ANALYSIS OF MARGINAL COSTS IN THE OPERATION OF INTERCONNECTED HYDROTHERMAL SYSTEMS

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Keywords: Hydrothermal dispatch, Lagrangian function, optimality conditions.

ABSTRACT

This paper presents a method to derive and to understand the meaning of marginal costs in the multi-area hydrothermal dispatch problem. Optimal economic Karush-Kuhn-Tucker conditions are derived for each system resource on any stage of the study period using the Lagrange theory. The economic role of thermal units, hydro turbines, reservoirs and interconnections is analysed in detail. This methodology has been applied to a simplified case derived from part of the Central American system. Numerical values illustrate relations of agents at the optimum.

1. INTRODUCTION

Most planners are familiar with global optimal operation conditions of an electric energy system. Nevertheless, there is no comprehensive framework to understand the economic relationships among every system agent, such as hydro and thermal generators, reservoirs, and interconnections. Multi-area hydro-thermal systems have two different types of constraints difficult to deal with:

- temporal links between the consecutive stages in which the study period is divided, imposed by reservoir management; and
- spatial links between the system areas imposed by tie-line interconnections.

Since inflows are stochastic parameters, advanced methodologies have been implemented to model explicitly their effects in the hydrothermal dispatch [1]. Obviously, stochastic behaviour of inflows is propagated to all operation results making the economical analysis a difficult task. In this paper, deterministic models are preferred because marginal analysis can be carried out in a more comprehensive way. Besides they give planners consistent and comparative knowledge about economical effects of changing parameters (e.g., exchange capacity between regions, fuel prices, etc.).

Energy spot markets use marginal pricing as the optimal economic signals that agents must exchange in order to achieve maximisation of global social benefits [2], [3]. Under perfect information and competition assumptions, decentralised maximisation of benefits is equivalent to centralised minimisation of costs. Keeping this dual principle in mind, this paper presents a method to derive and to understand the meaning of marginal costs in the hydrothermal dispatch problem among interconnected subsystems. Optimal economic Karush-Kuhn-Tucker conditions are derived for each system resource on any stage of the study period using the Lagrange theory [4]. The economic role of thermal generators, hydro turbines, reservoirs and inter-connections is analysed in detail, giving a clear understanding that how these economical signals could be used in a competitive environment based on marginal pricing.

Other works have been carried out in the same topic. Reference [5] used a deterministic linear programming model to derive marginal capacity benefits of some agents in an isolated hydrothermal system, exchanges were not included. Those benefits were defined for cost allocation and marginal investment signal purposes. They were not defined and generalised for all operating possibilities. Reference [6] considered a short-term model including aggregated transmission losses. Operating conditions of thermal generators, hydro turbines and reservoirs were used for defining some decision rules to be included into an algorithm.

This paper contributes with a step by step procedure to derive optimal economic conditions for each agent including effects of water value, spillages, non-served energy and interconnections. Clarity and simplicity of the method guarantee an easy understanding of the role of each agent in the system.

Section 2 describes the symbols used in the paper. Section 3 presents the mathematical model of the multi-area hydrothermal problem and its Lagrangian model to derive the general optimality conditions. The analysis of marginal costs for each agent is detailed in section 4. This method has been applied to a simplified case from part of the Central American system. Numerical values illustrate optimal relations of agents in section 5. The conclusions of the paper are in section 6.

2. NOTATION

In this section, the symbols used along the paper are listed according to their importance.

