AN ITERATIVE ALGORITHM FOR PROFIT MAXIMIZATION BY MARKET EQUILIBRIUM CONSTRAINTS

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Abstract – The electricity industry has undergone significant restructuring toward deregulation and competition during the last years. In this period, new methodologies have been appearing for helping in modeling the operation of the electric companies in market-based environments.

So far, Cournot-based equilibrium models are the most widely used. The approach used in this paper is intended for modeling the medium-term operation of the system. It is based on the Cournot conjecture, so the firms offer to the electricity market the quantity that maximizes their profit. This model is formulated as a classic production cost model where some market equilibrium constraints have been added. These constraints reproduce the first-order optimality conditions of strategic companies. All the constraints needed to represent the detailed operation of the generating units are also taken into account.

In this model an iterative algorithm has been implemented to accurately compute the system marginal cost included in the market equilibrium constraints. Besides this, a refinement algorithm for hydro scheduling has also been introduced.

Results for a case study of realistic size are presented.

Keywords:* *electricity market equilibrium, generation scheduling, medium-term, hydro scheduling.

1 INTRODUCTION

Kahn [4] and Hobbs [3] have recently made a classification and review of the current developments made in modeling the electricity markets. The two main roads followed by researchers are based on the Cournot approach or on the supply function equilibrium approach. Cournot-based models consider that firms compete only in quantities while the price is derived from the demand function. This assumption is called the Cournot conjecture, see [7]. In the second approach, firms not only compete in quantity but also in price.

A great number of oligopoly market models that try to represent firms' medium and long-term behavior are somehow based on the Cournot equilibrium.

Borenstein and Bushnell [1] were the first to explore the market equilibrium of an electricity market. They modeled the Californian market under the Cournot framework, where the companies were considered strategic or fringe depending on their characteristics. The market equilibrium was determined using an iterative algorithm that sets the strategic firm's production at its optimal level keeping constant the output of the other strategic firms. The process is repeated until a Nash equilibrium is reached, i.e., there is no incentive to any strategic firm, taking its competitors' outputs as given, to modify its output unilaterally. Later, Bushnell [2] extends this approach by considering hydro inflows management that involves multiple period decisions.

Scott and Read [6] developed another different approach that used a stochastic dynamic programming model to obtain the market equilibrium for a hydrothermal system. The state variables were the system marginal prices.

Recently, a new model has been proposed by Ventosa et al. [8] to determine the market equilibrium by direct formulation of the optimization problems of each firm plus the demand function that couples all these problems. This model can be solved as a mixed complementary problem (MCP) where, recently, solvers are becoming available. The contribution of this paper is the clear and compact formulation of the market equilibrium problem that allows to be used for realistic case studies.

This paper proposes the representation of the market equilibrium among firms by introducing a set of constraints into a detailed production cost model, see Ramos et al. [5]. These market constraints model the behavior of generation agents under an oligopoly competition. By introducing these constraints the generation agents maximize their profits (market revenues minus variable operation costs).

This approach has the advantage of using any classic production cost method for modeling electric systems. However it has a drawback, the market constraints depend on the system marginal cost, that cannot be, in general, simultaneously calculated within the optimization procedure. In this paper we further develop an iterative procedure to avoid it.

Another iterative procedure is presented to deal with the hydro inflows management for the strategic companies.

Under the Cournot approach there are still some issues that merit further investigation. The sensitivity of the results to parameters such as the slope and elasticity of the demand function should be examined. Moreover, further refinement of the solution will be needed to also consider stochasticity in the competitor behavior.

The paper is organized as follows. Section 2 describes the main characteristics of the model. Section 3 analyzes the advantages and disadvantages of the approach with respect to the mixed complementary problem. Section 4 presents the case study used to show the capabilities of the model to represent the market. Finally, section 5 provides the conclusions.

2 MODEL DESCRIPTION

Production cost models are used for many purposes: hydro and maintenance scheduling, fuel purchase, pumped-hydro operation, thermal unit commitment or economic planning. They are able to represent in detail the technical and economic constraints that influence the system operation under a cost minimization objective function. The mathematical methods used to solve the optimization problem range from dynamic programming, lagrangean relaxation, and Benders decomposition of a direct mixed integer problem formulation.

