

# A Benders' Decomposition Approach for Optimizing the Electric System of Offshore Wind Farms

S. Lumbreras, A. Ramos

**Abstract**— Optimizing the electric design of an offshore wind farm is a key issue given cost and reliability considerations. The model presented here has been developed to efficiently optimize the electric layout of a wind farm taking into account stochasticity in wind scenarios and component failures. Given the suitable structure of the problem, Benders' decomposition method has been applied to the model successfully finding optimal solutions in a reduced computation time.

**Index Terms**—Offshore installations, Wind Energy, Power System Reliability, Circuit Optimization, Integer Linear Programming, Stochastic Optimization.

## I. INTRODUCTION

OFFSHORE wind power is one of the technologies attracting the largest growth in the past few years. This growth is manifesting itself in the unparalleled research effort that is causing the technology to evolve quickly (average turbine size installed in 2009 was 2.9 MW [1], but the machines that will be used in coming projects will be considerably bigger, with the Siemens 3.6 MW model becoming a standard and even 5 MW turbines like the REPower ones that are planned to be installed in the Ormonde farm in the UK [2]. This size increase in generators has been matched by an increase in average farm installed power. Average size in 2009 was 72.1 MW [1], but there are now much bigger plants under construction, like Greater Gabbard in the UK with 504 MW, Bard 1 in Germany with 400 MW or Sheringham Shoal also in the UK with 315 MW. Indeed, there will be a twofold increase shortly if we look at the proposed projects, with Dogger Bank in the UK leading the list with 9000 MW and other ten projects over 1000 MW in the UK and Ireland. These developments must be grounded on investment: funds attracted in 2009 were approximately EUR 1.5 bln, and this number is expected to approximately double in 2010 [1].

There is also an understanding that offshore wind will be indispensable for the European Union target of reducing

carbon emissions by 20% by 2020 and 80% in 2050 [3]. Due to the shallow waters in the North Sea, the Baltic Sea and the Irish Sea present ideal conditions for this technology; the countries in the North of Europe are expected to export electricity generated in huge offshore wind farms. The UK and Denmark have already expressed their interest in leading the market and are backing their statements with substantial investment in a pipeline of projects that is already delivering competitive functioning plants.

The characteristics of the offshore environment have a profound impact on the plant design and cost. Firstly, all cables, transformers and other electrical equipment are, given the insulation requirements, much more expensive. It is estimated [4] that the cost of a plant onshore is 75% imputable to the turbines, while that number is reduced to 30-50% in their offshore counterparts. The electric components make up for around 8% of the total cost onshore and 18% offshore. This makes the selection of the best electric layout a key aspect of project cost. This design problem is complex and computationally expensive, and given the above mentioned growing trend in farm size can only be expected to get both more complex and more relevant.

A further repercussion of sea conditions is that the failure rates of components, both in the turbines and in the electrical system, are larger than onshore. Moreover, repairs take substantially longer, as it is impossible to gain access to the turbine in adverse weather conditions. These situations might arise unexpectedly, like storms (that is why Siemens decided in 2010 that they will install a small emergency refuge in all their new turbines), or be essentially the harsh months of the year. This long repair times mean that reliability is essential. Therefore, the electric design optimization must take into account the stochastic failures in the system components.

These stochastic failures, added up to the naturally present uncertainty of wind speed, make the problem very heavy computationally.

This paper is structured as follows. First, the state of the art of the problem will be reviewed. Then, the mathematical model that underlies OWL (Offshore Windfarm Layout optimizer) will be presented. After that, the Benders' decomposition technique applied is described, together with some improvements on the implementation. Finally, the results of the model in computation times will be reviewed and conclusions extracted.

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## II. STATE OF THE ART

Although the optimization of the electric design of offshore wind farms has only appeared in the literature relatively recently, the transmission expansion planning problem resembles it enough to make it worthwhile to refer also to this field.

