

# CHRONOLOGICAL STOCHASTIC SIMULATION OF MEDIUM- AND LONG-TERM OPTIMAL OPERATION USING A MULTILEVEL HIERARCHICAL MODEL

Efraim Centeno, Andrés Ramos, Fernando de Cuadra  
Instituto de Investigación Tecnológica  
Universidad Pontificia Comillas  
c/ Santa Cruz de Marcenado, 26  
28015 – Madrid SPAIN  
Tel: +34 91 542 28 00 Fax: +34 91 542 31 76  
e-mail: Efraim.Centeno@iit.upco.es

**Keywords:** Hydrothermal coordination, hydrothermal scheduling, hierarchical production planning.

## ABSTRACT

This paper presents the MADEA model. It uses chronological stochastic simulation techniques in order to emulate medium- and long-term optimal operation under uncertain conditions. Operator's decisions are internalised in the model to perform a realistic system representation. They are assumed to be optimal decisions taken without precise information about the future. MADEA reproduces the system operation through a hierarchical structure including yearly, weekly and hourly submodels. A representative states selection technique has been used to reduce computing time. A case study is presented showing the model behaviour in a large-scale electric power system.

## 1 .INTRODUCTION

Chronological stochastic simulation techniques, [1] and [2], have frequently been used in operation models, [3], [4] and [5], mainly oriented towards system reliability assessment [6], [7]. This technique allows a great level of detail, but frequently its application is limited to a short time horizon (about one month). This limitation is caused not only by computing time requirements but also by the scope of validity of the simulation scenario hypothesis.

The model presented here extends to longer periods the algorithmic capability of chronological stochastic simulation. It can be used for medium-term operation (about one year) or even long-term operation, which allows performing cost and reliability studies.

## 2 .HIERARCHICAL MODEL STRUCTURE

The time scope extension for the chronological simulation has been done by chaining short-term simulations (one week). Each simulation needs criteria to determine the simulation conditions. These conditions are the weekly operation criteria, for example, which generation units (thermal or hydro) have to supply the demand. Short-term models usually obtain this information from higher scope models that compute it from system initial conditions at the beginning of the simulation. Nevertheless, a chronological simulation model than chains several operation periods (weeks) needs to include the weekly

optimal decisions taken by the operators in order to be realistic. The MADEA model includes optimisation submodels with higher scope that provide those decisions, trying to resemble the actual circumstances in which they are taken (non-anticipativity principle).

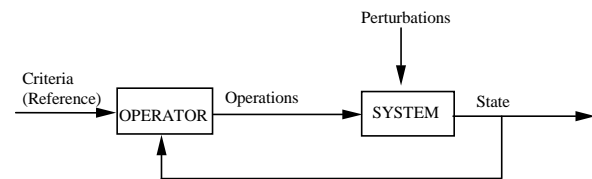


Figure 1. Basic Control Scheme.

A control theory viewpoint has been used to identify the system structure. Operation of an electrical power system has been analysed as a control problem, as shown in Fig. 1. The operator controls the system through commands (operations) that are chosen by comparing a measure of the system state against higher scope criteria. The system state evolution is determined by the operations, its current state and the external perturbations. This scheme can be found at the different scopes of the model.

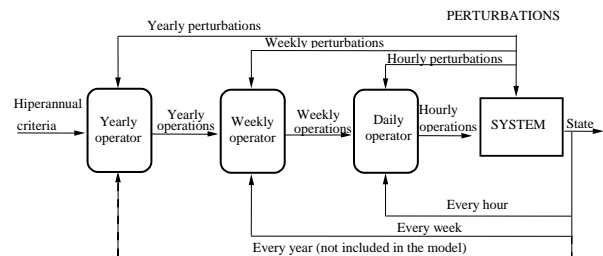


Figure 2. Hierarchical Control Scheme.

Three different levels have been identified to represent the system operation. They have been called hourly, weekly and yearly submodels. Other decision levels, e.g. daily, have been integrated into these ones. The different elements of a control problem (state, reference, perturbations) have been identified in the three levels of the model: variables that characterise the system state, perturbations that change the state evolution (unit failures, demand, inflows), decisions that are taken (operations) and the higher level criteria that are needed to take these decisions. The criteria needed by the operator in each level are operations calculated by a higher level operator (see Fig. 2).

