# MODELLING PROFIT MAXIMISATION IN DEREGULATED POWER MARKETS BY EQUILIBRIUM CONSTRAINTS

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### <u>ABSTRACT</u>

This paper presents a novel conceptual approach to model the new deregulated power markets. It combines powerful traditional tools related to the detailed system operation with techniques for modelling the economic market equilibria.

The proposed approach models the competitive behaviour of the electric firms by incorporating a set of constraints, namely the *Equilibrium Constraints*, into a traditional production cost model. These constraints reproduce the first order optimality conditions of the strategic companies. Thus, the approach achieves a profit maximisation objective while retaining the system operation details.

This model has been implemented in GAMS. An application to the large-scale Spanish electric power system is also presented.

#### 1 INTRODUCTION

The global electric industry is currently experiencing significant changes toward deregulation and competition. Spain is also immersed in deep changes, which have led to a completely new regulatory framework [7] beginning in January, 1998. In this new framework a Market Operator (MO) determines the actual operation of the generating units, based on a simple hour by hour merit order of their bids. The market clearing price is set hourly by the highest accepted bid.

Under the new framework, electric firms assume much more risk, becoming responsible for their own decisions. In particular, they have to estimate their own unit commitment in order to decide, based on costs, prices and quantities, what they will submit to the Market Operator. These bids will decide the actual operation of their units and their incomes. Therefore, utilities need original models that fulfil their new requirements. Such models should not only consider in detail the technical operation constraints still prevailing in the system, but should also represent the new competitive framework.

Recently, several papers have addressed the computation of the market equilibrium<sup>1</sup> in the electric sector. Green and Newbery tackle the issue using a simplified supply function equilibrium approach, see [4]. Borenstein and Bushnell [1] use a simulation model, which heuristically evaluates the market equilibrium under competition. Bushnell [2] extends this simulation model to include inter-period elements. He represents the equilibrium conditions analytically and his model achieves market equilibrium taking into account hydro scheduling decisions, which regard planning resources for multiple periods. Hogan [6] also models the profit maximisation objective of each firm. He does so by using a non-linear optimisation problem that considers the network constraints. Scott and Read [11] have developed a long term model with an emphasis on hydro operation. Most recently, Hobbs [5] utilises the Linear Complementarity Problem (LCP) to model imperfect competition among electricity producers. His model includes a congestion pricing scheme for transmission.

This paper addresses the evolution of a detailed traditional production cost model [9] to a new model that determines both the long term system operation and also the market equilibrium. The market equilibrium is obtained in a single shot optimisation procedure instead of the more commonly used iterative procedures. It models the competitive behaviour of the electric generation energy market by incorporating a set of constraints in a traditional production cost model. These constraints, namely the *Equilibrium Constraints*, reproduce the first order optimality conditions of the companies. Thus, the approach achieves a profit maximisation objective while retaining the system operation details.

This paper is organised as follows. Section 2 presents an overview of the model explaining the meaning of the equilibrium constraints and the way these constraints are incorporated into a traditional production cost model. Section 3 outlines the notation used for the mathematical expressions. Section 4 states the mathematical formulation of the model. Section 5 discusses the implementation. Section 6 describes an application of the model to the Spanish electric energy

<sup>&</sup>lt;sup>1</sup> The market equilibrium is a set of outputs such that no firm, taking its competitors' outputs as given, wishes to change its own output. In other words, each firm's strategy choice is the best response to the strategies actually played by its opponents.

market and finally, section 7 provides the conclusions drawn from the study.

## 2 MODEL OVERVIEW

This paper presents an original way to model the new deregulated power markets. It combines the powerful and well tested existing tools for modelling the detailed operation of thermal and hydroelectric units, with techniques devoted to the modelling of economic market equilibria.