The technical aspects of the electrical systems for offshore wind farms have attracted attention in the past few years. Several technologies are compared in [5], [6] and [7] with the aim of identifying the most competitive (AC or DC) depending on plant characteristics. AC in medium voltage seems to be better for small sizes and short distances to shore, with a transformer lowering total costs at distances above 20 km. From 130 km on, AC/DC appears to be the best option. Indeed, given the voltage drops in cables, it could be the only viable layout for very long distances. An alternative where the turbines would generate DC should be considered for the larger plants. Reference [8] highlights that HVDC can also present fewer losses for long distances to shore and present some other desirable qualities like avoiding resonance and fault propagation, as well as allowing for reactive power control in the case of VSC technology. The general connection patterns have also been studied from a theoretical viewpoint, like in [4]. They evaluate different options: radial, single sided ring and double sided ring, in increasing order of reliability and cost. They propose an alternative to the single sided ring by linking the turbines in extreme rows and connecting them with a cable of rated power equal to the power of one of the lines. However interesting this sort of general analysis might be as general considerations, they lose all the particularities of a specific plant, so a full optimization on the individual project like the one performed in OWL is still desirable.

Stochasticity was long ago taken into account for power system expansion planning. Already [9] acknowledges the importance of demand or fuel cost uncertainty. Stochasticity in the aggregate wind power production for a system was included in analysis like the one in [10].

Reliability can be taken into account using two very different approaches [11]. Deterministic techniques have the advantage of being very compact but can be oversimplifying. Such approaches have been used for example in [12]. Applying Monte Carlo techniques to reliability allows taking into account the chronological characteristics of wind patterns as in [13]. Reliability indices have been developed with the aim of quickly comparing different layouts. One of the most used [12] is the Generation Ratio, defined as the energy at the point of common connection divided by the total generated power.

Several optimization methods have been applied to the problem. Techniques applied include Mixed Integer Non Linear Programming [14] and Monte Carlo [15]. Metaheuristics have also been tested on the problem. References [16] and [17] apply a genetic algorithm where they introduce several methods to preserve diversity, as premature convergence is usually the most important problem of this method. Combined metaheuristic approaches have also been tested, as the hybrid of genetic and immune system in [18].

The structure of the problem is especially favorable for Benders' decomposition, with the decision variables representing the electric layout, being far less than the operation ones (for example, all power flows for every wind speed scenario and every failure scenario). Therefore, in our model OWL we aim to solve for the optimal electric layout of a wind farm using Benders' decomposition to keep solving times manageable even in the growing scales of plants seen today. In this model we consider stochasticity in wind generation and in the availability of system components. OWL follows a Benders' decomposition strategy that takes advantage of the favorable structure of the problem. Decomposition techniques have already been applied to the more general transmission expansion planning problem, like in [19], [20], [21] or [22].

## III. MODEL DESCRIPTION

### A. General

The OWL model considers a wind farm of variable size, turbine characteristics and location, and optimizes the electrical system to link the aerogenerators among them and to the point of common connection to the grid. Only AC technology is considered at this stage, with the installation or not of an offshore substation with a transformer being one of the decision variables. The extension of the model to allow for a choice of DC will be performed at a later stage, but it does not substantially vary the proposed approach as stated in [23].

Stochasticity in power generation is considered through a small set of wind speed scenarios calculated as a fan-shaped scenario tree as in [23] and [24].

For reliability, the state space method has been used as described in [25]. An N-1 criterion is established, so all the failures of the individual components are considered. The probabilities of those states are calculated from the failure and repair rates assuming a Markov discrete process.

