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# Strategic Bidding under Uncertainty in a Competitive Electricity Market

Álvaro Baíllo  Mariano Ventosa  Michel Rivier  Andrés Ramos

INSTITUTO DE INVESTIGACIÓN TECNOLÓGICA
Universidad Pontificia Comillas
Alberto Aguilera 23
28015 Madrid, SPAIN
alvaro.baillo@iit.upco.es

Abstract: The electricity industry is suffering an intense process of re-regulation throughout the world. In many cases generation companies are summoned to submit bids in some sort of organized market where demand and supply are matched and a clearing price for electricity results. In this context, bidding effectively the production of the generating units becomes a task of paramount importance. The uncertainty about the behavior of the competitors and the problems inherent in the operation of the generating units make bidding a very complex activity. In this paper a systematic and automatic bidding procedure is developed. It consists of a probabilistic optimization tool oriented to the construction of profit-maximizing hourly offer curves. This tool combines a probabilistic representation of the market with traditional production modeling techniques. Results of the application of the method to a numerical example are presented.

Keywords: Generation scheduling, competitive electricity market, bidding strategies.

## I. INTRODUCTION

The electricity industry is in the midst of a profound process of re-regulation in an increasing number of countries. These changes are intended to bring about competition in some of the electricity business activities so as to promote a higher level of efficiency in the provision of electric services.

Electricity generation companies have traditionally been subject to regulatory policies which, to a certain extent, guaranteed the recovery of their costs. In the new framework, generation firms have to compete to sell the electric services provided by their facilities. Therefore, they are now exposed to a higher degree of uncertainty and risk. New procedures and tools devoted to the maximization of the firm's profit that take into account the different market mechanisms and evaluate the degree of risk exposure, are needed.

In many cases, wholesale electricity markets are organized as daily uniform-price multi-unit auctions where suppliers have to bid their production in the form of blocks of energy at different prices [1]. Consequently, daily bidding becomes an activity that requires the maximum attention from generation companies, as their benefits are subject to the results obtained in these auctions.

In some cases, as in England and Wales, generation firms have to bid a unique offer curve for each whole day. This makes bidding a repeated daily game, which eventually may reach equilibrium. Green and Newbery [2] have looked into this possibility and have adapted Klemperer and Meyer's theory of supply function equilibrium (SFE, [3]) to the English case. Furthermore, Newbery analyzed the dependence of the shape of the supply function on the existence of contracts or the threat of market entry [4]. The most significant conclusion is that, given a set of firms with different cost structures, a variety of SFE exists. The range of equilibria goes from Cournot equilibrium, which results in the highest prices, to perfect competition, where prices are given by marginal costs. This range is wider as the number of competing firms decreases. Thus a potential usage of supply curves to exercise market power is possible. This matter has been studied by Rudkevich [5].

Anderson and Philpott [6] have addressed the daily bidding problem of a generation firm by proposing a model in which a generator constructs its profit-maximizing supply curve as a continuous function, considering that the opponents do not immediately react to this bid. In their approach the uncertainty of both the demand and the competitors' behavior is represented by means of a market probability distribution function which, for a certain bid, gives the probability that it won't be accepted. In this framework they develop the expression of the necessary optimality conditions for a supply function to be locally optimal. Their approach is very systematic and seems powerful, but no technical constraints are considered.

Wolak [7] proposes a very similar approach to that in [6] and uses it to analyze the influence of forward contracts on bidding behavior in the National Electricity Market in Australia. He describes how a generator tends to bid its production at lower prices when a part of it has been forwardly contracted at a fixed price. He also highlights the difficulty of obtaining optimal daily bids.

The paramount importance of daily bidding and its inherent complexity demand systematic and automatic bidding procedures. In this paper we present a probabilistic optimization tool oriented to the construction of profit-maximizing hourly offer curves. This tool combines a probabilistic representation of the competitors with traditional power generation modeling techniques. The method allows a generation company to take into account different factors such as fuel prices, available generating units, water reserves and positions in futures contracts.

This paper is organized as follows. Section II discusses the medium-term decisions that affect to the short-term activities of a generation firm. Section III gives a brief description of the optimization tool, while section IV presents the mathematical formulation. A numerical example has been solved, and the results are shown in section V. Finally, section VI outlines the conclusions drawn from this research.

## II. MEDIUM-TERM GUIDELINES

As in the past, a generation firm has to decompose its management decisions into different time scopes, so that the objectives fixed for a longer time horizon directly affect the goals of the short term (one day to a week). Two particularly important short-term decisions of the generation firm are the weekly unit commitment ([8], [9]) and the daily bids submitted to the market. Both of them strongly depend on the medium-term strategies followed by the firm. These medium-term strategies are a combination of operational and risk-hedging decisions that result from the market scenarios envisaged by the firm in a period ranging from one month to a year.

In this paper the influence of four medium-term decisions on the daily bidding process will be analyzed. In first place, electricity generation is strongly conditioned by the price of fuel for thermal plants. In the medium term, a generation firm will decide to sign certain fuel contracts by estimating the expected future levels of demand and electricity prices. In the short term, the firm must adapt its bids to the prices at which the fuel stocks have been purchased. Secondly, the unavailability of a generating unit reduces the volume of production the firm is able to offer in the market. Depending on the relative size of the unit this can cause a more or less slight price rise. Thirdly, managing hydro resources in the medium term means distributing the usage of water along the planning period (typically about a year). Once these medium-term decisions have been made, in the short term the generation company must choose how to bid the volume of hydro production assigned to that specific period. Finally, the progressive growth of electricity derivatives markets will allow firms to take medium-term financial positions so as to hedge their risk exposure in the electricity spot market ([10], [11]). If a generation company keeps an open position in a derivatives contract, this has to be considered in the bidding process.