The modelling of the electric system behaviour under the new regulatory framework takes advantage of two relevant characteristics of traditional production cost models. One characteristic is the detailed representation of the electric system operation, which keeps track of different technical and economic constraints affecting both the system operation and the market equilibrium. The second characteristic is that the main decision variables of these models are the generation output levels offered to the market (i.e., quantities). These variables fit with one of the most common approaches to model market equilibriu, the Cournot model [3]. The Cournot market equilibrium is based on quantities rather than prices.

On the other hand significant changes must be introduced into the classical production cost models in order to properly represent the market. The most important change is that the profit maximisation behaviour of each generation company must be considered. This is included in the model via a set of additional constraints, namely the equilibrium constraints. The maximum-profit optimal solution for each company is represented by its Karush-Kuhn-Tucker first order optimality conditions [8]. Keep in mind that the proposed approach maintains the traditional cost minimising objective function.

However, although maximising profits is usually the main objective of a firm, this is not always the case. Therefore, two kinds of firm behaviours are considered in this model. The *strategic* firms maximise their profits using their ability to affect the price, considering both the short-term wholesale market and their long-term contracts. The *leader in quantity* firms try to achieve a minimum market share goal.

Another relevant change is the introduction of the demand response to the energy price. In classic production cost models the demand is inelastic and has to be met. In the new model, the electricity demand is represented by a linear function of the system marginal price, modelling the consumers' marginal utility function. The traditional non-served demand costs included in the objective function represent consumer's costs.

In summary, the model minimises the sum of the producers' and consumers' costs, properly reproducing the market clearing output and price under a perfect competitive market. In perfect competition, both the generation and the demand bid their variable cost and marginal utility, respectively. Note that minimising the sum of producers' and consumers' costs (the common area below the generation variable cost and demand functions) is equivalent to maximising the net social welfare. The net social welfare is defined as the sum of the consumers' and producers' surplus (the common area on the left of the generation variable cost and demand functions). It is the equilibrium constraints which model the strategic behaviour of the companies.

The way in which the equilibrium constraints work within the traditional cost minimising framework is discussed later on in subsection 2.3 of this paper, and a more detailed description with an emphasis on the economical analysis can be found in [10].

## 2.1 System Model

This model considers a scope of one year, divided into *periods*, *subperiods* and *load levels*. Typically, periods will correspond to months, subperiods to working days and weekends of a month, and load levels to peak, plateau and off-peak hours.

The generation system is modelled in detail. Both thermal and hydro plants are represented, taking into account the specific characteristics of the Spanish system.

Thermal units are divided into two blocks, with the minimum load block first. A straight line with independent and linear terms specifies the heat rate. Random outages are deterministically modelled by derating the unit's full capacity by its equivalent forced outage rate. A physical plant consists of several thermal units. Mandatory fuel purchases concerning domestic coal are represented by fuel consumption constraints.

Hydro plants are grouped together into several relevant units. Each equivalent unit has a limited energy reservoir. For the scope of the model, two or three reservoirs per each company are enough to represent the hydro generation behaviour. Pumped-storege units are also considered. The pumping economic function includes both transference of energy between different load levels and the alleviation of minimum/maximum load conditions in off-peak/peak hours.

# 2.2 Market Model

The equilibrium constraints incorporate the maximisation of the producer surplus (difference between revenues<sup>2</sup> and costs) of each *strategic* firm into the classical variable cost minimisation problem. Each equilibrium constraint says that at the profit-maximum choice of output, marginal cost must equal marginal revenue for each firm.

An ascending stepwise function represents the firm's marginal cost as a function of its own generation. Each step represents the marginal costs of different committed generating units. As long as the marginal

<sup>&</sup>lt;sup>2</sup> In the Spanish electricity industry, generation companies receive incomes from the energy market and also from stranded costs, which can modify the firms' strategies. Stranded costs payments, as well as the revenues from contracts for differences, increase when system marginal price decreases. Such kinds of additional features can also be considered in this approach.

cost of each firm is greater than any marginal cost of each committed unit, it can be expressed as a function of the binary commitment variables.

