role of hydro generation in a market framework. In [8] it is shown that the Mixed Complementarity Problem (MCP) is a powerful and flexible methodology for addressing the long-term operation planning of a GENCO within a Cournot context. The MCP structure allows to consider the simultaneous firms' profit optimizations while representing the relevant operation constraints of generation units.

The network congestion influence in Cournot GENCO's behavior has also brought about interesting developments. Modeling electricity markets as spatial price discrimination process is proposed in [9], [10] and [11]. Recently, Complementarity Problem and Variational Inequality approach have been used to model imperfect competition among producers, including a congestion-pricing scheme for transmission [12] and [13].

3 CONJECTURAL VARIATIONS APPROACH

As can be seen in the previous section, there exists a number of electricity market models based on Cournot competition. However, it is well known that these models provide barely credible prices. These high prices are due to neglecting competitors' supply functions.

The conjectural variations approach considers the reaction of competitors when a firm is deciding its optimal production. This reaction comes out from firms' supply functions and demand curve.

This reaction can be modeled by the so-called firm's residual demand function. This function is different for each firm and relates each firm's production with the market price. It can be obtained by means of subtracting the remaining competitors' supply functions from the market demand curve.

In the next subsection, the optimization problem of each firm is stated as its profit maximization facing its own residual demand function. Hence, conjectural variations are considered in firms' strategic behavior.

3.1 Firms' optimization problem

In this section the optimization problem of each firm is described, whose objective function can be defined as its profit maximization

$$\Pi_e = \pi \cdot P_e - C_e \tag{1}$$

where Π_e is the firm's market profit, π is the system marginal price, P_e is the firm's total production and C_e represents firm's operational costs.

Since the system marginal price is set by the decisions made by the supply and demand side, it is possible to relate this price with the aggregated production by means of the demand function.

$$\pi = f\left(\sum_e P_e\right) \tag{2}$$

In order to calculate the eth firm optimal production, the derivative of the profit function with respect to the decision variable (P_e) is equalized to zero.

$$\frac{\partial \Pi_e}{\partial P_e} = \pi + \frac{\partial \pi}{\partial P_e} \cdot P_e - MC_e\left(P_e\right) = 0 \tag{3}$$

The first two terms of the equation make up the firms' marginal revenue MR_e. The marginal revenue measures how firm's revenue increases (or decreases) when the firm increases (or decreases) its production in one unit. Likewise, firm's marginal cost can be defined as how firm's cost increases (or decreases) when the firm increases (or decreases) its production in one unit. Therefore, firm's optimal production is achieved when its marginal revenue is equal to its marginal cost.

Firm's marginal revenue can be expressed in terms of its residual demand elasticity ε_e instead of the residual demand slope

$$MR_e(P_e) = \pi + \frac{\partial \pi}{\partial P_e} \cdot P_e = \pi \cdot \left(1 + \frac{1}{\varepsilon_e}\right) \tag{4}$$

where residual demand elasticity is defined as quotient between the unitary change on firm's production caused by an unitary change on market price.

$$\varepsilon_e = \frac{\dfrac{\partial P_e}{P_e}}{\dfrac{\partial \pi}{\pi}} \tag{5}$$

Note that residual demand elasticity is different for each firm. This parameter takes into account how the market price changes when each firm changes its production unilaterally.

To conclude, this parameter expresses the market conjecture of each firm. The variation of this parameter encompasses different type of competition, providing great flexibility of modeling.

3.2 Flexibility of modeling

Depending on the value of firms' residual demand elasticity, a different assumption is made about how firms' marginal revenue is conceived, and consequently, about how firms behave. In this way, some widely used market models can be stated in terms of the residual demand elasticity as follows.

- *Perfect competition*. Perfect competition can be defined as a market situation where firms are not able to change the market price by modifying their production uniterally. In terms of market modeling, each firm's residual demand function becomes horizontal. The elasticity value of a fixed-price residual demand function (see equation(5)) is infinity. From equation (4), firm's marginal revenue is equal to the market price. As expected, firm's optimal production takes place when its marginal cost reaches the market price.
- *Cournot model*. As previously mentioned, Cournot model is one of the most common approaches to represent competition among few firms. In contrast to perfect competition, firms are able to modify the market price by means of changing its own production. The market model conjecture that states a Cournot equilibrium can be written in terms of a demand elasticity equal to every firm competing in the market scaled by its market share ($\varepsilon_e = \varepsilon_d / \alpha_e$), where ε_d is the demand function elasticity and α_e is the eth firm market share. Cournot prices are usually far higher than real market prices.

