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Strategic Bidding in a Competitive Electricity Market: A Decomposition Approach

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***Abstract*--: Daily bidding is an activity of paramount importance for generation companies operating in day-ahead electricity markets. The authors have developed a strategic bidding procedure based on stochastic programming to obtain optimal bids. In this paper, the problem is decomposed under the Benders framework to permit the solution of large-size problems. A numerical example illustrates the advantages of the proposed approach.**

***Index Terms*: Competitive electricity market, Benders decomposition, generation scheduling, strategic bidding.**

I. Introduction

In an increasing number of countries, power generation is now considered an activity that must be carried out within a competitive framework. Although the debate of how competitive electricity markets should be organized is still in force [4], in several relevant cases a design based on a day-ahead market has been implemented [11]. This market mechanism is operated by a market coordinator and consists on a set of hourly electricity auctions. Generation companies willing to sell energy through this market are asked to offer blocks of energy at different prices for each hour. Wholesale energy buyers submit bids to express the price at which they are ready to consume at each hour. The market coordinator matches the hourly aggregate supply and demand curves and an hourly market clearing price results.

In this competitive environment, a generation company is subject to an intense short-term risk exposure. The operation of its generating units and the revenue obtained for their production depends strongly on the strategy followed by its competitors, which is uncertain. Thus, generation companies demand new decision-support tools specifically adapted to meet these new short-term risk-management requirements. With this purpose, a strategic bidding procedure (SBP) based on stochastic programming was developed by the authors [2]. In this paper this procedure is decomposed using the Benders technique to allow for the solution of real-size problems. Together with the formulation of the algorithm, a numerical case is solved to illustrate the possibilities of this approach.

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II. Model Description

The development of a bidding procedure for a generation company operating in a short-term electricity market requires not only the usage of traditional equations typical in unit-commitment or economic-dispatch models [3], but also new market-modeling equations have to be incorporated. In fact, the main difference among the variety of bidding procedures recently proposed in the literature lies in how market clearing prices are included in the model [5].

In our approach the influence on the price of both the company's own decisions and the competitors' bids is specifically represented by means of the residual demand curve. This curve determines how the market clearing price changes with the variations of the company's production [1], [6]. It is built using both the demand curve and the competitors' offer curves. A generation company ignores the shape of its residual demand curve for each of tomorrow's hours. However, as historic information on the competitors' offers is usually available, the company can build a set of hourly residual-demand-curve scenarios (Fig. 1).

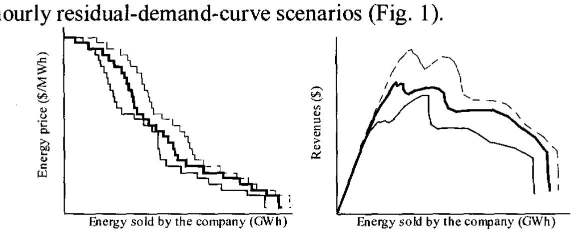

Fig. 1. Residual-demand-curve and revenue scenarios

Owing to the fact that the offers are decided simultaneously for all hours, it can be said that a recombining decision-tree is being considered (Fig. 2).

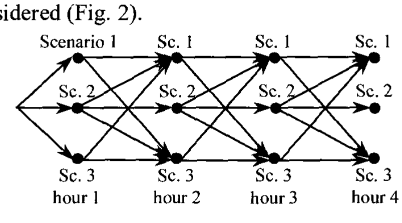

Fig. 2. Scenario tree.

Once we have chosen to model uncertainty using scenarios, the problem comes down to selecting an optimal bid (quantity, price) for each scenario. Fig. 3 shows an example with 10 scenarios.

