

A PROBABILISTIC PRODUCTION COST MODEL WITH OPTIMIZATION OF THE LOADING ORDER

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INTRODUCTION

A limitation of the classical probabilistic simulation models for production costing, see for instance Baleriaux (3), Booth (4), Finger (8) and MIT (10), is their inability to account properly for some practical constraints in the loading order of the generating units. For instance the ones resulting from operating reserve considerations, technical minima or the fact that only a subset of the available units are connected to the grid at a particular time. On the other hand, it is relatively simple to optimize the units loading order for any given availability scenario of the power system, but then the impact of the uncertainty in the availability of the generating units is lost.

These limitations of the presently available models result in numerical errors in the estimates of production costs and utilization factors of each unit, that can be of some significance. The errors are typically larger for the most expensive connected units and also for those generators that must be connected to the grid when some unit fails.

This paper presents a probabilistic production cost model that makes use of the optimal loading order of the thermal units for any availability state of the system. This method, although needing more computing time than purely probabilistic simulation models or deterministic optimization models, still has reasonable computational requirements for some applications and captures all the realistic constraints related to the loading order of the units. The model has been applied to the Spanish electric system, and it has been successfully used to identify and quantify the errors inherent to existing techniques.

The first section of the paper presents the overall approach and the detailed model formulation. The following section is dedicated to the simulation techniques that have been applied in this work, including variance reduction techniques. Then, the optimization method that is used within the model is presented. The validation results and, finally, the errors detected in classical production cost models that do not consider loading order variations are quantified and described.

MODEL FORMULATION

The model presented in this paper includes two major characteristics that are relevant and desirable for production cost models:

- uncertainty in generation availability.
- loading order dependence on the specific operating constraints and availability status.

An availability scenario (i.e., the availability status of each unit) is randomly generated using Monte Carlo techniques for a given period and its associated demand function. An optimization algorithm determines the subset of available thermal units to be connected to the grid so that the production cost of the period is minimized, see figure 1.

The decision variables in each simulated scenario are the connection status of the available units, the power output of each unit and the unserved demand.

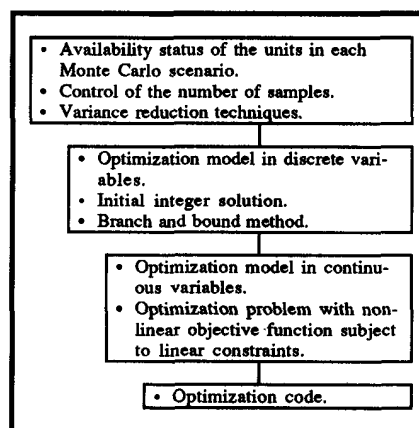


Figure 1. Overall framework of the model.

For any given set of connected units the algorithm takes into account:

- the nonlinearity of the demand, represented by the complementary distribution function (i.e., the load-duration curve) for each period.
- the economic loading of thermal units by several capacity blocks with different costs.
- the loading order constraints resulting from operating considerations.

With the adequate simplifications the model can easily be converted into a simpler one. For example, if no loading order constraints are considered the model becomes equivalent to the classical prob-

abilistic simulation model, but requiring more computing time.

The objective function to be minimized includes fuel and variable maintenance costs of the generating units, costs of unserved energy, connection cost of the units, and some penalties that account for violations in operating reserve requirements and technical minima constraints. Nonlinearity (in the first two terms of the objective function) is a consequence of the nonlinear characteristic of the load-duration curve. The constraints represent the loading order restrictions caused by technical minima, operating reserves and connection status of the units.

The model can be mathematically formulated as follows:

$$\min \sum_{i=1}^N \sum_{j=1}^{nb} (E_{ij} v_{ij}) + E_{N+1} v_{N+1} + \sum_{i=1}^N (A_i c_i T) + P_m^e c^{er} T + P_R^d c^R T \quad (1)$$

subject to:

i) used capacity by each connected unit must be between its technical minimum and its available capacity (partial availability is allowed)

$$P_{i1} A_i - P_i \leq 0 \quad i=1, \dots, N \quad (2)$$

$$P_{id} A_i - P_i \geq 0 \quad i=1, \dots, N \quad (3)$$

ii) sum of the technical minima of the connected units must be equal to the minimum demand plus an eventual violation to be penalized

$$\sum_{i=1}^N (P_{i1} A_i) + P_m^d - P_m^e = D_m \quad (4)$$

iii) upper bound for the excess of technical minima capacity over the minimum demand

$$P_m^e \leq P_m^e \quad (5)$$

iv) sum of used capacities of all units plus unserved demand must be equal to the maximum demand

$$\sum_{i=1}^N P_i + P_{N+1} = D_M \quad (6)$$

v) sum of available capacities of the connected units must be equal to the maximum demand plus a reserve margin (both an eventual deficiency to be penalized and a surplus are allowed)

$$\sum_{i=1}^N (P_{id} A_i) + P_R^d - P_R^e = (1+R) D_M \quad (7)$$

vi) discrete nature of the connection status of each unit

$$A_i \begin{cases} 0 & \text{disconnected} \\ 1 & \text{connected} \end{cases} \quad i=1, \dots, N \quad (8)$$

where

A_i variable indicating the connection status of unit i .

c_i connection fixed costs of unit i per unit of time.

c^m penalty applied to any technical minima total capacity exceeding the minimum demand, per unit of power and per unit of time.

c^R penalty applied to the deficiency in operating reserve, per unit of power and per unit of time.