The different values of the residual demand elasticity depending on the theoretical market model are summarized in Table 1.

In this section it has been presented a flexible approach that allows to represent different optimal market behaviors using firms' residual demand elasticity.

Model	Market Conjecture
Perfect Competition	$\varepsilon_e = \infty$
Cournot	$\varepsilon_e = \varepsilon_d / \alpha_e$
Conjectural Variations	$\varepsilon_e \neq \varepsilon_{e'}$

Table 1: Market Conjecture

4 MARKET EQUILIBRIUM MODEL

Since firms compete within a market context, firm's optimal energy schedule must consider not only its own optimization program but also the interaction among the optimization programs of the remaining firms.

The mathematical structure of a market equilibrium problem conceptually corresponds to various simultaneous optimizations (one for each firm) linked together through the market price resulting from the interaction of all of them. This scheme is shown in Figure 1, where Π represents the market profit of each company $e \in [1,...,E]$, x the decision variables and the set of constraints h and g are particularized for each company. The electricity market is modeled by the demand function that relates the supplied demand to the electricity price.

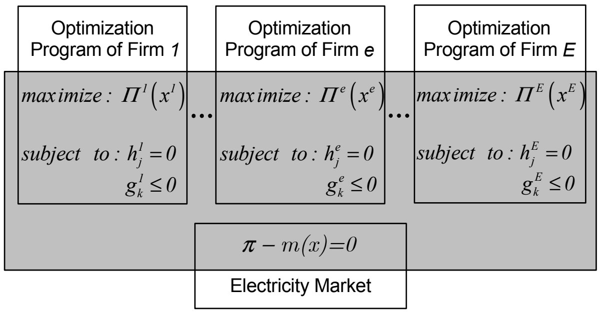

Figure 1: Market Equilibrium

There are several methodologies that are able to deal with this market equilibrium structure. One of these methodologies is based on the Mixed Complementarity Problem (MCP). Examples of these MCP-based models can be found in [8] and [12].

The market equilibrium stated in terms of an MCP scheme requires setting the first order optimality conditions of Karush-Kuhn-Tucker [14]. These optimality conditions are associated to the set of maximization programs, as shown in Figure 2.

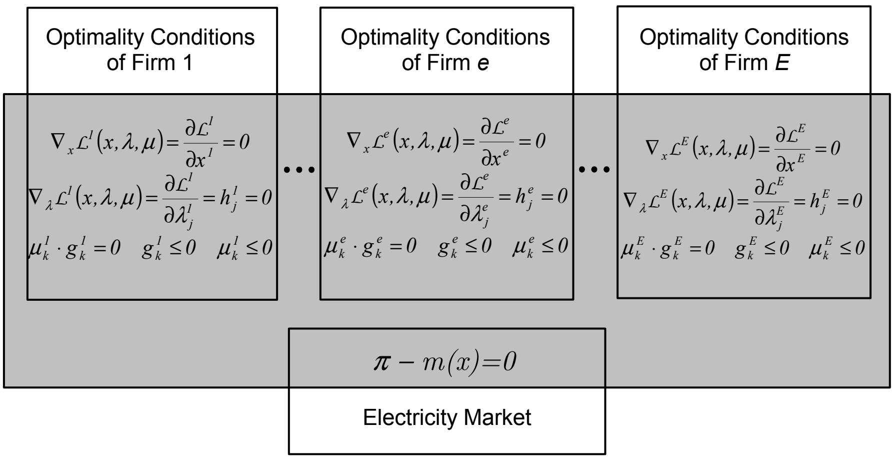

Figure 2: Market equilibrium as a Mixed Complementarity Problem

In Figure 2, $\mathcal{L}$ represents the Lagrangian function of the corresponding optimization problem and λ and μ represent the dual variables associated to the set of h and g constraints respectively. The optimality conditions can be stated as three sets of equations. The first one cancels the gradient of the Lagrangian function with respect to the decision variables x. The second set (the gradient of the Lagrangian function with respect to the dual variables λ) coincides with the equality constraints h. The third one is made up by the complementary slackness conditions associated to the inequality constraints g. Grouping together all companies' system of equations leads to a mixed complementarity problem formulation.