### B. Mathematical Programming Model

#### 1) Indices:

- $p$  : points in the grid considered, for turbine, substation or connection to the grid location
- $N$  : total number of turbines installed
- $wt$  : wind turbines
- $cp$  : point of connection to shore
- $ct$  : cable types
- $ctmv(ct)$  : medium voltage cable types
- $cthv(ct)$  : high voltage cable types
- $tt$  : transformer types
- $ws$  : wind generation scenario
- $ss$  : system state scenario
- $r$  : redundancy level (if one cable is installed between two given points it would be marked with level 1. A redundant conductor would have the mark 2, and so on)
- $rt$  : redundancy level for the transformer

2) *Parameters:*

- $D_{p,p'}$  : distance between two points [m]
- $\bar{P}$  : maximum output of turbines [MW]
- $CP_{ct}, CC_{ct}, CFR_{ct}, CRr_{ct}$  : rated power, cost, failure rate and repair rate respectively of cable type  $ct$  [MW, EUR, failures per km per year, repairs per year respectively]
- $TP_{tt}, TC_{ct}, TFR_{tt}, TRr_{tt}$  : rated power, cost, failure rate and repair rate respectively of transformer type  $tt$  [MW, EUR, failures per year and repairs per year respectively]
- $Dur^{ws}$  : period duration of wind scenario [h]
- $WTPower^{ws}$  : power generation of a turbine in a certain wind scenario [MW]
- $Prob^{ss}$  : probability of system state [p.u.]
- $L$  : financial life of the plant [years]
- $R$  : long term interest rate [p.u.]
- $CEns$  : cost of not served energy
- $FaC_{p,p',ct,r}^{ss}$  : failure in system state  $ss$  of the cable going from  $p$  to  $p'$ , of type  $ct$  and redundancy  $r$   $\{1, 0\}$
- $FaTf_{tt,r}^{ss}$  : failure in system state  $ss$  of the transformer of type  $tt$  and redundancy  $r$   $\{0, 1\}$

3) *Layout variables:*

- $vOs_p$  : location of substation or collecting point at  $p$   $\{0,1\}$
- $C_{p,p',ct,r}$  : binary variable that takes a value of 1 if there is a cable installed between two given points, of a certain cable type and with a specific redundancy level. Redundancy levels are defined for every possible connection.  $\{0,1\}$
- $vN$  : binary variable that reflects whether the substation is an electric node  $\{0,1\}$
- $Tf_{tt,r}$  : binary variables that take a value of 1 if there is a transformer of a specific type installed with a specific redundancy  $\{0,1\}$

4) *Operation variables:*

- $F_{p,p'}^{ws,ss}$  : power flow between two points in a specific scenario
- $WTPNS_p^{ws,ss}$  : power surplus curtailed from the output of a turbine
- $TotP^{ws,ss}$  : total power sold in an scenario
- $PNS_p^{ws,ss}$  : total energy not served

5) *Layout and investment constraints:*

- There must be exactly one substation or collector point:

$$\sum_p vOs_p = 1 \quad (1)$$

- If there is a central collection point for cables rather than a substation, the cables that go into the point must go out of it:

$$\sum_{p,p',r} C_{p,p',ct,r} - (1 - vOs_p) \cdot N - vN \cdot N \leq \sum_{p,p',r} C_{p,p',ct,r} \quad (2)$$

$$\forall p \in os_p, ct$$

where  $N$  represents the number of turbines and is therefore an upper bound of the cables that can be connected to the central collection point.

- There are only cables installed from a turbine, from a substation or a collection point:

$$C_{p,p',ct,r} \leq vOs_p \quad \forall p \notin wt, p \notin cp \quad (3)$$

- Only one transformer type and one cable type per connection are allowed:

$$\sum_{tt} Tf_{tt,r} \leq 1 \quad (4)$$

$$\sum_{ct} C_{p,p',ct,r} \leq 1$$

- Redundancy levels of cables and transformers must be used from the lowest to the highest :

$$C_{p,p',ct,r} \leq C_{p,p',ct,r'} \quad (5)$$

$$Tf_{tt,rt} \leq Tf_{rt'}$$

High voltage cables can be used only if there is a transformer:

$$C_{p,p',ct,r} \leq \sum_{tt,rt} Tf_{tt,rt} \quad \forall ct \in cthv \quad (6)$$