S, s	number of subsystems or areas, indexes areas.
J, j	number of thermal units, indexes thermal units.
I, i	number of hydro plants, indexes hydro plants.
T, t	number of stages, indexes time stages.
c_{ens}	non-supplied energy cost.

c_{js} incremental cost in thermal unit j in area s.
ρ_{is} production coefficient of hydro plant i in area s.
A_{ist} inflow volume to plant i in area s during stage t.
D_{st} energy demand in subsystem s in stage t.
$\overline{F}_{sk}$ maximum transfer capacity of line sk.
$\overline{g}_{js}$ generation capacity of thermal unit j, area s.
$\overline{q}_{is}$ maximum turbined volume of plant i, area s.
$\overline{V}_{is}$, $\underline{V}_{is}$ max. and min. storage volume of plant i, area s.
f_{skt} power flow in line sk to/from area s, stage t.
ff_{skt} power flow in line sk to/from area k, stage t.
g_{jst} generation of thermal unit j in area s in stage t.
pns_{st} non-supplied energy in area s in stage t.
q_{ist} turbined outflow volume in plant i, area s, stage t.
v_{ist+1} stored volume in hydro plant i in area s at the ending of stage t or beginning of stage $t+1$.
ve_{ist} spilled volume in plant i, area s, during stage t.
$s \blacklozenge k$ tie-lines specified with area s as sending.
$k \blacklozenge s$ tie-lines specified with area s as receiving.
$s \cap k$ all tie-lines (or border nodes) in the system.
π it refers to a Lagrange multiplier.

3. MATHEMATICAL MODEL

The multi-area hydrothermal dispatch can be formulated by the following linear program problem:

$$z = Min \sum_{t=1}^{T} \beta_t \sum_{j=1}^{J} c_j g_{jt} \quad \text{(a)}$$

subject to: multiplier (1)

$$\sum_{i=1}^{I} \rho_i q_{it} + \sum_{j=1}^{J} g_{jt} = d_t + pns_{st} = D_{st} \quad :\pi d_t \quad \forall_t \quad \text{(b)}$$

$$\sum_{i=1}^{I} \rho_i q_{it} + \sum_{j=1}^{J} g_{jt} = d_t \quad :\pi d_t \quad \forall_t \quad \text{(c)}$$

$$v_{i,t+1} + q_{it} + s_{it} - \sum_{m \in Mi} \left[q_{mt} + s_{mt}\right] = v_{it} + a_{it} \quad :\pi h_{it} \quad \forall_i, \forall_t \quad \text{(d)}$$

$$g_{jt} \leq \overline{g}_j \sum_{\tau=1}^{t} x_{jt} \quad :\pi g_{jt} \quad \forall_i, \forall_t \quad \text{(e)}$$

$$g_{jt} \leq \overline{g}_j \sum_{\tau=1}^{t} x_{jt} \quad :\pi g_{jt} \quad \forall_i, \forall_t \quad \text{(f)}$$

$$v_{i,t+1} \leq \overline{v}_i \sum_{\tau=1}^{t} x_{it} \quad :\pi v_{it} \quad \forall_i, \forall_t \quad \text{(g)}$$

$$v_{i,t+1} \leq \overline{v}_i \sum_{\tau=1}^{t} x_{it} \quad :\pi v_{it} \quad \forall_i, \forall_t \quad \text{(h)}$$

$$q_{it} \leq \overline{q}_i \sum_{\tau=1}^{t} x_{it} \quad :\pi q_{it} \quad \forall_i, \forall_t \quad \text{(i)}$$

$$q_{it} \leq \overline{q}_i \sum_{\tau=1}^{t} x_{it} \quad :\pi q_{it} \quad \forall_i, \forall_t \quad \text{(j)}$$

$$g_{jt} \leq \overline{g}_j \sum_{\tau=1}^{t} x_{jt} \quad :\pi g_{jt} \quad \forall_t \quad \text{(k)}$$

$$g_{jt} \leq \overline{g}_j \sum_{\tau=1}^{t} x_{jt} \quad :\pi g_{jt} \quad \forall_t \quad \text{(l)}$$

$$g_{jt} \leq \overline{g}_j \sum_{\tau=1}^{t} x_{jt} \quad :\pi g_{jt} \quad \forall_t \quad \text{(m)}$$

$$g_{jt} \leq \overline{g}_j \sum_{\tau=1}^{t} x_{jt} \quad :\pi g_{jt} \quad \forall_t \quad \text{(n)}$$

$$g_{jt} \leq \overline{g}_j \sum_{\tau=1}^{t} x_{jt} \quad :\pi g_{jt} \quad \forall_i, \forall_t \quad \text{(o)}$$

$$q_{it} \leq \overline{q}_i \sum_{\tau=1}^{t} x_{it} \quad :\pi q_{it} \quad \forall_i, \forall_t \quad \text{(p)}$$

For the sake of simplicity, the duration of each stage t is assumed as 1 time unit, the discount rate, run-of-river plants and hydro networks are not modelled here. An appendix in reference [7] completes problem (1) with run-of-river plants and hydro networks.