The market-clearing price, or system marginal price, which sets the marginal revenue, is represented by a linear decreasing function of the electricity demand. However, in order to avoid non-linearities in the objective function ---non-served demand costs--- the price is considered as a descending stepwise function (with the slope of the linear function) where each step is a fictitious demand bid.

#### How the Equilibrium Constraints Work 2.3

It is interesting to analyse the complementary features of the cost minimisation scheme (still respected and present in this model) and the profit maximisation (incorporated implicitly through the Cournot equilibrium constraints).

While the equilibrium constraints impose, for each firm, a global power output in order to maximise their profits, it is the cost minimisation that decides the specific unit commitment. The cost minimisation will look for the cheapest commitment of thermal units and the cheapest scheduling of hydro inflows, exactly as each firm would have done in the case that its total output had been previously set. The solution accounts for all the technical (thermal and hydro) operating constraints modelled, therefore achieving a realistic system dispatch.

Therefore, the proposed approach obtains an economic market equilibrium, which is both technically feasible and very close to the optimum profit of the firms.

#### **NOTATION** 3

In this section all the symbols used in this paper are identified and classified according to their use into indices, sets, parameters and variables. Indices and sets share the same letter using capitals for sets and lowercase for indices.

#### 3.1 Indices and Sets

- number of pumped-storage<sup>3</sup> units. В
- number of thermal plants. С
- number of demand bids. D
- number of firms F
- number of hydro and/or pumped-hydro Η units.
- number of load levels. Ν
- number of periods. Р
- Snumber of subperiods.
- Т number of thermal units.

- 3.2 Parameters
  - hydro inflows expressed in energy for hydro unit h in period p.
  - maximum and minimum capacity of  $\overline{b}_{b}, \underline{b}_{b}$ pumped-storage unit *b* when pumping.
    - $C_{cp}$  mandatory fuel purchase by thermal plant c at the beginning of period p.

power demand at price zero and constant  $d_{nsp}$ ,  $d'_{nsp}$  slope of the demand function in load level n of subperiod s of period p.

- duration of load level n of subperiod s of  $D_{nsp}$ period *p*.
- maximum and minimum capacity of  $\overline{e}_h, \underline{e}_h$ pumped-hydro unit h when pumping.
- maximum and minimum capacity of hydro  $\overline{h}_{\scriptscriptstyle hp}$  ,  $\underline{h}_{\scriptscriptstyle hp}$ unit h in period p.
  - Selfconsumption coefficient of thermal unit k.
  - $L_{fp}$  long term contract for power of firm f in period p.
- $\overline{M}_{fp}$ ,  $\underline{M}_{fp}$  maximum and minimum market share of firm f in period p.
  - $o_t, o'_t$  heat rate (independent and linear terms) of thermal unit *t*.
  - $\overline{p}_{t}, \underline{p}_{t}$  maximum and minimum rated capacity of thermal unit *t*.
    - $q_t$  EFOR of thermal unit t.
    - *R* power reserve margin.
  - $\overline{R}_h, \underline{R}_h$  maximum and minimum hydro energy reserve of hydro unit *h*.
    - $r_t$  start-up cost of thermal unit t.
  - $\overline{S}_c, \underline{S}_c$  maximum and minimum fuel storage capacity of thermal plant c.
  - $\bar{t}_b, \underline{t}_b$  maximum and minimum capacity of pumped-storage unit *b* when generating.
    - $u_t$  O&M variable cost of thermal unit t.
    - $V_b$  upper reservoir limit of pumped-storage unit *b*.
    - $v_t$  fuel cost of thermal unit *t*.
    - W penalty cost by power reserve unmet.
  - price of the demand bid d in load level n of  $W_{dnsp}$ subperiod s of period p.
    - $\eta_b$  performance of pumped-storage unit b.
    - $\eta_h$  performance of pumped-hydro unit h.

