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Modeling Inflow Uncertainty in Electricity Markets: A Stochastic MCP Approach

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Abstract: This paper proposes a new approach for addressing the long term hydrothermal coordination of a generation company operating in a competitive market, fully adapted to represent hydro-scheduling in an inflow uncertainty context. The proposed approach employs the traditional Stochastic Dynamic Programming (SDP) methodology. Since the modeling of the market behavior in these new conditions hardly fits with the traditional cost minimization scheme, the sub-models at each SDP stage are stated as Mixed Complementarity Problems (MCP) in order to properly represent the electricity market equilibrium. Each MCP sub-model sets up the electricity generation market equilibrium by formulating the equations that express the optimal behavior of the generation companies, considering both stochastic hydraulic inflows and technical constraints that affect the scheduling of their units. The hydrothermal coordination stochastic model has been developed and implemented in GAMS. A case study is also presented to show its successful application to a large-scale electric power system such as the Spanish one.

Keywords: Stochastic hydrothermal generation scheduling, Dynamic programming, Market equilibrium, MCP.

I. INTRODUCTION

In deregulated power markets, electricity generation becomes an unbundled and liberalized activity in which both expansion and operation decisions no longer depend on administrative and centralized procedures –usually based on cost minimization schemes–, but rather on the managerial decisions of the generation companies looking to maximizing their own profits. In this competitive market context, there are increased opportunities, and where there are increased opportunities, there are also increased risks. Generation companies must assume not only all the standard functions related to the operation planning of their power plants (start-ups, shut-downs, hydro-thermal coordination, maintenance, etc.) but also new functions associated to free market competition (bidding, hedging risk, etc.). Consequently, companies need specific and accurate decision support tools that fulfil these new market requirements.

As long as the market decides the actual operation of generation units, an important developmental effort is needed to design new models and tools that properly represent the electricity market equilibrium. Consequently, the optimal energy scheduling of each company will depend not only on companies' production cost profiles and companies' technical operation constraints but also on the market behavior resulting from the interaction of all market participants.

This paper addresses the long term hydrothermal coordination problem and proposes a methodology to incorporate the modeling of the market equilibrium. In order to properly tackle the long term operation planning, the source of uncertainty due to the stochastic nature of hydraulic inflows must be considered. This is crucial in power systems with a high level of hydro component, such as the Spanish one.

The inflow uncertainty influence on the power system operation has been widely studied, and a great number of probabilistic models have been developed. In the new competitive context, this uncertainty highly conditions the energy management decisions of those firms owning significant hydro resources. These firms must robustly manage their hydro resources to hedge against the risk arising from uncertain expected profit due to the variability in hydraulic inflows among different periods.

The interest of the research community regarding the development of hydrothermal scheduling models adapted to the new circumstances has grown and has been demonstrated as such in numerous publications [6]. Scott and Read [10] have developed a medium term model with emphasis on hydro operation, focused on the New Zealand conditions. The model utilizes dual dynamic programming, where at each stage the hydro optimization problem is superimposed on a Cournot market equilibrium. Bushnell [2] has developed a model that evaluates the California market equilibrium under competition. His model achieves market equilibrium taking into account hydro scheduling decisions. Although modeling advances have been notable, limitations persist in tackling realistic power systems and accurately modeling the stochastic inflows nature.

This paper proposes a new model for addressing the long term hydrothermal coordination of a generation company operating in a competitive market, fully adapted to represent hydro-scheduling in an inflow uncertainty context. The proposed approach employs the traditional Stochastic Dynamic Programming [3] methodology that was implemented in GAMS. As long as the traditional cost minimization scheme does not properly model the market behavior, the sub-models at each stage are stated as Mixed Complementarity Problems [8][5]. Each MCP sub-model sets up the electricity generation market equilibrium by formulating the equations that express the optimal behavior of the generation companies, considering both stochastic hydraulic inflows and technical constraints that affect the energy scheduling among periods. The structure of each MCP sub-model allows the use of special resolution methodologies incorporated nowadays in commercial software packages [1], such as MILES and PATH solvers within GAMS [9].

This paper is organized as follows: Section 2 provides an overview of the method proposed for the design of a probabilistic hydrothermal coordination model in a competitive environment. Section 3 outlines the notation used for the mathematical expressions. Sections 4 and 5 state the detailed mathematical formulation of the model. Section 6 discusses the SDP implementation. Section 7 describes an application of the model to the Spanish electric energy market and finally, section 8 provides the conclusions drawn from the study.