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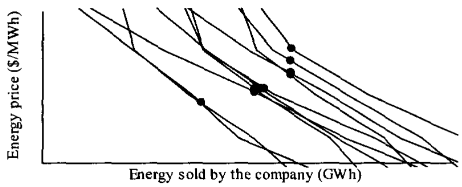

Fig. 3. One offer for each residual-demand scenario

The objective function used to search for optimal offers is the company's expected profit. This is calculated as the difference between expected revenues and expected costs. Revenues for each scenario are obtained by multiplying the energy price and the quantity offered by the company. Consequently, revenues constitute a non-linear function, as the price itself depends on the offered quantity. This is a drawback, because the most powerful commercial optimizers are those designed to solve linear programming problems. To overcome this difficulty we will use the linearizing technique described in [1]. This intuitive method divides the company's hourly revenue function into convex sections and approximates each one by a piecewise linear function. The slope obtained for each linear segment is the firm's marginal revenue at the corresponding energy output (Fig. 4). Each convex section is assigned a binary variable and each linear segment is assigned a continuous bounded variable.

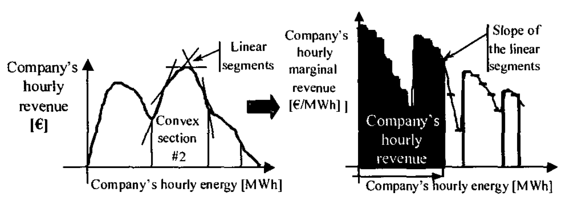

Fig. 4. Company's hourly marginal revenue function.

A group of consecutive segments with strictly decreasing marginal revenues defines a convex section in the revenue function. When we seek the optimum we select a specific convex section by switching its binary variable from zero to one. Once we have chosen a convex section we fill its segments with continuous bounded variables. In other words, we obtain the hourly revenue by integrating the marginal-revenue function. Prices are not explicitly used to calculate the company's revenue.

An important additional remark is that the bids chosen by the model for each generation unit must be increasing both in quantity and price. Thus, all the scenarios of a certain hour are linked by a set of increasing constraints. If A and B are two scenarios for hour n, the increasing constraint that links the quantities offered for unit g in both scenarios is:

$$\left[q_{gAn}-q_{gBn}\right]\left[p_{An}\left(\sum_g q_{gAn}\right)-p_{Bn}\left(\sum_g q_{gBn}\right)\right]\geq 0\ \forall g,n.$$

where p_{An}, the price expected for scenario A, is a function of the total energy offered by the company for that scenario, $\sum_g q_{gAn}$ (Fig. 5).

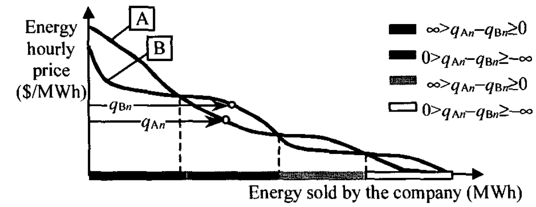

Fig. 5. Increasing offers.

To linearize this increasing constraint, a binary variable, x_{ABn}, has to be defined. When curve A runs above curve B, x_{ABn} is equal to zero. In other case, $x_{ABn} = 1$.

To summarize, the expression of the SBP problem would be as follows:

$$\max_{q_{gsn}} \sum_{n,s} \rho_{sn}\left[r_{sn}\left(\sum_g q_{gsn}\right)-\sum_g c_{gsn}\left(q_{gsn},u_{gsn}\right)\right] \quad (1)$$

$$\text{s.t. } \left\{q_{gsn},u_{gsn}\right\}\in X_g, \qquad \forall g, \ (2)$$

$$\begin{cases} q_{gsn}-q_{gs'n}\geq -x_{ss'n}M^{q_g}, \\ q_{gs'n}-q_{gsn}\geq -\left(1-x_{ss'n}\right)M^{q_g}, \end{cases} \qquad \forall g,(s,s'),n, \ (3)$$

$$\begin{cases} p_{sn}\left(\sum_g q_{gsn}\right)-p_{s'n}\left(\sum_g q_{gs'n}\right)\geq \\ \quad \geq -x_{ss'n}M^{p}, \\ p_{s'n}\left(\sum_g q_{gs'n}\right)-p_{sn}\left(\sum_g q_{gsn}\right)\geq \\ \quad \geq -\left(1-x_{ss'n}\right)M^{p}, \end{cases} \qquad \forall (s,s'),n. \ (4)$$

where:

