Computational Experience with Optimization for a Bulk Production Cost Model

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Abstract

The interior point or barrier method has made possible to solve in a workstation large-scale optimization problems that very recently could not be solved in reasonable time or even were unsolvable. In this paper the optimization problem arises from a medium term bulk production cost model developed for economic planning. The paper contains the computational experience and the performance results obtained using different solution methods, implementation of the methods, strategies of solution for various options of the models, and algorithmic approaches. The results demonstrate the superiority of the interior point method with respect to the simplex method. The production cost model has been coded in a flexible mathematical programming language allowing to do these tests very easily.

This large scale optimization capability and development flexibility can be a very significative aid by allowing to quickly create models at the time when the deregulation is establishing new operation rules for the electric sector.

1. Bulk Production Cost Model

Economic planning is a major concern of medium term operations planning studies devoted to predict the future system operation of an electric utility and determine their parameters (costs, fuel consumption, productions, etc.). This production cost model addresses all the operating decisions in an integrated framework and establishes a compromise and balance among different modeling decisions.

This model provides the minimum variable cost subject to operating constraints (generation, transmission and fuel constraints). *Generation constraints* include power reserve margin with respect to the system peak load, first Kirchhoff's law at each node, hydro energy scheduling, maintenance scheduling, and generation limitations. *Transmission constraints* cover the second Kirchhoff's law and transmission limitations. The generation and transmission economic dispatch is approximated by the linearized (also called DC) load flow equations. Network losses are included as a non linear approximation of the voltage angles at the nodes and resistance and reactance of the circuit. *Fuel constraints* include minimum consumption quotas and fuel scheduling for domestic coal thermal plants. The decisions can be classified into *interperiod decisions*, i.e., fuel, hydro and maintenance scheduling, and *intraperiod decisions*, i.e., generation outputs and transmission flows for each period. The mathematical formulation of the model has been reported elsewhere [5, 8].

The model is a very powerful and flexible tool that can be easily adapted to any electric energy system. On one hand, this characteristic is specially useful when deregulation is going to change the operation rules and the model can help in defining a new ones or has to be adapted to them, as is the case of the Spanish power system. On the other hand, in the internationalization process of the electric utilities a flexible model can be a valuable tool to easily represent different electric energy systems with different operation or remuneration rules.

By these reasons several *modeling options* have been provided to customize the use of the model to different needs, such as the following:

- production cost model for a single demand node with all the hydro units aggregated into a single reservoir or, alternatively, each hydro unit individually considered
- generation/transmission production cost model with or without losses
- linear, piecewise linear or quadratic heat consumption curves
- maintenance exclusively defined by the user, or integer optimization of maintenance decisions with or without partially prespecified data
- commitment decisions in continuous or binary variables
- specification of seasonal pumped-hydro and/or weekly/daily pumped-storage in any period, subperiod or load level

The paper contains the computational experience and the performance results obtained using different solution methods, several implementation of the methods, strategies of solution for various options of the models, and algorithmic approaches.

The paper is organized as follows. Firstly, the system description and its representation into the model for each type of generation and the transmission network is shown. In section 3 it is presented the case study based on the Spanish electric energy system. The implementation of the model and the algorithmic alternatives are discussed in section 4. Next, the detailed comparisons done for some options of the model are analyzed. Finally, further developments of the model are presented and extracted the conclusions.

2. System Description

A production cost model minimizes the variable cost for a time *scope*. In this medium term model, the scope is usually one year, divided into *periods*, *subperiods* and *load levels*. Typically, periods will correspond to months, subperiods to weekdays and weekends of a month, and load levels to peak, plateau and off-peak hours.

The load for each period is modeled as a staircase load duration curve, where an step is a load level. Hence, generation will be constant for each load level. Load is individually specified for each node.

Each thermal unit is divided into two blocks, being the minimum load block the first. Heat rate can be specified by a straight line, by a quadratic curve. Random outages are deterministically modeled by derating the unit's full capacity by its equivalent forced outage rate. A thermal plant consists of units in a physical plant. Fuel constraints currently affect the fuel consumption of some thermal plants. Multiple fuel consumption is allowed for each thermal unit because of environmental, economic or technical reasons. Also dual alternative fuel thermal units are modeled.

Very small hydro units are grouped. Spatial dependencies among hydro plants are considered irrelevant to the medium term thermal generation scheduling problem and ignored.