#### 3.3 **Decision variables**

- commitment decision of thermal unit t in  $a_{tsp}$ subperiod *s* of period *p*.
- power consumption by pumped-storage unit  $b_{\scriptscriptstyle bnsp}$ *b* in load level *n* of subperiod *s* of period *p*.
- power consumption by pumped-hydro unit  $e_{hnsp}$ *h* in load level *n* of subperiod *s* of period *p*.
- total power generation of firm f in load  $g_{fnsp}$  level *n* of subperiod *s* of period *p*.

<sup>&</sup>lt;sup>3</sup> In this paper the following convention is used. A *pumped-hydro* unit is a pump-turbine, which has a large upper reservoir with seasonal storage capability that receives water from pumping and also from natural hydro inflows. On the other hand, a pumped-storage unit has a small upper reservoir filled with pumped water that allows only a weekly or daily cycle.

- $h_{hnsp}$  power generation by hydro unit *h* in load level *n* of subperiod *s* of period *p*.
- $m_{fsp}$  marginal cost of firm f in subperiod s of period p.
- $n_{dnsp}$  non-accepted power to the demand bid *d* in load level *n* of subperiod *s* of period *p*.
- $p_{tmsp}$  power generation by thermal unit *t* in load level *n* of subperiod *s* of period *p*.
- $R_{hp}$  hydro energy reserve of hydro unit *h* at the beginning of period *p*.
- $S_{cp}$  fuel storage level of thermal plant *c* at the beginning of period *p*.
- $t_{bnsp}$  generation by pumped-storage unit b in load level n of subperiod s of period p.
- $z_{sp}$  margin reserve unmet in subperiod *s* of period *p*.
- $\pi_{nsp}$  system marginal price in load level *n* of subperiod *s* of period *p*.

### 4 MODEL FORMULATION

The model is formulated as a large-scale MIP optimisation problem. The objective function to be minimised is the total variable cost for the scope of the model, including non-served demand costs subject to operating and market constraints. The operating constraints can be classified into inter and intraperiod, according to the spanning periods. The market constraints model the behaviour of the strategic and leader in quantity firms in the electric power market as previously mentioned.

The following sections of this paper are the mathematical formulation of the objective function, the constraints and the variables involved in the problem.

#### 4.1 Objective Function

The objective function represents the sum of fuel costs (including independent and linear terms of the heat rate), O&M variable costs, start-up costs, penalty by reserve margin unmet and costs by non accepted demand bids for all load levels, subperiods and periods within the scope.

$$Min \quad \sum_{p=1}^{P} \sum_{s=1}^{S} \sum_{n=1}^{N} \sum_{t=1}^{T} D_{nsp} v_t \left( o_t a_{tsp} + o'_t \frac{p_{msp}}{k_t} \right) \\ + \sum_{p=1}^{P} \sum_{s=1}^{S} \sum_{n=1}^{N} \sum_{t=1}^{T} D_{nsp} u_t p_{msp} \\ + \sum_{p=2}^{P} \sum_{t=1}^{T} r_t \left( a_{t1p} - a_{tSp-1} \right) \\ + \sum_{p=1}^{P} \sum_{s=1}^{S} W z_{sp} \\ + \sum_{p=1}^{P} \sum_{s=1}^{S} \sum_{n=1}^{N} \sum_{d=1}^{D} D_{nsp} w_{dnsp} n_{dnsp}$$

$$(1)$$

### 4.2 Interperiod Operating Constraints

The interperiod operating constraints are those that regard resources planning for multiple periods. In particular, yearly hydro energy scheduling, seasonal operation of pumped-hydro units and fuel scheduling are represented.