The objective function (1a) is the minimisation of the variable costs in thermal production and non-served energy over all stages of the study period.

Constraints (1b & 1c) are the energy balance at each subsystem and each border node. Demand is a known parameter for all stages. Invariable hydro production coefficients are assumed. Interconnections between subsystems are split up by a fictitious border node. A classical transport model with no losses is used. Intra-area transmission network is not modelled.

Equation (1d) represents temporal coupling of reservoirs. Inflows are considered as deterministic parameters. The states of reservoirs at the beginning and ending of the study period are fixed data.

Constraints (1b-d) have been defined by the energy conservation principle: incoming flows are equal to outgoing ones. Inequality equations (1e-p) correspond to the technical and operational limits of primal variables.

As indicated in (1), each constraint has associated a dual variable called Lagrange multiplier. When problem (1) is solved, each multiplier indicates how much the total costs change with respect to a marginal change on the right hand side of the associated constraint while keeping the optimality conditions.

3.1 Relations Between Constraints and Multipliers

At the optimum, all constraints must be satisfied. Equality constraints must always be active, while inequality ones can be active or not. Table I resumes behaviour of Lagrange multiplier for each case.

Table I. Behaviour of Lagrange Multiplier

Type of Constraints		
$x \leq X$	$x ? \underline{X}$	$Ax = B$
$q_{it} \leq \overline{q}_i \sum_{\tau=1}^{t} x_{it}$	$q_{it} \leq \overline{q}_i \sum_{\tau=1}^{t} x_{it}$	$q_{it} \leq \overline{q}_i \sum_{\tau=1}^{t} x_{it}$

3.2 Lagrangian Function

The global optimisation problem (1) can be formulated as a Lagrangian function [4].

$$z = Min \sum_{t=1}^{T} \beta_t \sum_{j=1}^{J} c_j g_{jt} + \sum_{i=1}^{I} \rho_i q_{it} + \sum_{j=1}^{J} g_{jt} = d_t - \sum_{k \blacklozenge s} ff_{skt} - pns_{st} \Big] \pi d_{st} \quad (2)$$

$$+ \sum_{i=1}^{I} \rho_i q_{it} + \sum_{j=1}^{J} g_{jt} = d_t$$

$$+ v_{i,t+1} + q_{it} + s_{it} - \sum_{m \in Mi} \left[q_{mt} + s_{mt}\right] = v_{it} + a_{it}$$

$$+ \quad q_{it} \leq \overline{q}_i \sum_{\tau=1}^{t} x_{it}$$

- $q_{it} \le \overline{q}_i \sum_{\tau=1}^{t} x_{it}$

+ $q_{it} \le \overline{q}_i \sum_{\tau=1}^{t} x_{it}$

- $q_{it} \le \overline{q}_i \sum_{\tau=1}^{t} x_{it}$

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- $q_{it} \le \overline{q}_i \sum_{\tau=1}^{t} x_{it}$

- $q_{it} \le \overline{q}_i \sum_{\tau=1}^{t} x_{it}$

- $q_{it} \le \overline{q}_i \sum_{\tau=1}^{t} x_{it}$

At the optimum, the Lagrangian additional terms (constraints times multipliers) are equal to zero, because at least one of the two factors of the product is zero. The advantage of Lagrangian function (2) is that it allows to derive the following Karush-Kuhn-Tucker optimality conditions [4].