A generation firm also faces threats indirectly related to the spot price of electricity that can be triggered if the firm takes advantage of all the short-term profit opportunities that arise in the spot market. These threats include the punishing response of its competitors to a possible breakdown of the existing equilibrium, the entry of new participants encouraged by high prices and the fearsome intervention of regulatory authorities. Therefore, generation firms may have a strong incentive to ration their short-term greed.

## III. MODEL DESCRIPTION

### A. General Overview

The goal of the probabilistic bidding procedure is to build a set of offer curves, one for each hour or load level, that maximizes the expected profit of the firm, defined as the difference between the expected revenue and the expected production cost.

### B. Market representation

The bidding process is affected by the demand curve and the competitors' supply functions, both of which are uncertain. If the firm increases its energy output, lower prices will result. This is due to the combined effect of a decrease in the competitors' output and an increase in the energy consumption. Therefore, the firm is able to sell more energy at lower prices and less energy at higher prices. The amount of energy that the firm is able to sell at each price is given by the residual-demand function.

How to estimate the residual-demand curve is a complex issue. In the Spanish wholesale electricity market, generation firms learn their competitors' bids one month after they were submitted, while in California it takes three months. In spite of the delay, this is the available information and has to be squeezed.

One alternative is to select, for each of tomorrow's hours, a set of Z past residual-demand curves which are likely to appear. These representative curves, if relevant and numerous enough, can be used to estimate the results that the generation firm will obtain if a certain offer curve is submitted (Figure 1). Notice that, by using a finite set of residual-demand scenarios, we are assuming that the probability of crossing tomorrow's residual demand curve between two historic residual-demand curves is equal to zero. In other words, we are concentrating the probability density in Z residual-demand curves.

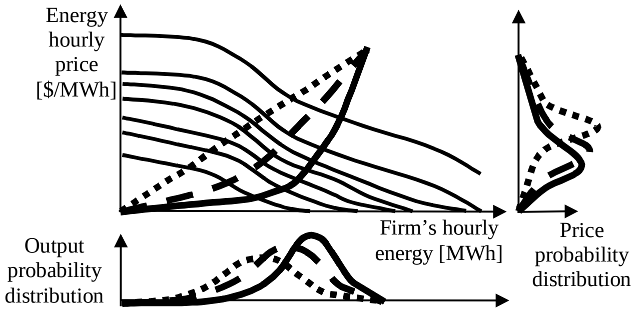


Figure 1. Residual-demand curves and probability distributions.

### C. Offer curve

With this assumption, building an offer curve is equivalent to choosing Z pairs quantity-price, $\left(q_{\zeta}, p_{\zeta}\right)$ keeping in mind that they must be increasing in both terms.

Each offer curve yields a probability distribution for the market outcome. Therefore, a certain offer curve will have associated probability distributions for the clearing price, the firm's production, its revenues and its costs.

### D. Linearized Revenues

Suppose that we decide to cross the $\zeta$-th residual-demand scenario of hour $n$, whose probability is $\rho_{\zeta n}$, at a point defined by quantity $q_{\zeta n}$. and price $p_{\zeta n}$. Then the expected revenues for the entire time scope are given by

$$r = \sum_{n \in N} \sum_{\zeta \in Z(n)} \rho_{\zeta n} p_{\zeta n} q_{\zeta n} .$$

The former is a non-linear function, as the price depends on the offered quantity, $q_{\zeta n}$. This is a drawback, because the most powerful commercial optimizers are those designed to solve linear programming problems. To overcome this difficulty we will use the linearizing technique described in [8]. This intuitive method divides the firm's hourly revenue function into convex sections and approximates each one by a piecewise linear function. The slope obtained for each linear segment is the firm's marginal revenue at the corresponding energy output (Figure 2). Each convex section, $i$, is assigned a binary variable, $v_{i\zeta n}$, and each linear segment, $j$, is assigned a continuous bounded variable, $q_{ji\zeta n}$.

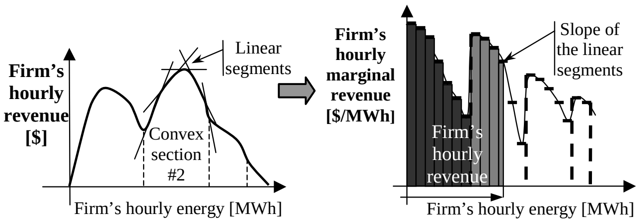


*Figure 2. Firm's hourly marginal revenue function.*

A group of consecutive segments with strictly decreasing marginal revenues defines a convex section in the revenue function. When we seek the optimum we select a specific convex section by switching its binary variable from zero to one. Once we have chosen a convex section we fill its segments with continuous bounded variables. In other words, we obtain the hourly revenue by integrating the marginal-revenue function. Prices are not explicitly used to calculate the firm's revenue.

## E. Open positions in derivatives contracts

Consider that a position in a contract for differences (CfDs) is left open for load level $n$ and for a certain quantity $q_{cn}$ at a fixed price $p_{cn}$. The expected revenues simply change in that less quantity has to be valued at the expected market clearing price. This requires using price as a explicit variable.