The market equilibrium formulation as an MCP allows to model firms competition as the conjectural variations approach previously detailed in section 2. For the sake of clarity, it is considered as each firm's decision variable its portfolio production (P_e). Therefore, equation (3) expressed in terms of residual demand elasticity (ε_e) corresponds to the first set of the KKT optimality conditions within the MCP scheme.

$$\nabla_x \mathcal{L}^c(\cdot) = \frac{\partial \mathcal{L}^c}{\partial P_e} = \pi \cdot \left(1 + \frac{1}{\varepsilon_e}\right) - CM_e(P_e) = 0 \qquad (6)$$

Note that in (6) appears the residual demand elasticity. Hence, an MCP-based model allows to consider explicitly the conjectural variations approach.

In more complex models, single units productions are stated as decision variables instead of firms' portfolio production. Still, firms' marginal revenue is always an element of the optimality condition associated to each decision variable.

5 IMPLICIT ELASTICITY ESTIMATION

In this section, a methodology suitable to estimate residual demand elasticity is presented. The resulting estimations will allow the fitting of electricity market models.

5.1 Overview

As has been previously detailed in section 2, the market conjectures selected to model the firms' behavior have a relevant impact on firms' productions and market prices. In section 2 it has also been presented a conjectural variations approach to model firms' behavior more flexibly than the traditional Cournot modeling. This approach is based on using firms' residual demand functions instead of market's aggregated demand curve. Therefore, it becomes necessary a methodology able to deal with the fitting and estimation process of the firms' residual demand.

The proposed methodology is based on fitting the residual demand elasticity by means of an evaluation of the conjectural variations model on past data. It is first supposed that firms behaved *via* a conjectural variations pattern during the fitting period. Therefore, the decisions (productions and market prices) they made can be assumed to be optimal. This optimal schedule can be expressed equalizing firm's marginal revenue with its marginal cost (see equation (3)).

When fitting with past data, firms' decisions have already been made. Consequently, we can infer the past residual demand elasticity by means of using these decision variables as acknowledged values. This procedure is known as implicit valuation. Thus, the residual demand elasticity obtained is called *implicit elasticity*.

It is important to remark that the implicit values of the elasticity measure firms' perception about their market positions in a conjectural variations context. In fact, this perception does not need to coincide with the real residual demand computed by the supply and demand curves submitted to the market by all the firms.

5.2 Implicit elasticity

The procedure to obtain the implicit value of firms' residual demand in a conjectural variations context is stated along these lines. Let π be the actual system market price and MR_e be the marginal revenue of the firm e. The implicit value of the residual demand elasticity ($\widehat{\varepsilon_e}$) is derived from expression (4) and can be written as follows.

$$\widehat{\varepsilon_e} = \frac{\pi}{MR_e - \pi} \qquad (7)$$

The value of firm's marginal revenue is impossible to estimate regarding market data. However, it can be approximated to firm's marginal cost by means of considering that each firms is generating optimally (3). Firm's marginal cost can easily be estimated as the generation cost of the most expensive committed unit[3]. Therefore, the implicit expression of the residual demand elasticity can be stated in terms of the market price and firm's marginal cost as follows.

$$\widehat{\varepsilon_e} \simeq \frac{\pi}{MC_e - \pi} \qquad (8)$$

5.3 Estimation

By means of valuing the implicit residual demand elasticity, it is possible to infer firms' long-term behavior. The estimation procedure uses only public market information (prices, production, estimation of firms' marginal costs) hourly provided. Applying equation (8) to this information, the hourly time series of the implicit residual elasticity of each firm is directly acknowledged. These implicit values not only consider long-term firms' behavior, but also are very influenced by short-term uncertainty sources and operational constraints.

Some relevant short-term uncertainty sources that have some bearing on the implicit demand elasticity are plants outages, water spills, deviations of the forecasted demand and supply curves submitted by the rest of the firms. Short-term operational constraints as plant ramps, minimum stable output, limited reservoir capacity and

[3] A more precise marginal cost estimation requires considering units' generation constraints (*i.e.* ramps and minimum stable output) and other relevant market issues (*i.e.* capacity payments).

network constraints also have a significant effect on implicit values.

