BIDDING IN A DAY-AHEAD ELECTRICITY MARKET: A COMPARISON OF DECOMPOSITION TECHNIQUES

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Abstract – Daily bidding is an activity of paramount importance for generation companies operating in day-ahead electricity markets. The authors have developed a strategic bidding procedure based on stochastic programming to obtain optimal bids. In this paper, this large-scale mathematical programming problem is solved under the Benders and Lagrangian relaxation frameworks to determine the adequacy of these techniques to solve the optimal bidding problem. Numerical examples illustrate the conclusions of this research.

Keywords: Competitive electricity market, bidding, Benders decomposition, Lagrangian relaxation.

1 INTRODUCTION

Competitive electricity markets that operate on a daily basis have been organized in many regions throughout the world so as to enhance the efficiency of the bulk electric industry. This has triggered an intense research effort to devise new optimization tools devoted to the maximization of the companies' long-term profits in the new framework.

In particular, the development of optimal bids for a generation company that participates in a day-ahead electricity market has attracted much attention. Li *et al.* [1] propose an iterative model in which each generation company solves a self-unit commitment problem based on a set of hourly price scenarios and builds an offer curve formed by a fixed number of bids. Zhang, Wang and Luh [2] first define a model for the day-ahead market and obtain a closed-form expression both for the marginal clearing price and for the energy sold by the company as functions of the company's quadratic offer curve parameters (which have to be optimized) and of the competitors' parameters (which are estimated). They then formulate the company's optimal bidding problem and decompose it using Lagrangian Relaxation, which leads to a set of individual generation unit subproblems and an additional bidding subproblem.

Thus, the general trend indicates that offering electric energy to a day-ahead market must be considered as a twofold problem.

On the one hand, the company's revenues are subject to the future price of energy, which depends on the unknown behavior of its competitors. Consequently, the problem has a stochastic nature and a set of market scenarios has to be considered, which enlarges the size of the problem. In addition to this, due to the fact that electricity cannot be stored and to the frequent lack of response from the demand side, the company might have the ability to affect the price of electricity by changing its market position. However, increasing prices above long-term average costs might not be a good policy, as it leads to the entry of new competitors. In this context, the company's revenues can be expressed as a non-concave function of its production [3].

On the other hand, the strategy followed by the company results in a generation schedule. Hence, both fuel costs and operating constraints must be contemplated, which emphasizes the important role still played by traditional short-term planning tools such as unit-commitment or economic-dispatch models.

The development of profit-maximizing bidding strategies requires the usage of specific mathematical programming methods due to the non-concave nature of this large problem. In our paper two powerful techniques (Benders decomposition and Lagrangian relaxation) are evaluated and compared.

Benders algorithm ([4] [5]) is a straightforward approach to this problem by formulating a master market problem and a generation scheduling subproblem. This procedure has given excellent results when solving the stochastic day-ahead-bidding problem [6].

Lagrangian relaxation (LR) is the alternative for large non-convex problems with complicating constraints [7]. In the bidding problem the decisions taken by the company for each market scenario are used to build offer curves, which must be increasing (low-cost generation should be offered beforehand). This establishes a link between each pair of scenarios, which would be independent otherwise. By relaxing these increasing constraints the problem naturally decomposes into as many subproblems as scenarios are being considered. The iterative solution of the Lagrangian dual problem yields a very good approximation for the company's optimal weekly bidding strategy.

In this paper, computational results are reported about the performance of Benders decomposition and LR in the solution of the optimal bidding problem. We begin by formulating the complete optimal bidding problem and identifying complicating variables and complicating constraints. The problem structure is then exploited to decompose it under the Benders and LR frameworks. Two numerical cases are solved with both approaches to illustrate the features of these techniques.

2 PROBLEM STATEMENT

We consider the problem of developing optimal bidding strategies for a power generation company operating in a day-ahead electricity market, which consists of 24 hourly auctions. Assuming risk neutrality, the company decides its offers in order to maximize its expected profits, given as the difference between expected revenues and expected costs.

After the day-ahead market clears, the company is informed about its net energy sales in MWh for each time period (typically one hour). The company then has to decide how to produce this energy. This results in an hourly generation schedule for the company's generation system, which must observe the units' technical constraints. The company must also consider the possibility of changing its net energy sales by participating in the subsequent hour-ahead market, which also consists of 24 hourly auctions

2.1 Day-ahead market model

In our model, the uncertain market outcome is characterized by a collection of hourly residual-demand realizations. We assume that the randomness of this collection of residual-demand curves has finite support. Index *k* is used to denote the possible realizations or scenarios. Thus, market scenario *k* is represented by a collection of hourly residual-demand curves both for the day-ahead and the hour-ahead market.