- High voltage cables can only be installed from the substation to the point of connection to the grid:

$$C_{p,p',ct,r} \leq vOs_p \quad (7)$$

6) *Constraints that link the layout decisions with the system operation:*

- The flow between two points is limited by the capacity of their interconnection:

$$\begin{aligned}
fC_{p,p'}^{ws,ss} &\leq \sum_{ct} C_{p,p',ct,r} \cdot (1 - FaC_{p,p',ct,r}^{ws,ss}) \\
fC_{p,p'}^{ws,ss} &\geq -\sum_{ct} C_{p,p',ct,r} \cdot (1 - FaC_{p,p',ct,r}^{ws,ss}) \\
&\forall ws, ss
\end{aligned} \quad (8)$$

- The flow through the transformer is limited by its rated power:

$$\begin{aligned}
TotP^{ws,ss} &\leq \sum_{tt,rt} Tf_{tt,rt} \cdot TP_{tt} \cdot (1 - FaT_{tt,r}^{ws,ss}) \\
&+ \bar{P} \cdot N \cdot (1 - \sum_{tt} Tf_{tt,rt}) \\
&\forall ws, ss
\end{aligned} \quad (9)$$

### 7) Constraints of the system operation:

- Balance of active power:

$$\begin{aligned}
\sum_{p',p} fC_{p',p}^{ws,ss} + WTPower_{p \in wt}^{ws} &= \sum_{p,p'} fC_{p,p'}^{ws,ss} \\
&+ WTPNS_{p \in wt}^{ws,ss} - PNS_{p \in cp}^{ws,ss} \\
&\forall p, ws, ss
\end{aligned} \quad (10)$$

The objective value to be minimized corresponds to the total annualized cost and is expressed as:

$$\begin{aligned}
ObjTot = & \frac{R \cdot (1+R)^L}{(1+R)^L - 1} \cdot \left[ \sum_{p,p'} C_{p,p',ct,r} \cdot CC_{ct} + \sum_{tt} Tf_{tt,rt} \cdot TC_{tt} \right] \\
& + \sum_{p \in cp, ss, ws} Dur^{ws} \cdot Prob^{ss} \cdot PNS_p^{ws,ss} \cdot CENS
\end{aligned} \quad (11)$$

## IV. BENDERS' DECOMPOSITION APPROACH

### A. Introduction

The solution of the optimization problem above is computationally very expensive given the amount of equations and variables present (see the *Conclusions* section). Benders' decomposition method was developed by Benders [26], then further developed by Geoffrion [27], [28]. This approach decomposes a two-stage linear programming problem in two parts: a *master* problem and a *subproblem* that are solved iteratively until convergence is reached. This section provides a brief outline of this method. For the sake of simplicity, we refer only to the case where all the possible solutions for the first stage are feasible at the second stage (*complete recourse*), which holds for the problem under study. For a more complete description, see [29].

The general two-stage linear problem can be expressed in what we will refer to as the *complete problem* as:

$$\begin{aligned}
\min_{x_1, x_2^\omega} & c_1^T x_1 + c_2^T x_2^\omega \\
A_1 x_1 &= b_1 \\
A_1 x_1 + A_2 x_2^\omega &= b_2 \\
x_1, x_2^\omega &\geq 0
\end{aligned} \quad (12)$$

We can also interpret this as:

$$\begin{aligned}
\min_{x_1} & c_1^T x_1 + \sum_{\omega \in \Omega} \theta_2^\omega(x_1) \\
A_1 x_1 &= b_1 \\
x_1 &\geq 0
\end{aligned} \quad (13)$$

Where  $\theta_2^\omega(x_1)$  represents the objective function of the second stage as a piecewise linear function of the decisions of the first stage. We can now express it as:

$$\begin{aligned}
\theta_2^\omega(x_1) &= \min_{x_2^\omega} c_2^T x_2^\omega \\
A_2 x_2^\omega &= b_2 - B_1 x_1 \quad : \pi_2^\omega \\
x_2^\omega &\geq 0
\end{aligned} \quad (14)$$

