Risk assessment in a contract of energy exchanging

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Abstract: This paper presents an alternative for the performing of risk management analysis when the variables of study cannot be considered as complying with the conditions of the Binomial and Black-Scholes models. The method used is based on the calculation of the discrete convolution. The objective is making possible the valuation of an energy export contract under conditions of uncertainty.

Key words: Risk management, interconnection contracts, discrete convolution.

I. INTRODUCTION

This paper presents a method to evaluate the risk associated with the signature of an energy exchange contract between Argentina and Brazil. The conditions under which the contract is to be developed are uncertain, both for hydro conditions and for the energy prices in the two countries.

At present, with electricity markets growing significantly, the complexity both in contracts and in derivatives is notably increasing. In [1] and [2] the more common derivative products in the electricity market are described. In [3] and [4] some of the new contract types currently being applied are also presented.

The type of contract being studied in this paper includes an additional income for the Argentinean producer as a capacity payment, while the producer is committing itself to supply energy at a fixed price whenever the Brazilian system decides it.

Consequently, this can be considered as the signature of a Call option in which the premium is known, as it corresponds to the additional charge per capacity payment.

The valuation of options is normally performed through the Binomial method if their variations are discrete, or through the Black-Scholes if they are considered to be continuous [5].

Once it has been verified that the variables studied do not comply with the hypothesis of these models, the discrete convolution is used as a method to determine the benefit along the scope of the contract.

II. CONTRACT CHARACTERISTICS

This chapter presents a comprehensive description of the energy exchange contract under valuation. First, the main characteristics of the Argentinean system are described, going then to the full characterisation of the contract.

A. Characteristics of the Argentinean system

This system is characterised by the long distance between the generation sites and the consumption centres, as well as a remarkably radial network.

The system works as an electricity spot market, where a system marginal price is calculated in every node. Each producer perceives the system marginal price of the node it is connected to, and has to cover the transmission cost of selling its energy at the so-called *market node*, where the purchase and sale of energy is taking place. This cost includes a fixed term and a variable term, which is the difference between the price of both nodes.

Energy supply is not performed through a bidding procedure, as in other countries, but through a system of audited production costs. In this way, the final supply is obtained trough a central dispatch, in light of existing technical constraints.

Additionally, each producer perceives a capacity payment, although actually this is representing extra income for each MWh produced during the peak hours.

B. Characteristics of the model used

In order to perform the study, a model that determines the spot prices across all the Argentinean system as well as the flow in all lines has been used. This is a generation and transmission model, including the lines of 400, 220 and 132 kV. It works with a total of 330 nodes and 450 lines, 158 thermal and 15 hydro groups. The model is an optimal power flow solved through mixed integer programming, as described in [6].

The operational constraints considered are those of the annual hydro management (reservoirs and groups, including pumping), thermal management (minimum and maximum group power, as well as start-up and shut-down constraints), gas consumption constraints and the power balance in the system for every node (losses, flow in the lines, and international exchanges).

The one-year horizon has been divided in periods (months), subperiods (working and non-working days) and blocks (peak, off-peak1 and off-peak2).

C. Characteristics of the export contract

The contract to be valuated is an export of energy at a fixed price from Argentina to Brazil. This export might take place depending on the pricing at each moment in the Argentinean and the Brazilian markets, and the price at which the contract has been signed.

If ***a*** and ***b*** are, respectively, the prices in the Argentinean and Brazilian nodes through which the interconnection is taking place, and ***c*** is the price agreed in the contract, we have the following possibilities:

a) If $b > a$, and $c<a$, energy exchange will take place at the price c.

b) If $b>a$, and $c>a$, the energy exchange will take place. The contract pricing will not be applied, and Brazil will purchase directly from the Argentinean market at price a.

c) If $b<a$, energy exchange will not be taking place.

In summary, when the price in Brazil is higher than in Argentina, energy will be exported to Brazil. In this case, the contract will be applied or not depending on the rapport between the price in Argentina and the price in the contract, as presented in the attached table.

	c>a	**c<a**
b>a	Exchange: price **a**	Exchange: price **c**
b<a	There is **no** exchange	

Table II.1. Possibilities in relation to pricing

On the other hand, if the price in Brazil is lower than in Argentina, the energy flow will take place from Brazil to Argentina. This flow is significantly limited vs. export due to the characteristics of the Argentinean system.