## F. Ovelapping residual-demand curves

When the residual-demand curves overlap as in Figure 3, guaranteeing the increasing property of the offer curve that links the Z bids may be a little less obvious. If a residual-demand curve, A, intersects with other residual-demand curve, B, then a binary variables, $x_{\mathrm{AB}n}$ has to be defined such that when curve A runs above curve B $x_{\mathrm{AB}n}$ is equal to zero. In other case, $x_{\mathrm{AB}n}$ = 1. Four constraints have to be introduced. Two of these constraints refer to quantities and the other two refer to prices:

$$q_{\mathrm{A}n} - q_{\mathrm{B}n} \geq -x_{\mathrm{AB}n} M^q ,$$
$$q_{\mathrm{B}n} - q_{\mathrm{A}n} \geq -(1 - x_{\mathrm{AB}n}) M^q ,$$
$$p_{\mathrm{A}n} - p_{\mathrm{B}n} \geq -x_{\mathrm{AB}n} M^p ,$$
$$p_{\mathrm{B}n} - p_{\mathrm{A}n} \geq -(1 - x_{\mathrm{AB}n}) M^p ,$$

where $M$ is a very big number. This requires using price as a explicit variable.

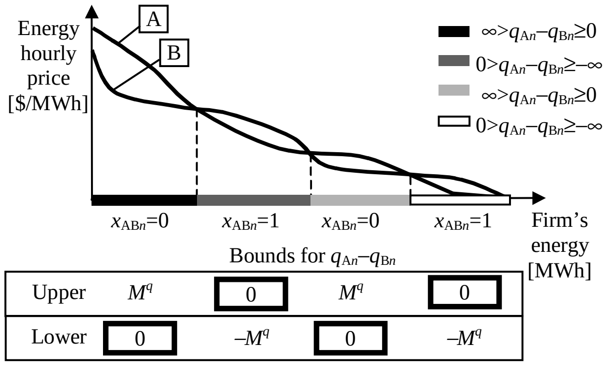


Figure 3. Overlapping residual-demand curves in hour $n$.

## G. Multiperiod extension

If $N$ hours are considered, then $N$ sets of Z residual demand curves must be used. This suggests using a scenario tree representation. Figure 4 shows an example for $N$=4 hours, Z=2 scenarios in each hour.

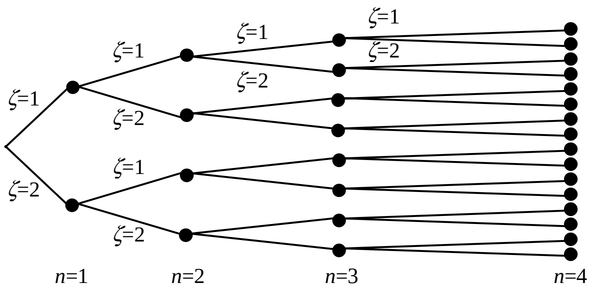


*Figure 4. Residual-demand-curve scenario tree.*

However, it must not be forgotten that all decisions (offers) are made simultaneously. Therefore, the offer for scenario $\zeta$ in hour $n$ will be unique and independent of the scenario that takes place in hour $n-1$. A recombining scenario tree is proposed as the best way to represent this decision process (Figure 5).

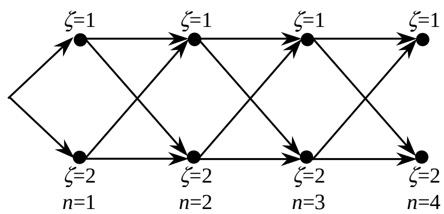


Figure 5. Residual-demand-curve scenario tree.

When several hours are considered, interperiod relations must be incorporated to the model. These include thermal units' ramping constraints, start-up and shutdown decisions or the management of hydro reserves among different hours. In this model the expected daily usage of hydro resources is fixed according to the information received from medium-term hydrothermal coordination models.

# IV. MATHEMATICAL FORMULATION

## A. Notation

In this section the symbols used in this paper are identified and classified according to their use. Table 1 shows the indices and sets considered, being capitals used for sets and lower-case for indices. Table 2 defines the information given to the model as fixed data. Table 3 includes the auxiliary variables. Decision variables are shown in Table 4.

*Table 1. Indices and sets.*

| Notation | Definition |
|---|---|
| $b, B$ | Pumped-storage units. |
| $c, C$ | Contracts for differences. |
| $g, G$ | Generating units. |
| $h, H$ | Hydro units. |
| $i, I$ | Convex sections for the approximation of the revenue function. |
| $j, J$ | Segments for the approximation of the firm's revenue function. |
| $n, N$ | Load levels. |
| $t, T$ | Thermal units. |
| $\zeta, \xi$ | Scenarios of residual demand. |
| $\mathrm{Z}(n)$ | Residual-demand scenarios in load level $n$. |