Only the economic utilization of pumped units is considered. This economic function includes both the transference of energy from off-peak hours to peak hours and the alleviation of minimum (maximum) load conditions in off-peak (peak) hours.

Each circuit of the high voltage transmission network is modeled individually by its resistance and reactance and a thermal capacity limit.

3. Case Study

Some data extracted from 1994 statistical records are given next to present the size and complexities of the mainland Spanish electric energy system.

The scope of the model has been split into 12 periods (months) with 2 subperiods each (weekdays and weekends) and 3 and 2 load levels per each subperiod respectively, that gives a total of 60 load levels per year.

The system met a maximum peak load of 25336 MW and a yearly energy demand of 145670 GWh. The installed generation capacity is 42096 MW (16110 MW are hydro, 10675 MW coal, 7910 MW oil/gas and 7401 MW nuclear).

There are about 71 thermal generators (8 nuclear, 36 coal and the remaining oil/gas) grouped in 16 thermal plants. Their production is about 80 % of the total generation.

There are 70 hydro units with capacity greater than 5 MW and annual energy production greater than 100 GWh, that can be grouped into about 10 basins. Other 52 additional and smaller hydro units have been considered. The maximum hydro capacity at the same plant is 915 MW. They produce as an average about 20 % of the total generation, ranging in between 13 % and 28 %, depending on the hydrology.

There are 8 pumped-storage units, but their impact on annual energy production is minimum (about 1 %).

The high voltage transmission network has been simplified to 138 nodes and 263 lines.

In summary, the characteristics of the system regarding time division and number of elements are presented in table 3.

Depending on the modeling option being chosen the problem can be a large-scale LP with very different sizes (10000 rows and 9000 columns for single demand node with the hydro aggregation option or 36000 rows and 65000 variables for the generation/transmission option), NLP (due to quadratic heat consumption curves or to network losses) or MIP (with around 2500 binary maintenance and commitment variables). Not all combinations are directly resolvable with the current optimization technology for the sizes involved in the Spanish case so different strategies or algorithms have to be used.

4. Implementation

> "The technology improvements in algorithms, modeling languages, software, and hardware have made the methodology accessible, easy to use, and fast. So the *Age of Optimization* has arrived" [7].

This production cost problem is implemented in GAMS version 2.25 [2]. This language allows the creation of large and complex problems in a concise and reliable manner. It lets the user to concentrate on the modeling problem by eliminating the writing details of special code in the preliminary stages of algorithmic development. GAMS is flexible and powerful. This flexibility is crucial in the development and test of new algorithms. Several solvers can be used in conjunction with the GAMS language, for example, CPLEX and OSL as LP and MIP solvers and MINOS as a NLP solver.

The methodological implementation of this model and its resolution using this compact and elegant algebraic language takes only 1775 lines of code. Exactly the same model is portable to any hardware platform where GAMS and the solvers were available. Currently, a workstation is being used.

Careful attention when solving a large-scale optimization problem should be paid to the scalation of constraints and variables in order to keep them around 1. So constraints and variables have been scaled accordingly, GW is taken as the natural unit for power, TWh for energy, Tpta for monetary unit and kTcal for heat consumption.

Associated to the execution of a model written in GAMS two very different types of time consumption should be distinguished:

- *time for preparation and generation* of the problem to be solved from the equations describing it and writing the interface with the optimizer. This time is needed each occasion an optimization problem is to be solved. In the model this time ranges from 4 to 40 seconds depending on the problem size.

- *solution time* required by the solver for obtaining the optimal solution. This depends on the type and size of the problem, on the initial point provided, and on the solution method and optimizer used, among other issues. Only CPU time is consumed during this step.

Any model requires a combination of both but the solution time is the main part of the total execution time required by the model and can be dramatically reduced by several approaches:

1. Using different *solution methods*

Periods	12
Subperiods/period	2
Load Levels/subperiod	3 and 2
Nodes	138
Circuits	263
Thermal Units	71
Thermal Plants	16
Hydro / Pumped-Hydro Units	122
Pumped-Storage Units	8

Cuadro 1: Characteristics of the case study

> "In the last decade, new advances in algorithms have been as important as the impressive advances in computer technology" [7].