- n load level (e.g. 1 hour),
- s,s' residual-demand-curve scenarios,
- g generating unit,
- ρ_{sn} probability of scenario s in hour n,
- r_{sn} company's revenues in hour n and scenario s,
- q_{gsn} quantity produced by unit g offered for scenario s in hour n,
- u_{gsn} commitment state (0/1) of unit g in hour n and scenario s (input data),
- c_{gsn} cost of unit g in hour n and scenario s,
- X_g set of feasible schedules for unit g,
- p_{sn} energy price for scenario s in hour n,
- $x_{ss'n}$ binary variable that links the offers for scenarios s and s' in hour n.

Obtaining a solution for a real-size SBP problem is extremely hard due to the presence of the binary variables used to linearize the revenue function and the increasing constraints. We suggest a decomposition approach to overcome this difficulty.

III. Decomposition Strategy

One of the most popular ways of addressing the solution of large-scale short-term generation-scheduling problems is through their dual, using the Lagrangian relaxation approach (LR) [9]. This is justified by the fact that a non-linear and non-concave maximization problem like this has a dual minimization problem that is convex. In traditional UC problems, the non-convexities were caused by start-up costs, fixed costs and minimum-up-and-down times. In the SBP problem, in contrast, the start-up decisions are considered as given data obtained from a weekly model similar to the one described in [1]. On the contrary, non-concavities are found in the revenue functions.

Two are the major disadvantages of the LR approach. The first one is algorithmic and is related to the oscillations in the value of the Lagrange multipliers during the first iterations. This problem is due to the lack of information about the dual function and has been partially overcome by progressive improvements in the algorithms that update these multipliers [6]. The second drawback is that the solution obtained for the dual problem is not feasible because the coupling constraints that have been relaxed are not satisfied. Therefore, a postprocessing of the dual solution is required to obtain a feasible schedule. This is usually done through heuristics.

In the Benders framework [10], the problem is decomposed into a master problem (MP) and a subproblem (SP), by breaking up the complicating constraints. These constraints, that link some variables of the MP, x_{MP}, with those of the SP, are eliminated from the MP. In their place, the MP includes a linear approximation of the SP's objective function (recourse function) formed by Benders' cuts. Once a solution is obtained for the MP, the value given to the x_{MP} variables is evaluated in the SP. The SP returns a Lagrange multiplier expressing how much its objective function would improve if the x_{MP} variables were slightly changed.

Benders decomposition is not suitable for UC problems because this technique requires that the subproblems be convex (or concave, in a maximization context), which is not the case of the UC generation subproblems. In contrast, the SBP problem does not make start-up decisions. Hence, the decomposition strategy shown in Fig. 6 is suggested:

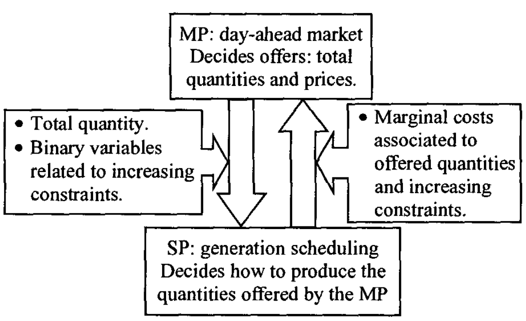

Fig. 6. Benders decomposition strategy

IV. FORMULATION OF THE PROBLEM

The coupling constraints present in traditional short-term generation-scheduling problems (the demand and reserve constraints) disappear in the new profit-maximization context. The reason is that the company is free to decide the amount of energy and reserve that should be sold in order to maximize its profit.