*Table 2. Parameters.*

| Notation | Definition |
|---|---|
| $\overline{d}_b,\ \underline{d}_b$ | Maximum and minimum pumping capacity of unit $b$ [GW]. |
| $D_{\zeta n}$ | System hourly demand in scenario $\zeta$ and load level $n$ [GW] |
| $f_t$ | Fuel cost of thermal unit $t$ [k\$/kTcal]. |
| $k_t$ | Self consumption coefficient of thermal unit $t$ [p.u]. |
| $l_t$ | Ramp rate limit for unit $t$ [GW/h]. |
| $m_{ji\zeta n}$ | Marginal revenue of the segment $j$ of convex section $i$ in scenario $s$ and load level $n$ [k\$/GW]. |
| $o_t$ | O&M variable cost of thermal unit $t$ [k\$/GW]. |
| $p^0_{\zeta n}, p'_{ji\zeta r}$ | Linear approximation (independent [k\$/GWh] and linear [k\$/(GW·GWh)] terms) of residual demand $\zeta$ in load level $n$. |
| $q_c, p_c$ | Quantity [GWh] and price [k\$/GWh] of CfDs $c$. |
| $\overline{q}_g,\ \underline{q}_g$ | Maximum and minimum generating capacity of unit $g$ [GW]. |
| $\overline{q}_{ji\zeta n}$ | Maximum power generation valued with segment $s$ of convex section $c$ in scenario $\zeta$ and load level $n$ [GW]. |
| $r^0_t,\ r'_t$ | Heat rate (independent [kTcal] and linear [kTcal/GW] terms) of thermal unit $t$. |
| $S_n$ | Firm's minimum market share in load level $n$ [p.u.] |
| $s_t$ | Start-up cost for thermal unit $t$ [k\$]. |
| $W^0_g$ | Available energy for hydro or pumped-storage unit $g$ [TWh]. |
| $W^{N+1}_g$ | Final energy for hydro or pumped-storage unit $g$ [TWh]. |
| $\overline{w}_b$ | Upper reservoir limit of pumped-storage unit $b$ [TWh]. |
| $\eta_b$ | Performance of pumped-storage unit $b$ [p.u.]. |
| $\rho_{\zeta n}$ | Probability of scenario $\zeta$ in load level $n$ [p.u.]. |
| $\rho_{\zeta\xi n}$ | Probability of scenario $\zeta$ in load level $n$ conditioned to the occurrence of scenario $\xi$ in load level $n-1$ [p.u.]. |

*Table 3. Auxiliary variables.*

| Notation | Definition |
|---|---|
| $c_{t\zeta n}$ | Operating costs due to thermal unit $t$ in scenario $\zeta$ and load level $n$ [k\$]. |
| $c_T$ | Expected total thermal operating costs [k\$]. |
| $r$ | Expected total revenue [k\$]. |

*Table 4. Decision variables.*

| Notation | Definition |
|---|---|
| $d_{b\zeta n}$ | Power consumption by pumped-storage unit $b$ in scenario $\zeta$ and load level $n$ [GW]. |
| $q_{g\zeta n}$ | Power generation by unit $g$ in scenario $\zeta$ and load level $n$ [GW]. |
| $q_{ji\zeta n}$ | Power generation valued with segment $j$ of convex section $i$ of scenario $\zeta$ and load level $n$ [GW]. |
| $u_{t\zeta n}$ | Commitment decision (0/1) of thermal unit $t$ in scenario $\zeta$ and load level $n$. |
| $v_{i\zeta n}$ | Decision variable (0/1) corresponding to convex section $i$ of scenario $\zeta$ and load level $n$. |
| $w_{b\zeta n}$ | Expected available energy for pumped-storage unit $b$ in scenario $\zeta$ at the beginning of load level $n$ [TWh]. |
| $w_{h\zeta n}$ | Expected available energy for hydro unit $h$ in scenario $\zeta$ at the beginning of load level $n$ [TWh]. |
| $x_{\zeta\xi n}$ | Binary variable (0/1) that counts the number of intersections of curves $\zeta$ and $\xi$ in load level $n$. |
| $y_{t\zeta n}$ | Fuzzy start-up decision of thermal unit $t$ in load level $n$. |
| $z_{t\zeta n}$ | Fuzzy shutdown decision of thermal unit $t$ in load level $n$. |
| $p_{\zeta n}$ | Price in scenario $\zeta$ and load level $n$. |

## B. Model Formulation

### Objective Function

The objective function represents the firm's expected profit defined as the difference between the firm's expected revenue and the firm's expected operating costs:

$$Max \quad r - c_T \,. \tag{1}$$

### Thermal generation constraints

Total thermal operating costs include fuel costs, O&M costs, start-up costs and shutdown costs:

$$\begin{aligned} c_T &= \sum_n \sum_{\zeta\in \mathrm{Z}(n)} \rho_{\zeta n}\left(\sum_{t\in T} c_{t\zeta n}\left(q_{t\zeta n}, u_{t\zeta n}, y_{t\zeta n}\right)\right) = \\ &= \sum_n \sum_{\zeta\in \mathrm{Z}(n)} \rho_{\zeta n}\left(\sum_{t\in T}\left\{\begin{matrix} f_t\left(r_t u_{t\zeta n} + o'_t \dfrac{q_{t\zeta n}}{k_t}\right) + \\ o_t q_{t\zeta n} + s_t y_{t\zeta n}\end{matrix}\right\}\right). \end{aligned} \tag{2}$$

For each committed thermal unit the maximum generation is less than the maximum available capacity, and the minimum generation is greater than the minimum stable load:

$$\underline{q}_t k_t u_{t\zeta n} \le q_{t\zeta n} \le \overline{q}_t k_t u_{t\zeta n}\,, \qquad \forall t,\zeta,n\,. \tag{3}$$

The quantity offered must be strictly increasing:

$$q_{t\zeta n} - q_{t\xi n} \ge -x_{\zeta\xi n} M^q\,, \qquad \forall t,\xi > \zeta,\zeta,n\,, \tag{4}$$

$$q_{t\xi n} - q_{t\zeta n} \ge -\left(1 - x_{\zeta\xi n}\right) M^q\,, \qquad \forall t,\xi > \zeta,\zeta,n\,. \tag{5}$$

The hourly change in the output of each thermal unit for each residual-demand trajectory is limited by the ramp rates:

$$-l_t \le q_{t\zeta n} - \sum_{\xi\in \mathrm{Z}(n-1)} \rho_{\zeta\xi n} q_{t\xi n-1} \le l_t\,, \qquad \forall t,\zeta,n\,. \tag{6}$$