The system evolution is simulated under different scenarios (perturbations) that are randomly generated.

Nevertheless, “optimal” decisions are always taken under the assumption of unknown future (non-anticipativity principle).

The yearly submodel is a mixed-integer programming problem representing system operation with a one-year horizon that determines the generation units’ maintenance schedule. The weekly submodel uses this result as input. It is another mixed-integer programming problem, that obtains weekly dispatch and unit commitment. The hourly submodel uses these results as input. This submodel represents the system operation using chronological stochastic simulation with one-hour as the minimum time step. It represents unit production ramp rates, failures, stochastic demand variation, units’ start-up and shutdown and other operation aspects with a high level of detail. At the end of an hourly full-week simulation, the weekly submodel is executed again to obtain a new dispatch and unit commitment for the next week. This process is repeated until the desired number of weeks is reached.

Hourly simulation of each week is performed for several scenarios. This leads to so many different states at the end of the week as scenarios have been simulated. Each one of these states would require an execution of the weekly submodel to be able to simulate next week. In order to decrease the number of required optimisations, a selection of representative states is made by using clustering techniques and it reduces dramatically the running time of the model.

This paper describes first system representation, then describes the submodels separately giving more details of weekly and hourly ones. After, it explains uncertainty representation. Finally a case study is presented.

### 3 . SYSTEM REPRESENTATION

Each element of the model is represented by a set of parameters. Each submodel uses a part of these parameters.

#### 3.1 Time Representation

Time is divided in several intervals, each one is characterised by a subscript.

- $p$  Period (usually one week).
- $s$  Subperiod (weekdays or weekend).
- $b$  Load level (peak, plateau, off-peak).
- $i$  Hour.

Besides:

- $d_{psb}$  Interval duration.
- $N_i$  Hours in a week.

#### 3.2 Thermal Unit Model

A thermal unit is characterised as a set of generating units. A generating unit uses a single fuel and has a linear heat rate.

Each fuel is characterised by:

- $m_c$  Fuel price.
- $S_{MAXc}, S_{MINc}$  Maximum (minimum) stock.
- $C_{MAXc}$  Yearly maximum quota.
- $C_{MINc}$  Yearly minimum quota.
- $K_{pc}$  Fuel purchase of fuel  $c$  in period  $p$ .

Each generating unit  $g$  is characterised by:

- $\bar{p}_g, \underline{p}_g$  Maximum (minimum) power.
- $c_{ag}$  Start-up consumption.
- $q_g$  EFOR.
- $COMB_g$  Unit fuel.
- $A_g$  Linear term of heat rate.
- $B_g$  No load term of heat rate.

Thermal units are characterised by:

- $\bar{p}_t, \underline{p}_t$  Maximum (minimum) power.
- $q_t$  EFOR.
- $GDT_t$  Set of generating units of the thermal unit.
- $MTTF_t$  Mean time to failure.
- $MTTR_t$  Mean time to reparation.
- $COMBUS_t$  Set of fuels of the thermal unit.
- $c_t(p)$  Consumption function for each fuel.

#### 3.3 Hydro Subsystems Model

Hydro equipment is represented as hydro subsystems. A hydro subsystem represents a set of hydro units with pumping capability. Maximum power is a linear function of subsystem energy reservoir level. A hydro subsystem  $h$  is characterised by:

- $U_h$  Constant term of power-reserve function.
- $V_h$  Linear term of power-reserve function.
- $i_h$  Lower subsystem.
- $\alpha_h$  Production performance to lower subsystem.
- $\bar{R}_h, \underline{R}_h$  Maximum (minimum) reservoir limit.
- $\bar{b}_h$  Maximum pumping power.
- $\rho_h$  Pumping performance.
- $EV_h$  Set of upper subsystems.
- $\omega_h$  Security coefficient for spillage.

#### 3.4 Pumped-storage Units Model

Each unit  $n$  is characterised by:

- $\bar{p}b_n$  Maximum pumping power consumption.
- $\rho_n$  Pumping performance.

#### 3.5 Demand

Demand is represented by a single value for each level and also by its hourly average value for each period. These values must be coherent.

- $D_{psb}$  Average demand for period, subperiod and level.
- $k(T)$  Temperature-dependent demand factor.

$dppi$  Average demand for period  $p$  and hour  $i$ .  
 $F$  Random factor with distribution  $N(0, \sigma^2)$

### 3.6 Inflows

Inflows are represented by its medium value and its correlation with temperature. Run-off-the-river inflows are computed separately.