For LP problems primal simplex, dual simplex or interior point methods can be used. Using the new interior point algorithms or advanced implementations of simplex methods, we can now solve very large linear problems with thousands of constraints and variables on a PC. However, only few papers appear in power systems literature that mention such a use. References [3, 6, 10] present variants of Karmarkar's interior point method applied to NLP problems in the case of VAR planning or to LP problems in fuel planning, but with problems sizes lower than the smallest presented in this paper. In reference [11] a non linear predictor-corrector primal-dual interior point method is used for an OPF. Comparisons among different solution methods for other type of problems can be found in [4].

2. Using different *implementations* of previous solution methods in different optimization packages

   This model has been solved using CPLEX, MINOS and OSL as LP solvers, CPLEX and OSL as MIP solvers and MINOS as NLP solver. CPLEX and OSL both use a predictor-corrector primal-dual interior point method.

   The optimizers performance can be controlled by a set of parameters. By default their values are appropriate for most of the problems as has been this case. However in general, it can be convenient to test different options of certain parameters, especially those related to pricing and preprocessing.

3. Implementing an *hierarchical solving strategy*

   This strategy solves firstly a much easier to solve problem as starting point for the complex one. For example, solving a NLP problem providing the solution for a close LP problem as starting point.

4. Making the *solution algorithm* more complex

   For the NLP generation/transmission dispatch problem with losses a generalized Benders decomposition method [1] has been also implemented in GAMS. The master problem determines the interperiod decisions, i.e., fuel, hydro and maintenance scheduling, and each subproblem solves the intraperiod decisions, i.e., generation and transmission economic dispatch for one period.

As important as the previous approaches to improve solution performance can be the correct formulation of the problem itself, especially in MIP problems. A good formulation is not as trivial as can be thought and is not measured simply in terms of numbers of constraints and variables. Several techniques have been found on the literature [9] than improve the representation of a MIP problem and successfully applied to the model:

- fixing of binary variables
- logical implications due to other binary variables
- improvement of bounds

5. Comparisons

In this section the different comparisons done to study the previous performance improvements are discussed and analyzed in detail.

5.1. Different optimizers and solution methods in LP problems

The same test cases are solved with three optimizers, CPLEX version 3.0, MINOS version 5.3 and OSL release 2. The problem NUHA corresponds to a single demand node with aggregation of hydro units, neither pumped-storage nor pumped-hydro, linear heat consumption curves, maintenance specified by the user, and continuos commitment decisions. The objective value is 338.423352511 Gpta. The problem NUHDBB differs from the previous in the individual treatment of hydro units, allowance of pumped-storage and pumped-hydro, and piecewise linear heat consumption curves. The objective value is 336.536352784 Gpta. Default parameters have been used for the optimizers.

In table 5.1 it is shown the solution time in seconds required by the optimizers for the model executed in a workstation Axil 311 Model 5.1 (with 83 SPECfp92 and 65 SPECint92, around 3 times quicker than a PC

	PROBLEM NUHA 8977 r, 7416 v, 32827 e		PROBLEM NUHDBB 12805 r, 32817 v, 79357 e	
	time	iter	time	iter
CPLEX				
Primal simplex	169	11549	1069	31880
Dual simplex	258	7712	1782	24361
Barrier + crossover	55	27	133	28
MINOS				
Primal simplex	1498	10670	7362	33764
OSL				
Primal simplex	290	6427	2249	22653
Dual simplex	845	10829	5242	34878
Interior3 + crossover	72	2453	302	7765

Cuadro 2: Comparison among CPLEX, MINOS and OSL in LP problems.

486DX2 at 50 MHz) and the number of iterations for the different alternatives. For each problem the number of constraints (r), variables (v) and non zero elements (e) of the constraints matrix is presented.

Crossover is the transition from the interior point to the simplex method once a nearly optimal interior point solution has been reached. This is specially important when an optimal basis is required as is the case when dual information is needed or when the LP problem is the relaxation of a MIP problem.

The results show that the best solution method is the interior point or barrier method. For the test cases, the solution time required is 3 to 8 times lower that the time required by the primal simplex method, both with the same optimizer. The dual simplex even requires more time than the primal simplex.

With the primal simplex method the solution time increases quadratically with the size of the problem. However, with the interior point method this increment is approximately linear.

Regarding optimizers, CPLEX obtains the best performance decreasing the solution time in a 24 to 56 % with respect to OSL, both using the interior point method, or in a 42 to 52 % when using the primal simplex. With respect to MINOS, the primal simplex method of CPLEX requires around 10 times less than the implemented in MINOS.

The first conclusion derived from this test and the performance obtained with CPLEX and OSL is that large-scale LP problems can be solved in very reasonable time.