Two important features of these models are kept in the current market oriented approach:

- A detailed representation of system operation
- Use of generator output levels as decision variables

While keeping the above characteristics, in this approach we introduce the market equilibrium constraints that represent the profit maximization objective of each firm.

2.1 Production cost models

The objective function to be minimized corresponds to the total variable costs subject to the operation constraints. These can be classified into inter- and intra-period depending on the periods that are involved in. *Inter-period constraints* are associated with the coordination of limited production resources (minimum quotas of domestic fuel consumption, hydro inflows, and seasonal pumping, storage and generation). *Intra-period constraints* deal with the system operation in each period (balance between generation and demand, thermal unit commitment, weekly/daily pumping and storage and all the generation limits).

Schematically, this classical production cost model is outlined in the following table, considering only the white areas. The introduction of market equilibrium constraints, which are to be discussed in the following section, implies only some minor modifications to the previous optimization problem. The shaded areas correspond to the new market equilibrium constraints.

Minimization of

Sum of total variable costs for each period, subperiod and load level + costs of unserved demand

Subject to

Inter-period constraints • Maintenance scheduling • Hydro scheduling + seasonal pumping • Domestic fuel scheduling
Intra-period constraints • Balance between generation and demand • Thermal unit commitment constraints • Weekly/daily pumping and storage • Generation limits
Market equilibrium constraints • Marginal revenues = marginal cost for each firm • Variable cost of each firm as a function of committed units • System marginal price as a function of the demand

Under a market competition framework the demand reaction is modeled by the demand function, i.e., the demand response to the energy marginal price. Then, the market equilibrium is obtained by maximizing the total surplus (consumer's plus producer's surplus). That is equivalent to minimize the area under the supply curve on the left of the equilibrium output and the demand curve on the right of this quantity, as can be seen in figure 1.

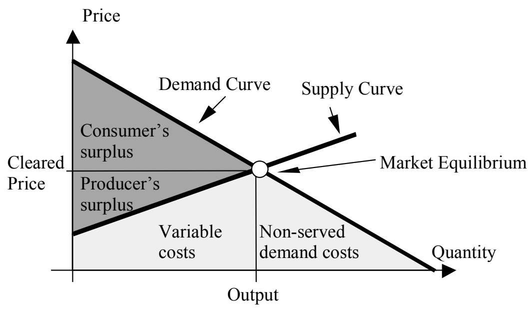

Figure 1. Utility function.

2.2 Market equilibrium constraints

The market equilibrium constraints model the behavior of the market generation agents. Their objective is to maximize their profits. The producer' surplus for a given load level is calculated as the difference between revenues and costs. Revenues for each firm are calculated as the short run marginal price (SMP) times the power produced by the firm i, P_i.

$$\text{profit}_i = SMP \cdot P_i - C_i(P_i) \tag{1}$$

where $C_i(P_i)$ is the firm's total variable cost as a function of P_i.

The market equilibrium constraints represent the first-order optimality conditions of each firm under its profit maximization objective. For each firm in each load level the derivative of the profit with respect to the power generated by the firm is equal to zero, $\partial \text{profit}_i / \partial P_i = 0$.

$$SMP + P_i \frac{\partial SMP}{\partial P} - MC_i(P_i) = 0 \tag{2}$$

Where $MC_i(P_i)$ is the firm's marginal cost as a function of P_i, $\partial SMP / \partial P$ is the change in the SMP due to a change in the output of the firm, corresponding to the slope of the price-demand curve, which is negative.

The first two terms of (2) form the marginal revenue of the firm and the last term correspond to the marginal cost. So (2) is equivalent to

$$\text{marginal revenue} = \text{marginal cost} \tag{3}$$

Equation (2) can be alternatively expressed as the generation level that, for each firm, maximizes its profit as a function of the SMP, its marginal cost and the slope of the demand function

$$P_i = \frac{SMP - MC_i(P_i)}{-\partial SMP / \partial P} \tag{4}$$

These constraints limit the power offered by each company i as a function of the system marginal cost SMP, the own firm marginal cost $MC_i(P_i)$ and the slope of the demand function $\partial SMP/\partial P$.