3.3 First Derivatives Respect to Dual Variables

$$\sum_{i=1}^{I} \rho_i q_{it} + \sum_{j=1}^{J} g_{jt} = d_t - \sum_{k \neq s} ff_{skt} - pns_{st} \qquad \forall_i, \forall_t \ (3)$$

$$\sum_{i=1}^{I} \rho_i q_{it} + \sum_{j=1}^{J} g_{jt} = d_t \qquad \forall_i, \forall_t \ (4)$$

$$q_{it} \le \overline{q}_i \sum_{\tau=1}^{t} x_{it} \qquad \forall_i, \forall_t \ (5)$$

$$q_{it} \le \overline{q}_i \sum_{\tau=1}^{t} x_{it} \qquad \forall_i, \forall_t \ (6)$$

$$q_{it} \le \overline{q}_i \sum_{\tau=1}^{t} x_{it} \qquad \forall_i, \forall_t \ (7)$$

$$q_{it} \le \overline{q}_i \sum_{\tau=1}^{t} x_{it} \qquad \forall_i, \forall_t \ (8)$$

$$q_{it} \le \overline{q}_i \sum_{\tau=1}^{t} x_{it} \qquad \forall_i, \forall_t \ (9)$$

$$q_{it} \le \overline{q}_i \sum_{\tau=1}^{t} x_{it} \qquad \forall_i, \forall_t \ (10)$$

$$q_{it} \le \overline{q}_i \sum_{\tau=1}^{t} x_{it} \qquad \forall_i, \forall_t \ (11)$$

$$q_{it} \le \overline{q}_i \sum_{\tau=1}^{t} x_{it} \qquad \forall_i, \forall_t \ (12)$$

$$q_{it} \le \overline{q}_i \sum_{\tau=1}^{t} x_{it} \qquad \forall_i, \forall_t \ (13)$$

$$q_{it} \le \overline{q}_i \sum_{\tau=1}^{t} x_{it} \qquad \forall_i, \forall_t \ (14)$$

$$q_{it} \le \overline{q}_i \sum_{\tau=1}^{t} x_{it} \qquad \forall_i, \forall_t \ (15)$$

$$q_{it} \le \overline{q}_i \sum_{\tau=1}^{t} x_{it} \qquad \forall_i, \forall_t \ (16)$$

$$q_{it} \le \overline{q}_i \sum_{\tau=1}^{t} x_{it} \qquad \forall_i, \forall_t \ (17)$$

Satisfying optimality conditions (3-17) implies satisfying constraints of the problem (1).

3.4 First Derivatives Respect to Primal Variables

$$\sum_{i=1}^{I} \rho_i q_{it} + \sum_{j=1}^{J} g_{jt} = d_t \qquad \forall_i, \forall_t \ (18)$$

$$\sum_{i=1}^{I} \rho_i q_{it} + \sum_{j=1}^{J} g_{jt} = d_t \qquad \forall_i, \forall_t \ (19)$$

$$\sum_{i=1}^{I} \rho_i q_{it} + \sum_{j=1}^{J} g_{jt} = d_t \qquad \forall_i, \forall_t \ (20)$$

$$\sum_{i=1}^{I} \rho_i q_{it} + \sum_{j=1}^{J} g_{jt} = d_t \qquad \forall_i, \forall_t \ (21)$$

$$\sum_{i=1}^{I} \rho_i q_{it} + \sum_{j=1}^{J} g_{jt} = d_t \qquad \forall_i, \forall_t \ (22)$$

$$\sum_{i=1}^{I} \rho_i q_{it} + \sum_{j=1}^{J} g_{jt} = d_t \qquad \forall_i, \forall_t \ (23)$$

$$\sum_{i=1}^{I} \rho_i q_{it} + \sum_{j=1}^{J} g_{jt} = d_t \qquad \forall_i, \forall_t \ (24)$$

Since derivatives (18-24) must always be satisfied at the optimum, they are more restrictive than primal constraints (3-17). The optimality conditions (18-24) define relations among dual variables of the problem. Therefore, they allow a comprehensive economical interpretation.