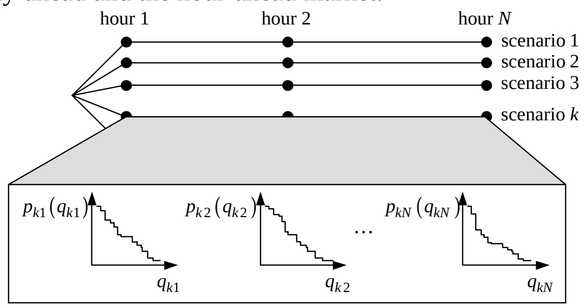

Figure 1: Market scenarios characterized by residual-demand collections.

The company faces a finite set of possible hourly residual-demand scenarios and has to decide the hourly offer curves that must be submitted to the day-ahead market in order to maximize its expected profit, assuming that the shape of the offer curve between each two residual-demand scenarios is irrelevant. Therefore, selecting a bidding strategy is limited to deciding the position of the intersection points between the offer curve and the residual-demand scenarios.

Accordingly, bidding strategies will be characterized hereafter by a set of quantities and their corresponding prices. If the company decides to bid the pair (q_{kn}, p_{kn}) and the residual-demand curve corresponding to scenario *k* actually occurs in hour *n*, then the company will sell q_{kn} MWh and the market clearing will be p_{kn}.

Bidding strategies must be formed by increasing offers to be admissible. Thus, the following conditions hold for each pair of offers (k, k') submitted for hour *n*:

$$q_{kn} - q_{k'n} \geq -x_{kk'n} M^q, \quad k, k' \in K, \tag{1}$$

$$q_{k'n} - q_{kn} \geq -(1 - x_{kk'n}) M^q, \quad k, k' \in K, \tag{2}$$

$$p_{kn} - p_{k'n} \geq -x_{kk'n} M^p, \quad k, k' \in K, \tag{3}$$

$$p_{k'n} - p_{kn} \geq -(1 - x_{kk'n}) M^p, \quad k, k' \in K, \tag{4}$$

where $x_{kk'n}$ is a binary variable and M^q and M^p are a big quantity and a big price, respectively. These constraints link the different market scenarios and complicate significantly the solution of the problem.

The company's revenues, r_{kn}, for scenario *k* and hour *n* are approximated by a piecewise linear function using binary variables. As a result, expected revenues in the day-ahead market are estimated as follows:

$$\mathbb{E}R(x) = \sum_{k \in K} \pi_k \sum_{n \in N} r_{kn}(q_{kn}), \tag{5}$$

2.2 Modeling the generation system

A typical generation system is formed by a combination of thermal units and hydro plants [8].

The cost of producing with a thermal unit, $t \in T$, in scenario *k* and hour *n*, c_{tkn}, has several contributions. Every thermal unit has a fuel consumption characteristic that can be approximated as a linear function of its power output, q_{tkn}. Additional costs are due to operation and maintenance, self-consumption and start-ups, though we assume that unit commitment decisions are assessed with a weekly model. Thermal units also have a maximum capacity, a minimum stable output, and ramp rate limits.

A hydro unit, $h \in H$, transforms a water flow into electric energy, q_{hkn}. We consider a constant energy/flow ratio for the current day. This is equivalent to modeling hydro reserves in terms of stored energy, expressed in MWh. Some hydro units can also operate in pumping mode, consuming electric energy, b_{hkn}, to drive water back to the upstream reservoir. Hydro units have also upper and lower bounds for their operation variables, and reservoirs have minimum and maximum levels. We assume that a certain amount of energy is available for the considered day.