The previous market equilibrium model implicitly assumes that there are no operating constraints. Constructing the lagrangean and then formulating the Karush-Kuhn-Tucker (KKT) first order optimality conditions can solve the firm's profit maximization problem subject to the operating constraints. In the KKT equation corresponding to the first order derivative of the lagrangean with respect to the firm's output we can neglect the terms —all positive— associated to the derivative of the operating constraints and then the constraint (4) becomes

$$P_i \le \frac{SMP - MC_i(P_i)}{-\partial SMP/\partial P} \qquad (5)$$

The inequality sign can be understood intuitively. The objective function of cost minimization (or, equivalently, perfect competition) leads each firm's output to levels greater than those of the profit maximization problem that appears in an oligopoly market. The inequality in (5) acts therefore as a constraint on the output levels of the firms, keeping them below perfect competitive levels.

The market-clearing price SMP is represented in two different ways. As a constraint, it is modeled by a linear function of the electricity demand. However, in the objective function (i.e., the term of non-served demand costs), SMP is transformed into a decreasing stepwise function (with the same slope of the linear function) where each step is a fictitious demand bid. This second modeling approach is used to avoid nonlinearities in the objective function.

$$SMP = SMP_0 + \frac{\partial SMP}{\partial P}\sum_i P_i \qquad (6)$$

2.3 Iterative computation of system marginal cost

However, the firm's marginal cost $MC_i(P_i)$ involved in equation (5) cannot be directly calculated in the optimization problem. The reason is based on the discrete nature of the commitment decisions and on the minimum load of thermal units that has to be produced once the unit is committed. An iterative algorithm is implemented over the production cost model to determine it. This algorithm achieves the simultaneous profit maximization for all the firms.

The algorithm begins computing analytically the $MC_i(P_i)$ at each iteration (taking into account all the operation details) as the lowest marginal cost of each generating unit committed of company i in this period. Then, the profit maximization problem with market equilibrium constraints model is updated and solved using this $MC_i(P_i)$. After that, a new $MC_i(P_i)$ is computed. When the difference among two successive $MC_i(P_i)$ in under a threshold the algorithm stops, see figure 2.

Graphically, the effect of the iterative algorithm in the producer surplus can be seen in figure 3. In general although not for every load level, the firm's output under market competition is lower than under cost minimization so the algorithm begins with the competitive market levels. Then, the output of each firm is reduced in each iteration.

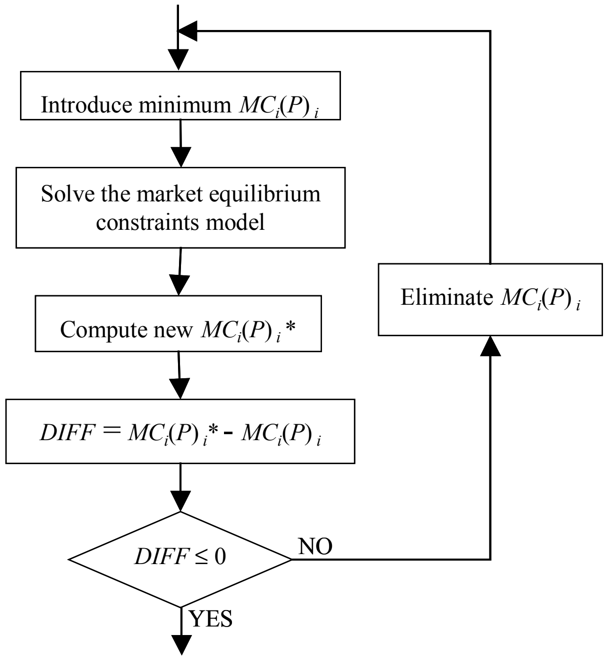

Figure 2. Iterative algorithm for computing the $MC_i(P_i)$.