4. ANALYSYS OF MARGINAL COSTS

In this section, the marginal capacity benefit (called marginal benefit) is defined for each variable combining all the Karush-Kuhn-Tucker conditions (3-24).

$$\sum_{i=1}^{I} \rho_i q_{it} + \sum_{j=1}^{J} g_{jt} = d_t$$

(30abc)

When a reservoir is at its lower level (30a) then the value of water in the current stage is greater than that value in the next stage. For example, if current inflows are scarce and the expected future incomes would be abundant, benefits for depleting the reservoir now would be higher than in the future.

When a reservoir operates within bounds then the value of water in the current stage is equal to that value in the next one (30b).

When a reservoir operates at its maximum level (30c) then the value of water in the current stage is lower than that value in the next one. For example, wet hydrology on current stage and dry one in the future.

The role of a hydro reservoir is as an agent that buys/sells water on the current stage at the price πh_{ist} and sells/buys it in the next stage at the price πh_{ist+1}. The difference between those prices are the incentive for storage capacity expansion of the reservoir, πv_{ist+1}.

4.2.1 Value of water between stages

In consecutive stages, the value of water for each reservoir can be defined matching (28) and (29).

$$\sum_{i=1}^{I} \rho_i q_{it} + \sum_{j=1}^{J} g_{jt} = d_t \quad (31abc)$$

According to (31b), if the reservoir is within bounds at every stage, the value of water is unique at all stages. In practice, condition (31b) can be applied to large reservoirs with high regulation capacity, since it is difficult to reach their limits during a certain period.

If the reservoir reaches any of its limits at some stage (31ac), then its water would have different values in the separated common stages.

4.3 Marginal Benefit for Hydro Turbine

Marginal benefit for each hydro turbine in each subsystem at any stage can be derived for both bounds using (20) and taking into account (10)-(11) while satisfying (3) and (5).

$$\sum_{i=1}^{I} \rho_i q_{it} + \sum_{j=1}^{J} g_{jt} = d_t$$

(32abc)

$$\sum_{i=1}^{I} \rho_i q_{it} + \sum_{j=1}^{J} g_{jt} = d_t$$

(33abc)

Matching (32abc) and (33abc), the marginal benefit for each hydro turbine for any stage is defined as:

$$\sum_{i=1}^{I} \rho_i q_{it} + \sum_{j=1}^{J} g_{jt} = d_t \quad \text{(34abc)}$$

A turbine should not be dispatched when the value of water at its own reservoir is higher than SMC times the production coefficient (34a). For example, a possible application case is to compute the value of water in a subsystem simulating isolated operation. This water value would reflect high thermal production costs avoided into this local area. Then, the hydro turbine would bid with that price in a possible more efficient regional market where all the subsystems compete.

When a hydro turbine operates within bounds then the value of water at its own reservoir is equal to SMC times the production coefficient (34b). That means the hydro turbine is the marginal of the system.

When a hydro turbine operates at its maximum capacity then the value of water at its own reservoir is lower than SMC times the production coefficient (34c). This is common at peak load hours where the turbine operates at top capacity below the system marginal generator.

The role of a hydro turbine is as an agent that buys water to its own reservoir at the price πh_{ist} and sells it at the market price πd_{st} times ρ_{is}. Again, the marginal benefit of a turbine, πq_{ist}, is the incentive signal for increasing its capacity.

4.3.1 Value of water

From (34abc) the optimal water value relations in any stage can be defined as:

$$\sum_{i=1}^{I} \rho_i q_{it} + \sum_{j=1}^{J} g_{jt} = d_t \quad \text{(35abc)}$$

When the hydro turbine is within bounds (35b) in some stage t, the value of water in its reservoir can be directly checked relating (35b) and (27b):

$$\sum_{i=1}^{I} \rho_i q_{it} + \sum_{j=1}^{J} g_{jt} = d_t \quad \text{(36)}$$

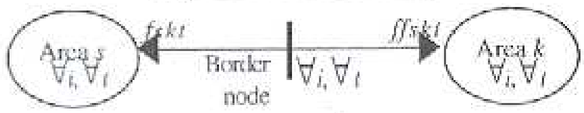
If the marginal cost of the system is imposed by another hydro plant instead of a thermal plant, the condition (35b) is preferred. As indicated in (31b), the water value can be propagated to other stages if the reservoir does not reach its storage limits. That effect does not mean that the SMC in the next stage will have the same value than in the current one, since the current marginal generator might be dispatched at top capacity or put out of the system (minimum capacity) in the next stage.