D_M maximum demand.

D_m minimum demand.

E_{ij} energy generated by block j of unit i . It can be obtained from the load duration curve once the values of the P_i are specified, see figure 2.

E_{N+1} unserved energy.

i index of a unit.

j index of a capacity block of a unit.

N number of thermal units.

nb number of capacity blocks of the units.

P_m^d deficiency in technical minima total capacity with respect to the minimum demand.

P_m^e excess of technical minima total capacity with respect to the minimum demand.

P_R^d deficiency in available capacity of connected units with respect to the maximum demand plus a reserve margin.

P_R^e excess of available capacity of connected units with respect to the maximum demand plus a reserve margin.

P_i power output by unit i .

P_{i1} technical minimum capacity of unit i .

P_{id} available capacity of unit i .

P_{N+1} unserved demand.

R reserve margin.

T duration of the considered time period.

v_{ij} variable cost of block j of unit i .

v_{N+1} variable cost of the unserved energy.

A branch and bound algorithm has been used because of the discrete nature of the optimization problem in equation (8). The optimization problem has been formulated as the minimization of a nonlinear objective function subject to linear constraints, see figure 1.

The number of variables is $2N+5$, the number of constraints is $2N+3$ and the number of non zero coefficients of the constraints is $7N+5$. These numbers

indicate the sparsity of the constraints matrix, which will be exploited.

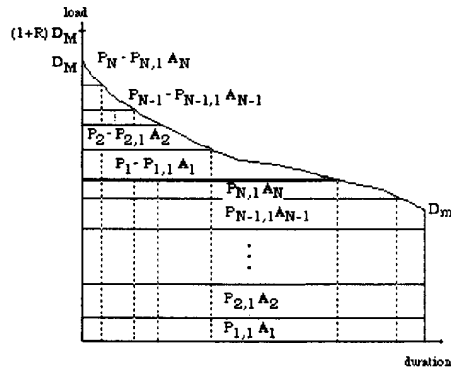


Figure 2. Loading of the connected units under the load-duration curve.

SIMULATION TECHNIQUES

Two basic methods have been proposed in power systems to study the uncertainty in generation: *state enumeration* and *simulation*. It can be concluded, see Wang (15) and Endrenyi (7) in the context of reliability of power systems, that the enumeration method has disadvantages for systems with numerous units, because the computation time grows exponentially for the enumeration method and linearly for the simulation method with the number of units. The growth rates depend on the availability of the units but the advantage of simulation holds for the typical value range of availabilities and number of units. For example, in enumeration with 50 units and a contingency level of 4 units the analyzed probability space is 43 % if the unavailability of each unit is 0.1 (and 90 % if the unavailability of each unit is 0.05) and the number of states that should be evaluated is 230300. For this reason the Monte Carlo simulation method has been used in conjunction with the proposed model.

For each simulated scenario the current availability state of each unit is determined. A random number uniformly distributed in the interval $[0,1]$ is generated by means of a multiplicative congruential algorithm. Depending on the value of this random number each unit can be totally or partially unavailable or fully available. Once the availability status of each unit has been determined the optimization problem is solved.

The main results of a production cost model are the total operating cost and the utilization factors of all units. Both are random variables depending among other factors on the units availability. The expected value and other statistical properties of these variables are obtained by repeated evaluations of the optimization problem for different availability scenarios generated by Monte Carlo simulation. The

sample mean and sample variance can be progressively computed as follows:

$$\mu_n = \frac{1}{n} [(n-1)\mu_{n-1} + z_n] \quad (9)$$

$$s_n^2 = \frac{1}{n-1} \left[(n-2)s_{n-1}^2 + \frac{n}{n-1} (\mu_n - z_n)^2 \right] \quad (10)$$

where

n current sample size (number of observations).