However, some new coupling constraints are present, such as those used to guarantee that offers are increasing both in quantity and price, (3) and (4). Additionally, the hourly revenues for each scenario, r_{sn}, are calculated as a function of the total quantity offered by the company in the corresponding bid, $\sum_g q_{gsn}$. This means that the generating units are also linked through the objective function. The best way to eliminate this difference with the traditional problem structure is to add a new variable, the company's total offered quantity, q_{sn}, and a new constraint, expressing that this total output must be equal to the sum of the productions of each of the company's units, $\sum_g q_{gsn} = q_{sn}$.

Thus, the SP evaluates the cost of producing the quantities decided in the MP in iteration t, q_{sn}^t:

$$\theta^t = \underset{q_{gsn}}{\mathrm{Min}} \sum_{n,s} \rho_{sn} \sum_g c_{gsn}\left(q_{gsn}, u_{gsn}\right) \tag{5}$$

$$\text{s.t.} \sum_g q_{gsn} = q_{sn}^t : \lambda_{ns}^t, \qquad \forall s, n, \tag{6}$$

$$\left\{q_{gsn}, u_{gsn}\right\} \in X_g, \qquad \forall g, \tag{7}$$

$$\begin{cases} q_{gsn} - q_{gs'n} \geq -x_{ss'n}^t M^{q_g} : \mu_{ss'n}^t, \\ q_{gs'n} - q_{gsn} \geq -\left(1 - x_{ss'n}^t\right) M^{q_g} : \mu_{s'sn}^t, \end{cases} \qquad \forall g, (s, s'), n, \tag{8}$$

where:

λ_{ns}^t dual variable associated to constraint (6),

$\mu_{ss'n}^t, \mu_{s'sn}^t$ dual variables associated to constraints (8).

Typically, λ_{ns}^t will take negative values, expressing that a decrease in the offered quantity, q_{sn}^t, permits a cost reduction.

The MP suggests new offers in iteration t+1 according to the cost signals returned by the SP in the previous t iterations:

$$\underset{q_{sn}}{\mathrm{Max}} \sum_{n,s} \rho_{sn}\left[r_{sn}\left(q_{sn}\right)\right] - \theta \tag{9}$$

$$\text{s.t. } \theta \geq \theta^t - \sum_{n,s} \begin{Bmatrix} \lambda_{sn}^t \left(q_{sn} - q_{sn}^t\right) + \\ \sum_{s'>s,g} \left(\mu_{gss'n}^t - \mu_{gs'sn}^t\right) M^{q_g} \left(x_{ss'n} - x_{ss'n}^t\right) \end{Bmatrix} \tag{10}$$

$$\begin{cases} p_{sn}\left(q_{sn}\right) - p_{s'n}\left(q_{s'n}\right) \geq -x_{ss'n} M^p \\ p_{s'n}\left(q_{s'n}\right) - p_{sn}\left(q_{sn}\right) \geq -\left(1 - x_{ss'n}\right) M^p \end{cases} \qquad \forall (s, s'), n. \tag{11}$$

$$\begin{cases} q_{sn} - q_{s'n} \geq -x_{ss'n} M^q \\ q_{s'n} - q_{sn} \geq -\left(1 - x_{ss'n}\right) M^q \end{cases} \qquad \forall (s, s'), n. \tag{12}$$

Equation (10) approximates the recourse function θ by means of the Benders' cuts obtained from the SP.

Equation (12) has been incorporated to guarantee that the total quantities decided in the MP verify the increasing constraints.

The total quantities decided by the MP may constitute an infeasible schedule for the company's generation system. There at least are two ways to deal with this difficulty: generating feasibility cuts or assuming that there is an hour-ahead market where a feasible schedule can be attained.

According to our results, the majority of the offers proposed by the MP lead to expected unfeasible schedules. This is either because the company's installed capacity is not sufficient or because its units are not flexible enough.

Actually, it is not strange that the day-ahead generation schedule returned to the company by the market operator be infeasible. The company can then try to correct this situation in the subsequent hour-ahead market. The clearing price for a certain hour in the day-ahead market should not differ much from the clearing price for the same hour in the hour-ahead market. The residual demand curve considered in our model for the hour-ahead market is represented in Fig. 7.