III. MODEL DESCRIPTION

A. Representation of the interconnection

One way of approaching the problem is obtaining separate pricing for the Argentinean and the Brazilian systems. From these prices, energy exchange will take place (at a maximum value) when the price in Brazil is higher than in Argentina.

This approach is, however, incorrect, because the exchange of energy represents a significant amount for the Argentinean system. Therefore, whether the export is taking place or not will modify considerably the prices in the Argentinean market, especially in the area closest to the interconnection. For this reason, the method should be taking into account these aspects, to obtain intermediate values for the export of energy to Brazil.

The model developed for the interconnection is based on introducing both a fictional demand and a fictional generation unit in the Brazilian node in which the interconnection is made. These two elements of the exchanging node in Brazil are representing exactly the behaviour of the Brazilian system in front of the energy exchange.

If we assume that the export of energy to Brazil has a maximum value of ***E***, and that the maximum import of energy from Brazil has a value of ***I***, the resulting scheme is shown in the next figure.

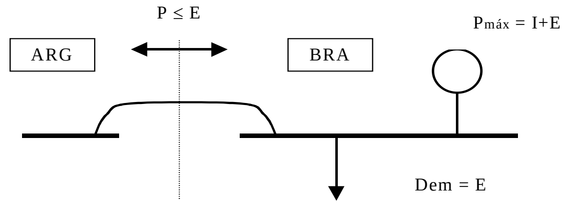


Fig. III.1 Representation of the interconnection with Brazil

As it can be observed, the demand introduced corresponds to the maximum value of the export of energy from Argentina to Brazil. It has also been introduced a generator that bids at a variable price and has a limited generation capacity given by the addition of the maximum values of import and export. In this way, if the generator's variable price equals the price in the node in which it is connected, the behaviour of the interconnection will be the one expected in the characteristics of the contract, as represented in the two figures presented next:

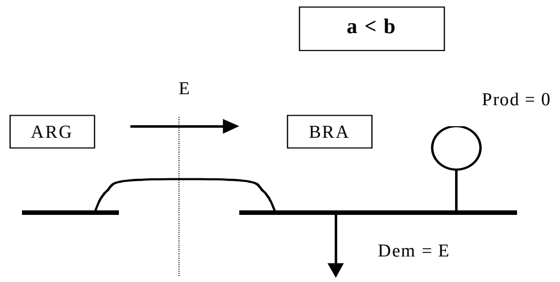


Fig. III.2 Case in which it exists export to Brazil

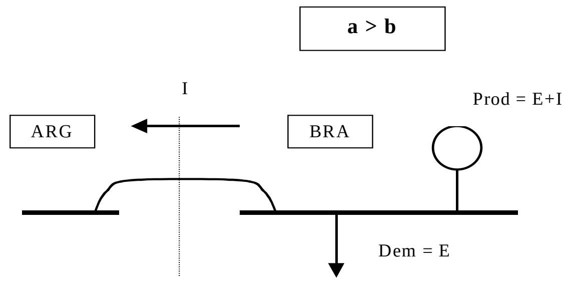


Fig. III.3 Case in which it exists import from Brazil

An intermediate situation it is also possible, with prices in Argentina and Brazil becoming equal through an energy exchange lower than the maximum possible.

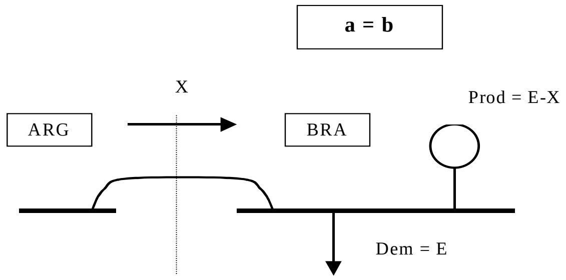


Fig. III.4 Case in which prices become equal

When the price in Brazil is higher than contract price, an export will take place, limited by its maximum value or by the value which causes that the price in the exchanging Argentinean node becomes equal to the Brazilian price. When the price in Brazil is lower than in Argentina, an exchange of energy will not be taking place, as the fictional generator will cover all the Brazilian demand.