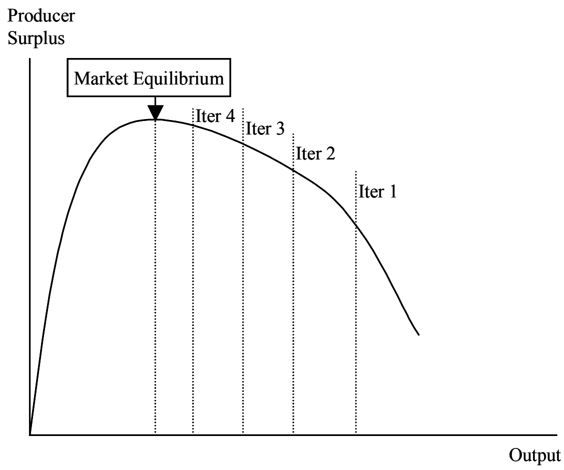

Figure 3. Evolution of producer surplus in the iterative algorithm.

2.4 Hydroelectric scheduling

The hydro operation determined by the model may still have multiple optima from the point of view of cost minimization. In this model, the hydro production is considered to have zero cost for any hydro plant. But the strategic companies may increase their profits by exchanging energy from load levels of low system marginal cost to other with higher marginal values without violating the market equilibrium constraints. This situation may happen when the market constraints are not binding in all the load levels.

Then, a refining iterative procedure has been implemented to achieve market equilibrium for the strategic companies taking care of hydro operation details.

The algorithm can be divided in the following steps:

1. Obtain an initial market equilibrium by solving the optimization problem
2. Select a load level L1 with high system marginal price where the market equilibrium constraints are not binding for some strategic company
3. Select a load level L2 with lower system marginal price where the same company can decrease its production
4. Find a fringe company that can decrease its production in load level L1 and simultaneously decrease in load level L2
5. Exchange the hydro generation of the strategic and fringe companies among load levels taking into account the technical hydro constraints (i.e., maximum and minimum output, maximum and minimum reserve levels, etc.) and any other constraint (i.e, firm market share)
6. If two load levels with different system marginal prices can be selected go to step 2. In other case go to end

This algorithm refines the hydro units operation of the strategic companies. It has observed that this algorithm introduces only minor changes in the output of the strategic companies in the cases tested so far.

3 COMPARISON WITH A MCP APPROACH

The approach presented has two main advantages. One is the realistic modeling of the electric system that allows including binary unit commitment decisions. The other is the use of robust and efficient solution methods, those of MIP problems. Convergence of the proposed method is not theoretically guaranteed. However, cases tested so far have proven to be robust.

Qualitatively, it is interesting to analyze the complementary features of the cost minimization side, still explicitly represented in the model, and the firm's profit maximization objective, which are incorporated implicitly through the market equilibrium constraints. While the later determines, for each strategic firm considered, an output level that maximizes its profits, it is the cost minimization, which decides the specific unit commitment that achieves that output level. It will do so by looking for the cheapest commitment of their thermal units and the cheapest hydro scheduling, exactly as each firm would have done if its output requirements had been set exogenously to the model. With this approach, where market behavior and operating constraints are simultaneously considered, the equilibrium solution accounts for all the technical operating constraints modeled, thereby achieving a realistic system dispatch.

The MCP approach has alternative advantages. One is a compact problem formulation. The other is the possibility of introducing nonlinear constraints. However, as an NLP-based approach it cannot introduce binary variables and the solution procedure is less efficient and robust than LP-based methods. Theoretically, optimallity is guaranteed only with linear constraints.

Both algorithms achieve the same results under the same underlying modeling assumptions (i.e., continuos variables and linear constraints).

Table 1 summarizes the previous comparison between the alternative methods.

	MIP Approach	MCP Approach
Qualitative	Optimal solution not guaranteed in every case	Optimality guaranteed and solution uniqueness in case of linear constraints
Algorithmic	Binary variables are allowed. Constraints must be linear	Only continuous variables. Constraints can be non linear
Solution process	Solution method is efficient and robust	Solution method is slower and depending on the initial value

Table 1. Comparison between MIP and MCP approaches.

4 CASE STUDY

In this section we present a representative case study, which was developed to test the model results.

4.1 Time scope

The time scope is divided into 3 periods, each one representing one month, 2 subperiods corresponding to week and weekend days and 3 load levels for weekdays and 2 load levels for weekend days.