II. MODEL OVERVIEW

As long as market mechanisms decide the actual operation of the generation units, the market equilibrium[1] should be considered in order to properly model the optimal energy scheduling of each specific company. In a competitive framework each firm looks toward maximizing its own profit, considering other firms' behavior. The simultaneous consideration of the profit maximization objective of each company, that is to say, the calculation of the market equilibrium, constitutes the newest and most complicated issue of the electricity market models.

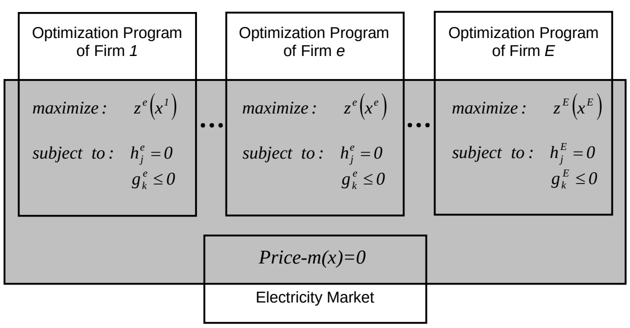

Fig. 1. Market Equilibrium.

The modeling of the market behavior in these new conditions hardly fits with the traditional cost minimization scheme, although there are models that give reasonable approximations based on this kind of approach [11][7]. Conceptually, the new structure better corresponds to various simultaneous optimizations –for each one of the participating companies, the maximization of its expected profit subject to stochastic inflows and its particular technical constraints– linked among themselves by the market price resulting from the interaction of all of them. This scheme is shown in Fig. 1, where z represents the expected operation profit (market revenues minus production costs plus expected future profit) of each company $e \in [1,...,E]$, x the decision variables and the set of constraints h and g are particularized for each company. The electricity market is modeled by the demand function that relates the supplied demand to the electricity price.

MCP methodology is appropriate for solving this kind of optimization problem as is described in detail [8]. Subsection C below summarizes the guidelines of this approach.

A. Modeling uncertainty

As previously mentioned, the market equilibrium model must consider the hydraulic inflow uncertainty. In this paper we assume that natural inflows random occurrences may be represented by a finite set of scenarios. Therefore, the inflow stochasticity can be modeled by means of a scenario tree[2]. Each branch represents a different realization of hydro inflows with its associated probability (see Fig. 2).

In this framework, in firms' hydro scheduling it is necessary to decide how much water to release in each period, considering the current hydro reserve level and the expected future profit over the inflows scenarios.

B. The stochastic dynamic programming algorithm

Given the large scale of the stochastic problem, we have chosen the traditional Stochastic Dynamic Programming methodology in order to decompose the whole market equilibrium problem in separable sub-models. The entire scope is divided into periods (stages of the SDP). The sub-model at each SDP stage is stated as a Mixed Complementarity Problem, which sets up the electricity generation market equilibrium.

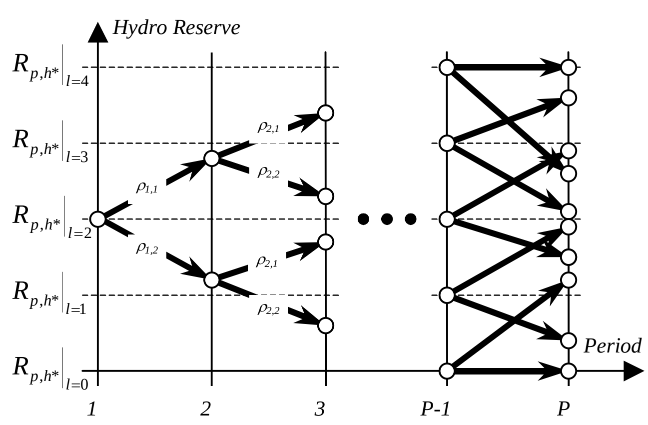

Fig. 2. Scenario tree within dynamic programming structure.

[1] The market equilibrium defines a set of outputs such that no firm, taking its competitors' outputs as given, wishes to change its own output unilaterally. In other words, each firm's strategic choice is the best response to the strategies actually played by its opponents.

[2] The tree intrinsically considers the nonanticipativity nature of the hydrothermal scheduling decisions.