It has been occasionally observed, however, that the optimal solution of the load flow does not correspond to the actual behaviour of the interconnection. It has been observed that in spite of a lower price in Brazil than in Argentina, the export of energy did not reached the maximum possible value. In order to avoid this distortion in the results, a second execution of the load flow was introduced. In this second execution, the value of the interconnection flow is fixed at the actual value related to the current prices.

B. Benefit calculation

The calculation necessary to determine the benefit (or loss) obtained by signing the contract will be now described. The model has been built as deterministic for a series of scenarios and includes the pricing for the whole Argentinean system, as well as the flow through the interconnecting line across the full valuation horizon.

The signature or not of the contract by an Argentinean producer is a purely financial fact, not affecting future pricing nor energy exported. The benefit obtained by a specific producer when signing the contract is computed taking into account that the contract will be subscribed for the same total amount of energy independently from the involvement of this producer.

In this way, the potential benefit gained by the producer signing and not signing the contract for a specific power can be calculated for different contract prices. The difference between these values will represent the additional benefit produced by signing the contract.

Subtracting total cost from total income performs the benefit calculation. First, the equation without a contract will be presented:

- Income:

  Power in Argentina: As indicated before, once the energy supply has been determined, the income can be calculated for the peak hours. If $\boldsymbol{P_{ph}}$ is the total energy supplied to the generator in peak hours and $\boldsymbol{p_{PA}}$ is the price at which is paid (which is a fixed and known value) we will get:

$$I_{PA} = P_{ph} \cdot p_{PA} \tag{1}$$

  Energy in Argentina: is the multiple of the energy produced by the generator, $\boldsymbol{P}$, by the price in its node $\boldsymbol{\lambda}$.

$$I_{EA} = P \cdot \lambda \tag{2}$$

- Costs:

  Production costs: they are calculated by multiplying energy produced by the generator by the unitary production cost $\boldsymbol{C_p}$.

$$C_P = P \cdot C_p \tag{3}$$

  Transmission from generator to market node: if $\boldsymbol{F}$ is the fixed fee, and $\boldsymbol{\lambda_m}$ the price in the market node, we will get:

$$C_{TA} = F + P \cdot (\lambda - \lambda_m) \tag{4}$$

Then the benefit generated without a contract $\boldsymbol{B}$ will be:

$$B = I_{PA} + I_{BA} - C_P - C_{TA} \tag{5}$$

If the contract is signed, new elements are to be added to the calculation of the benefit, and some of the ones presented until now are modified, as seen below.

- Income:

  Power in Argentina: in this case, this concept represents an income in Argentina only for the amount of energy produced beyond the amount agreed in the contract with Brazil. If $\boldsymbol{P_C}$ is the energy associated to the contract with Brazil, the equation will be:

$$I'_{PA} = (P_{ph} - P_C) \cdot p_{PA} \tag{6}$$

  Power in Brazil: a fixed monthly income $\boldsymbol{I'_{PB}}$ will be received.

  Energy in Argentina: this concept is representing the energy produced by the generator less the one supplied through the contract $\boldsymbol{P_D}$. If at any moment the energy supplied through the contract is bigger than the one produced by the generator, this income will be zero.

$$I'_{EA} = \text{máx}\ \{(P - P_D) \cdot \lambda, 0\} \tag{7}$$

  Energy in Brazil: it is corresponds to the energy supplied through the contract at the price $\boldsymbol{\lambda_C}$ fixed in it.

$$I'_{EB} = P_D \cdot \lambda_C \tag{8}$$

- Costs:

  Production costs: they are identical to the one calculated without the signature of the contract.

$$C'_P = P \cdot C_p \tag{9}$$

  Transmission from the generator to the market node: in the same way, the producer has to cover the transmission cost, independently from the signature of the contract.

$$C'_{TA} = F + P \cdot (\lambda - \lambda_m) \qquad (10)$$

Transmission from the market node to the interconnecting node with Brazil: it includes a fixed fee $\boldsymbol{F_B}$ multiplied by the maximum available power (which is the one agreed in the contract) and a variable part proportional to the price $\boldsymbol{\beta}$ per unit of energy supplied.

$$C'_{TB} = F_B + P_D \cdot \beta \qquad (11)$$

Energy bought in Argentina: if, in any moment, the energy supplied through the contract is bigger than the amount produced by the generator, this generator will have to buy the part not supplied at the price $\boldsymbol{\lambda_m}$ in the market node.