A logical relationship exists between the start-up, shutdown and commitment variables:

$$y_{t\zeta n} - z_{t\zeta n} = u_{t\zeta n} - \sum_{\xi \in Z(n-1)} \rho_{\zeta\xi n} u_{t\xi n-1}, \qquad \forall t,n. \quad (7)$$

Since the commitment decision variables are binary, both the start-up and the shutdown decision variables can be continuous but must have upper and lower bounds:

$$0 \le y_{t\zeta n} \le 1, \qquad \forall t,\zeta,n, \quad (8)$$

$$0 \le z_{t\zeta n} \le 1, \qquad \forall t,\zeta,n. \quad (9)$$

**Hydro generation constraints**

The available energy for hydro and pumped-storage units is determined by a longer scope model:

$$w_{h\zeta 1} = W_h^0, \qquad \forall h,n, \quad (10)$$

$$w_{b\zeta 1} = W_b^0, \qquad \forall b,n. \quad (11)$$

The contents of the reservoirs in each node of the scenario tree depend on the energy produced or stored at the preceding node and have upper and lower bounds:

$$w_{h\zeta n} = \sum_{\xi \in Z(n-1)} \rho_{\zeta\xi n} \left( w_{h\xi n-1} - q_{h\xi n-1} \right), \qquad \forall h,\zeta,n>1, \quad (12)$$

$$W_h^{N+1} = \sum_{\xi \in Z(N)} \rho_{\zeta N} \left( w_{h\xi N} - q_{h\zeta N} \right), \qquad \forall h, \quad (13)$$

$$0 \le w_{h\zeta n}, \qquad \forall h,\zeta,n, \quad (14)$$

$$w_{b\zeta n} = \sum_{\xi \in Z(n-1)} \rho_{\zeta\xi n} \left( w_{b\xi n-1} - q_{b\xi n-1} + \eta_b d_{b\xi n-1} \right), \forall b,\zeta,n>1, \quad (15)$$

$$W_b^{N+1} = \sum_{\xi \in Z(N)} \rho_{\zeta N} w_{b\zeta N} - q_{b\zeta N} + \eta_b d_{b\zeta N}, \qquad \forall b, \quad (16)$$

$$0 \le w_{b\zeta n} \le \overline{w}_b, \qquad \forall b,\zeta,n. \quad (17)$$

Each unit has an upper and a lower limit for its output:

$$\underline{q}_h \le q_{h\zeta n} \le \overline{q}_h, \qquad \forall h,\zeta,n, \quad (18)$$

$$0 \le q_{b\zeta n} \le \overline{q}_b, \qquad \forall b,\zeta,n, \quad (19)$$

$$0 \le d_{b\zeta n} \le \overline{d}_b, \qquad \forall b,\zeta,n. \quad (20)$$

The quantity offered must be strictly increasing:

$$q_{h\zeta n} - q_{h\xi n} \ge -x_{\zeta\xi n} M^q, \qquad \forall h,\xi>\zeta,\zeta,n, \quad (21)$$

$$q_{h\xi n} - q_{h\zeta n} \ge -\left(1 - x_{\zeta\xi n}\right) M^q, \qquad \forall h,\xi>\zeta,\zeta,n. \quad (22)$$

$$q_{b\zeta n} - q_{b\xi n} \ge -x_{\zeta\xi n} M^q, \qquad \forall h,\xi>\zeta,\zeta,n, \quad (23)$$

$$q_{b\xi n} - q_{b\zeta n} \ge -\left(1 - x_{\zeta\xi n}\right) M^q, \qquad \forall h,\xi>\zeta,\zeta,n. \quad (24)$$

$$-d_{b\zeta n} + d_{b\xi n} \ge -x_{\zeta\xi n} M^q, \qquad \forall h,\xi>\zeta,\zeta,n, \quad (25)$$

$$-d_{b\xi n} + d_{b\zeta n} \ge -\left(1 - x_{\zeta\xi n}\right) M^q, \qquad \forall h,\xi>\zeta,\zeta,n. \quad (26)$$

**Market constraints**

Each segment of the firm's net hourly energy output is valued at a different marginal revenue. The sum of all the segments must equal the sum of the power produced by thermal and hydro units minus the power consumed by pumped-storage units:

$$\sum_{i \in I} \sum_{j \in J} q_{ji\zeta n} = \sum_{g \in G} q_{g\zeta n} - \sum_{b \in B} d_{b\zeta n}, \qquad \forall \zeta,n. \quad (27)$$

The power offered for a certain residual-demand curve determines the relative position of the rest of curves

$$\sum_{i \in I} \sum_{j \in J} q_{ji\zeta n} - \sum_{i \in I} \sum_{j \in J} q_{ji\xi n} \ge -x_{\zeta\xi n} M^q, \qquad \forall \xi>\zeta,\zeta,n, \quad (28)$$

$$\sum_{i \in I} \sum_{j \in J} q_{ji\zeta n} - \sum_{i \in I} \sum_{j \in J} q_{ji\xi n} \ge -\left(1 - x_{\zeta\xi n}\right) M^q, \quad \forall \xi>\zeta,\zeta,n. \quad (29)$$

Each segment has an upper and a lower bound and the convex sections must be chosen in order:

$$q_{ji\zeta n} \le v_{i\zeta n} \overline{q}_{ji\zeta n}, \qquad \forall j,i,\zeta,n, \quad (30)$$

$$v_{i\zeta n} \overline{q}_{ji-1\zeta n} \le q_{ji-1\zeta n}, \qquad \forall j,i>1,\zeta,n, \quad (31)$$

$$v_{i\zeta n} \le v_{i-1\zeta n}, \qquad \forall i>1,\zeta,n. \quad (32)$$

Price for each scenario is determined with a linear approximation of the residual-demand curve:

$$p_{\zeta n} = p_{\zeta n}^0 + \sum_{i \in I} \sum_{j \in J} p'_{ji\zeta n} q_{ji\zeta n}, \qquad \forall \zeta,n. \quad (33)$$