The SDP algorithm goes backward from the last stage or period to the first one. Within each stage, the expected future profit curve is settled by means of solving the market equilibrium for each hydro reserve level –state variable–. Afterwards, the expected future profit function is approximated as a piecewise linear function.

C. Market equilibrium sub-model

As previously mentioned, the sub-models at each SDP stage are Mixed Complementarity Problems which represent the electricity expected market equilibrium. This expected market equilibrium is achieved when each firm maximizes its own expected profit considering its expected future profit.

The equations that define the expected market equilibrium at each stage, schematically represented in Fig. 2, are the first order Karush-Kuhn-Tucker optimality conditions of associated to the set of maximization programs represented in Fig. 1. In this figure, $\mathcal{L}$ represents the Lagrangian function of the corresponding optimization problem and λ and μ represent the dual variables associated to the set of h and g constraints respectively. The optimality conditions can be written down as three sets of equations. The first one states the gradient of the Lagrangian function with respect to the decision variables x. The second set (the gradient of the Lagrangian function with respect to the dual variables λ) coincides with the h equality constraints themselves. The third one is formed by the complementary slackness conditions associated to the inequality constraints g.

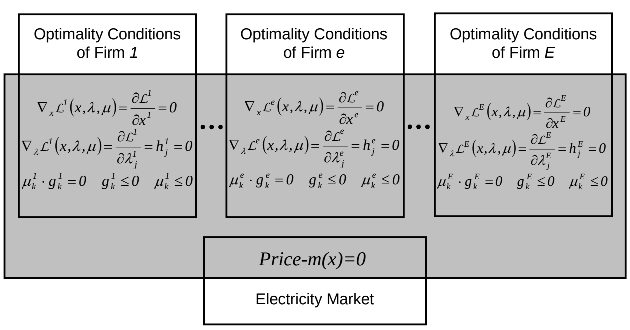

Fig. 3. Market equilibrium as a mixed complementarity problem.

When grouping together all companies' first order optimality conditions, a mixed complementarity problem is formed [4]. It can be solved directly taking advantage of its structure, which allows the use of special complementarity methods implemented in commercial software such as MILES and PATH solvers.

Section 4 states in detail the optimization problem that defines the operation of each firm within a period (Fig. 1). Section 5 analytically derives the associated Karush-Kuhn-Tucker (KKT) optimality conditions, which form the mentioned system of non-linear equations to be solved (Fig. 3).

D. Model assumptions

This hydrothermal coordination model considers a hyperannual scope divided into periods and load levels. Each SDP stage represents one sole period, which usually coincides with a month, while the grouping of the peak, plateau, and off-peak hours makes up the load levels.

For hydro plants, it is necessary to take into account both the rated power output and the reservoir limited capacity. A single seasonal hydro plant is considered, with a large reservoir that has seasonal storage capability[3]. The rest of hydro plants do not have enough storage capability, thus all the hydraulic inflows within a period must be used or spilled. The performance in the pumping and generation cycle is also considered for pumped units.

In order to provide an accurate hyperannual hydrothermal coordination modeling, a detailed representation of the thermal sub-system is regarded. For each thermal unit the rated power output and the quadratic fuel consumption are specified. Each type of fuel is characterized by its price. Fuel purchases constraints are also considered.

Traditional operation models consider a fixed demand for each load level. Nevertheless, in new operation models the representation of the demand should model the reaction of the quantity demanded to changes in price. In the proposed model the total demand at each load level is a linear function of the market price.

The previous model assumptions together with h and g being linear constraints make each MCP sub-model a mixed linear complementarity problem[4], which assures solution existence and uniqueness if the thermal units' marginal costs are strictly monotone increasing [12].

A convex expected future profit curve is guaranteed when only one firm owns seasonal hydro resources. This also guarantees that the SDP approach achieves the whole market equilibrium solution [3].

III. NOTATION

In this section all the symbols used in this paper are identified and classified according to their use into indices, parameters, variables and dual variables.

Table 1. Indices.

Index	Description
c	Thermal plants.
e	Firms.

[3] In a competitive market, firms do not have enough information to model properly their competitors' seasonal hydro plants. Therefore, the firm using this model is also the firm that is able to consider its seasonal hydro plants.

[4] The Mixed LCP is a mixture of a LCP and a system of linear equations.

Index	Description
e^*	Firm with seasonal pumped-hydro unit.
h	Hydro and/or pumped-hydro[5] units.
h^*	Seasonal hydro and/or pumped-hydro unit.
n	Load levels.
l	Energy reserve level of seasonal pumped-hydro unit.
p	Periods or stages.
s	Hydro scenario
t	Thermal units.