$$C'_{EA} = \text{máx}\ \{(P_D - P)\cdot \lambda_m, 0\} \qquad (12)$$

In this case, the income ***B'*** obtained when the contract is signed will be.

$$\begin{aligned} B' = I'_{PA} + I'_{PB} + I'_{BA} + I'_{BB} \\ - C'_{P} - C'_{TA} - C'_{TB} - C'_{EA} \end{aligned} \qquad (13)$$

As shown in the equation, the signature of the contract is equivalent to the selling of a Call option by the Argentinean generator. The premium of this option will be the addition of the fixed incomes perceived by the producer as capacity payment in Brazil, I'_{PB}. On the other hand, Brazil can execute at any moment its option of energy purchase, and this will be causing that the losses associated to the selling of the option are depend not only on the prices in Argentina (underlying asset) but also on the prices in Brazil.

C. *Uncertainty assessment*

This is described considering that in every scenario, the benefit can be calculated with and without the signature of the contract.

C.1 Scenario definition

The study is considering a series of scenarios.

As both the plan for installation of new generators in the Argentinean market and the new transmission lines to be built during the scope of the study are known, the two more important stochastic variables remaining are the demand evolution and the hydro inflows in Argentina, as well as pricing at the exchanging node in Brazil.

After considering diverse studies of the evolution of the Argentinean market, the demand was decided to be considered as a deterministic variable. It was also checked that deviations in the demand from the projected value were not influencing significantly future pricing.

The hydro conditions in Argentina and Brazil has, however, a much stronger influence, as the variation on prices when years are "wet" or "dry" have shown to be very important.

The estimation of the stochasticity in the Brazilian market has been made through price scenarios in the exchanging nodes under different hydro conditions. Thus, it was decided to generate the scenarios to valuate the contract as a function of the two key variables: pricing in Brazil and hydro conditions in Argentina.

By performing a study of the river basins and hydro inflows of the Argentinean generators, it was confirmed that the most important hydro generators are located in two areas, southwest and northeast of the country.

As energy exchanging with Brazil is taking place in the northeastern region of Argentina, it was studied how different hydro conditions in both river basins are affecting prices in the exchanging nodes, amount of energy exchanged, and benefit obtained with and without the contract. The consequence is that these variables are very sensitive to the hydro conditions in the northeastern region, but much less in the southwest. Specifically, taking into account average hydraulic inflows in the southwest and calculating related deviation in prices in the eastern and northeastern regions, it is confirmed that in more than 90% of the hydraulic scenarios in the southwest, prices are oscillating less than 5%. Even in the case of extreme hydro conditions, prices will never change in 10%.

After this evaluation, determinist hydraulic inflows were considered for this region, and the study was performed with the hydro conditions in the northeastern region of Argentina and pricing in Brazil as stochastic variables. A sensitivity study performed a posteriori is confirming this hypothesis.

Considering the geographic proximity between these areas, it is necessary to check if there is any kind of correlation between both variables before performing a stochastic study.

It was confirmed through this analysis that a high negative correlation does exist (higher hydraulic inflows correlates with lower prices, and the opposite), and this fact conditioned the elaboration of scenarios.

Finally, a series of joint scenarios with hydro conditions and pricing that considered the existing relationship (with a probability estimated through historical data) were prepared. The table III.1 shows the scenarios selected and their associated probability in percentage.

Hydro conditions in Argentina	**Prices in Brazil**				
	Very expensive	Expensive	Average	Cheap	Very cheap
Very dry	7.69	5.77	0	0	0
Dry	7.69	11.54	13.46	0	0
Average	0	3.85	7.69	11.54	0
Wet	3.85	0	0	5.77	11.54
Very wet	0	0	0	1.92	7.69

Table III.1. Probabilities (in %) associated to the different scenarios

C.2 Discrete convolution

The benefits obtained with and without signing the energy export contract were calculated after preparing these scenarios. This calculation was performed considering a time horizon, so benefit in both possibilities was calculated for a series of years, each one associated to its probability.

Afterwards, of all these future incomes were updated, in order to prepare a distribution of probability of the benefit with and without the contract. After this step was done, it is enough to evaluate these distributions with the criteria considered more important (average value expected, value at risk...) in order to take a decision.

Due to the size of the problem, the calculation of these distributions is not immediate. If we have ***n*** scenarios (corresponding with n possible benefit values) during ***i*** years, the total number of possible values in the distribution of income will be $\mathbf{n^i}$.