$APOR_{Fhp}$  Average run-off-the-river inflows.  
 $\mu_{ph}$  Average inflows (except run-off-the-river).  
 $\Sigma T$  Covariance matrix total inflows-temperature.  
 $TMp$  Average temperature in each period.  
 $\underline{ph}_{ph}$  Minimum hydro power.  

$$\underline{ph}_{ph} = \frac{APOR_{Fhp}}{\sum_s \sum_b d_{psb}} \quad (1)$$

### 3.7 Hyperannual Operation Criteria

Hyperannual operation criteria are represented by future cost functions that will be provided to the model by an external source. These functions determine the expected future cost associated to reservoir levels for each hydro subsystem, and to stock levels for each fuel. As we are using MIP for solving the model these functions must be convex. Let  $p^*$  be the last operation period, the expression for these criteria is:

$$CF = \sum_h CF(r_{p^*h}) + \sum_c CF(stk_{p^*c}) \quad (2)$$

### 3.8 General Parameters

$RR$  Spinning-reserve coefficient.  
 $CENS$  Non-supply energy cost.  
 $PNP$  Penalty for spinning-reserve defect.  
 $PNE$  Penalty for energy excess.

### 3.9 Initial Values

$stk0c$  Fuel stock at the beginning of a period.  
 $r0h$  Reservoir level in a hydro subsystem at the beginning of a period.

## 4 .YEARLY SUBMODEL

Yearly operations are computed with a MIP model with a time scope of one year and obtains the optimal maintenance schedule for thermal units. These are the criteria used by the weekly model.

The outputs of this model are binary variables that indicate whether a unit is available or not:

$drgpg$  Maintenance availability of a thermal unit.

## 5 .WEEKLY SUBMODEL

The weekly submodel computes weekly operation criteria. It uses as inputs the yearly operations, information about past perturbations, and the system state. The system state for this submodel is the value of reservoir levels in hydro subsystems and fuel stocks.

This submodel is a MIP model, similar to that described in [9]. It performs the hydrothermal coordination and also the unit commitment. The hourly submodel uses these results after some -described later-postprocessing. The scope of this submodel is one year, but only the results for the first period are used for the simulation of one week. The weekly submodel is executed once for each week (and each initial state).

### 5.1 Decision Variables

$agpsg$  Commitment decision of generation unit  $g$  in a period and subperiod. (1-0)  
 $apst$  Commitment decision of thermal unit  $t$  in a period and subperiod. (1-0)  
 $ptpsbg$  Power generated by a thermal unit in a period, subperiod and level.  
 $stkpc$  Fuel stock in a period.  
 $phpsbh$  Power generated by a hydro subsystem in a period, subperiod and level.  
 $bepsbh$  Power consumption by pumping in a hydro subsystem in a period, subperiod and level.  
 $pbpsbn$  Power generated by a pumped-storage unit in a period, subperiod and level.  
 $bspsbn$  Power consumption by a pumped-storage unit in a period, subperiod and level.  
 $rph$  Energy storage in a hydro subsystem in a period.  
 $veph$  Spillage in a hydro subsystem in a period.  
 $nspsb$  Non served power in a period, subperiod and level.  
 $wpsb$  Excess of generated power in a period, subperiod and level.  
 $zps$  Spinning reserve defect in a period, subperiod and level.

### 5.2 Constraints and Objective Function

The main constraints in this submodel are: generation-demand balance, minimum spinning reserve, relation between commitment decisions and generated power, start-up and shutdown decisions, generated power limits, fuel balance, maximum and minimum fuel quotas, relations between maximum power and energy storage in hydro subsystems, and hydro energy balance.

The objective function to be minimised represents the operation cost in one year plus the future cost due to the final state. Operation costs consider generated power, start-up, shutdown and penalties.

## 6 .WEEKLY OPERATION CRITERIA

These criteria are calculated from the results of the weekly submodel. Only results for the first period ( $p=1$ ) are used.

### 6.1 Marginal Values for Pumping Decisions

These data are directly taken from the weekly submodel results. The marginal value is the dual variable in the generation demand balance constraint.

$CMVs$  Off-peak marginal value for each subperiod.  
 $CV_t$  Variable cost for each thermal unit when producing maximum power.