The relations (34) or (35) indicate that the value of water is the economic signal for dispatching the hydro turbine. That it is why the water value is used as a key price to bid hydro generation in a competitive market.

4.4 Marginal Benefit for Interconnections

The power flow through interconnection between subsystems s and k is modelled as indicated in Figure I.

Figure I. Transport model



4.4.1 Marginal benefit for line sk for subsystem s

Marginal benefit for the interconnection sk from point of view of subsystem s at any stage can be derived for both bounds using (21) and taking into account (12)-(13) while satisfying (3) and (4).

$$\sum_{i=1}^{I} \rho_i q_{it} + \sum_{j=1}^{J} g_{jt} = d_t \quad \text{(37)}$$

$$\sum_{i=1}^{I} \rho_i q_{it} + \sum_{j=1}^{J} g_{jt} = d_t \quad \text{(38)}$$

Matching (37) and (38), the marginal benefit for the interconnection sk for the subsystem s at any stage is:

$$\sum_{i=1}^{I} \rho_i q_{it} + \sum_{j=1}^{J} g_{jt} = d_t \quad \text{(39)}$$

When the power flow through the interconnection sk is at its lower capacity (maximum exports of the subsystem s) then the marginal cost at the border node is higher than the marginal cost at the subsystem s (39a). When the flow is within bounds then both marginal costs are the same (39b). When the flow is at the top capacity (maximum imports of the subsystem s) then the subsystem marginal cost is higher than the one at the border (39c).

The role of the interconnection sk is as an agent that buys/sells energy at the border node at a price πf_{skt} and sells/buys it to the subsystem s at the price πd_{st}. The difference between those prices (πF_{skt}) is the incentive for transfer capacity expansion at the interconnection.

4.4.2 Marginal benefit for line sk for subsystem k

Analogously, the marginal benefit for the interconnection sk for the subsystem k at any stage is defined as:

$$\sum_{i=1}^{I} \rho_i q_{it} + \sum_{j=1}^{J} g_{jt} = d_t \quad \text{(40)}$$

Similar conclusions of (39) are valid for area k (40).

4.4.3 System marginal cost

If there is no network congestion during any stage, the relations (39b) and (40b) establish that all areas and border nodes operate at the same marginal cost.

$$\sum_{i=1}^{I} \rho_i q_{it} + \sum_{j=1}^{J} g_{jt} = d_t \qquad \forall_i, \forall_t \quad (41)$$

If there were some network congestion, then the global system is spatially separated in economical islands with different SMC, see (39ac) and (40ac).

4.5 Effects of Non-Supplied Energy

In problem (1), the Non-Supplied Energy -NSE- is modelled as a fictitious thermal generator with infinite capacity and a high unitary cost, *Cens*. The marginal benefit of NSE at any stage can be derived from the optimality condition (23) taking into account (16) and satisfying (3).

$$\sum_{i=1}^{I} \rho_i q_{it} + \sum_{j=1}^{J} g_{jt} = d_t \qquad (42ab)$$

Relation (42a) means that there is no NSE when *Cens* is higher than the SMC. NSE appears when *Cens* is equal to SMC (42b). That is, the fictitious generator NSE is the marginal of the system. This happens when installed thermal generation capacity plus hydro production plus imports are not enough to supply load.

4.6 Effects of Spillages

Marginal benefit of spillages for each hydro plant in each subsystem at each stage can be defined from (24) taking into account (17) and satisfying (5).

$$\sum_{i=1}^{I} \rho_i q_{it} + \sum_{j=1}^{J} g_{jt} = d_t \qquad (43ab)$$

The economical interpretation of relations (43ab) is straightforward. At each stage and in each subsystem, if a hydro plant does not spillage, then its water has a value higher than zero (43a). If there are spillages, then the value of water is null (43b).