1. Fuel scheduling.

For each thermal plant, the stock level at the beginning of each period is a function of the previous stock, the purchase and consumption done during the period. The user specifies the initial and final stock levels. It represents takeor-pay contracts and must-buy fuel purchases mandated by socio-economic and political considerations for domestic coal plants, although their cost can be more expensive than other available fuels.

$$\sum_{s=I}^{S} \sum_{n=I}^{N} \sum_{t \in C} D_{nsp} \left( o_t a_{tsp} + o'_t \frac{p_{msp}}{k_t} \right)$$

$$\geq C_{cp} + S_{cp} - S_{cp+I}$$

$$(2)$$

2. Hydro scheduling.

For each hydro unit, the reservoir level at the beginning of each period is a function of the previous level, the hydro inflow and the amount of pumping and generation during that period. The user specifies the initial and final reservoir levels.

$$\sum_{s=1}^{S} \sum_{n=l}^{N} D_{nsp} \left( h_{hnsp} - \rho_h e_{hnsp} \right)$$

$$\leq A_{hp} + R_{hp} - R_{hp+l}$$
(3)

## 4.3 Intraperiod Operating Constraints

These constraints are internal to each period and represent balance between generation and demand, thermal generation constraints, weekly/daily operation of pumped-storage units and security constraints based on reserve margin.

1. Balance generation-demand.

This set of constraints provides balance between generation and demand for any load level including non accepted demand bids.

$$\sum_{t=1}^{T} p_{msp} + \sum_{h=1}^{H} \left( h_{hnsp} - e_{hnsp} \right)$$

$$+ \sum_{b=1}^{B} \left( t_{bnsp} - b_{bnsp} \right) + \sum_{d=1}^{D} n_{dnsp} \ge d_{nsp}$$
(4)

2. Thermal generation constraints.

For each committed thermal unit the maximum generation is less than the maximum available capacity, and the minimum generation is greater than the minimum stable load.

$$\underline{p}_{t}k_{t}(1-q_{t})a_{tsp} \leq p_{tnsp}$$

$$(5)$$

$$p_{tnsp} \le \overline{p}_t k_t (l - q_t) a_{tsp} \tag{6}$$

3. Pumped-storage units.

Balance between pumped and generated energy in a period is imposed onto pumped-storage units, and a reservoir limit is imposed onto the pumped energy.

$$\sum_{s=1}^{S} \sum_{n=1}^{N} D_{nsp} \left( t_{bnsp} - \rho_b b_{bnsp} \right) = 0$$
 (7)

$$\sum_{s=l}^{S} \sum_{n=l}^{N} D_{nsp} \rho_b b_{bnsp} \le V_b \tag{8}$$

4. Reserve margin.

A capacity reserve margin for the peak load level of each subperiod must be met. This constraint represents the necessity to provide some amount of power available to account for increments in demand or failures of committed generation units.

$$\sum_{t=1}^{T} \overline{p}_{t} k_{t} a_{tsp} + \sum_{h=1}^{H} \overline{h}_{hp} + \sum_{b=1}^{B} \overline{t}_{b} + z_{sp}$$

$$\geq (I+R) \cdot \left( d_{1sp} - \sum_{d=1}^{D} n_{d1sp} \right)$$
(9)

## 4.4 Market Constraints

These constraints are internal to each period and they model the market behaviour. In a market model, the price equation that relates the price with the demand is fundamental. The equilibrium constraints model the strategic behaviour while the market share constraints model the leader in quantity behaviour.

While in traditional cost models the basic element is the generation unit, in market models the basic element is the company. Therefore, aggregated firm values are both required and meaningful. Auxiliary constraints are needed in order to obtain new variables, such as marginal cost and total output of each firm.

1. Price equation.

Classic production cost models compute the system marginal price as the dual variable of the balance between generation and demand. However, the equilibrium constraints need the price as a primal variable. The system marginal price is represented by a linear function of the electricity demand so that the model considers the demand response to the price.

$$\pi_{nsp} = d'_{nsp} \sum_{d=1}^{D} n_{dnsp}$$
(10)

### 2. Firm's marginal cost.

As long as the marginal cost of each firm f is greater than any variable cost of a committed unit, it and can be expressed as a function of the binary commitment variables.

$$m_{fsp} \ge \left(\frac{v_{t}o'_{t}}{k_{t}}\right) a_{tsp} \quad \forall t \in f$$
(11)

#### 3. Firm's power generation equation.

The total power generation of the firm f is explicitly computed in order to simplify the equilibrium constraints formulation.