## 6.2 Maximum and Minimum Power for Hydro Subsystems.

This data is directly taken from the weekly submodel results.

$\underline{p}_h$  Minimum hydro power.

$$\underline{p}_h = \frac{APOR_{FH1}}{\sum_s \sum_b d_{1sb}} \quad (3)$$

$\overline{p}_h$  Maximum hydro power.

$$\overline{p}_h = U_h + V_h * r_h \quad (4)$$

## 6.3 Start-up Demand Level for Hydro Generation

$PHID_{ps}$  Start-up demand level for hydro generation for each subperiod.

$$PHID_{ps} = \sum_g \frac{pt_{ps1g}}{q_g} + \sum_h \underline{p}_h \quad (5)$$

## 6.4 Available Thermal Units

$DISP$  List of available thermal units without fuel quota.

$DISP_c$  List of available thermal units with fuel quota for each fuel.

This is not a result from the weekly model but from the yearly model.

## 6.5 Committed Thermal Units in a Subperiod

This list is directly created from decision variable  $ag$ . There is one list for weekday subperiod and other one for weekend subperiod.

## 6.6 Thermal Units for Night Shut-Down

$PNOct_s$  Thermal units for night shutdown in a subperiod.

Night shutdown will be applied if start-up cost is lower than saving. Saving is computed as the cost of the thermal unit -generating its minimum power- minus the cost of the marginal unit generating the same power. Criteria for shut-down is:

$$\underline{p}_t * C_t(\underline{p}_t) * d - PT_{MINt} * C_{MV_s} * d > C_{ARRt} \quad (6)$$

Where:

$C_t(\underline{p}_t)$  Cost of the thermal unit  $t$  generating its minimum power.

$d$  Night shutdown duration.

$C_{ARRt}$  Thermal unit  $t$  start-up cost.

Minimum shut-down duration is obtained from the same equation.

$$d_{MINt} = \frac{C_{ARRt}}{PT_{MINt} * (C_t(PT_{MINt}) - C_{MV_s})} \quad (7)$$

Shutdown will be done if this duration is below a previously fixed value.

## 6.7 Fuel Rate

$Q_{tpc}$  Fuel rate for a thermal unit in a period.

Since it is calculated as a single value for each period, it is computed as the energy rate for each fuel. If  $c=COMB_g^*$ :

$$Q_{tpc} = \frac{\sum_s \sum_b pt_{psbg} * d_{psb}}{\sum_{g \in GDT} \sum_s \sum_b pt_{psbg} * d_{psb}} \quad (8)$$

This value is calculated only for  $p=1$ .

## 6.8 Thermal Units Load Order

$ORDT_t$  Thermal units load order.

The load order is computed from  $pt$  decision variable. First a provisional load order is computed using weekend period values. Then the final load order is computed with weekday values.

**Provisional load order** selects first the thermal units generating over its minimum power in off-peak level. Then, priority rules are applied:

- Thermal units generating their maximum power (before those generating less than the maximum).
- In each one of these groups, those with minimum fuel quota (before those without fuel quota).
- In each one of the four groups, units are ordered by average variable cost.

Next, it chooses from the rest of the units those that are generating over its minimum power in plateau level and orders it with the same criteria.

Then, it does the same with the units that are generating over its minimum power in peak level. And finally it includes the rest of the units ordered by average variable cost.

**Final load order** is computed the same way, but using provisional order as additional criteria.

## 7 . HOURLY SUBMODEL

Hourly submodel simulates system operations with a one-hour time step. It considers ramp rates, failures, stochastic demand variation, start-up and shutdown of units, and other operation aspects with a high level of detail. At the end of an hourly full-week simulation, the weekly submodel is executed again to obtain a new dispatch and unit commitment for next week. This process is repeated until the desired number of weeks is reached.

Hourly simulation is repeated for different scenarios that are randomly generated.

### 7.1 Failures Scenario

A single scenario of failures is represented by:

$FALLOS_t$  Failure hour of the week (if unit fails).

$REPARA_t$  Repair hour of the week (if unit is repaired).

The scenarios are generated from the parameters  $q$ ,  $MTTF$  and  $MTTR$  of each thermal unit.

### 7.2 Inflows

A single scenario of inflows is represented by:

$WTOT$  Total inflow.

$T$  Temperature.