This value is very high, so a procedure of calculating the distribution without the need of calculating every value was developed. The method used was the *discrete convolution*.

Generally, when there are two random variables X_1 and X_2 with distribution of probabilities $f_1(x)$ y $f_2(x)$, the distribution of the variable addition $Y = X_1 + X_2$ is obtained by the composition of both distributions through the operation called *convolution*:

$$f_Y(y) = f_1(x) \oplus f_2(x) \quad (14)$$

The convolution $\oplus$, is defined as:

$$f_1(x) \oplus f_2(x) = f_Y(y) = \int_{-\infty}^{\infty} f_1(u) \cdot f_2(y-u)du \quad (15)$$

A formal definition of convolution can be found in [7]. In case of X_1 and X_2 being discrete variables, the calculation should be performed on the probability mass function. Let $P_1(x)$ and $P_2(x)$ the probability mass functions of X_1 and X_2, with n and m possible values, respectively:

$$P_1(x) = P[X_1 = x] \quad \text{for } x = x_{11}, x_{12},..., x_{1n} \quad (16)$$

$$P_2(x) = P[X_2 = x] \quad \text{for } x = x_{21}, x_{22},..., x_{2m} \quad (17)$$

Then, the random variable addition $Y = X_1 + X_2$ will have a maximum of $n{\cdot}m$ possible values, to be calculated as follows:

$$P_Y(y) = P[Y = y] = \sum_{u=x_1}^{x_n} \left(P[X_1 = u] \cdot P[X_2 = y-u]\right) \quad (18)$$

If n and m are very large, or if many random variables are to be added, a discrete convolution by intervals of equal size can be used instead of all the discrete values. In this case, intervals as small as desired are selected, and every value included in each one of them is characterised by its average value. This value will be assigned the addition of the probability of all of them. An example will clarify this point.

Let us assume that the benefit of a specific contract in two consecutive years (i.e. 2001 and 2002) in two scenarios (wet and dry) has been studied. Let us now imagine that the benefit (in US$M) with its probability are the ones reflected in the following table:

	Year 2001		Year 2002	
Scenario	Benefit	Probability	Benefit	Probability
Wet	0.75	0.3	0.5	0.4
Dry	1.5	0.7	1	0.6

Table III.2. Probabilities of benefit for a hypothetical contract in two years

The total benefit for the two years is the result of considering all the possible combinations of the scenarios (four in total). This is presented in the following table.

	Years 2001 and 2002	
Scenario	Benefit (US$M)	Probability
2001 wet 2002 wet	1.25	0.12
2001 wet 2002 dry	1.75	0.18
2001 dry 2002 wet	2	0.28
2001 dry 2002 dry	2.5	0.42

Table III.3. Probabilities of total benefit for a hypothetical contract.

This operation is solved in a systematic way by the discrete convolution, representing probabilities in intervals of equal size. If we are selecting intervals of US$ 0,5M, and considering for the distribution values from 0 to US$ 3M, its distribution would have six intervals. In the first one the probability of benefit from 0 (included) to 0,5 (excluded) is represented, with the second having from 0,5 (included) to 1 (excluded) and so on. The distributions would be:

	0-0.5	0.5-1	1-1.5	1.5-2	2-2.5	2.5-3
Year 2001	0	0.3	0	0.7	0	0
Year 2002	0	0.4	0.6	0	0	0
Total	0	0	0.12	0.18	0.28	0.42

Table III.4. Probabilities associated to discrete distributions of probability

C.3 Control of errors

An important decision, at the time of using the discrete convolution, is the size of the interval used in the random variable. In the former example, an interval of 1M$ will obtain different results from intervals of US$ 0,5M or US$ 0,25M.

In the case studied, in which the time horizon is broad and there are a considerable number of scenarios for each year, the decision about the number of intervals is specially important.