2.3 Modeling the hour-ahead market

The hour-ahead market allows agents to adjust the positions taken in previous markets. For example, a generation company that has suffered the outage of a unit after the clearing of the day-ahead market can buy in the hour-ahead market the energy that this unit was supposed to produce. Thus, the hour-ahead market guarantees that any position adopted by the company in the day-ahead market is feasible. In our model hour-ahead-market residual-demand curves are represented as linear functions of the company's net energy sales, q_{kn}^h. Expected revenues in the hour-ahead market are calculated based on the revenues estimated for each scenario and each hour, r_{kn}^h:

$$\mathbb{E}R^h = \sum_{k \in K} \pi_k \sum_{n \in N} r_{kn}^h(q_{kn}^h), \tag{6}$$

2.4 Modeling contracts

Frequently, generation companies have the possibility of selling part of their energy through bilateral contracts, as an alternative to the spot market. If we consider a contract for q_n^c MWh at p_n^c \$/MWh, where $n \in N^C$ and N^C is the set of hours affected by the contract, the expected revenues associated to this contract are given by:

$$\mathbb{E}R^c = \sum_{k \in K} \pi_k \sum_{n \in N^c} \left(p_n^c - p_{kn} \right) q_n^c \tag{7}$$

2.5 Energy balance equation

The strategies chosen by the company for the day-ahead and the hour-ahead market, its bilateral contracts and its scheduling decisions are linked by an energy balance equation of the form:

$$q_{kn} + q_{kn}^h + q_n^c = \sum_{g \in G} q_{gkn} - \sum_{h \in H} b_{hkn}, \quad k \in K, n \in N . \tag{8}$$

2.6 Problem formulation

The optimal bidding problem, as a result of previous comments and assumptions, has the following formulation:

$$\text{Max} \sum_{k \in K} \pi_k \sum_{n \in N} \left\{ r_{kn}\left(q_{kn}\right) + r_{kn}^h\left(q_{kn}^h\right) + \left(p_n^c - p_{kn}\right) q_n^c - \sum_t c_{tkn} \right\}$$

s.t.: day-ahead-market constraints,
generation-system constraints,
hour-ahead-market constraints,
energy balance equation.

3 SOLUTION PROCEDURES

3.1 Problem structure

The optimal bidding problem comprises two distinct decision levels. On the one hand, the expected revenues of bidding strategies for the day-ahead market are evaluated by means of a MIP model. On the other hand, generation costs and revenues in the hour-ahead market are estimated using a LP model.

Obviously, the difficulty arises from the first part, due to the existence of binary variables introduced to deal with the non-concave nature of the problem. However, binary variables would not constitute a problem if certain constraints, such as those that establish a link between the offers presented for different market scenarios, were not present.

Two solution procedures can be considered as natural approaches given this problem structure: Benders decomposition and Lagrangian relaxation.

3.2 Benders decomposition

Benders approach [4] is based on partitioning the given problem into a master problem, which includes the complicating variables, and a LP subproblem, which results from fixing the complicating variables. At each iteration, the master problem suggests values for the complicating variables. The subproblem then returns the corresponding optimal value of the objective function and a linear outer approximation of the influence of those variables on the objective function. This method has become a standard in stochastic programming to address multiple-stage problems in which some decisions have to be made before some uncertain events take place, but where subsequent recourse actions are also possible [9], [10]. First-stage decisions are kept in the master problem, while second-stage recourse actions for each scenario are evaluated in the subproblem. If first-stage decisions never yield infeasible subproblems the recourse is said to be complete. The optimal bidding problem can be expressed as a two-stage stochastic program with complete recourse.

The master problem takes the following form:

$$\text{(MP)} \ \underset{q_{kn},\theta}{\text{Max}} \sum_{k \in K} \pi_k \sum_{n \in N} \left\{ r_{kn}\left(q_{kn}\right) + \left(p_n^c - p_{kn}\right) q_n^c \right\} + \theta$$

s.t.: day-ahead-market constraints,

$$\theta \le \theta^v + \sum_k \sum_n \lambda_{kn}{}^v \left(q_{kn} - q_{kn}{}^v \right), \quad v = 1,\dots,V$$

where θ is the outer linearization of the recourse function, θ^v is the value of the recourse function given by the subproblem at iteration *v*, $\lambda_{kn}{}^v$ is the dual variable of the energy balance equation and $q_{kn}{}^v$ is the energy decided in (MP) at iteration *v*.

The corresponding subproblem is expressed as:

$$\text{(SP)} \ \underset{q_{kn}^h, q_{gkn}, b_{hkn}}{\text{Max}} \sum_{k \in K} \pi_k \sum_{n \in N} \left\{ r_{kn}^h\left(q_{kn}^h\right) - \sum_t c_{tkn} \right\}$$

s.t.: generation-system constraints,
hour-ahead-market constraints,
energy balance equation.

At each iteration, the master problem decides a bidding strategy for the day-ahead market in the form of a quantity for each scenario *k* in each hour *n*, q_{kn}. The subproblem then evaluates the cost of producing this energy and chooses a strategy for the hour-ahead market. As a result, an optimality cut is returned to the master problem. A commercial solver can be used to iteratively solve both the MIP master problem and the LP subproblem.