Therefore, the previous short-term issues must be sieved in order to properly estimate implicit values that will model firms' long-term behavior. It is necessary to process all the information about the implicit residual demand evolution looking for a trend value. Statistics make available several data-analysis methodologies that are able to deal with long-term behavior estimation from short-term data. Depending on the nature of the variables used to do so, statistical methods can be classified as follows.

- Relational models. By means of these models, the implicit elasticity of the residual demand is related to one or several relevant variables (explanatory variables) whose long-term values are easy to infer. This relationship is expressed by a function among the variables. Having a long-term estimation of the set of explanatory variables (*e.g.* demand, hydraulic inflows) a trend value of the implicit elasticity can easily be obtained by evaluating the former function. Some of the statistical methods that can be defined as *relational models* are regression, neural networks and decision trees.
- Classification models. In contrast to relational models, the set of values of implicit elasticities are categorized by the different levels of several discrete factors. The long-term value of the implicit elasticity for each factor combination (*i.e.* class) can be expressed as a statistical measure (average, median or mode) of the past data distribution. Variance analysis and clustering are some statistical methodologies based on classification processes.
- Time series models. These models consider implicitly the evolution of firm's elasticity and infers a trend value of the time series. Time series methodology allows not to take into account other variables or factors when estimating the long-term values of the implicit residual demand elasticity. Some of the time series methodologies devoted to deal with long-term trends are time series decomposition and exponential smoothing methods.

6 CASE STUDY

An MCP-based equilibrium model [8] in which firms compete considering conjectural variations has been applied to the Spanish Electricity Market to validate the estimation and fitting procedures detailed in sections 3, 4 and 5. A comparison with firms' Cournot behavior is also shown in this section.

6.1 System description

The Spanish Electricity Market was established in 1998 and since then four firms compete: Endesa, Iberdrola, Unión Fenosa and Hidrocantábrico. The system meets a maximum peak load close to 30000 MW and a yearly energy demand of 175770 GWh, and the average hydro energy available is 30176 GWh. The annual scope of the model has been divided into twelve periods (months) with 5 load levels for each one.

Type	# Units	Power (MW)	Ibedr.	Endesa	Unión	Hidroc.
Thermal	82	32107	29 %	54 %	13 %	5 %
Hydro	38	16628	51 %	36 %	3 %	10 %

Table 2: Spanish firms' production structure

There are 82 thermal generators grouped into 42 thermal plants. The hydro units have been grouped into 28 equivalent units plus 10 pumped-storage units.

All the information that has been used in this case study is public. Units' productions and market prices have been obtained through the Spanish Market Operator. Fuel costs and units' thermal rates are respectively based on international fuel prices and standard technology rates.

6.2 Numerical results

Two different Cournot scenarios have been considered by changing the aggregated demand's elasticity value. For the first one, ε_d is equal to 0.3, while for the second one ε_d is equal to 0.5. A conjectural variations approach has also been implemented. The value of firms' residual demand elasticity has been estimated using a classification model where each value was labeled depending on the firm, demand level and the expected hydraulic inflows. Data used to fit this estimation model is based on the market results of the year 1999.

The model-obtained prices (€/MWh) and the real market results can be seen in Table 3. Note that conjectural variations provide a realistic estimation of the real market prices for every demand level. In contrast, Cournot equilibrium gives deficient results when demand elasticity decreases.

Model Assumption	On-On-Peak	On-Peak	Plateau	Off-Peak	Off-Off-Peak
Cournot ε_d 0.3	281.21	258.80	231.34	168.47	116.76
Cournot ε_d 0.5	79.09	74.24	68.45	54.61	39.99
Conjectural Variations	**43.92**	**38.01**	**31.59**	**19.56**	**13.77**
Market Prices (2000)	***46.08***	***40.21***	***32.61***	***22.16***	***16.53***

Table 3: Model's average prices (€/MWh) for each demand level

For the first Cournot scenario, estimated prices are even eight times higher than the real ones. It is important to remark that a demand elasticity value of 0.3 is very high in electricity markets. For instance, the average demand elasticity in the Spanish Electricity Market in 1999 was 0.03. Therefore, the use of a Cournot model in which its demand elasticity estimation is based on the submitted demand curves is completely inappropriate.