Table 2. Parameters.

Parameter	Description
$A_{p,h}$	Hydro inflows for hydro unit h in period p [TWh].
$A_{p,h^*,s}$	Hydro inflows for hydro unit h^* in period p in scenario s [TWh].
$\bar{b}_h, \underline{b}_h$	Maximum and minimum capacity of pumped-hydro unit h when pumping [GW].
$C_{p,c}$	Mandatory fuel purchase by thermal plant c at the beginning of period p [TWh].
$d_{n,p}, d'_{n,p}$	Power demand at price zero [GW] and constant slope of the demand function in load level n of period p [(k\$/TWh)/GW]
$D_{n,p}$	Duration of load level n of period p [kh].
$FP_{p,e^*}\big\|_l$	Future profit of firm e^* in period p at reserve level l [k\$].
$FP'_{p,e^*}\big\|_l$	Partial derivate of future profit respect to energy reserve of firm e^* in period p at reserve level l [k\$/TWh].
$\bar{h}_{p,h}, \underline{h}_{p,h}$	Maximum and minimum capacity of (seasonal) pumped-hydro unit h (h^*) in period p [GW].
k_t	Self-consumption coefficient of thermal unit t [p.u.].
$L_{p,e}$	Long term contract for power of firm e in period p [GW].
o'_t, o''_t	Heat rate (linear [kTcal/TWh] and quadratic [kTcal/(GW^2·kh)] terms) of thermal unit t.
$\bar{p}_t, \underline{p}_t$	Maximum and minimum rated capacity of thermal unit t [GW].
q_t	EFOR of thermal unit t [p.u.].
$\bar{R}_h$	Maximum hydro energy reserve of pumped-hydro unit h [TWh].
$\bar{R}_{p,h^*,s}; \underline{R}_{p,h^*,s}$	Maximum and minimum hydro energy reserve of hydro unit h^* in period p in scenario s [TWh].
$R_{p,h^*}\big\|_l$	Hydro energy reserve of hydro unit h at reserve level l [TWh].
u_t	O&M variable cost of thermal unit t [k\$/GW].
v_t	Fuel cost of thermal unit t [k\$/kTcal].
$\rho_{p,s}$	Probability of inflow scenario s in period p.
η_h	Performance of pumped-hydro unit h [p.u.].

[5] In this paper the following convention is used. A *seasonal pumped-hydro* unit is a pump-turbine, which has a large upper reservoir with seasonal storage capability that receives water from pumping and also from natural hydro inflows. On the other hand, a *pumped-hydro* unit has a small upper reservoir that allows only a weekly or daily cycle.

Table 3. Decision variables.

Variable	Description
$b_{n,p,h}$	Power consumption by (seasonal) pumped-hydro unit h (h^*) in load level n of period p [GW].
$FP_{p,e,s}$	Future Profit of firm e in period p in scenario s [k\$].
$h_{n,p,h}$	Power generation by (seasonal) pumped-hydro unit h (h^*) in load level n of period p [GW].
$p_{n,p,t}$	Power generation by thermal unit t in load level n of period p [GW].
$Profit_{p,e}$	Operation profit of firm e in period p [GW].
$R_{p,h^*,s}$	Hydro energy reserve of hydro unit h^* at the end of period p in scenario s [TWh].

Table 4. Auxiliary variables.

Variable	Description
$g_{n,p,e}$	Total power generation of firm e in load level n of period p [GW].
$\pi_{n,p}$	System marginal price in load level n of period p [k\$/TWh].

Table 5. Dual variables.