4.2 Demand

The demand function is represented by a linear function of the price with a slope of 0.5 Mpta/GWh/GW at each load level.

4.3 Suppliers

The thermal generation system is composed of 18 units, each one with different variable costs. Their main characteristics are presented in table 2.

The last two columns of table 2 represent the constant and linear terms, respectively, of the straight line that models the heat consumption of each thermal unit.

	Firm	Pmax [MW]	Pmin [MW]	Fuel cost [pta/Mcal]	Constant Variable cost [Mcal/h]	Linear Variable cost [cal/Wh]
T1	E1	140	50	1.38	20	3.21
T2	E2	140	80	1.38	20	3.26
T3	E3	140	50	1.38	30	3.28
T4	E1	140	40	1.38	36	3.30
T5	E2	140	50	1.38	30	3.33
T6	E3	110	50	1.62	25	3.35
T7	E1	100	45	1.62	25	2.85
T8	E2	100	45	1.62	25	2.90
T9	E3	100	60	1.62	20	2.91
T10	E1	130	50	1.62	22	3.01
T11	E2	110	50	1.62	25	3.09
T12	E3	110	60	1.62	28	3.12
T13	E1	140	120	1.68	20	2.92
T14	E2	140	110	1.68	27	3.00
T15	E3	140	75	1.68	35	2.89
T16	E1	250	55	1.68	35	2.90
T17	E2	270	90	1.68	26	2.95
T18	E3	270	85	1.68	26	3.23

Table 2. Thermal generation system.

The maintenance scheduling program has determined that thermal unit 1 will be on maintenance in period 1, unit 5 in period 2 and unit 9 in period 3.

The hydro generation system is composed by 6 plants. Their main characteristics are presented in table 3. The columns represent the maximum power available for consumption as pump or production as generator, the initial and maximum and minimum reserve levels of each hydro reservoir[1] and the performance of the pumping-generation cycle for a pumping-hydro unit. According with the table only the last three hydro units are pumping-hydro.

The natural hydro inflows, expressed in GWh, received in each hydro reservoir in each period are represented in table 4.

	Firm	Pmax pump [MW]	Pmax gener [MW]	Initial reserve [TWh]	Max reserve [TWh]	Min reserve [TWh]	Perfrm [p.u.]
H1	E3		300	80	150	35	
H2	E2		270	70	132	33	
H3	E1		250	60	123	31	
H4	E3	176	220	50	90	22	0.65
H5	E2	160	200	45	82	20	0.70
H6	E1	112	140	30	58	15	0.68

Table 3. Hydro generation system.

	P1	P2	P3
H1	35	30	33
H2	30	25	35
H3	25	25	32
H4	25	20	25
H5	25	20	20
H6	50	10	10

Table 4. Natural hydro inflows.

4.4 Analysis of results

The model has been run with and without market equilibrium constraints. In the second case, the model represents a cost minimization framework or a perfectly competitive market. In the first case, it incorporates the profit maximization objective of strategic firms by means of the market equilibrium constraints.

As it can be observed in figure 4 the price for the cost minimization problem moves between 4.86 and 5.16 pta/kWh while in the profit maximization problem the price range is between 5.00 and 5.22 pta/kWh for the 15 load levels.

The following figures represent the power generated by each company and the total system production, expressed in GW, under cost minimization or market equilibrium constraints.

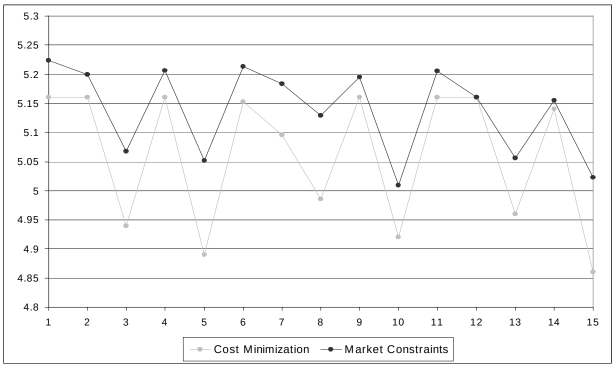

Figure 4. System marginal price.