$$g_{fnsp} = \sum_{t=1}^{T} p_{msp} + \sum_{h=1}^{H} (h_{hnsp} - e_{hnsp})$$

$$+ \sum_{b=1}^{B} (t_{bnsp} - b_{bnsp}) \quad \forall t, h, b \in f$$
(12)

### 4. Equilibrium constraints.

The equilibrium constraints model the behaviour of the strategic companies by reproducing their Karush-Kuhn-Tucker first order optimality conditions. Each first order condition reflects that the maximum-profit quantity choice is achieved when the marginal revenue is equal to the marginal cost.

By explicitly solving for the generation output, this constraint can also express the maximum generation of each strategic firm that maximises its profits. It is a function of the price, the firm's marginal cost and the slope of the demand curve.

$$g_{fnsp} \le \frac{\pi_{nsp} - m_{fsp}}{d'_{nsp}} + L_{fp}$$
(13)

Note that several Lagrangian terms associated with the operating constraints have been neglected, because these constraints will be addressed by the production cost model. Therefore, the equal sign of the optimality conditions is replaced by a less than or equal to sign.

5. Market share constraints.

The leader in quantity companies look for a minimum market share. These constraints model this goal. In the electricity industry the market share of each firm changes between peak, plateau and off-peak hours. Therefore, because the market share goal can not be fixed for each load level, it is established period by period.

$$\sum_{s=l}^{S} \sum_{n=l}^{N} D_{nsp} g_{fnsp} \ge \underline{M}_{fp} \left( d_{nsp} - \sum_{d=l}^{D} n_{dnsp} \right)$$
(14)

$$\sum_{s=1}^{S} \sum_{n=1}^{N} D_{nsp} g_{fnsp} \le \overline{M}_{fp} \left( d_{nsp} - \sum_{d=1}^{D} n_{dnsp} \right)$$
(15)

#### 4.5 Variable Bounds

The vast majority of the variables involved in the previous formulation are subject to the following bounds:

- 1. Commitment decision of thermal units,  $a_{isp} = \{0, l\}$
- 2. Power consumption of pumped-storage units,  $\underline{b}_b \leq b_{bnsp} \leq \overline{b}_b$
- 3. Power consumption of pumped-hydro units,  $\underline{e}_h \le e_{hnsp} \le \overline{e}_h$
- 4. Power generation of hydro units,  $\underline{h}_{hp} \leq \underline{h}_{hnp} \leq \overline{h}_{hp}$
- 5. Generation by pumped-storage units,  $\underline{t}_b \leq t_{bnsp} \leq \overline{t}_b$
- 6. Non served power to the demand bids,  $W_{dnsp}$

$$0 \le n_{dnsp} \le \frac{dnsp}{d'_{nsp}}$$

- 7. Power generation of thermal units.  $0 \le p_{msp} \le \overline{p}_t k_t (1 - q_t)$
- 8. Hydro energy reserves,  $\underline{R}_h \leq R_{hp} \leq \overline{R}_h$
- 9. Fuel stock levels,  $\underline{S}_{c} \leq S_{cp} \leq \overline{S}_{c}$

### 5 **IMPLEMENTATION**

The resulting optimisation problem is a mixed integer programming (MIP) problem, which is difficult to solve for a large-scale electric energy system. The model has been implemented in GAMS version 2.50, a mathematical specification language specially suited for the solution of optimisation problems, and solved by CPLEX or OSL, well-known solvers.

When solving a large-scale optimisation problem, careful attention should be paid to the scalation of constraints and variables in order to keep them around one. Constraints and variables have been scaled accordingly. GW is used as the natural unit for power, TWh for energy and kTcal for heat consumption.

The main market equilibrium results obtained from the model are the system marginal price and revenues, the operating costs and the power and energy production for each firm. Other significant results presented from the production cost model are power, energy production, utilisation, commitment and shutdown hours, operating costs for each generating unit and fuel consumption of thermal plants for each plant. These results can be grouped together for each load level, period and company.



