Auxiliary material allowed during the exam: All the teaching material available in the Moodle site of the course printed, any handwritten notes of the course, any non-photocopied book and a non-programmable calculator. Each student must bring her or his own auxiliary material, as this cannot be shared during the exam. The use of any device with internet connection is prohibited and must remain turned off and out of the reach of the student during the exam.

Name: Group:

THE ORCHESTRA

The producer of a concert venue is organizing the next four concerts. Each concert is devoted to one composer and requires a different number of musicians, as shown in the following table (concerts are listed in chronological order):

Concert	violins required	violas required	cellos required
Beethoven	3	2	1
Bach	2	2	1
Debussy	2	2	0
Brahms	3	3	1

After a previous selection, they are considering a group of 10 candidates. Each of these candidates plays a different instrument and they have a different performance level depending on the composer they have to play. They also demand a different amount of money depending on the composer. In the following table, all this information is shown:

Musician	Instrument	pay [k€] / performance	pay [k€] / performance	pay [k€] / performance	pay [k€] / performance
		Beethoven	Bach	Debussy	Brahms
Alice	violin	1.3 / 9.2	1.4 / 9.7	1.0 / 8.5	1.1 / 8.8
Brian	violin	1.1 / 8.6	1.2 / 9.0	1.4 / 9.8	1.3 / 9.5
Carol	violin	1.2 / 8.9	1.5 / 10.0	1.2 / 9.1	1.2 / 9.1
Dave	violin	1.0 / 8.4	1.3 / 9.4	1.6 / 9.9	1.0 / 8.7
Ernest	viola	1.0 / 9.1	1.1 / 9.3	1.3 / 9.6	1.2 / 9.4
Fabiola	viola	1.2 / 9.6	1.2 / 9.5	1.1 / 9.1	0.9 / 8.2
Gabrielle	viola	0.9 / 8.1	1.0 / 8.4	1.2 / 9.0	1.1 / 8.7
Helen	viola	1.1/ 9.3	1.3 / 9.7	1.0 / 8.9	1.0 / 8.5
Ignatius	cello	1.0 / 9.0	0.9 / 8.8	Not required	1.3 / 9.7
Jane	cello	1.3 / 9.8	1.2 / 9.4	Not required	0.9 / 8.2

The producer has a budget of 27 k€ to hire musicians for the total of these four concerts.

Formulate a mixed integer **linear** program to help the producer decide which musicians to hire for each concert so that the total performance across the four concerts (sum of performance points of all the musicians hired for each concert) is maximized.

- Define clearly sets, parameters and variables (2.5 points).
- Define clearly the objective function (1.5 points).
- Define clearly the constraints to consider the following conditions that have to be fulfilled:
  1. The required number of musicians for each instrument is hired for each concert (1 point).
  2. The total budget is not exceeded (1 point).
  3. If Alice, Dave and Fabiola, the three together, are hired for the same concert, Ernest has to be hired for that concert too (1 point).
  4. When Jane and Fabiola play together in the same concert, as they have played many times before in the past, they increase their performance in 0.2 points each (1.5 point).
  5. The difference between the average performances of the concert with the highest average performance and the concert with the lowest average performance cannot be greater than 0.5 performance points (1.5 points).

Name: Group:

SOLUTION.

Sets

c	Concerts {Beethoven, Bach, Debussy, Brahms}
m	Musicians {Alice, Brian, Carol, Dave, Ernest, Fabiola, Gabrielle, Helen}
i	Instruments {violin, viola, cello}

Parameters

$conc\_instr_{ci}$		number of instruments i required in concert c
mus_inst_{mi}		which istrument i is played by each musician m
$price_{mc}$		price of each musician m for each concert c [k€]
$perf_{mc}$		performance of each musician m for each concert c
$budget$		total budget available [k€]
$extra\_perf$		Additional performance points for Jane and Fabiola when they play together
dif		Maximum difference in average performance among concerts
n_c		Number of musicians required for concert c

Variables

X_{mc}	whether musician m is selected for concert c (binary)
Z_c	whether Jane and Fabiola play together in concert c (binary)
B	Best average performance of a concert
W	Worst average performance of a concert

Equations

0. Objective Function. Maximize sum of total performance points:

$$\max \sum_{m,c} perf_{mc}X_{mc} + extra\_perf \sum_{c} Z_c$$

1. Number of instruments required in each concert

$$\sum_{m} mus\_inst_{mi}X_{mc} = conc\_instr_{ci} \quad \forall c,i$$

2. Total budget available

$$\sum_{m,c} price_{mc}X_{mc} \leq budget$$

3. If Alice, Dave and Fabiola, the three together, are hired for the same concert, Ernest has to be hired for that concert too

$$X_{Alice,c} + X_{Dave,c} + X_{Fabiola,c} \leq 2 + X_{Ernest,c} \quad \forall c$$

4. When Jane and Fabiola play together, as they have played many times before in the past, they increase their performance in 0.2 points each.
Jane and Fabiola play together in concert c:

$$X_{Jane,c} + X_{Fabiola,c} \geq 2Z_c \quad \forall c$$

4º GITI/GITT – Operations Research
October 2, 2019
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$$X_{Jane,c} + X_{Fabiola,c} \leq 1 + Z_c \quad \forall c \quad (not\ necessary)$$

5. The difference between the average performances of the concert with the highest average performance and the concert with the lowest average performance cannot be greater than 0.5 points

   5.1. Best average performance
$$B \geq \frac{1}{n_c}\left(\sum_m perf_{mc}X_{mc} + extra\_perfZ_c\right) \quad \forall c$$

   5.2. Worst average performance
$$W \leq \frac{1}{n_c}\left(\sum_m perf_{mc}X_{mc} + extra\_perfZ_c\right) \quad \forall c$$

   5.3. Maximum difference
$$B \leq W + dif$$

6. Natural constraints
$$X_{mc} \in \{0,1\} \quad \forall m,c$$
$$Z_c \in \{0,1\} \quad \forall c$$
$$B, W \geq 0$$