Prices must also be increasing:

$$p_{\zeta n} - p_{\xi n} \ge -x_{\zeta\xi n} M^q, \qquad \forall \xi>\zeta,\zeta,n, \quad (34)$$

$$p_{\xi n} - p_{\zeta n} \ge -\left(1 - x_{\zeta\xi n}\right) M^q, \qquad \forall \xi>\zeta,\zeta,n. \quad (35)$$

We calculate the total expected revenue by valuing the different segments of the net energy output at their corresponding marginal revenues. In other words, the expected revenue is obtained by integrating the marginal revenue function. CfDs are also considered.

$$r = \sum_{n \in N} \sum_{\zeta \in Z(n)} \rho_{\zeta n} \left( \sum_{i \in I} \sum_{j \in J} m_{ji\zeta n} q_{ji\zeta n} + \left( p_c - p_{\zeta n} \right) q_c \right). \quad (36)$$

**Strategic constraints**

We define a set of hourly minimum-market-share constraints. In the numerical example we investigate the influence of this constraint on hourly prices and on the firm's short-term benefit. In our formulation we suppose that demand is stochastic, but also perfectly inelastic. Consequently, the only variations of demand we allow are those introduced by pumping:

$$\sum_{\zeta \in Z(n)} \rho_{\zeta n} \left\{ S_n \left( D_{\zeta n} + \sum_{b \in B} d_{b\zeta n} \right) - \sum_{g \in G} q_{g\zeta n} + \sum_{b \in B} d_{b\zeta n} \right\} \le 0, \forall n \quad (37)$$

## V. NUMERICAL EXAMPLE

The probabilistic bidding procedure has been implemented in GAMS [12]. A study case has been solved with the optimiser CPLEX 6.6.

### A. Study case

The firm's generating equipment is described in Table 5.

*Table 1. Firm's generating units.*

| Type of unit | Number of units | Installed capacity (MW) |
|---|---|---|
| Nuclear | 3 | 2860 |
| Coal-fueled | 6 | 1440 |
| Oil/Gas-fueled | 4 | 1995 |
| Hydro | 1 | 2000 |
| Pumped-storage | 1 | 200 |

Three hourly load levels are studied: an off-peak hour, an intermediate ramping hour and an on-peak hour. In each hour we will consider three residual-demand scenarios (Figure 6). Also, three trajectories are defined. Trajectory $\tau$ is formed by the $\tau$-th scenario of each of the three hours.

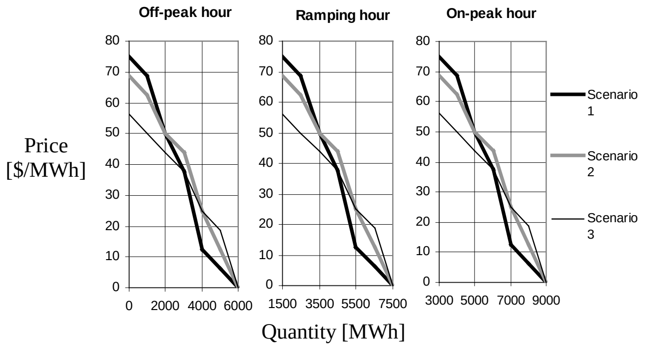


Figure 6. Residual demand probability distribution.

A expected hydro production of 1500 MWh has been assigned to these three hours. Additionally, a expected pumped-storage net production of 100 MWh will be achieved.

### B. Results

The study case was solved in a PC Pentium III 550 MHz 256 MB in 1.89 seconds. The offers shown in Figure 7 were given by the model. Circular offers were obtained when the increasing constraints were not used. As can be seen, circular offers could not be used as real offers, as they do not constitute an increasing curve. When the increasing constraints were included the square offers were obtained.

The procedure chooses to bid in a strategic manner, partly deviating from marginal costs and trying to reach high prices. This is consistent with the portfolio bidding strategy Although the firm may not produce with certain units whose marginal costs are somewhat lower than the market-clearing price, the model correctly interprets that this yields higher benefits in the short term. In the long term, however, this would lead to a loss of market share.

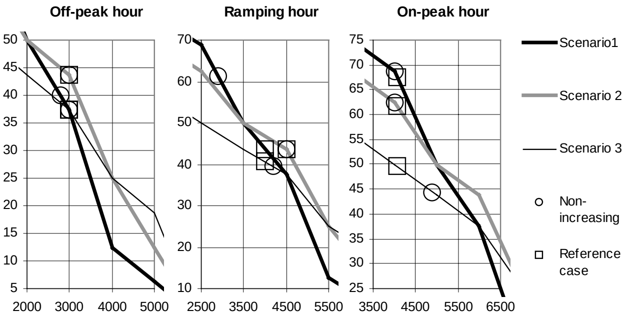


Figure 7. Offer curves for three different hours.

In Figure 8 the power offered at each scenario and load level has been classified according to the type of unit. It can be observed that the model is able to manage water reservoirs among the three hours depending on the price levels expected. In particular, the model suggests pumping water in the off-peak hour and using it in the other two hours.