5.2. Different optimizers and solution methods in MIP problems

The MIP problems appear when maintenance and/or commitment decisions of thermal units are required to be binary.

The test consists of solving the same MIP problems with two optimizers. The problem NUHAMNUC corresponds to a single demand node with aggregation of hydro units, neither pumped-storage nor pumped-hydro, linear heat consumption curves, optimization using *binary maintenance and commitment decisions* (2556 binary variables for the case study). The problem NUHDBBMNUC differs from the previous in the individual treatment of hydro units, pumped-storage and pumped-hydro in any time interval, and piecewise linear heat consumption curves.

In both cases a relative termination tolerance of $3 \cdot 10^{-3}$ which means that the optimizer will stop with a integer solution whose objective function is within this tolerance of the best possible solution.

In table 5.2 it is shown the solution time in seconds, the number of iterations, the number of nodes evaluated, the objective function for the current integer solution, and the relative termination tolerance. For each problem the number of constraints (r), variables (v), non zero elements (e) of the constraints matrix and the number of binary variables (i) is presented.

The node selection strategies chosen for CPLEX and OSL, depth-first and probing only on satisfied binary variables respectively, have been found to be the best for these problems.

Regarding the MIP problems again CPLEX obtains the best results for both cases, but it takes around one order of magnitude more to solve a MIP problem of about the same size than the LP problem.

5.3. Hierarchical strategy

Two hierarchical solving strategies have been tested to improve the optimization performance of the model. Both are described next.

	PROBLEM NUHAMNUC 11299 r, 9204 v, 39145 e, 2556 i				
	time	iter	nodes	objective function	relative gap
CPLEX Barrier + crossover + nodeselect 0	489	18291	225	337.824835429	0.001034482
OSL Interior3 + crossover + strategy3	992	10041	309	337.659354440	0.00053
	PROBLEM NUHDBBMNUC 15127 r, 34605 v, 85675 e, 2556 i				
	time	iter	nodes	objective function	relative gap
CPLEX Barrier + crossover + nodeselect 0	1840	32672	235	338.077770303	0.001455915
OSL Interior3 + crossover + strategy3	5695	48822	588	337.820747655	0.00066

Cuadro 3: Comparison between CPLEX and OSL in MIP problems.

1. NLP problems in the option single demand node production cost model

   As it is well known NLP problems are difficult to solve and a good stating point should be provided. In this model the solution with non linear heat consumption curves is obtained using MINOS from a starting point obtained with CPLEX or OSL for the LP problem with the piecewise linear approximation of the heat consumption curves.

   The NLP problem size corresponds to the NUHA problem in table 5.1 and the LP problem previously solved corresponds to the NUHDBB problem in the same table.

2. single node solution as starting point

   Another test of hierarchical strategy has been done regarding the use of the single demand node production cost model (much smaller) as starting point for the bulk (generation/transmission) model. However, the time saved seems to be negligible in the cases studied when using an interior point method for solving the bulk model.

5.4. Decomposition for very large NLP problems

When the size of a NLP problem is greater than the current capability of available NLP optimizers, a decomposition method has to be used. For example, the option generation/transmission production cost problem reaches 36000 rows, 51000 variables and 150000 non zero elements, far beyond the capability of MINOS.

This optimization problem has an special staircase structure that can be exploited. Decomposition techniques are useful in solving this problem by two reasons. First, the global problem is divided into smaller and easier to solve problems. Second, it allows the use of different solution algorithms for master and subproblem. In particular, Benders method is applied when some variables “complicate” the solution of the problem. In this model the complicating variables are those regarding to maintenance, fuel and hydro scheduling, that are decided in the master problem. Thermal operation and flows through the network are determined in the subproblem. Generalized Benders decomposition method is required by the non linearity of the losses appearing in the constraints of balance between generation and demand in each node.

For solving this very large NLP problem (due to losses) it is provided as initial point the optimal solution of the global LP problem (with no losses). In this model the global LP problem and the master problem are solved using an interior point method and the non linear subproblems (one for each period) are solved using MINOS.

The computational advantage of solving smaller problems is partially decreased by the drawback of using GAMS for decomposition. Time for preparation and problem generation for master and subproblems is added in each iteration.

For the case study, size, time (expressed in seconds) and number of iterations required for solving the different problems are presented in table 5.4.