5. CASE STUDY

The linear programming technique implemented in GAMS language [8] has been used for solving the linear optimisation problem (1). The concepts on marginal costs developed in this paper will be illustrated in a case study.

5.1 System Configuration

The studied system is formed by three simplified subsystems corresponding to Guatemala -GU-, El Salvador -ES- and Honduras -HO-. They are connected in cascade by a tie-line with a transfer capacity of 200 MW. The study period has been divided in 3 stages. In each stage, the demand is modelled by two constant load levels: i) peak -n1-, and ii) off-peak -n2-. The peak level lasts 17.8% of the stage time duration.

Each subsystem has one large hydro plant with a reservoir. Hydro reserves are considered as 75% of maximum capacity at the beginning and ending of the study period. Inflows reflect average historical data for each reservoir taking into account temporal and spatial correlations.

Table II. Data of Hydro Plants

Plant - Country	Minimum Storage hm^3	Maximum Storage hm^3	Maximum Turbined Flow m^3/s - MW	Production Coefficient $MW/(m^3/s)$
Chixoy-GU	128	451	69 - 275	3.974
5Nov-ES	787	2180	293 - 135	0.461
Cajón-HO	2139	5653	239 - 300	1.257

Table III. Hydro Inflows

Plant	Inflows m^3/s		
	Stage *t1*	Stage *t2*	Stage *t3*
Chixoy	55.4	36.1	95.7
Cajón	292.5	163.8	312.0
5Nov	115.4	95.0	225.7

Diesel -DS- and Bunker -BK- are the basic fuels used by the thermal plants in the region. So, each country is modelled with two equivalent thermal plants.

Table IV. Data of Thermal Plants

Plant - Country	Maximum Capacity MW	Incremental Cost $/MWh
BK-GU	150	35
DS-GU	335	55
BK-ES	185	45
DS-ES	210	65
BK-HO	160	60
DS-HO	110	75

The incremental cost of those plants reflects: i) real average efficiencies of state-own plants, autoproducers and private contracts, and ii) the same fuel price in all regions, BK 18.5 $/bbl and DS 27.3 $/bbl.

5.2 Energy Results

The optimal power solution for the case study is presented in Table V. The values in "bold" indicate that maximum limits have been reached.

Table V. Power Balance (MW)

Area	Plant	Stage *t1*		Stage *t2*		Stage *t3*	
		n1	n2	n1	n2	n1	n2
GU	BK	**150**	**150**	**150**	**150**	**150**	**150**
	DS	201	0	190	0	215	0
	Chixoy	**275**	**275**	**275**	176	**275**	**275**
	Import	-176	-125	-155	-16	**-200**	-95
	Load	*450*	*300*	*460*	*310*	*440*	*330*
ES	BK	**185**	89	**185**	**185**	**185**	**185**
	DS	0	0	0	0	0	0
	5Nov	**135**	**135**	**135**	**135**	**135**	73
	Import	55	26	65	-110	45	-18
	Load	*375*	*250*	*385*	*210*	*365*	*240*
HO	BK	0	0	0	0	15	0
	DS	0	0	0	0	0	0
	Cajón	**300**	171	**300**	94	**300**	207
	Import	121	99	90	126	155	113
	Load	*421*	*270*	*390*	*220*	*470*	*320*

GU exports and HO imports in every load level at all stages. ES exports to HO during the off-peak levels of stages *t2* and *t3* and imports from GU in the rest of the study period. Network congestion appears only between GU and ES at peak hours in stage *t3*.

All hydro plants have been dispatched at their maximum capacity during peak hours.

The BK thermal plant in GU is the most competitive of the region, it has been always dispatched at maximum capacity along the study (base-load plant). BK in ES is also competitive. BK in HO has very expensive costs; it is only dispatched in stage *t3* due to the network congestion. The DS plants in ES and HO have not been dispatched due to low efficiency.