Dual variables	Associate constraint
$\mu^R_{p,h}$	Hydro energy reserve scheduling of hydro unit h in period p [k\$/TWh].
$\mu^R_{p,h^*,s}$	Hydro energy reserve scheduling of hydro unit h^* in period p in scenario s [k\$/TWh].
$\mu^{FP}_{p,e^*,l,s}$	Piecewise linear segment l of the future profit of firm e^* in period p in scenario s.
$\mu^S_{p,c}$	Fuel storage scheduling of thermal plant c in period p [k\$/kTcal].
$\mu^{\bar{R}}_{p,h}$	Maximum hydro energy reserve of pumped-hydro unit h in period p [k\$/TWh].
$\mu^{\underline{R}}_{p,h^*,s}; \mu^{\bar{R}}_{p,h^*,s}$	Maximum and minimum hydro energy reserve of hydro unit h^* in period p in scenario s [k\$/TWh].
$\mu^{\underline{p}}_{n,p,t}; \mu^{\bar{p}}_{n,p,t}$	Maximum and minimum rated capacity of thermal unit t in load level n of period p [k\$/GW].
$\mu^{\underline{h}}_{n,p,h}; \mu^{\bar{h}}_{n,p,h}$	Maximum and minimum capacity of (seasonal) pumped-hydro unit h (h^*) in load level n of period p [k\$/GW].
$\mu^{\underline{b}}_{n,p,h}; \mu^{\bar{b}}_{n,p,h}$	Maximum and minimum capacity of (seasonal) pumped-hydro unit h (h^*) in load level n of period p when pumping [k\$/GW].

IV. STOCHASTIC HYDROTHERMAL SCHEDULING PROBLEM FOR EACH GENERATION COMPANY

This section states the detailed mathematical formulation of the optimization problem that defines the generation companies' behavior within SDP stage p. Their goals are to maximize their own expected profits (market revenues minus production costs plus expected future profit) subject to the set of constraints that limit their operation decisions.

The proposed methodology can be directly applied and extended to any type of constraint considered relevant. However, only the critical constraints, in the sense of the scope considered (long term), have been included in this article.

A. Objective function

The objective of each generation firm consists of the maximization of its expected profits within each SDP stage. The expected profit is equal to the obtained market revenue minus the incurred costs plus expected future profit, which is related to the inflow scenario and the hydro energy level at the end of the stage.

Maximize:

$$\begin{aligned} Profit_{p,e} = & \sum_n D_{n,p} \cdot \pi_{n,p} \cdot g_{n,p,e} \\ & - \sum_n \sum_{t \in e} D_{n,p} \cdot u_t \cdot p_{n,p,t} \\ & - \sum_n \sum_{t \in e} D_{n,p} \cdot v_t \left(o'_t \frac{p_{n,p,t}}{k_t} + o''_t \left(\frac{p_{n,p,t}}{k_t} \right)^2 \right) \\ & + \sum_s \rho_{p,s} \cdot FP_{p,e,s} \end{aligned} \qquad \forall e \qquad (1)$$

B. Inter-stage constraints

The inter-stage constraints are those that regard resource planning for multiple periods. In particular, yearly hydro energy scheduling of seasonal pumped-hydro units and the expected future profit as a function of the hydro energy reserve at the end of the stage are represented.

Stochastic hydro scheduling

For each scenario *s*, the hydro inflows and the initial and final levels of the period limit the energy available during each period. The initial level of the period is fixed as given by the hydro reserve state loop of SDP algorithm.

$$\begin{aligned} & \sum_n D_{n,p} \left(h_{n,p,h^*} - \eta_h \cdot b_{n,p,h^*} \right) \\ & \le A_{p,h^*,s} + R_{p-1,h^*}\big|_l - R_{p,h^*,s} \end{aligned} \qquad : \mu^R_{p,h^*,s} \;\; \forall\, h^* \in e^*, e^*, s \qquad (2)$$

Future profit constraints

The future profit at each stage is represented by the following piecewise linear function, which is an outer approximation of the future profit (see Fig. 4).

$$\begin{aligned} & FP'_{p,e^*}\big|_l \cdot \left(R_{p,h^*,s} - R_{p,h^*}\big|_l \right) \\ & \ge FP_{p,e^*,s} - FP_{p,e^*}\big|_l \end{aligned} \qquad : \mu^{FP}_{p,e^*,l,s} \qquad \forall\, e^*, l, s \qquad (3)$$

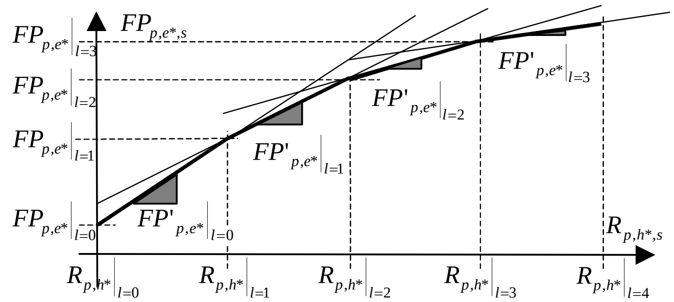

Fig. 4. Piecewise linear approximation of the future profit.