Where  $\pi_2^\omega$  are the dual variables of the restrictions of the second stage for every scenario. Expressed in dual form we have:

$$\begin{aligned}
\theta_2^\omega(x_1) &= \max_{\pi_2} (b_2 - B_1 x_1)^T \pi_2^\omega \\
A_2^T \pi_2^\omega &\leq c_2
\end{aligned} \quad (15)$$

As the optimal solution must be in one of the vertices of the problem we could solve it by enumeration:

$$\theta_2^\omega(x_1) = \max \left\{ (b_2 - B_1 x_1)^T \pi_2^{wl} \right\} \quad l = 1, \dots, v \quad (16)$$

If we express this again as a linear problem we obtain:

$$\begin{aligned}
\theta_2^\omega(x_1) &= \min_{\theta_2} \theta_2^\omega \\
\theta_2^\omega &\geq (b_2 - B_1 x_1)^T \pi_2^{wl} \quad l = 1, \dots, v
\end{aligned} \quad (17)$$

Finally, we get the expression below:

$$\begin{aligned}
\min_{x_1} & c_1^T x_1 + \sum_{\omega \in \Omega} \theta_2^\omega \\
A_1 x_1 &= b_1 \\
\theta_2^\omega &\geq f_2^{wl} + \pi_2^{wlT} B_1 (x_1^l - x_1) \quad l = 1, \dots, j \\
x_1 &\geq 0
\end{aligned} \quad (18)$$

Where  $l$  represents the iteration index until  $j$ , the current iteration. This formulation encodes all the information of the problem and is denominated *complete master problem*. However, rather than incorporating all the conditions at once, the algorithm adds a new condition to the master problem from each resolution of the subproblem.

The solution of this problem, where  $\theta_2^\omega$  has not been completely defined yet it is known to be a convex function of  $x_1$ , represents a lower bound for the result of the optimization problem. The subproblem minimizes the costs of the second stage for the first stage variables provided by the master problem in that iteration  $x_1^l$ . As such, it provides with a feasible solution  $x_2^\omega$ :

$$\begin{aligned}
f_2^{\omega j} &= \min_{x_2^\omega} c_2 x_2^\omega \\
A_2 x_2^\omega &= b_2 - B_1 x_1^j \quad : \pi_2^{\omega j} \\
x_2^\omega &\geq 0
\end{aligned} \quad (19)$$

The algorithm terminates when the upper and lower bounds converge to the optimal solution given a relative tolerance:

$$\frac{|\bar{z} - z|}{\bar{z}} \leq \text{Tolerance} \quad (20)$$

where the upper bound is calculated from the latest iteration, and the upper bound is the minimum feasible solution found until the present iteration:

$$\begin{aligned} \bar{z} &= \min(c_1^T x_1 + \sum_{\omega \in \Omega} c_2^T x_2^{\omega}) \\ \underline{z} &= c_1^T x_1 + \sum_{\omega \in \Omega} \theta_2^{\omega} \end{aligned} \quad (21)$$

Benders' decomposition approach works best when the number of variables that link the two stages is small compared to the total number of variables. In addition, the method is interesting when the master problem and the subproblem are of different nature. In our case, the only variables that appear in the equations of both the first and the second stage are  $C_{p,p',ct,r}$  (cables installed) and  $Tf_{tt,r}$  (transformer installed). Moreover, the master problem is a MIP problem while the subproblem contains no integer variables. Therefore, it seems like the problem is well suited for this method.

The decomposition can be accomplished in different ways as detailed in the sections below and schematically represented in Figure 1.