An interesting feature of the master problem in this particular case is that the bids chosen for different hours are only linked by the recourse function, θ. The recourse function can also be expressed as the sum of hourly recourse functions, as follows:

$$\theta = \sum_n \theta_n \le \sum_n \left\{ \theta_n{}^v + \sum_k \lambda_{kn}{}^v \left(q_{kn} - q_{kn}{}^v \right) \right\}, \quad v = 1,\dots,V \tag{9}$$

where $\theta_n{}^v$ is the value of the recourse function for hour *n* given by the subproblem at iteration *v*.

It is because of the recourse function that the master problem is not separable into hourly problems of the form:

$$(\text{MP}_n) \ \text{Max} \sum_{k \in K} \pi_k \left[r_{kn}\left(q_{kn}\right) + \left(p_n^c - p_{kn}\right) q_n^c \right] + \theta_n$$

s.t.: day-ahead-market constraints,

$$\theta_n \le \theta_n{}^v + \sum_k \lambda_{kn}{}^v \left(q_{kn} - q_{kn}{}^v \right), \quad v = 1,\dots,V$$

However, it can be easily proven that any feasible solution for (MP_n, $n \in N$) is also a feasible solution for (MP). Therefore, a feasible startpoint can always be easily obtained by solving (MP_n, $n \in N$).

3.3 Lagrangian relaxation

Lagrangian relaxation is a natural approach for large-scale mixed-integer programs with complicating constraints. Instead of addressing the primal problem, which is non-convex and has complicating constraints, the Lagrangian dual is constructed in such a form that in each step of its iterative solution easier problems have to be solved. The main disadvantage of this approach is that the optimal solution for the Lagrangian dual is not a feasible solution for the primal maximization problem, though it constitutes an upper bound and a good approximation. In addition to this, it provides correct dual information about the MIP primal problem, which cannot be obtained otherwise.

In the optimal bidding problem two sets of complicating constraints can be identified: the one that forces offers to be increasing and the energy balance equation. The Lagrangian dual is formulated as follows:

$$\text{(LD)}\quad \min_{\lambda_{kn},\mu_{kk'n}} \left\{ \max_{\substack{q_{kn},q^h_{kn}\\ q_{gkn},b_{hkn}}} \mathcal{L}\left(q_{kn}, q^h_{kn}, q_{gkn}, b_{hkn}, \lambda_{kn}, \mu_{kk'n}\right) \right\}$$

s.t.: day-ahead-market constraints,
generation-system constraints,
hour-ahead-market constraints,

where λ_{kn} is the Lagrange multiplier associated to the energy balance equation, $\mu_{kk'n}$ are the Lagrange multipliers associated to the constraints that force offers to be increasing and $\mathcal{L}\left(q_{kn}, q^h_{kn}, q_{gkn}, b_{hkn}, \lambda_{kn}, \mu_{kk'n}\right)$ is the Lagrangian, defined as:

$$\begin{aligned}
&\mathcal{L}\left(q_{kn}, q^h_{kn}, q_{gkn}, b_{hkn}, \lambda_{kn}, \mu_{kk'n}\right) = \\
&\sum_{k\in K}\sum_{n\in N}\Bigg\{\pi_k\left[r_{kn}\left(q_{kn}\right) + r^h_{kn}\left(q^h_{kn}\right) + \left(p^c_n - p_{kn}\right)q^c_n - \sum_t c_{tkn}\right] \\
&\quad + \lambda_{kn}\left[\sum_{g\in G} q_{gkn} - \sum_{h\in H} b_{hkn} - q_{kn} - q^c_n - q^h_{kn}\right] \\
&\quad + \sum_{k'>k} \mu^q_{kk'n}\left[q_{kn} - q_{k'n} + x_{kk'n}M^q\right] \\
&\qquad + \mu^{-q}_{kk'n}\left[q_{k'n} - q_{kn} + \left(1 - x_{kk'n}\right)M^q\right] \\
&\qquad + \mu^p_{kk'n}\left[p_{kn} - p_{k'n} + x_{kk'n}M^p\right] \\
&\qquad + \mu^{-p}_{kk'n}\left[p_{k'n} - p_{kn} + \left(1 - x_{kk'n}\right)M^p\right]\Bigg\}
\end{aligned}$$