C. Intra-stage constraints

These constraints are internal at each stage and represent weekly/daily scheduling of pumped-hydro units, fuel scheduling and upper and lower limits of each generation unit.

Hydro scheduling

The first constraint establishes a balance among natural inflows, pumped and generated energy, while the second limits the energy that can be pumped in each period.

$$\sum_n D_{n,p} \left(h_{n,p,h} - \eta_h \cdot b_{n,p,h} \right) \le A_{p,h} \qquad : \mu^R_{p,h} \qquad \forall\, h \in e, e \qquad (4)$$

$$\sum_n D_{n,p} \cdot \eta_h \cdot b_{n,p,h} \le \overline{R}_h \qquad : \mu^{\overline{R}}_{p,h} \qquad \forall\, h \in e, e \qquad (5)$$

Fuel scheduling

This constraint models take-or-pay fuel purchasing contracts. It also allows for the enforcing of domestic fuel consumption for strategic energy policy or socioeconomic reasons.

$$\sum_n \sum_{t \in C} D_{n,p}\, p_{n,p,t} \ge C_{p,c} \qquad : \mu^S_{p,c} \qquad \forall\, c \in e, e \qquad (6)$$

Variable bounds

The vast majority of the variables involved in the previous formulation are subject to the following bounds:

$$\underline{R}_{p,h} \le R_{p,h,s} \le \overline{R}_{p,h} \qquad : \mu^{\underline{R}}_{p,h^*,s} ; \mu^{\overline{R}}_{p,h^*,s} \qquad \forall\, h \in e, e, s \qquad (7)$$

$$\underline{p}_t \le p_{n,p,t} \le \overline{p}_t \qquad : \mu^{\underline{p}}_{n,p,t} ; \mu^{\overline{p}}_{n,p,t} \qquad \forall\, n, t \in e, e \qquad (8)$$

$$\underline{h}_{p,h} \le h_{n,p,h} \le \overline{h}_{p,h} \qquad : \mu^{\underline{h}}_{n,p,h} ; \mu^{\overline{h}}_{n,p,h} \qquad \forall\, n, h \in e, e \quad (9)$$

$$\underline{b}_{h} \le b_{n,p,h} \le \overline{b}_{h} \qquad : \mu^{\underline{b}}_{n,p,h} ; \mu^{\overline{b}}_{n,p,h} \qquad \forall\, n, h \in e, e \quad (10)$$

D. Auxiliary equations

For the sake of clarity, the following equations have been stated. However, in the implementation the total power generation and the price have been substituted by these expressions:

Total power generation of each firm

The total power generation of each firm represents the output that is sold in the market without considering the long term contract production as it is sold at a fixed price.

$$g_{n,p,e} = \sum_{t \in e} p_{n,p,t} + \sum_{h \in e} h_{n,p,h} - \sum_{h \in e} b_{n,p,h} - L_{p,e} \qquad \forall\, n, e \quad (11)$$

Price equation

The price is represented as a linear function of the total supplied output.

$$\pi_{n,p} = d'_{n,p} \cdot \left(d_{n,p} - \sum_{e} g_{n,p,e} \right) \qquad \forall\, n \quad (12)$$

V. FORMULATION OF THE ELECTRICITY MARKET EQUILIBRIUM PROBLEM OF EACH STAGE

This section states the mathematical formulation of the Karush-Kuhn-Tucker optimality conditions associated to the previous stochastic hydrothermal scheduling problem. When grouping together all companies' systems of equations a mixed LCP is formed, which defines the expected market equilibrium of SDP stage *p*.

A. Optimality conditions

Deriving each Lagrangian function associated to optimization problem of each firm with respect to its decision variables, we obtain the optimality conditions of each firm *e*:

$$\begin{aligned} \frac{\partial \mathcal{L}_{e,p}}{\partial p_{n,p,t}} = & -D_{n,p} \cdot \left(\pi_{n,p} - g_{n,p,e} \cdot d'_{n,p} - u_t \right) \\ & - D_{n,p} \cdot v_t \cdot \left(\frac{o'_t}{k_t} + 2 \cdot \frac{o''_t}{k_t^2} \cdot p_{n,p,t} \right) \\ & + D_{n,p} \cdot \mu^{S}_{p,c} \\ & - \left(\mu^{\overline{p}}_{n,p,t} - \mu^{\underline{p}}_{n,p,t} \right) = 0 \end{aligned} \qquad \forall\, n, t \in e, e \quad (13)$$