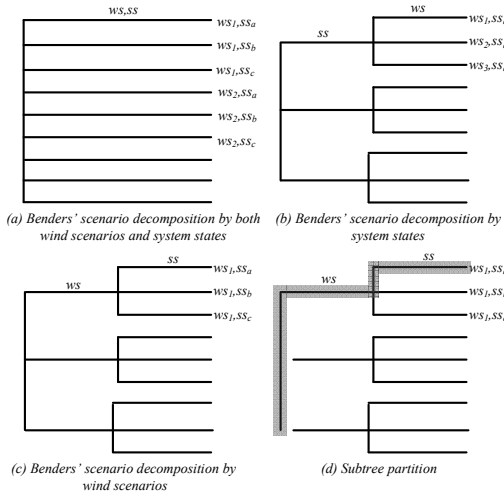


Figure 1. Schematic representation of the different approaches followed

### B. Benders' scenario partition

There is some flexibility on the implementation of the decomposition, depending on the choice for an elemental problem to be solved. See [30] for more details. In this case the second stage solutions for different wind scenarios and system states are independent. This means that we can split the problem by:

- wind scenario, so that we have to solve as many subproblems per iteration as wind scenarios are contemplated; each subproblem containing all the system states
- system state, so that we have to solve as many subproblems per iteration as system states considered, each subproblem containing all the wind scenarios
- both, so that we have to solve as many subproblems per iteration as there are combinations of wind scenarios and system states, each subproblem corresponding to only one specific wind scenario and system state.

This decision will have a double impact on the resulting calculation time. On the one hand, given that calculation times are larger than linear functions of the size of the problem, it would be desirable to split the problem in as many pieces as possible. This creates more cuts per iteration, which defines better the Benders' master problem and so the expected needed number of iterations decreases. However, the more cuts are included the slower the master iterations are. This tradeoff makes it worthwhile to analyze the suitability of a particular partitioning approach. We have tried the different alternatives and compared the results, as shown in the results section.

### C. Subtree partition

A modification in the Benders' algorithm that modifies the first stage function has been implemented to speed up performance. It consists of adding some of the most probable combinations of system and wind states to the master problem [31]. In this sort of problems we will find that, in general, the *all-components-OK* state is much more probable than the rest. In addition, the failure rates of some components are much larger than the others. For instance, transformers or HV cables connecting the substation to the point of common connection have a much larger failure probability than shorter turbine-to-turbine cables [32]. Furthermore, we can assume that the impact they will have on reliability will be much higher too. Therefore, it makes sense to calculate these high probability and high impact scenarios independently. The formulation of the problem will result in the expression below:

$$\begin{aligned} \min c_1^T x_1 + c_2^T x_2^{\omega_0} + \sum_{\omega \in \Omega, \omega \neq \omega_0} \theta_2^{\omega} \\ A_1 x_1 = b_1 \\ A_{2\omega} x_2^{\omega} = b_2^{\omega} \\ \theta_2^{\omega} \geq f_2^{\omega,l} + \pi_2^{\omega,l,T} B_1(x_1^l - x_1) \quad l = 1, \dots, j \\ x_1 \geq 0 \end{aligned} \quad (22)$$

Where the sub index  $\omega_0$  denotes the second stage scenarios that are included in the master problem –the most probable ones– and the sub index  $\bar{\omega}_0$  is used to indicate the remaining scenarios. The Benders' cuts will now approximate the recourse function associated with these remaining scenarios only. The following subsections describe the improvements obtained when these modifications were applied to the case study.

## V. RESULTS

### A. Case Study description

The OWL model above has been applied to a real case example in the Barrow Offshore Wind Farm (BOWF), a project completed in 2006 by Centrica and Dong Energy in the East Irish Sea. The full description of the plant and the technical specifications of its components can be found in [33]. Failure rates have been taken from [14]. Similar values have been used in [34].

BOWF consists of a total of 30 turbines in four rows, two with seven turbines and two with eight. The turbines are spaced 500 meters apart. The rows are spaced 750 meters apart. An offshore substation elevates the voltage so that the

energy generated is taken to shore in HV.