The Lagrangian dual has been formulated so that the complicating constraints are relaxed. At each iteration v of the solution process, given a set of Lagrange multipliers, the inner maximization can be performed by separately solving the following set of subproblems. Firstly, a MIP day-ahead market subproblem (MSP_{kn}) is obtained for each scenario k and each hour n.

$$\text{(MSP}_{kn}\text{)}\quad \begin{aligned}
&\max_{q_{kn}} \pi_k\left[r_{kn}\left(q_{kn}\right) + \left(p^c_n - p_{kn}\right)q^c_n\right] \\
&- \lambda_{kn}{}^v\left[q_{kn} + q^c_n\right] \\
&+ \sum_{k'\neq k}\Bigg\{\mu^q_{kk'n}{}^v\left[q_{kn} + x_{kk'n}M^q\right] \\
&\qquad + \mu^{-q}_{kk'n}{}^v\left[-q_{kn} + \left(1 - x_{kk'n}\right)M^q\right] \\
&\qquad + \mu^p_{kk'n}{}^v\left[p_{kn} + x_{kk'n}M^p\right] \\
&\qquad + \mu^{-p}_{kk'n}{}^v\left[-p_{kn} + \left(1 - x_{kk'n}\right)M^p\right] \\
&\qquad + \left[-\mu^q_{k'kn}{}^v + \mu^{-q}_{k'kn}{}^v\right]q_{kn} \\
&\qquad + \left[-\mu^p_{k'kn}{}^v + \mu^{-p}_{k'kn}{}^v\right]p_{kn}\Bigg\}
\end{aligned}$$

s.t.: day-ahead-market constraints.

Notice that the binary variable $x_{kk'}$ is calculated when solving for scenario k, while $x_{k'k}$ is calculated when solving for k'.

LP scheduling subproblems are formulated for each thermal unit (GSP_t) and each hydro unit (GSP_h):

$$\text{(GSP}_t\text{)}\quad \max_{q_{tkn}} \sum_{k\in K}\sum_{n\in N} \lambda_{kn}{}^v q_{gkn} - \pi_k c_{tkn}$$

s.t.: unit t's constraints.

$$\text{(GSP}_h\text{)}\quad \max_{q_{hkn},b_{hkn}} \sum_{k\in K}\sum_{n\in N} \lambda_{kn}{}^v\left[q_{hkn} - b_{hkn}\right]$$

s.t.: unit h's constraints.

Finally, a LP problem for each scenario k and each hour n is formulated for the hour-ahead market (HSP):

$$\text{(HSP}_{kn}\text{)}\quad \max_{q^h_{kn},q_{gkn},b_{hkn}} \sum_{k\in K}\sum_{n\in N} \pi_k r^h_{kn}\left(q^h_{kn}\right) - \lambda_{kn}{}^v q^h_{kn}$$

s.t.: hour-ahead-market constraints.

Lagrange multipliers are updated at iteration v by solving an outer linearization of the Lagrangian dual problem:

$$\text{(DP)}\quad \min_{\lambda_{kn},\mu_{kk'n}\geq 0} w$$

s.t.:

$$\begin{aligned}
w \geq &\sum_{n\in N}\pi_k\left[r_{kn}{}^v + r^h_{kn}{}^v + \left(p^c_n - p_{kn}{}^v\right)q^c_n - \sum_t c_{tkn}{}^v\right] \\
&+ \lambda_{kn}\left[\sum_{g\in G} q_{gkn}{}^v - \sum_{h\in H} b_{hkn}{}^v - q_{kn}{}^v - q^c_n{}^v - q^h_{kn}{}^v\right] \\
&+ \sum_{k'>k} \mu^q_{kk'n}\left[q_{kn}{}^v - q_{k'n}{}^v + x_{kk'n}{}^v M^q\right] \\
&\qquad + \mu^{-q}_{kk'n}\left[q_{k'n}{}^v - q_{kn}{}^v + \left(1 - x_{kk'n}{}^v\right)M^q\right] \\
&\qquad + \mu^p_{kk'n}\left[p_{kn}{}^v - p_{k'n}{}^v + x_{kk'n}{}^v M^p\right] \\
&\qquad + \mu^{-p}_{kk'n}\left[p_{k'n}{}^v - p_{kn}{}^v + \left(1 - x_{kk'n}{}^v\right)M^p\right]
\end{aligned}$$

4 NUMERICAL RESULTS

4.1 Implementation

The optimal bidding procedure has been implemented in GAMS language [11], together with the Benders and Lagrangian relaxation algorithms. All subproblems have been solved with CPLEX 7.0.