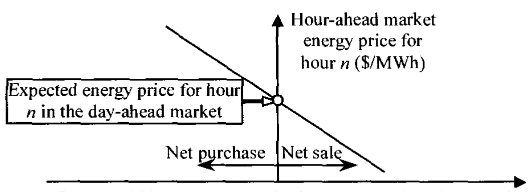

Fig. 7. Hour-ahead market representation.

If the company does not participate in the hour-ahead market, the hour-ahead clearing price of energy will be the same as in the day-ahead market. If the company sells energy in the hour-ahead market it will be paid at a lower price than in the day-ahead market. Conversely, if the company buys energy in the hour-ahead market it will be at a higher cost than in the day-ahead market. In this fashion the quantities decided by the MP will always be feasible, because the net energy sold in the hour-ahead market acts as a slack variable.

The formulation of the SP including the hour-ahead market would then be:

$$\theta^t = \underset{q_{gsn},q_{sn}^h}{\text{Max}} \sum_{n,s} \rho_{sn}\left[r_{sn}^h\left(q_{sn}^h\right)-\sum_g c_{gsn}\left(q_{gsn},u_{gsn}\right)\right] \quad (13)$$

$$\text{s.t.} \sum_g q_{gsn} = q_{sn}^h + q_{sn}^t : \lambda_{ns}^t, \qquad \forall s,n, (14)$$

$$\left\{q_{gsn},u_{gsn}\right\} \in X_g, \qquad \forall g, (15)$$

$$\begin{cases} q_{gsn}-q_{gs'n} \geq -x_{ss'n}^t M^{q_g} : \mu_{ss'n}^t, \\ q_{gs'n}-q_{gsn} \geq -\left(1-x_{ss'n}^t\right) M^{q_g} : \mu_{s'sn}^t, \end{cases} \qquad \forall g,(s,s'),n, (16)$$

where:

- q_{sn}^h net quantity sold in the hour-ahead market in scenario s and hour n,
- r_{sn}^h company's revenues in the hour-ahead market in scenario s and hour n.

The company will submit the quantities given by the MP, $\{q_{sn}\}$, though the net quantities that are expected to be produced are $\{q_{gsn}\}$, which in some of the forecast scenarios are different, as can be seen in Fig. 8. This can cause an infeasibility in the resulting day-ahead schedule. The company will then have to optimize its strategy in the hour-ahead market, given the outcome of the day-ahead market.

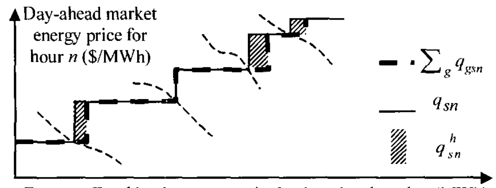

Fig. 8. Gap between the day-ahead sales and the expected generation.

V. Numerical Examples

In this section the results obtained for a generation company with an installed capacity of 4500 MW of thermal power and 1620 MW of hydro power are presented. The mathematical model has been formulated in GAMS language and solved with CPLEX 7.0. In all cases a 24-hour 15-scenario problem has been considered.

A. Case with no hydro energy

To better analyze the results given by the model, a case with no hydro or pumped-storage production is first studied.

Table I shows the evolution of the MP and the SP objective functions during the Benders algorithm. The MP objective function decreases as the recourse function is better approximated by the Benders cuts. The SP objective function has an increasing tendency, although with some oscillations.

TABLE I
Evolution of the Benders Algorithm

Iter	MP Obj. Function (M€)	SP Obj. Function (M€)
1	3.10609613	-2.544732004
2	2.23018316	-0.83130483
3	1.478897132	-0.850319364
4	1.459686021	-0.829655907
5	1.459667484	-0.829663986
6	1.459644697	-0.829663896
7	1.459640604	-0.829663932
8	1.459639203	-0.829663956
9	1.459639017	-0.829663944

The MP has 48000 eqs. and 24000 vars., 14500 of which are binary. The SP has 140000 eqs. And 55000 vars. Each iteration takes 1000 s in a Pentium III 1GHz 384 MB.