μ_n mean of n samples.

s_n^2 variance of n samples.

z_n value of the variable of interest in sample n .

and the confidence interval (assuming that the random variables are normally distributed) is

$$\left[\mu_n - t_{n-1, (1-\alpha)/2} \frac{s_n}{\sqrt{n}}, \mu_n + t_{n-1, (1-\alpha)/2} \frac{s_n}{\sqrt{n}} \right] \quad (11)$$

where

100α the confidence level

$t_{n-1, (1-\alpha)/2}$ the value of the t-Student distribution with $n-1$ degrees of freedom

A convergence criterion can be defined as the ratio between the confidence interval size and the sample mean:

$$cc = 2 t_{n-1, (1-\alpha)/2} \frac{s_n}{\mu_n \sqrt{n}} \quad (12)$$

A number of techniques have been used to reduce the computational effort of the simulation approach, mainly by trying to decrease the number of samples that are needed to attain a prespecified level of accuracy in the results.

Among the methods and criteria that have been explored and used in an integrated fashion the following can be mentioned:

- simulation with a *fixed sample size*, which is subdivided into batches of prespecified length.
- sequential simulation* with a prespecified convergence criterion.
- variance reduction techniques* such as anti-thetic variables, based on complementary successive random numbers.

A. FIXED SAMPLE SIZE SIMULATION.

In this type of simulation, see Law (9) and Bratley (5), the total sample size is divided into batches of fixed length. Then each batch mean can be considered an independent and identically distributed normal random variable if the batch size is large enough. The mean of each batch is now converted into an observation of the simulation process. The model calculates two means and variances in the

simulation process corresponding to each batch and to the set of batches.

After a detailed exploration of alternatives, in which several batch lengths have been tested for different total number of samples, some recommendations have been obtained for the use of the model. The results indicate that the best convergence is achieved with 30 batches of 30 observations each.

B. SEQUENTIAL SIMULATION.

This procedure, see Law (9) and Bratley (5), stops the simulation process when the confidence interval size is small enough with respect to the sample mean (both values are calculated for every batch), i.e., when the convergence criterion reaches certain prespecified value.

The variables chosen to satisfy the convergence criterion, equation (12), are the total operating cost and the technical minimum generation of nuclear units. The value chosen for the convergence criterion is 2 % for a 99 % confidence level.

C. VARIANCE REDUCTION TECHNIQUES.

Variance reduction techniques try to reduce the confidence interval width for a certain simulation effort or alternatively reduce the effort to achieve the same confidence interval, see Law (9), Bratley (5) and Rubinstein (14). There are many and diverse techniques: common random numbers, antithetic variables, control variables, stratification, importance sampling, indirect estimation, etc. Some of these techniques have been used in the context of stochastic chronological simulation models, see Breipohl (6), and composite power system reliability, see Allan (1), Anders (2) and Pereira (12).

The model uses the antithetic variables technique, based on complementary successive random numbers. This introduces a negative correlation between successive observations, decreasing the variance of the couple of observations with respect to the variance of the independent observations.

The application of this technique achieves up to a 20 % reduction in the confidence interval size.

OPTIMIZATION METHOD

The optimization problem becomes deterministic once the availability status of the units has been obtained by the Monte Carlo simulation. Then the problem can be solved by a mixed programming procedure, the branch and bound technique. This method resorts to successive relaxed nonlinear minimization problems to achieve the optimal integer solution.

Certain especial characteristics have been implemented into the branch and bound technique to reduce the size of the tree search, see Rao (13). For

example, a cutoff tolerance, LIFO ordering criteria for non explored branches, etc.

For each node of the tree the problem formulated in equations (1) to (7) is solved by relaxing the connection decision (8), which becomes

$$0 \leq A_i \leq 1 \quad i=1, \dots, N \quad (13)$$

This problem can be directly input to a general purpose optimization code such as MINOS (11), which can compute the solution very efficiently. MINOS is a well-known large-scale optimization package for the solution of sparse linear and nonlinear optimization problems.

The derivatives of the objective function with respect to the decision variables are provided to increase the accuracy and speed in obtaining the solution.

VALIDATION OF THE MODEL

The model has been applied to a realistic example based on the Spanish generating system, considering 65 thermal units (hydro units have been ignored).

Only 900 samples (30 batches of 30 observations each one) have been required to attain a convergence criterion of 1.3 % in the total production cost. The utilization factors of the units have absolute errors between +0.02 and -0.02, see figure 3.

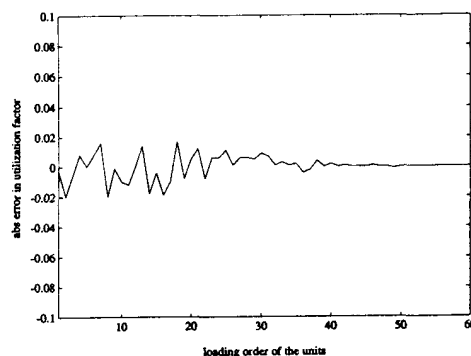


Figure 3. Absolute error in utilization factors of the units.