4.2 Study case 1

A small study case for a generation company facing two residual-demand scenarios in each hour is solved in this section to illustrate the performance of both algorithms. Residual-demand data have been obtained from the Spanish Market Operator [12] to replicate the problem faced by Iberdrola on July 18th, 2001. The historic information used to build the two residual-demand scenarios corresponds to July 13th and July 17th. Residual-demand curves are represented by piecewise linear approximations formed by 25 segments between 0 €/MWh and 100 €/mWh (Figure 2).

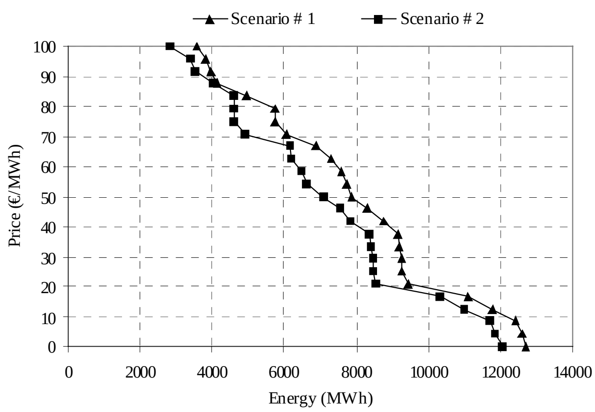

Figure 2: Residual-demand scenarios for hour 21.

This case has 12399 equations and 10813 variables, 1910 of which are binary. It has been solved in 20.89 s and the resulting expected profits are 6.822952 M€.

Using Benders decomposition, the same solution is obtained in 41 s and 30 iterations. The master problem is kept partitioned in hourly problems until iteration 28. As can be seen in Figure 3, the partitioned master problem is more restrictive and cannot be considered as an upper bound for the original problem. However, any feasible solution for the partitioned master problem is also a feasible solution for the master problem.

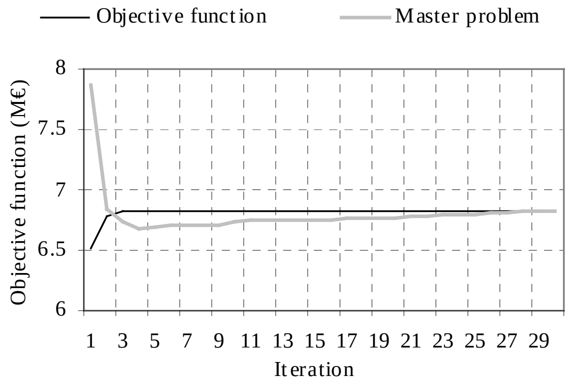

Figure 3: Evolution of the objective function in Benders.

The strategy suggested by the model for scenario # 1 is shown in Figure 4. Net sales in the day-ahead market are constrained by a minimum-market-share limit. Additional sales are carried out through the hour-ahead market, mainly during off-peak hours. Energy purchases for pumping take place in the day-ahead market.

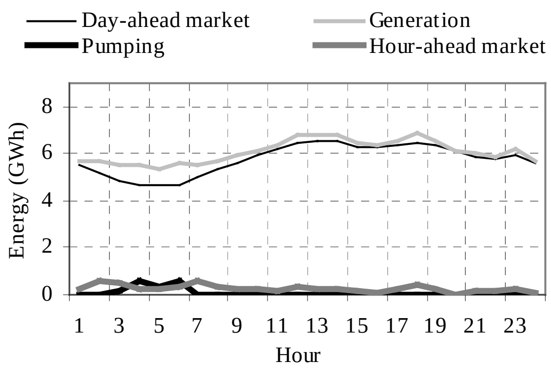

Figure 4: Strategy for market scenario #1 in Benders.

The dual variables obtained for both scenarios are shown in Figure 4. Marginal revenues for scenario #1 are higher than for scenario #2.

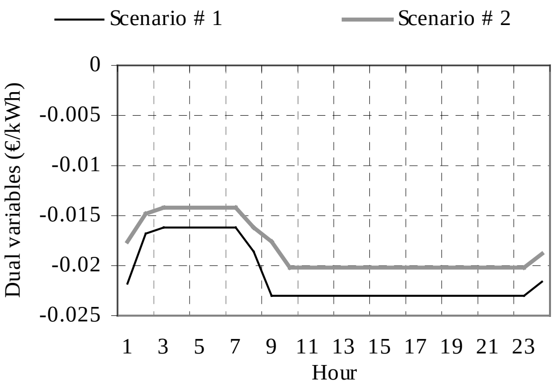

Figure 5: Dual variables returned by Benders algorithm.