When using the optimal solution of the LP problem as initial point for the decomposition algorithm 13 iterations are needed to achieve convergence under a tolerance of 10^{-3} with one cut per period added to the master problem for each iteration. The difference in variable costs due to losses is approximately 0.2 %.

	time	iter
GLOBAL PROBLEM 36625 r, 51335 v, 149217 e		
OSL Interior3	1185	39
MASTER 1477+12J r, 8675 v, $\approx$ 13917+300J e		
OSL Interior3	12	22
SUBPROBLEM 2932 r, 3487 v, 10887 e		
MINOS Primal simplex	19	450

Cuadro 4: Benders decomposition for a very large NLP problem. J is the number of Benders iterations.

6. Further Developments

The extraordinary attention paid in the development of the model has allowed to get a very complete, sophisticated and flexible model and there are planned several extensions to it:

- Monte Carlo simulation of the demand and natural inflows stochasticity and availability of elements of the system.
- piecewise approximation of non linearities in network losses.
- environmental dispatching strategies and externalities.
- multiyear evaluation of production costs.

7. Conclusions

The previous algorithmic approaches to improve performance have been tested for several options of the bulk production cost model presented when applied to the Spanish electric energy system. Detailed computational experience and conclusions have been shown in this paper.

The results obtained clearly show that interior point methods dramatically decrease the solution time, specially with large problems. CPLEX has been found to be the best solver for both LP and MIP problems. The NLP problems are solved providing the solution of the LP problem as initial point. The use of generalized Benders decomposition is not worthwhile for LP problems when they can be directly solved by an optimization package. However, for very large NLP problems is not possible to find optimization packages powerful enough and advantages can be obtained from the separation between master problem (linear) and subproblems (non linear). In that case, decomposition has to be used.

In the end, a problem that some time ago could be thought unsolvable can now been efficiently solved in a workstation allowing to face the modeling of new characteristics of the power system.

Referencias

[1] Benders, J.F. "Partitioning Procedures for Solving Mixed-Variable Programming Problems" *Numerische Mathematik*. Vol 4, pp 238-252. 1962.

[2] Brooke, A., Kendrick, D. and Meeraus, A. *RELEASE 2.25 GAMS A Users Guide*. The Scientific Press. South San Francisco, USA. 1992.

[3] Cazzol, M.V., Garzillo, A., Inmorta, M., Losignore, N. and Marannino, P. "The Solution of Voltage/Reactive Security Problems in VAR Planning and in Operation Scheduling by the Dual Affine Karmarkar Algorithm" *11th Power System Computation Conference*. pp 403-409. August 1993.

[4] Lusting, I.J., Marsten, R.E. and Shanno, D.F. "Interior Point Methods for Linear Programming: Computational State of the Art" *ORSA Journal on Computing*. Vol. 6, No. 1, pp. 1-14. Winter 1994.

[5] Mart'inez-C'orcoles, F. et al. "SEGRE: A Yearly Production Cost Model for Economic Planning" *Power-Gen Europe 95.* pp 295-307. May 1995.

[6] Momoh, J.A., Guo, S.X., Ogbuobiri, E.C. and Adapa, R. "The Quadratic Interior Point Method Solving Power System Optimization Problems" *ORSA Journal on Computing.* Vol. 9, No. 3, pp. 1327-1336. August 1994.

[7] Nemhauser, G.L. "The Age of Optimization: Solving Large-Scale Real-World Problems" *Operations Research.* Vol. 42, No. 1, pp. 5-13. January-February 1994.

[8] Ramos, A. et al. "A Medium Term Bulk Production Cost Model Based on Decomposition Techniques" *Stockholm Power Tech.* pp. 110-116. June 1995.

[9] Savelsbergh, M.W.P. "Preprocessing and Probing Techniques for Mixed Integer Programming Problems" *ORSA Journal on Computing.* Vol. 6, No. 4, pp. 445-454. Fall 1994.

[10] Sherkat, V.R. and Ikura, Y. "Experience with Interior Point Optimization Software for a Fuel Planning Application" *IEEE Transactions on Power Systems.* Vol. 9, No. 2, pp. 833-840. May 1994.

[11] Wu, Y.-Ch., Debs, A.S. and Marsten, R.E. "A Direct Predictor-Corrector Primal-Dual Interior Point Algorithm for Optimal Power Flows" *IEEE Transactions on Power Systems.* Vol. 9, No. 2, pp. 876-883. May 1994.