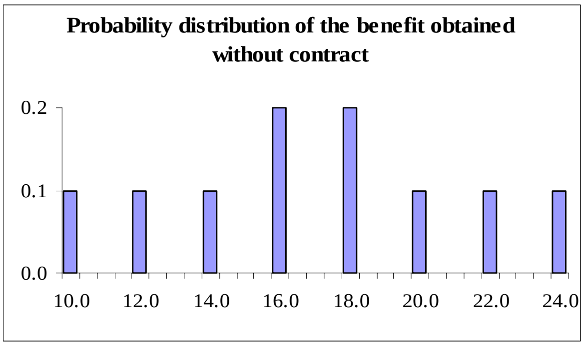
This is due to the fact of not being enough introducing as width of the interval the maximum value acceptable as error in the final result, because actual error obtained will be considerably bigger. As the period of years becomes longer, the errors of each discrete convolution are progressively accumulating. For this reason, the shorter possible interval viable under existing possibilities of computational calculation must be selected.

It is possible verifying the error being obtained by comparing the maximum value of theoretical benefit (addition of the maximum yearly values) with the value obtained through the discrete convolution.

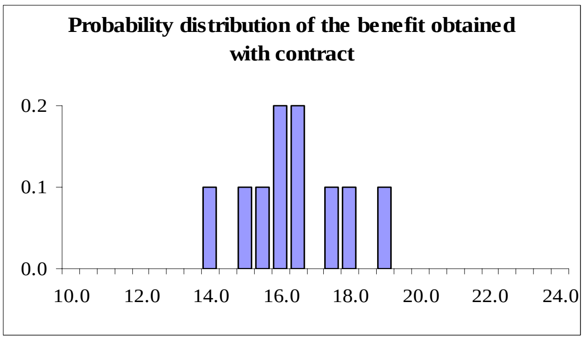
Once the distribution of probability of the expected benefit for the valuation horizon has been obtained, the expected value, its standard deviation, its values at risk at different levels of confidence and any other statistical or risk management measure required can be calculated.

IV. CASE STUDY

As an example, a convolution has been performed in a case with a horizon of five years. It has been assumed that the possible distribution of benefit is the same across the five years, although an interest rate of 10% has been considered. The two graphs IV.1 and IV.2 present the probability mass function used, both for benefit without contract and with its signature.

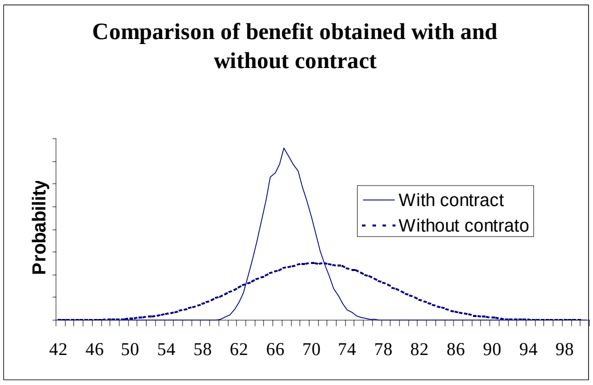


Graph IV.1. Scenarios for benefits without contract during the five-year period.



Graph IV.2. Scenarios for benefit obtained with contract during the five-year period

The next graph is detailing the distribution of probabilities obtained in each of the two situations.



Graph IV.3. Distribution of probability obtained with and without contract

It can be observed that, while the expected benefit without contract is a symmetrical distribution without peaks, the case with contract has an important and asymmetrical peak around the more frequently expected values of benefit. This is related to the data introduced.

The average and standard deviation of both distributions are presented next, including values at risk for 95% and 99% confidence.

	Without contract	With contract
Average	70.88	67.74
Std. deviation	7.76	2.68
VaR - 95	58.10	63.51
VaR - 99	53.17	62.07

Table IV.1. Comparison of statistic parameters of both distributions

After the observation of these results, the decision of signing or not the export contract depends on the producer's strategy. The signature of the contract would reduce the expected level of benefit, but is also reducing very significantly the associated risks.

An independent generator not willing to assume risk would sign the contract, obtaining a hedge. Inversely, a diversified company, with broad activities and investments could decide not to sign the contract in order to obtain a higher benefit.

V. CONCLUSIONS

A method, which allows a realistic representation of some contract of energy exchanging as a Call option, has been presented. In order to study the profitability of this option, a method different from the standard used in the valuation of options has been used.

This methodology is making possible working with options even when the probability distribution obtained cannot be approached through Normal distributions.

The practical example presented shows that this is an useful tool in obtaining simultaneous measures of risk and expected benefit, so investors can take an informed decision depending on their business approach to risk taking.

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VII. BIOGRAPHIES

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