Solving the dual problem using Lagrangian relaxation requires an intense computational effort. Among the variety of methods commonly used to update Lagrange multipliers, we have chosen to solve an outer linearization of the dual problem. Convergence is accelerated by dynamically updating the multipliers' feasibility region [13].

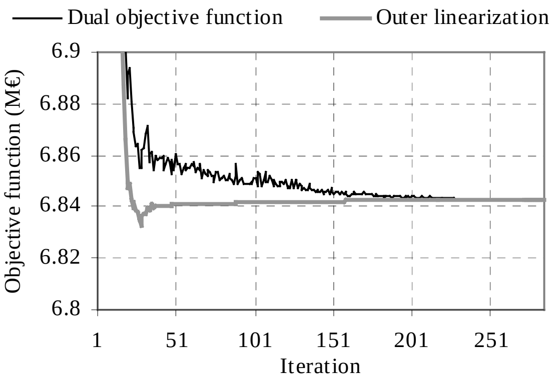

Figure 6: Evolution of the objective function in LR.

284 iterations and 261 s are needed to reach a dual solution of 6.842557 M€ with a tolerance of 10^{-6} M€. Figure 7 shows the resulting Lagrange multipliers. Table 1 summarizes the performance of both algorithms for this numerical case.

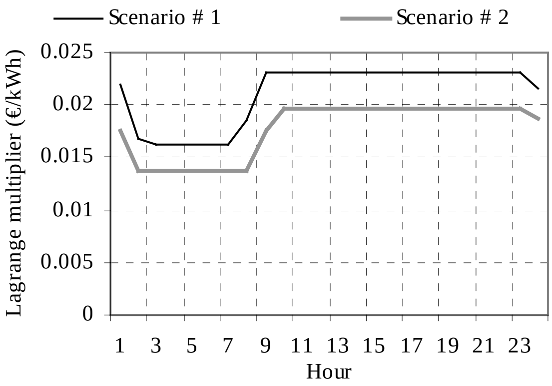

Figure 7: Lagrange multipliers reurned by LR.

	Benders	LR
Time (s)	41	614
Iterations	30	284

Table 1: Performance of the algorithms for study case 1.

Due to non-concavities of the primal maximization problem, in general the dual solution is not a feasible primal solution, because the relaxed constraints do not usually hold. In our case the energy balance equation and the increasing constraints have been relaxed. Figure 8 shows that energy production for scenario #º does not correspond to energy sales in the day-ahead and hour-ahead markets. In addition to this, the production profile seems rather unstable. This effect has already been reported by Guan *et al.* [14]. However, Table 2 shows that both solutions are almost equivalent in terms of energy.

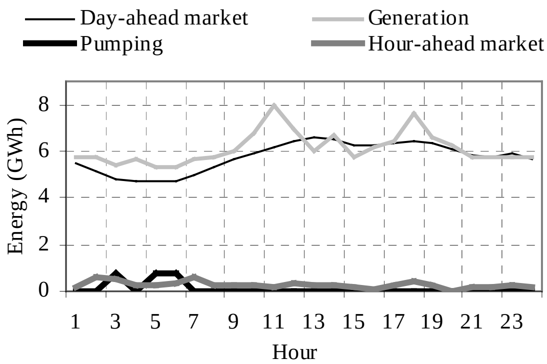

Figure 8: Strategy for market scenario #1 in LR.

	Benders		LR	
	Sc. 1	Sc. 2	Sc. 1	Sc. 2
Generation	146.11	144.73	146.39	144.77
Pumping	1.67	1.99	2.40	2.40
Day-ahead market	137.99	135.64	137.99	135.07
Hour-ahead market	6.45	7.20	6.37	7.25

Table 2: Energy balance in GWh for study case 1.

4.3 Study case 2

We now consider a six-scenario version of the previous case. This problem has 38552 equations and 33007 variables, 6259 of which are binary. CPLEX 7.0 is unable to obtain a feasible solution for it.

Benders algorithm gives a feasible solution of 5.688405 M€ to this problem in 360 s and 19 iterations, keeping the master problem partitioned until iteration 17 (Figure 9). In iterations 18 and 19 the execution time of the branch and bound algorithm used to solve the MIP master problem was limited to 60 s, so optimality cannot be guaranteed. However, the quality of this solution has been evaluated by using it as initial point to solve the original MIP problem with CPLEX 7.0. After 4000 s of execution, CPLEX is unable to improve this solution and its branch and bound algorithm reports a best possible solution of 5.7160 M€. This means that, at worst, Benders algorithm solution is suboptimal in 0.49 %.