5.3 Marginal Results

The ouputs about marginal costs are detailed in Table VI for each area and border node. The water value in each hydro plant is presented in Table VII.

The system marginal costs are 55 and 45 $/MWh during peak and off-peak hours, respectively. Due to the congestion between GU and ES during the peak hours in *t3*, the system is broken into two economical islands. Under that congestion, the marginal cost in ES and HO is 60 $/MWh as stated by relation (39a).

Table VI. Marginal Costs ($/MWh)

Area or Border Node	Stage *t1*		Stage *t2*		Stage *t3*	
	n1	n2	n1	n2	n1	n2
GU	55	45	55	45	55	45
GU-ES	55	45	55	45	60	45
ES	55	45	55	45	60	45
ES-HO	55	45	55	45	60	45
HO	55	45	55	45	60	45

Table VII. Value of Water ($/MWh)

Plant	Stage *t1*	Stage *t2*	Stage *t3*
Chixoy	45	45	45
5Nov	45	45	45
Cajón	45	45	45

The value of water is 45 $/MWh for all hydro plants during all stages. This value coincides with the system marginal cost during off-peak hours.

As stated by (27b) the system marginal generator at the peak hours in every stage is the DS thermal plant from GU. Meanwhile the off-peak demand has different marginal generators in each stage. Using (27b) the BK thermal plant from ES is the system marginal at the off-peak load in stage *t1* as well as the Cajón plant from HO (34b). According to (36), the water value in Cajón is derived there. According to (31b), that water value can propagate to the stages *t2* and *t3* if the Cajón's reservoir does not reach its storage limits as it happens in this example. Then, applying (34b), Cajón is the system marginal in the off-peak hours in stage *t2* as well as Chixoy from GU. Cajón is also the system marginal in the off-peak hours in stage *t3* as well as 5Nov from ES. The water values in Chixoy and 5Nov are derived at the off-peak hours in stages *t2* and *t3*, respectively, by means of (35b).

Table VIII. Marginal Benefit ($/MWh)

Plant	Stage *t1*		Stage *t2*		Stage *t3*	
	n1	n2	n1	n2	n1	n2
BK-GU	-20	-10	-20	-10	-20	-1
DS-GU	0	10	0	10	0	10
Chixoy	-10	0	-10	0	-10	0
BK-ES	-10	0	-10	0	-15	0
DS-ES	10	20	10	20	5	20
5Nov	-10	0	-10	0	-15	0
BK-HO	5	15	5	15	0	15
DS-HO	20	30	20	30	15	30
Cajón	-10	0	-10	0	-15	0

In Table VIII, the marginal capacity benefit is presented for thermal plants and hydro turbines. In this example, any reservoir reaches storage limits during the study period, except at the beginning and ending stages where a 75% reserve level was imposed. There are no spillages on any hydro plant. There is not non-supplied energy in any subsystem.

The marginal benefits for interconnections GU-ES and ES-HO are zero for all load levels and for all stages except GU-ES during peak of stage *t3*. Due to the congestion, marginal benefit of tie-line GU-ES is 5 $/MWh. According to (39a), that benefit comes from the difference between the system marginal costs at the end points of the line.

The marginal benefits for each agent indicated in Table VIII can be contrasted with all the theoretical conditions derived in this paper. For example, the marginal benefit for marginal plants is zero, for those plants operating at maximum capacity is negative, and for non-dispatched plants is positive.

6. CONCLUSIONS

In this paper, a simple method to derive the optimality conditions for operating multi-area hydrothermal systems has been presented. As a by-product, the economical implications of marginal costs and prices on the operating conditions of each single system agent have been analysed.

The understanding and computation of these marginal costs and prices can be a valuable information in the new framework of competition where several interconnected systems will form an integrated wholesale energy market. In addition, the computation of marginal capacity benefits for each one of the system resources appear as a relevant economical signal to drive new facility investments.

For the sake of simplicity, electrical and hydraulic losses, hydro networks and run-of-river plants were not modelled. Nevertheless, their economical effects and implications can be deduced using the presented methodology.

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