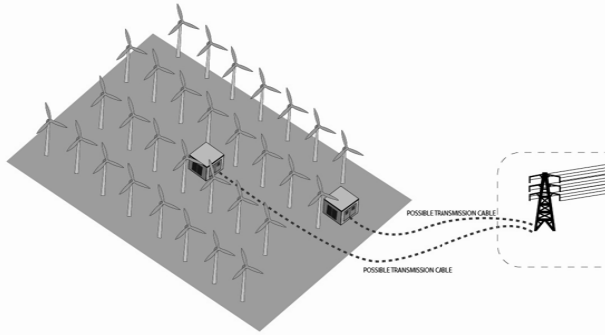


Figure 2. Schematic representation of the case study

In OWL we consider the alternative possibility of the power to be transmitted by MV cables gathered at a collection point, allowing for not installing a transformer (as detailed in section III. ). In addition, we consider two different locations for the substation from which the algorithm can choose the best candidate.

### B. Results

OWL has been applied to the real case study of BOWF and compared the computation times resulting of the complete problem and the decomposition approach, splitting the problem by wind scenario, system state and both.

In addition to the case study described, a smaller case, around the size where the decomposition starts to be profitable in terms of computation time, has been included in the analysis for comparison purposes. The number of wind scenarios and system states is of 5 and 93 respectively for the case study and 5 and 51 for the small case.

The actual layout of BOWF is represented in Figure 3. The generators are linked in four rows with MV120 cables that get upgraded to MV300 cables closer to the extremes of the rows. These four extreme points are connected to the offshore substation by a 120MVA transformer and a HV400 line.

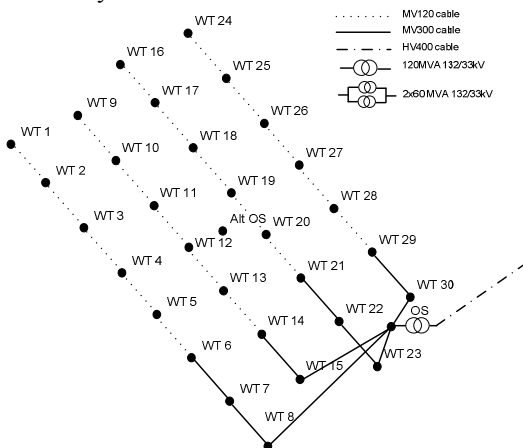


Figure 3. Actual layout of BOWF

The layout proposed by OWL differs from the above in two main ways. Firstly, instead of using a single 120MVA transformer, it installs two 60 MVA ones. This redundancy lowers the power not served appreciably, as can be seen in Table 1. In addition, the connections are changed. Notably, the

symmetry of the design is lost. For instance, WT16 is linked to WT24 instead of WT17 in its same row. WT23 is linked directly to the offshore substation and WT7 is connected to WT15. All these differences prove to be beneficial in terms of cost, as they enable to use fewer of the more expensive MV300 cables.

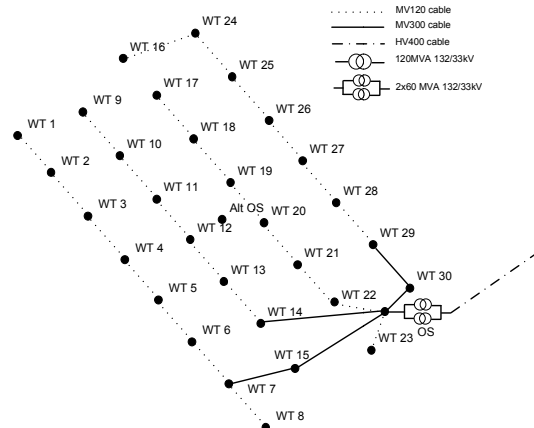


Figure 4. Proposed Layout

The layout proposed taking into account only the stochasticity of the wind, with no failures, is also provided for comparison purposes. It is identical to the final proposal except for the fact that it uses a single 120MVA transformer instead of two 60MVA ones.

The total investment cost as well as the cost of the non served power are provided in Table 1.