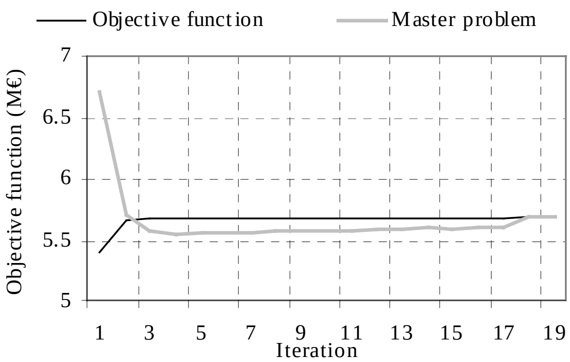

Figure 9: Evolution of the objective function in Benders.

Solving this problem with the LR algorithm requires tuning the multipliers' dynamic feasibility region [13]. When the multipliers are allowed a maximum variation of 0.01 c€/kWh, 14 hours of LR execution are insufficient to reach the solution. As shown in Figure 10, the outer linearization gives an approximation of 5.801604 M€ for a point where the dual objective function is 5.827910 M€. On the contrary, if the multipliers' maximum variation is narrowed in each iteration according to the formula $\text{max. variation} = \frac{0.02\ \text{c€/kWh}}{\text{number of current iteration}}$ until iteration number 500 and then is kept fixed, the dual solution, 5.807063 M€ is reached in 643 iterations.

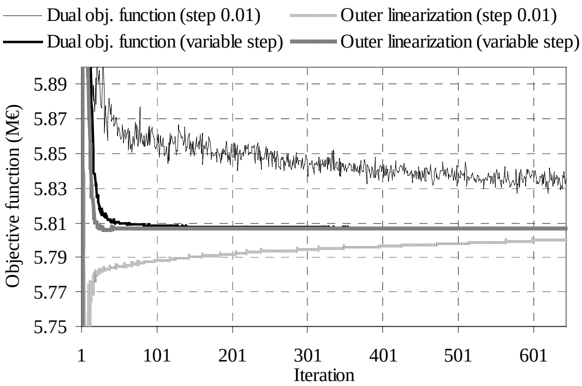

Figure 10: Evolution of the objective function in LR.

Figure 11 shows the evolution of the LR algorithm when the maximum variation that Lagrange multipliers are allowed is reduced in each iterarion until iteration 500. It seems clear that Benders algorithm is a better choice for this problem, given that it returns a primal feasible solution in less time and does not require tuning to improve its performance.

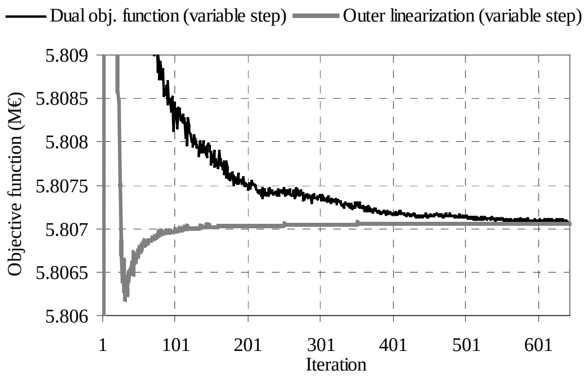

Figure 11: Detail of the objective function's evolution in LR.

5 CONCLUSIONS

In this paper two solution strategies have been tested to address the optimal bidding problem of a generation company operating in a day-ahead electricity market.

According to our results, Benders decomposition is more suitable than Lagrangian relaxation for solving this particular problem. Benders decomposition guarantees a feasible solution in significantly less time. Additionally, an upper bound can always be obtained by solving the LP problem that results from relaxing the integrality constraints.

The major drawback of Benders algorithm lies in its inability to deal with the weekly stochastic unit-commitment problem, where the size of the master problem becomes unmanageable and the scheduling subproblem is non-concave due to start-up costs. On the contrary, Lagrangian relaxation seems the best approach to make start-up decisions on a weekly basis taking into account residual-demand scenarios. Future research will be oriented to the development of such a model and the usage of advance techniques to update the multipliers.

Eventually, we conclude that both techniques happen to be complementary and can be coordinated to address the optimal bidding problem.

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