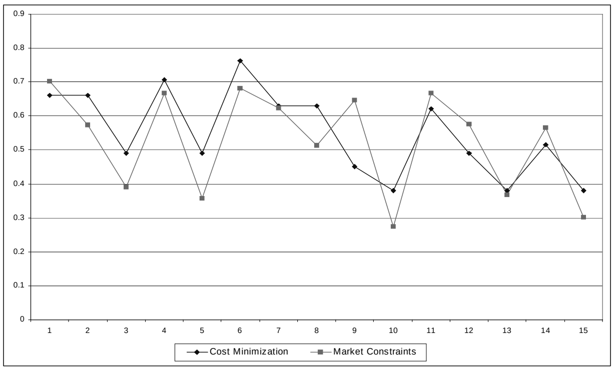

Figure 5. Output of firm E1.

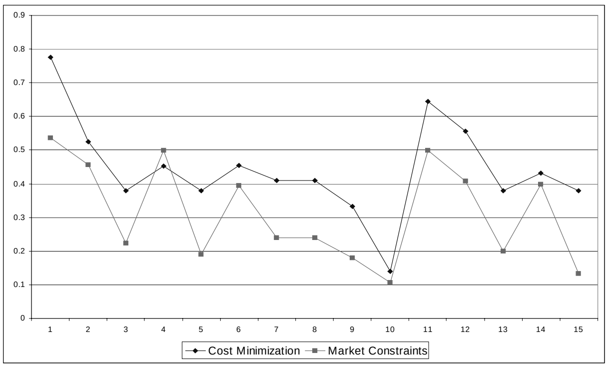

Figure 6. Output of firm E2.

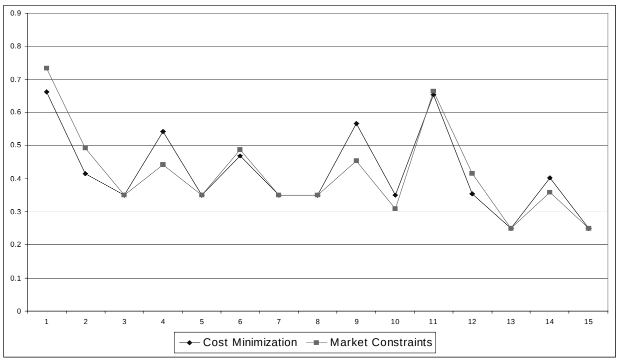

Figure 7. Output of firm E3.

[1] Each hydro unit is supposed to have an associated reservoir.

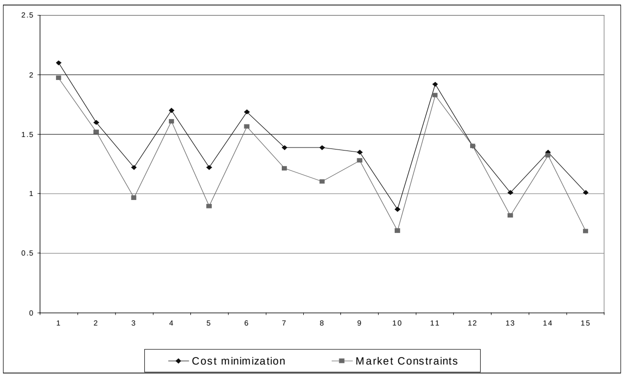

Figure 8. System generation.

It can be observed from previous figures that the energy produced by the companies in the profit maximization model is lower than those produced in the cost minimization model. However, not every load level follows this tendency.

5 CONCLUSIONS

We have presented a practical approach to model the market equilibrium under an oligopoly market competition based on the Cournot conjecture. The model presented in the paper includes the so-called market equilibrium constraints in a detailed production cost model. By this approach all the technical operation constraints can be considered including the binary commitment decisions of thermal units.

The resulting optimization model is a MIP problem so conventional, efficient and robust solvers can be used.

Two iterative algorithms, for obtaining the system marginal price and refining the hydro operation of strategic companies, have been added to the previous market equilibrium model. The results with this MIP problem formulation are equal than those obtained by the MCP approach under the same modeling assumptions.

The model has been tested with a case study and the results are shown in the paper.

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