$$\begin{aligned} \frac{\partial \mathcal{L}_{e,p}}{\partial h_{n,p,h^*}} = & -D_{n,p} \left(\pi_{n,p} - g_{n,p,e} \cdot d'_{n,p} \right) \\ & - D_{n,p} \sum_{s} \mu^{R}_{p,e^*,s} \\ & - \left(\mu^{\overline{h}}_{n,p,h^*} - \mu^{\underline{h}}_{n,p,h^*} \right) = 0 \end{aligned} \qquad \forall\, n, h^* \in e, e \quad (14)$$

$$\begin{aligned} \frac{\partial \mathcal{L}_{e,p}}{\partial b_{n,p,h^*}} = & D_{n,p} \left(\pi_{n,p} - g_{n,p,e} \cdot d'_{n,p} \right) \\ & + D_{n,p} \sum_{s} \eta_h \cdot \mu^{R}_{p,e^*,s} \\ & - \left(\mu^{\overline{b}}_{n,p,h^*} - \mu^{\underline{b}}_{n,p,h^*} \right) = 0 \end{aligned} \qquad \forall\, n, h^* \in e, e \quad (15)$$

$$\begin{aligned} \frac{\partial \mathcal{L}_{e,p}}{\partial h_{n,p,h}} = & -D_{n,p} \left(\pi_{n,p} - g_{n,p,e} \cdot d'_{n,p} + \mu^{R}_{p,h} \right) \\ & - \left(\mu^{\overline{h}}_{n,p,h} - \mu^{\underline{h}}_{n,p,h} \right) = 0 \end{aligned} \qquad \forall\, n, h \in e, e \quad (16)$$

$$\begin{aligned} \frac{\partial \mathcal{L}_{e,p}}{\partial b_{n,p,h}} = & D_{n,p} \begin{pmatrix} \pi_{n,p} - g_{n,p,e} \cdot d'_{n,p} - \\ + \eta_h \cdot \mu^{R}_{p,h} - \eta_h \cdot \mu^{\overline{R}}_{p,h} \end{pmatrix} \\ & - \left(\mu^{\overline{b}}_{n,p,h} - \mu^{\underline{b}}_{n,p,h} \right) = 0 \end{aligned} \qquad \forall\, n, h \in e, e \quad (17)$$

$$\frac{\partial \mathcal{L}_{e,p}}{\partial FP_{p,e^*,s}} = -\rho_{p,s} - \sum_{l} \mu^{FP}_{p,e^*,l,s} = 0 \qquad \forall\, p, e^*, s \quad (18)$$

$$\begin{aligned} \frac{\partial \mathcal{L}_{e,p}}{\partial R_{p,h^*,s}} = & -\sum_{l} \mu^{FP}_{p,e^*,l,s} \cdot FP'_{p,e^*}\Big|_{l} - \mu^{R}_{p,h^*,s} \\ & - \left(\mu^{\overline{R}}_{p,h^*,s} - \mu^{\underline{R}}_{p,h^*,s} \right) = 0 \end{aligned} \qquad \forall\, h^* \in e, e, s \quad (19)$$

B. Complementarity slackness conditions

There are no equality constraints. Therefore, as was previously established in section 2, in order to complete the set of non-linear equations that define the optimization problem of each company, the inequality constraints (2, 10) multiplied by their corresponding dual variable μ must be added.

C. Water value in power markets

Optimality conditions provide useful information about the role of hydro power plants in firms' optimal energy scheduling policy.

The gradient of the Lagrangian function with respect to the thermal power generation (13) together with the gradient of the Lagrangian function with respect to the hydro power generation (14, 16) show that the hydro scheduling tries to equalize the firm's marginal cost for high load levels within a period. This marginal cost is the dual variable of the hydro scheduling constraint, which defines the water value of each hydro unit. Obviously, this value is different for each firm and depends on the fuel cost structure of the hydro plant owner.

As long as the derivate of the future profit with respect to the reserve level is equal to the next stage water value, the gradient of the Lagrangian function with respect to the hydro energy reserve (19) shows that the hydro scheduling tries to equalize the water value between consecutive periods.

VI. SDP IMPLEMENTATION

The SDP algorithm has been implemented in GAMS version 2.50, a specific mathematical language designed to efficiently create and solve optimization problems. It can be written as follows.