Both results indicate a good performance of the model with a reasonable simulation effort because the absolute errors are:

- very small.
- homogeneously distributed for the set of units.
- both positive and negative.

LOADING ORDER OPTIMIZATION

The model has also been used as a benchmark to provide interesting information concerning the impact of the simplifications introduced in the existing production cost models, for example in the treatment of the technical minima of the units.

The loading order (which is used here in a broad sense that includes the specification of the set of units to be connected to the grid) depends on the set of available units and on the operation constraints that are imposed. Two types of errors can appear because of the simplifications that are typically introduced:

- I) if only one loading order is considered.
- II) if the constraints associated with the loading order are ignored.

The first type of error is caused by the utilization of only one loading order (usually the most frequent or one just based on simple economic considerations) instead of the multiple loading orders that would result when considering all the sets of available units. This simplification is intrinsic to probabilistic simulation models, very commonly used in production costing and in generation expansion planning models. Note for instance that the technical minima of all the available connected units must be fully base loaded. Just for economic reasons a decision has to be made regarding the choice of connected units, which depends on the availability scenario. In probabilistic simulation models at most a unique choice of connected units can be made, with the consequence that the technical minima of the non connected units are not base loaded. Extensive comparisons have been done to quantify the errors.

The main consequences caused by this assumption, as presented in figure 4, are:

- important errors in utilization factors of the units (up to 0.44). Technical minima of connected units have too large utilization factors and technical minima of non connected units have too small utilization factors.
- it can be shown that the a priori division of the set of units into two subsets of connected and non connected units introduces an artificial step in the utilization factors of the units.

With respect to the second simplification, i.e. ignoring the operating constraints (4) and (7), the principal detected errors are:

- units with higher technical minima actually produce less energy than what is obtained when the constraints are ignored. The differences in utilization factors of the technical minima of the units can reach 0.8 if the constraints are very restrictive, see figure 5 for numerical results.

- total production costs increase exponentially with respect to the allowed excess of technical minima total capacity over the minimum demand, reaching 21 % if no excess is allowed.
- in an approximate manner it can be concluded that the utilization factor of a unit is inversely proportional to the product of its technical minimum coefficient times its variable cost if the technical minima constraint is very tight. Units with very similar variable costs can produce very different values of energy because of the operating constraints, see figure 5.

These errors can only be detected with the presented model, since it includes multiple loading orders and their optimization for each scenario.

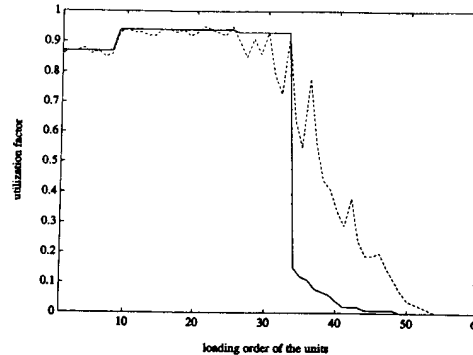


Figure 4. Utilization factors of the technical minima of the units.

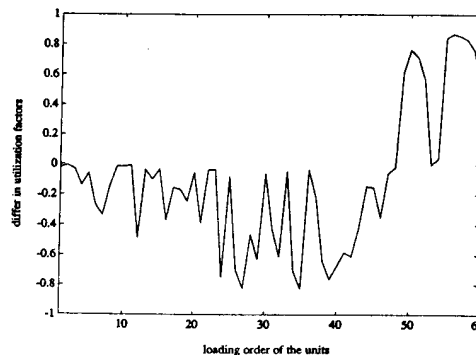


Figure 5. Differences in utilization factors of the technical minima of the units.

CONCLUSIONS

The paper has presented a new probabilistic production cost model developed as a benchmark to establish comparisons with classical production cost models. The importance of the operating constraints and loading order optimization has been emphasized. The impact in the utilization factors of

the units, as a consequence of the assumptions made in probabilistic simulation models, has been evaluated and shown to be very significant in certain cases.

The model explicitly considers uncertainty in the available generation and loading order optimization including operating constraints. For a given period and its associated demand function, availability scenarios (i.e., the availability status of each unit) are randomly generated using Monte Carlo techniques. Then an optimization algorithm determines the subset of the available thermal units that must be connected to the grid so that the production cost of the period is minimized. Because of the discrete nature of the optimization problem a branch and bound algorithm has been used. The final optimization problem has been formulated as the minimization of a nonlinear objective function subject to linear constraints.

A number of techniques have been used to reduce the computational effort of the simulation approach, mainly by trying to decrease the number of samples that are needed to attain a prespecified level of accuracy. Among the methods that have been explored and used successfully in an integrated fashion the following can be mentioned: simulation with a fixed sample size, sequential simulation and variance reduction techniques.

Illustrative numerical results have been presented using a realistic example based on the Spanish electric system.

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