TABLE 1. COMPARISON OF SOLUTIONS

	Actual layout	Stochasticity in wind scenarios	Stochasticity in wind scenarios and system states
Investment Cost (MEUR)	19.10	18.59	18.85
Cost of non served energy (MEUR)	0.66	0.66	0.61

The solution obtained taking into account wind scenarios only is the one with the lowest investment cost. Notably, the cost associated to non served energy is almost identical to the one of the actual layout of the plant. In addition, the solution incorporating both wind scenarios and component failures is still more economical in investment cost terms (0.85%) but it also presents less non served power costs (6.7%).

The CPU times were calculated using GAMS in a PC at 2.80 GHz with 4 GB RAM running Microsoft XP 32 bits. The opter value was constant for all the executions and equal to a value of  $1e-6$  and the convergence tolerance for the Benders' decomposition was fixed to  $1e-3$

TABLE 2. PROBLEM SIZE

	Equations	Continuous variables	Binary variables	Non-zero Elements
Small case	10332	6664	62	26090
Case study	45165	27904	126	112691

TABLE 3. CALCULATION TIMES FOR THE COMPLETE PROBLEM

	CPU time (s)
Smaller case	2.83
Case study	1697.00

TABLE 4. CALCULATION TIMES FOR DECOMPOSED PROBLEM. SMALLER CASE.

	By wind scenario	By system state	By both
Iterations	25	42	25
CPU time per iteration (Master problem)	0.07	0.18	0.15
CPU time per iteration (Subproblem)	0.05	0.15	0.11
CPU time per iteration (Total)	0.11	0.33	0.26
Total CPU time (s)	2.83	13.86	6.50

The decomposition by wind scenario seems to be more efficient. We therefore apply this strategy to the case study, and check the effect of the enhancements described in section IV. C.

Benders' decomposition technique greatly improves performance in this case. Furthermore, the scenario partition approach proves to be 18% more efficient than the Benders' decomposition for this problem.

TABLE 5. CALCULATION TIMES FOR DECOMPOSED PROBLEM. CASE STUDY.

	Benders' decomposition by wind state	Scenario tree partition
Iterations	363	49
CPU time per iteration (Master problem)	0.23	10.56
CPU time per iteration (Subproblem)	1.55	0.27
CPU time per iteration (Total)	1.79	10.84
Total CPU time (s)	648.73	531.31

## VI. CONCLUSIONS

The offshore wind industry is experiencing an accelerated growth that manifests itself among other factors in the number and size of the plants installed. The optimization of their electrical design is a key issue given that it accounts for a relatively high proportion of the total cost of the project. In addition, repair times are much longer offshore, so that reliability considerations are essential. Therefore, stochasticity in component failures should be added to the already present uncertainties of wind inputs to allow for a suitable problem representation. OWL (Offshore Windfarm Linear optimizer) has been developed as a MIP model that takes into account both effects. Uncertainty in the wind is considered through wind scenarios, while system component failure is contemplated via a system state method with an N-1 criterion.

The problem presents characteristics that make it very suitable to be decomposed by Benders' method. This approach splits the optimization problem in two parts solved iteratively. A master problem includes the first stage and a description of

the second stage as a polygonal function of the variables of the first stage. The subproblem deals with the second stage of the problem and feeds the master with an increasingly more detailed description of its objective function. This method presents a computational advantage when the number of variables of the first stage -corresponding to the investment decisions- is reduced compared with the total number of variables. This condition is met in the problem as modeled in OWL. Moreover, the second stage (corresponding to the operation of the wind farm) is a LP problem, with all the binary variables confined to the investment stage, so decomposing allows for using more efficient methods for that part of the problem.

We apply this approach to the problem in the three possible ways the problem is suited for: decomposing by wind scenario, system state and both. In addition, a mechanism that incorporates the most probable states to the first stage objective function was implemented with the aim of speeding convergence. The results show that OWL can give optimal layouts for real-sized farms in affordable times and that these optimal layouts can differ substantially from the most intuitive solutions. In addition, both the decomposition strategy and the enhancement proposed appreciably improve performance in real sized problems.

Next steps of research include the study of the efficiency of the resolution method for growing problem sizes and the generalization of the model to include HVDC connections.

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## VIII. BIOGRAPHIES



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