Step 0: ***Solve*** *a deterministic minimisation cost model*

Save *the current value of both decision variables and dual variables to initialise the next model*

Step 1: ***Solve*** *deterministic market equilibrium using MCP without SDP*

Save *the current value of both decision variables and dual variables to initialise all MCP sub-models*

Step 2: ***Do a loop*** *sweeping in stages from the last one to the first one (backward loop in* ***p****)*

Do a loop *sweeping in states of the seasonal hydro unit initial reserve* $R_{p-1,h^*}\big|_l$ *(state loop in* ***l****):*

Solve *the electricity market equilibrium problem of the current stage considering the hydro inflow scenarios and the expected future profit*

Update *the parameters of the future profits piecewise linear approximation for the next stage (****p-1****) to be solved:*

$$FP_{p-1,e^*}\big|_l = Profit_{p,e^*} + \sum_s \rho_{p,,s} \cdot FP_{p,e^*,s}$$

$$FP'_{p-1,e^*}\big|_l = \sum_s \mu^R_{p,h^*,s}$$

End loop *in states*

End loop *in stages*

VII.CASE STUDY

The stochastic model presented in this article has been applied to the Spanish electricity market, in which four firms compete: Endesa (END), Iberdrola (IBD), Union Fenosa (UF) and Hidrocantábrico (HCN). The system met a maximum peak load over 32000 MW and a yearly energy demand of 200000 GWh. Demand is modeled as a linear function of the price with 70 (euro/MWh)/GW slope. The annual scope (2000) of the model has been divided into twelve periods (months) with 5 load levels for each one. Inflow uncertainty within a period is considered by five inflow scenarios (extreme wet, wet, medium, dry and extreme dry) with different associated values and probabilities. The energy associated to the extreme branches of the hydro scenario tree ranges from 10341 GWh to 23971 GWh.

Table 6. Firms' production structure.

Type	Number	Power	IBD	END	UF	HCN
Thermal	82	32107 MW	29 %	54 %	13 %	5 %
Hydro	38	16628 MW	51 %	36 %	3 %	10 %

Fig. 5 shows the water value curves of the Iberdrola's seasonal hydro reservoir obtained from the model for each period. As should be, each water value curve is monotone decreasing and, for the same reservoir level, water value is higher when low inflow periods take place (*i.e.* 7th, 8th and 9th periods).

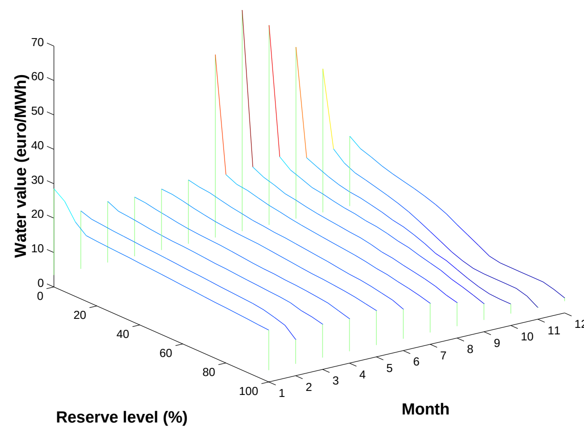

Fig. 5. Water value surface.

The size of the deterministic problem (*Step 1*) is 8691 variables and it is solved in approximately 120 seconds on a PC Pentium-II 233 MHz with 64 MB. Each sub-model's size is 719 variables and its average solving time is about 1 second. Since SDP algorithm sweeps in 12 stages and 20 states, the total time to solve the stochastic problem amounts to 240 seconds.

VIII. CONCLUSION

This paper presents a new approach for the development of stochastic inflows hydrothermal coordination models within the context of deregulated electricity markets. Because of the large scale of the stochastic problem, we have chosen the traditional Stochastic Dynamic Programming methodology in order to decompose the whole market equilibrium problem in separable sub-models. These sub-models are formulated as MCP whose structure allows the use of special resolution methodologies.

Since modeling of both the physical behavior of a power system and the inflow uncertainty are provided while the profit maximization goal of each company participating in the market is considered, the proposed approach allows for sufficient complexity and flexibility to properly represent the hydro generation in a competitive frame.

The mathematical model has been developed and implemented in GAMS taking advantage of the specific solvers designed to deal with complementarity problems. The successful application to the large-scale Spanish market presented in this article shows that the model is capable of representing real electric power systems.

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