This market model has been designed to represent the yearly operation of a large-scale electric power system and it has been applied to the Spanish electricity market. The scope of the model has been split into 12 periods (months) with 2 subperiods each (working days and weekends) and 5 and 4 load levels respectively per each subperiod.

There are 4 main firms competing in the Spanish power market. The approximate market share in energy is as follows: ENDESA 50 %, IBERDROLA 30 %, UEFSA 15 % and HC 5 %.

Firms	Thermal Power	Hydro Power
ENDESA	14614 MW	5803 MW
IBERDROLA	7842 MW	8228 MW
UEFSA	3494 MW	1693 MW
HC	1292 MW	408 MW

The system met a maximum peak load of 27219 MW and a yearly energy demand of 162204 GWh. The installed generation capacity is 43374 MW (16132 MW are hydro and 27242 MW thermal) and the average hydro energy available is 26553 GWh.

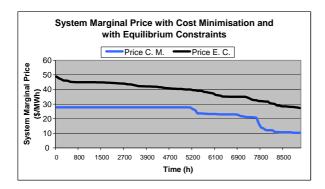
There are 73 thermal generators grouped into 43 thermal plants. The hydro units have been grouped into 20 equivalent units. Finally, there are 10 pumped-storage units.

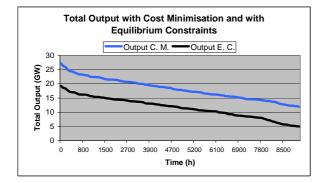
Units	Number	Power
Nuclear Units	9	7622 MW
Coal Units	37	11209 MW
Oil Units	15	4597 MW
Gas Units	12	3814 MW
Hydro Units	20	13731 MW
Pumped Units	10	2401MW

The size of the MIP in this case study is 25198 continuous variables, 1752 binary commitment variables and 33359 constraints. A workstation Sun Ultra 1 160 MHz spends 984 seconds to solve this problem.

The model has been run with and without equilibrium constraints. The slope of the demand curve<sup>4</sup> is 4 \$/MWh/GW. When run without equilibrium constraints, the model represents the former regulatory framework based on cost minimisation. When run with equilibrium constraints, it corresponds to the firm's profit maximisation objective. As can be observed from the figures, in the first case the average SMP is 25.3 \$/MWh and the SMP is around 27.7 \$/MWh for peak hours. However, in the second case, the generation is retracted to increase the SMP and therefore the producer surplus also increases. In the second case, the demand met is about 70 % of the original demand. The resulting SMP reaches an average value of 41.1 \$/MWh and 48.8 \$/MWh for peak hours.

<sup>&</sup>lt;sup>4</sup> The prices predicted by Cournot-based models are very sensitive to variations in the demand curve specification. For this reason, this model is more useful in comparing different strategic behaviours rather than in forecasting the absolute level of the market prices.





## 7 CONCLUSION

We have developed a novel and practical approach based on Mathematical Programming with Equilibrium Constraints for obtaining the electric power market equilibrium.

This paper addresses the addition of a set of equilibrium constraints to a traditional production cost model, in order to simultaneously determine both the long term system operation and the market equilibrium. The result is a model, which is able to maximise the producer surplus of each firm for a large power system, taking into account all kinds of operation constraints and also the multiple hydro reservoir interperiod scheduling.

This model is a useful sensitivity tool for economic planning in deregulated power markets. It can be used to study system marginal prices and the different market equilibria achieved under various demand characterisations and assumptions about the firm's behaviour.

The results obtained by this annual model may be used to provide clues for analysing and designing optimal bidding strategies in both the long term and the short term. In the long term, it is essential to find specific quantities, depending on the behaviour of market competitors, in order to maximise the profit of individual firms. In the short term, resultant prices suggest at what prices firms are willing to sell their energy for each load level. Therefore, these forecasted prices can be helpful in defining bidding tactics in the short term wholesale market.

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