An inflow value is generated for each day. Temperature is generated simultaneously because it is correlated to inflows level. This random generation correspond to the expression of a binormal distribution.

$$\begin{bmatrix} W_{TOT} \\ T \end{bmatrix} = C_T * U + \begin{bmatrix} \sum_h \mu_{ph} \\ T_{Mp} \end{bmatrix} \quad (9)$$

$$\Sigma_T = C_T * (C_T)^T$$

$C_T$  Lower triangular matrix. It is computed through Cholesky transformation.

$U$  Two dimensional random vector sampled from a normal distribution(0, 1).

$\mu_{ph}$  Average value for inflows in a hydro subsystem for a period.

$T_{Mp}$  Average temperature in a period.

### 7.3 Demand

A single demand scenario is represented by:

$D_i$  Demand for  $i$  hour.

It is computed as:

$$D_i = k(T) * dp_i + F \quad (10)$$

### 7.4 System State

The hourly system state is defined by fuel stocks, reservoir levels, generation unit productions, and availability state.

$Sci$  Hourly stock level for each fuel.

$Rh$  Hourly reservoir level for each subsystem.

$p_{ti}$  Hourly generated power of a thermal unit.

$p_{hi}$  Hourly generated power of a hydro unit.

$b_{bi}$  Hourly power consumption by pumped-storage unit.

### 7.5 System Hourly Operation

The hourly simulation submodel objective is to determine the system state in one hour to meet the demand. It uses the state in the previous hour and the weekly criteria operation as inputs. In order to simplify the model, it is assumed that every unit is available and committed.

The symbol “ $\leftarrow$ ” has been used to express assignment of a value to a variable. This assignment has to be understood in an algorithmic context. A variable takes different values in simulation time.

$N_t$  Number of thermal units.

$N_h$  Number of hydro units.

Algorithm:

- 1) Initialise total values of thermal and hydro generated power and power consumption by pumped-storage units.

$$\begin{aligned} PT_{i-1} &\leftarrow \sum_{t=1}^{N_t} p_t & PH_{i-1} &\leftarrow \sum_{h=1}^{N_h} p_{ht} \\ PB_{i-1} &\leftarrow \sum_{h=1}^{N_h} b_{ht} & PH_i &\leftarrow 0 \end{aligned} \quad (11)$$

- 2) Assign total minimum hydro power.

$$PH_i \leftarrow PH_i + \sum_{h=1}^{N_h} p_h \quad (12)$$

- 3) If demand is greater than start-up demand level for hydro generation, assign as hydro generated power the excess.

$$\begin{aligned} \text{If } : D_i > PHID_{ps} \\ PH_i &\leftarrow PH_i + D_i - PHID_{ps} \end{aligned} \quad (13)$$

- 4) If there is any hydro subsystem close to spillage, add its maximum production to total hydro generation.

$$PH_i \leftarrow \sum_{h=1}^{N_h} p_h + \sum_{R_h > \omega_h * R_h} \bar{p}_h - p_h \quad (14)$$

$$\text{If } : PH_i < \sum_{h=1}^{N_h} p_h + \sum_{R_h > \omega_h * R_h} \bar{p}_h - p_h$$

- 5) Check the non-served demand in these conditions.

$$\Delta DEM \leftarrow D_i + PB_{i-1} - PT_{i-1} - PH_i \quad (15)$$

- 6) If non-served demand is positive ( $\Delta DEM > 0$ ), complete it with the following sequence of procedures:

- a) Decrease pumping consumption.

$$\text{If } : PB_{i-1} > \Delta DEM \begin{cases} PB_i \leftarrow PB_{i-1} - \Delta DEM \\ \Delta DEM \leftarrow 0 \end{cases} \quad (16)$$

$$\text{If } : PB_{i-1} \leq \Delta DEM \begin{cases} PB_i \leftarrow 0 \\ \Delta DEM \leftarrow \Delta DEM - PB_{i-1} \end{cases}$$

- b) Increase thermal units generated power.