Fig. 9 represents the offers decided by the bidding procedure for three of the 24 hours.

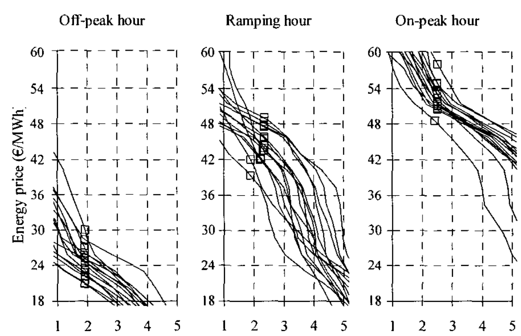

Fig. 9. Offer curves given by the model for three of the 24 hours.

In this case an hourly minimum-expected-market-share constraint has been defined to prevent the model from trying to increase prices by reducing the energy offered by the company. This leads to vertical offer curves both for the on-peak and the off-peak hours. The ramping hour requires an offer curve that permits the generating groups increase their output without exceeding their ramp rate limits.

Fig. 10 shows the detail of the offer curve designed by the model for the ramping hour (square dots). It includes the expected generation decided by the SP that constitutes a feasible schedule (circles). All the quantities decided by the SP coincide with the quantities offered in the day-ahead market except for the last one. If the residual-demand scenario associated to this offer occurred, the company would sell the quantity offered by the MP, which is lower than the expected generation decided in the SP. This could lead to some infeasibility which would have to be addressed in the hour-ahead market.

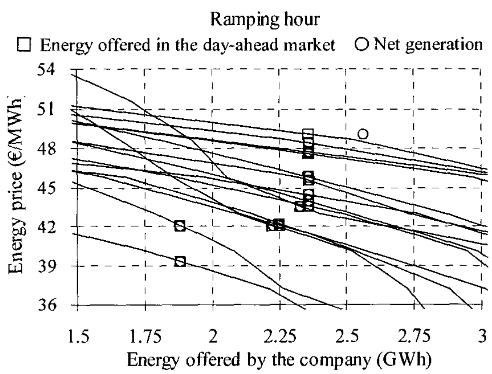

Fig. 10. Gap between the MP and the SP quantities in the ramping hour.

To better understand the flexibility introduced by the hour-ahead market, Fig. 11 depicts the profile of the expected amount of energy sold in the day-ahead market (given by the MP) and the expected net generation after the hour-ahead market (solution of the SP). The major differences appear in those hours where the company's power output changes abruptly (hours 9 and 23 are good examples).

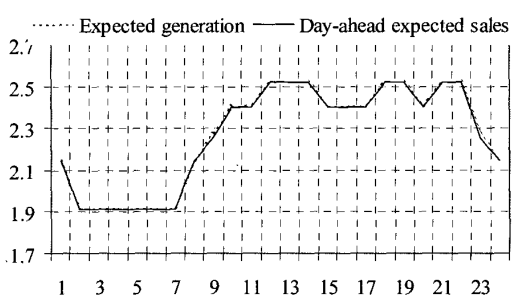

Fig. 11. Comparison between the MP and the SP quantities, in GWh.

The generation facilities available for this case were nuclear and coal-fueled units. Fig. 12 illustrates the expected usage of the different technologies for each hour.

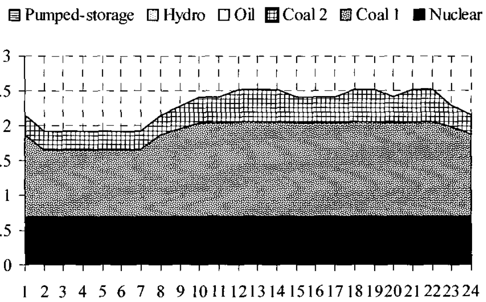

Fig. 12. Expected usage of the generation technologies.