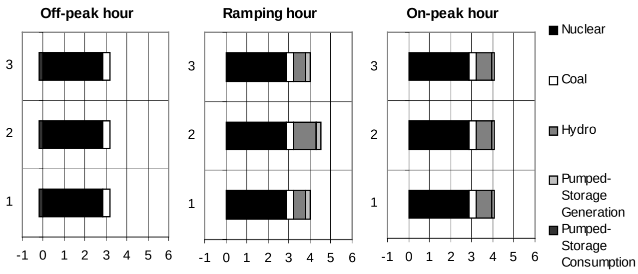


Figure 8. Power offered by type of unit.

Henceforth several changes in the problem data will be introduced to analyze the influence of different factors on the resulting offer curves.

### C. Influence of the strategic constraints

A minimum-market-share constraint has been introduced. This illustrates the influence of medium-term strategic guidelines on the construction of the offer curves (Figure 9). The model changes the offer curve significantly in the off-peak and on-peak hours. This indicates that trying to keep the firm's market-share in the short term may require a reduction of expected profits. The firm must be able to decide to what extent short-term opportunities must be lost in order to keep the position of the firm in the market.

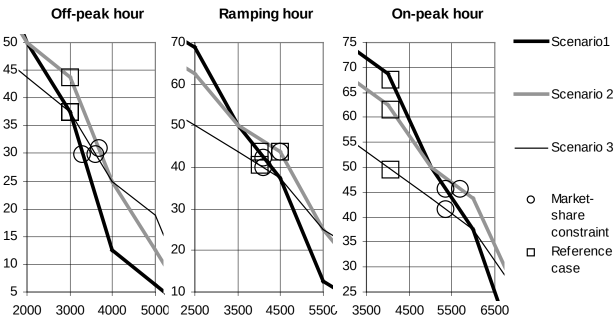


Figure 9. Influence of minimum-market-share constraints.

### D. Influence of fuel prices

A reduction has been introduced in the price of coal. The results are shown in Figure 10. The offer curves obtained are almost the same as in the original case. This confirms that the offer curves obtained for the reference case do not depend strongly on marginal costs.

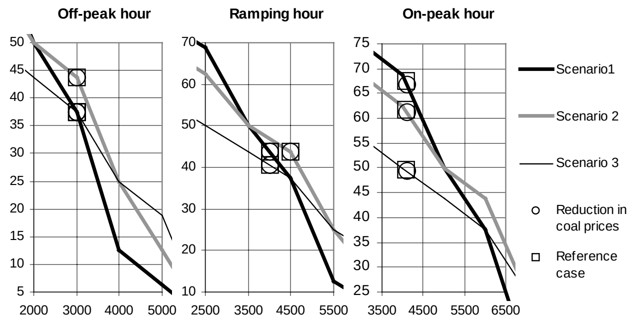


Figure 10. Influence of fuel prices.

### E. Influence of unit availability

The offer curves have also been obtained assuming that a nuclear unit was not available. As shown in Figure 11, this causes a substantial change in two of the three hours. An increase of prices is expected in the ramping hour.

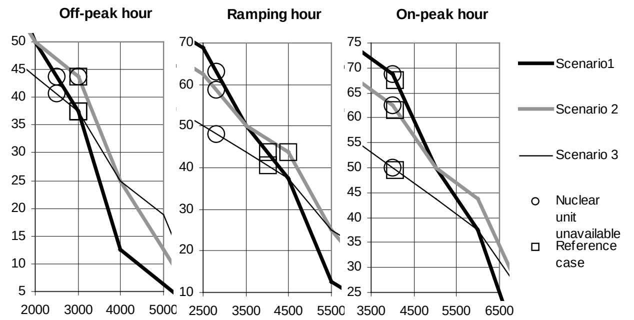


Figure 11. Influence of unit availability.

### F. Influence of available hydro resources

In the original case 1500 MWh of hydro energy were available for the three hours. Two additional cases, with 500 MWh and 2500 MWh respectively, have been studied. The offers given by the model are shown in Figure 12.

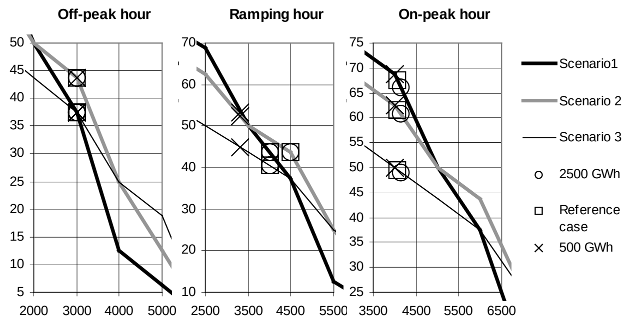


Figure 12. Influence of available hydro resources.

When only 500 MWh are available, the model provides a steeper offer curve for the ramping hour. In contrast, when up to 2500 MWh have to be produced, the model substitutes thermal production with water in the on-peak hour. This shows how the model is able to reallocate hydro resources among the different hours and highlights the flexibility of hydro units.

### G. Influence of open positions in CfDs

An open position in a 5000 MWh CfDs at a price of 37.5 \$/MWh for the ramping hour has been considered. This causes an increase of production in that specific hour (Figure 13). This is consistent with the analysis performed by Wolak [7], who concludes that when generation companies sell part of their production using long-term contracts, they tend to bid more competitively in short-term markets. It can be seen that the clearing price envisaged for the ramping hour is even lower than the one expected for the off-peak hour.