$$\text{If } : \sum_{t=1}^{N_t} \bar{p}_t - PT_{i-1} > \Delta DEM \begin{cases} PT_i \leftarrow PT_{i-1} + \Delta DEM \\ \Delta DEM \leftarrow 0 \end{cases} \quad (17)$$

$$\text{If } : \sum_{t=1}^{N_t} \bar{p}_t - PT_{i-1} \leq \Delta DEM$$

$$\left\{ \begin{array}{l} PT_i \leftarrow \sum_{t=1}^{N_t} \bar{p}_t; \quad \Delta DEM \leftarrow \Delta DEM - \left( \sum_{t=1}^{N_t} \bar{p}_t - PT_{i-1} \right) \end{array} \right.$$

c) Increase hydro units generated power.

$$PH_i \leftarrow PH_i + \Delta DEM \quad (18)$$

7) If non served demand is positive ( $\Delta DEM < 0$ ), complete it with the following sequence of procedures:

a) Decrease thermal units until pumping is profitable.

$$PT_{MIN} \leftarrow \sum_{t=1}^{N_t} \underline{p}_t + \sum_{C_{Vi} < C_{MVs}} (\bar{p}_t - \underline{p}_t) \quad (19)$$

$$\text{If } : PT_{MIN} > \Delta DEM \begin{cases} PT_i \leftarrow PT_i - \Delta DEM \\ \Delta DEM \leftarrow 0 \end{cases}$$

$$\text{If } : PT_{MIN} \leq \Delta DEM \quad (20)$$

$$\left\{ \begin{array}{l} PT_i \leftarrow PT_{MIN} \\ \Delta DEM \leftarrow \Delta DEM - (PT_i - PT_{MIN}) \end{array} \right.$$

b) Start pumping.

$$PB_i \leftarrow PB_{i-1} + \Delta DEM \quad (21)$$

8) Compare with the previous hour and check if the total thermal generated power has increased or not. Increase (or decrease) individual thermal generated power following load order. For example, a single thermal unit only increases its production when all the preceding units in the load order have reached their maximum increase.

$p^{t(i-1)}$  Previous hour thermal units generated power.

$\Delta PT$  Generated thermal power increase.

$$\Delta PT = PT_i - PT_{i-1} \quad (22)$$

Deaggregation must satisfy:

$$\sum_t p_{it} - \sum_t p_{t(i-1)} = \Delta PT \quad (23)$$

$$\underline{p}_t < p_{it} < \bar{p}_t$$

If there is any shutdown or failure in the previous hour, its generated power must be subtract from  $PT_{i-1}$ .

## 7.6 System Events

The hourly submodel considers the following events: start-up, shutdown, night shutdown, unit failures and restorations, and inflows. All the events that occur in the same hour are considered together before computing the next-hour state.

The hourly simulations results are recorded for its statistical processing and analysis.

## 8 . UNCERTAINTY REPRESENTATION

Uncertainty is considered at three different levels in the model: hourly, weekly and yearly.

At **hourly** level, the set of randomly generated scenarios allows the estimation of production variables (utilisation hours, fuel consumption, production cost) for each week. They are aggregated with those of the following weeks to obtain medium- and long-term results. As hourly simulation of each week is performed for several scenarios, it leads to so many different states at the end of the week as scenarios have been simulated. Each one of these states would require an execution of the weekly submodel to be able to simulate next week. In order to decrease the number of required optimisations, a selection of representative states is made by using clustering techniques [8] and it reduces dramatically the model execution time. Practical model usage requires deciding the number of representative states to choose each week. This parameter is very important to determine execution time.

At **weekly** level, the optimisation submodel treats inflows as a random variable. At **yearly** level, uncertainty is considered by selecting different scenarios of natural inflows and performing a whole yearly simulation for each one.

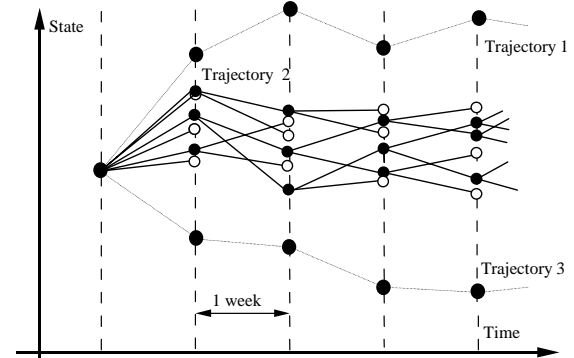


Figure 3. Uncertainty Representation.