B. Case with hydro energy

The proposed model is also designed to manage water resources. This adds the complexity of deciding in which hours this hydro production should be offered, although hydro units are more flexible than thermal ones.

Fig. 13 shows the offers for the same three hours as the ones represented in Fig. 9. It can be observed that more energy is bid and lower prices expected.

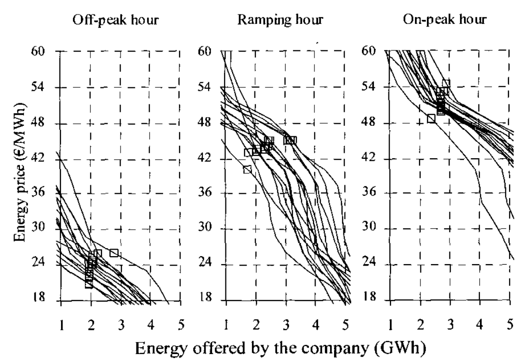

Fig. 13. Offer curves given by the model for three of the 24 hours.

An interesting representation of the offers is given in Fig. 14. It can be seen that, with this offers, the company is facing an extremely uncertain day-ahead schedule. In certain hours, the company's production may vary between 1.7 and 3.1 GWh, which is a great risk.

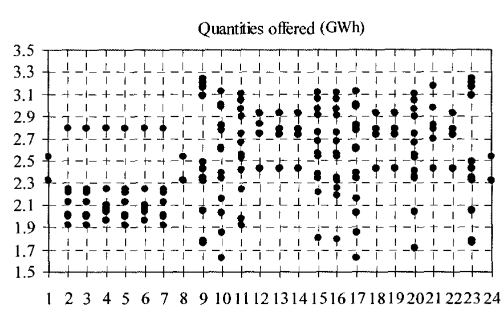

Fig. 14. Quantities offered in the day-ahead market.

In contrast, the probability distribution for prices is relatively narrow (Fig. 15). This suggests that the variance of the results should be an attribute for selecting optimal bids, to achieve an adequate short-term risk management.

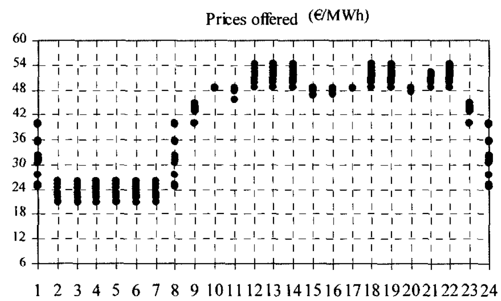

Fig. 15. Prices offered in the day-ahead market.

VI. Conclusions

In this paper a strategic bidding procedure based in stochastic programming is decomposed using the Benders technique. In this fashion the bidding problem separates naturally into a market master problem and a generation scheduling subproblem. This permits the solution of real-size problems and allows the user to keep a closer control on the optimization process.

The bidding procedure is enriched with the modeling of both the day-ahead and the hour-ahead markets. This tolerates certain infeasibilities in the day-ahead schedule, making it easier for the model to obtain a solution. Additionally, it suggests arbitraging opportunities between both markets.

Future research will be devoted to the reduction of the procedure's execution time and to the analysis of its results under different market and technical conditions.

VII. Acknowledgment

The authors would like to thank the support, experience and valuable comments of Iberdrola's Planning and Offers Department.

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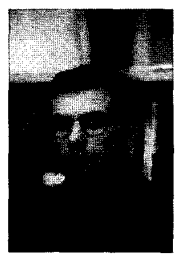
IX. Biographies

Alvaro Baillo (S '2000) was born in Madrid, Spain, in 1974. He graduated in Electrical Engineering in 1998 from the Universidad Pontificia Comillas, Madrid, Spain. He is currently a Ph.D. candidate at the Instituto de Investigación Tecnológica. His areas of interest include the application of operations research to the operation, planning and management of electric energy systems..



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