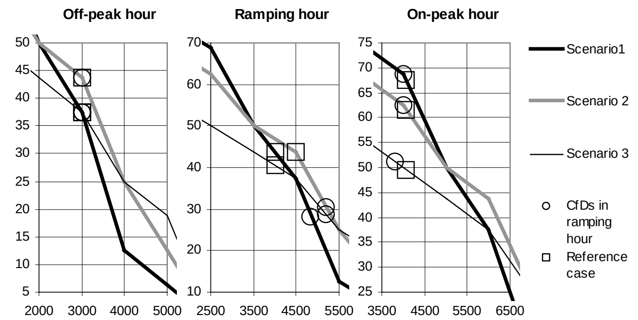


Figure 13. Influence of an open position in a CfDs.

## VI. CONCLUSION

In this paper a probabilistic procedure designed to provide hourly offer curves that maximize the expected benefits of a generation company operating in a daily electricity market has been developed. This optimization tool incorporates a probabilistic representation of the behavior of both the firm's competitors and the buyers of energy. The model also includes equations that take into account operating issues of the generating units, such as technical constraints, production costs, availability of the units and the management of hydro resources.

The model has been implemented in GAMS language as a mixed integer linear programming problem. A three-hour study case has been solved to prove the adequacy of the proposed approach. Several analysis have been performed to illustrate the sensitivity of the solution to certain factors. The results obtained are encouraging.

## VII. ACKNOWLEDGEMENTS

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## VIII. REFERENCES


[1] M.H. Rothkopf, "Daily Repetition: A Neglected Factor in the Analysis of Electricity Auctions". The Electricity Journal. April 1999, pp. 60-70.

[2] R.J. Green, D. M. Newbery, "Competition in the British Electricity Spot Market", Journal of Political Economy, Vol. 100 No. 5 pp 929-953, 1992.

[3] P. D. Klemperer, M. A. Meyer, "Supply Function Equilibria in Oligopoly under Uncertainty". Econometrica 57 (November 1989). pp.1243-77.

[4] D. Newbery, "Competition, contracts and Entry in the Electricity Market". Working Paper No. 9707. Department of Applied Economics, Cambridge University, January 1997.

[5] A. Rudkevich, M. Duckworth and R. Rosen. "Modeling electricity pricing in a deregulated generation industry: The potential for oligopoly pricing in a Poolco", *The Energy Journal*, Vol. 19 No. 3. pp 19-48. 1998.

[6] E. J. Anderson, A. B. Philpott, "Optimal offer curve construction in electricity markets". Available at http://www.esc.auckland.ac.nz/People/Staff/Philpott/

[7] F. A. Wolak, "An Empirical Analysis of the Impact of Hedge Contracts on Bidding Behavior in a Competitive Electricity Market". Draft prepared for the 4th Annual Power Research Conference. Berkeley, CA, March 1999.

[8] A. Baillo, M. Ventosa, A. Ramos, M. Rivier, A. Canseco, "Strategic Unit Commitment for Generation Companies in Deregulated Electricity Markets". DIMACS/EPRI Workshop *The Next Generation of Unit Commitment Models*. Rutgers University, September 1999. Under review for the book *The Next Generation of Unit Commitment Models*.

[9] J. García, J. Román, J. Barquín, A. González, "Strategic Bidding in Deregulated Power Systems". 13th PSCC Conference, Norway. Vol. 1 pp. 258-264. June 1999.

[10] A. Eydeland, H. Geman, "Fundamentals of Electricity Derivatives". *Energy Modelling and the Management of Uncertainty.* pp. 35-43. Risk Books. 1999.

[11] S. Stoft, T. Belden, C. Goldman, S. Pickle, "Primer on Electricity Futures and Other Derivatives". LBNL-41098, UC-1321. Ernest Orlando Lawrence Berkeley National Laboratory. University of California. January 1998.

[12] A. Brooke, D. Kendrick, A. Meeraus, *GAMS A User's Guide.* Boyd and Fraser, 1992


## IX. BIOGRAPHIES

**Alvaro Baillo** (S '2000) was born in Madrid, Spain, in 1974. He obtained a degree in Electrical Engineering degree in 1998 from the Universidad Pontificia Comillas, Madrid, Spain. He is currently a Research Fellow at the Instituto de Investigación Tecnológica (IIT). His areas of interest include the operations, planning and economics of energy systems and the application of Operations Research to the energy industry.

**Mariano Ventosa** was born in Madrid, Spain, in 1967. He obtained a degree in Electrical Engineering in 1989 and a degree in Electronic Engineering in 1993, from the Universidad Pontificia Comillas, Madrid, Spain. He is a Lecturer of Electric Machines at the Industrial Engineering School (ICAI) of the same University and a Research Fellow at the Instituto de Investigación Tecnológica (IIT). His areas of interest include operations, planning and economics of electric energy systems and application of operations research in electric energy markets.

**Michel Rivier** obtained a degree in Electrical Engineering in 1986 and a Ph.D. degree in Electrical Engineering in 1998, from the Universidad Pontificia Comillas, Madrid, Spain. He is Deputy Director at IIT and Professor at the Electrical Engineering Department of ICAI. His areas of interest include operations, planning, economy and regulation of electric energy systems, and application of operations research to electric energy systems. He has participated in several consultancy and research projects concerning these subjects for several Spanish and foreign firms related with the electricity industry.

**Andrés Ramos** obtained a degree in Electrical Engineering, from the Universidad Pontificia Comillas, Madrid, Spain in 1982 and a Ph.D. degree in Electrical Engineering from Universidad Politécnica de Madrid, Madrid, Spain in 1990. He is Head of Industrial Engineering and Operation Research Department at ICAI and a Professor at IIT. His areas of interest include operations, planning and economy of electric energy systems, application of operations research to electric energy systems, and software development.