Fig. 3 depicts uncertainty representation. Yearly simulation has been performed for three different inflow scenarios. This has led to three different trajectories for the system state. Hourly uncertainty treatment has been represented for trajectory 2. Black circles represent the states that have been used as initial states for next week simulation and white ones represent the remaining states. In this trajectory, 6 weekly scenarios have been simulated and only 3 representative states have been chosen among them at the end of each week.

## 9 . CASE STUDY

A case study representing the Spanish peninsular electrical power system during six months is presented.

Hourly simulation submodel has been written in C while optimisation submodels have been written in GAMS. There are 86 thermal units and 20 hydro subsystems in the system. Operation results have been classified in four types with different simulation behaviour: type I, yearly total productions and cost, type II, weekly fuel stocks and fuel consumption, type III, yearly unit production and type IV, reliability measures (not presented in this case study).

The number of weekly scenarios is determined depending on size of the confidence interval and also on the type of result. Table I shows the relationships between the relative size of the confidence intervals, number of weekly scenarios and number of representatives for a type I result.

Table I. Confidence Intervals (%)

Representatives number	3	6	10
100 scenarios	0.221	0.223	0.223
200 scenarios	0.161	0.159	0.161
400 scenarios	0.114	0.113	0.114

Table II shows average relative size of the confidence intervals for the different types of results.

Table II. Confidence Intervals and Result Type

Type	Interval
I	0.22 %
II	2 %
III	5 %

The number of weekly representative states is decided after deciding the number of weekly scenarios. A comparison between simulations with and without reducing the number of initial weekly states is presented in order to analyse the sensitivity of the results with respect to number of representatives.

A simulation with 100 weekly scenarios and 100 weekly representatives has been taken as reference. Maximum percentages of variation with respect to these simulations are shown in table III.

Table III. Variations and Representatives (100 scenarios)

Result type	3 representatives	10 representatives
I	1 %	0.65 %
II	10 %	5 %
III	5 %	3 %

An increment on the number of representatives improves the results. Higher deviations are those of results affected by representatives selection (type II).

Table IV. Computing time (% relative to reference case)

Representatives	3	6	10	100
100 scenarios	2.2	4.1	8.3	100
200 scenarios	2.4	4.6	9.7	-
400 scenarios	2.9	5.4	11.2	-

Table IV shows execution times for reference and simplified cases. Increasing number of scenarios has little influence on execution time. On the other hand, increasing the number of representatives reduces deviation with respect to the reference case but it has a higher impact on simulation time.

## 10 . CONCLUSIONS

The presented hierarchical model allows a clear and highly detailed representation of a power system operation, allowing extending it to medium- and long-term horizons. It has been applied to the Spanish power system with suitable results.

## 11 . ACKNOWLEDGEMENT

This research has been supported and technically assisted by ENDESA.

## 12 . REFERENCES

- [1] R.Y. Rubinstein, "Simulation and the Monte Carlo Method" John Wiley & Sons Inc., New York, 1981.
- [2] A.M. Law, W.D. Kelton, "Simulation Modelling and Analysis" Mc Graw Hill Book Company, New York, 1991.
- [3] R.A. Babb, P.R. Schrader, "The POWRSYM production cost model" EPRI Proceeding of the 1985 Chattanooga Conference on Production Simulation of Electric Power Production, Palo Alto, California, USA, 1987.
- [4] "BENCHMARK A computer program for simulation of hourly generation" EPRI Palo Alto, California, USA, 1988.
- [5] S.T. Lee, "Comparison of Planning Codes for Energy Storage Evaluation" Electric Power Consulting, Inc. San José, CA, USA. April, 1993.
- [6] J. Román, R.N. Allan, "Sequential Simulation Applied to Composite System Reliability Evaluation", IEE Proc.-C, Vol. 139, No. 2.
- [7] R. Billinton, A. Jonnavithula, "Application of Sequential Monte Carlo Simulation to Evaluation of Distributions of Composite System Indices" IEE Proceedings on Generation, Transmission and Distribution, Vol. 144, No. 2, Marzo 1997.
- [8] L. Kauffman, P.J. Rousseeuw. "Finding Groups in Data" John Wiley & Sons Inc., Nueva York, 1990.
- [9] T.S. Dillon, K.W. Edwin et al. "Integer

**Programming Approach to the Problem of Optimal Unit Commitment with Probabilistic Reserve Determination”** IEEE Transactions on Power Apparatus and Systems. Vol. PAS-97, No. 6, pp. 2154-2166. November/December 1978.