Note that although only four firms compete in the Spanish Electricity Market, actual clearing prices are much lower than those provided by a theoretical Cournot model. In fact, price-shocks as occurred in California have not been observed in the Spanish Electricity Market.

Regarding the results obtained of the case study, the conjectural variations approach and the proposed implicit estimation methodology of the residual demand elasticities provides a flexible and accurate tool to infer realistic firms' productions and market prices. The advantages with respect to Cournot-based models are notable.

7 CONCLUSIONS

In this paper it has been presented a conjectural variation approach to fit electricity market models. This approach allows a more flexible and accurate market description than Cournot-based models when modeling firms' behavior in a liberalized context. The different firms' behaviors, ranging from perfect competition to Cournot, are considered by means of its residual demand elasticity.

Due to the importance of an accurate estimation of firms' residual demand function, an inference procedure of their elasticity has been developed. It is based on an implicit estimation process that can easily be computed regarding only public market information. These implicit values of residual demand elasticity somehow model firms' perception about their market position.

A case study applied to the Spanish Electricity Market has been presented to illustrate how the proposed methodology is able to provide credible results. These results also show that behavior of firms competing in the Spanish Market clearly differs from the theoretical Cournot approach.

8 ACKNOWLEDGEMENTS

This work has been developed under a research project funded by Iberdrola, S.A. and with the support of a grant from the Ministerio de Educación, Cultura y Deporte (*Ref.* AP2001-2668).

REFERENCES

[1] Vives, X., *Oligopoly Pricing*. The MIT Press 1999.

[2] Day, C. J., Hobbs B.F. "Oligopolistic Competition in Power Networks: A Conjectured Supply Function Approach". *Tentatively accepted, IEEE Trans. P.S.*

[3] Borenstein, S., Bushnell, J., Kahn, E. and Stoft, S. "Market Power in California Electricity Markets". *Utilities Policy*. Vol. 5 No. 3/4 pp. 219-236. 1995.

[4] Borenstein, S. and Bushnell, J. "An Empirical Analysis of the Potential for Market Power in California's Electricity Industry". *Journal of Industrial Economics*, Vol. 47, No. 3, September, 1999.

[5] Ramos, A., Ventosa, M., Rivier, M. "Modeling Competition in Electric Energy Markets by Equilibrium Constraints". *Utilities Policy*, Vol. 7 Issue 4 pp. 233-242, Dec. 98.

[6] Scott, T.J. and Read, E.G. "Modelling Hydro Reservoir Operation in a Deregulated Electricity Market". *International Transactions in Operational Research*. Vol. 3 pp. 243-253. 1996.

[7] Bushnell, J. "Water and Power: Hydroelectric Resources in the Era of Competition in the Western US". *POWER Conference on Electricity Restructuring*. University of California Energy Institute. Berkley. 1998.

[8] M. Ventosa, M. Rivier, A. Ramos and A. García-Alcalde, "An MCP Approach for Hydrothermal Coordination in Deregulated Power Markets", IEEE-PES Summer Meeting Proceedings, Vol 4, pp 2272-2277, Seattle WA, 2000.

[9] Hogan, W.W. "A Market Power Model with Strategic Interaction in Electricity Networks". *The Energy Journal* 18 (4), pp. 107-141, 1997.

[10] Oren, S. "Economic Inefficiency of Passive Transmission Rights in Congested Electrical System with Competitive Generation", *The Energy Journal* 18 (1), pp. 63-83, 1997.

[11] Hobbs B. F., Metzler C., Pang J. S. "Strategic Gaming Analysis for Electric Power Networks: An MPEC Approach". *IEEE Trans. Power Systems*. 1998.

[12] Hobbs B. F. "LCP Models of Nash-Cournot Competition in Bilateral and POOLCO-Based Power Markets". *Proceedings, IEEE Winter Power Meeting*, NY City, Feb. 1999.

[13] Wei. Jing-Yuan, Y. Smeers. "Spatial Oligopolistic Electricity Models with Cournot Generators and Regulated Transmission Prices". Operations Research, Vol. 47, No. 1, January - February 1999, pp. 102 - 112.

[14] W.W. Cottle, J.S. Pang, and R.E. Stone. "The Linear Complementarity Problem". Academic Press, Boston, 1992.