

## Queueing Theory Problems

### PROBLEM 1

The counter of a bank branch performs the transactions with a mean time of 2 minutes. The customers arrive at a mean rate of 20 customers/hour. If we assume that arrivals follow a Poisson process and that the service time is exponential, determine:

- a) Percentage of the time the bank teller is idle
- b) Mean waiting time of the customers
- c) Percentage of customers that wait in a queue

### PROBLEM 2

A grocery shop is attended by one person. Apparently, the arrival pattern of customers during Saturdays follows a Poisson process with an arrival rate of 10 persons/hour. Customers are attended following a FIFO order and, due to the prestige of the shop, once they arrive, they are willing to wait for the service. The service time is distributed exponentially, with a mean time of 4 minutes. Determine

- a) Probability of waiting in line
- b) Average length of the waiting line
- c) Average waiting time

### PROBLEM 3

Customers arrive at a hair salon with a mean rate of 5 customers/hour and inter-arrival times distributed exponentially. There is always one hairstylist present in the salon at any moment and there are 4 chairs for the customers that arrive when the hairstylist is busy. The law regarding fire prevention limits the total number of customers in the salon, at any moment, to a maximum of 5. Customers that arrive when the salon is full can't enter in it. Service time is distributed exponentially with a mean that depends on the number of customers. Determine:

- a) Average number of waiting customers
- b) Average waiting time
- c) Percentage of time that the hairstylist is idle

Number of customers in the salon	1	2	3	4	5
Mean service time per customer (min.)	9	10	10	13	20

### PROBLEM 4

In a factory, there is a Social Security office where the workers can go during working hours. The Chief of Staff, who has observed the inflow of workers to the office, has asked to perform a study of this service. A specialist has been appointed to determine the average waiting time of the workers and the average duration of the conversation with

the office employee. This analyst reaches the conclusion that in the first and last half hours of the shift the inflow is very reduced and fluctuating, but during the remaining seven hours of the shift the phenomenon can be considered in steady state. From the analysis of 100 periods of 5 minutes, consecutive or not, but in the steady state phase, it can be deduced that the mean number of workers going to the office was 1.25 per period and the inter-arrival time followed an exponential distribution. A similar study about the duration of the conversations concluded that its distribution was exponential with a mean duration of 3.33 minutes. Determine:

- a) Average number of workers waiting
- b) Average waiting time
- c) Compare the time spent by the workers and the time spent by the office employee. Determine the cost for the company if an hour of the office employee is 250 € and one hour of the worker 400 €. Would it be profitable to open another office?

PROBLEM 5

Let us assume that the fleet of *Iberia* has four planes of type Airbus A350. Since 2015 the turbine failures of these planes have been observed. Data indicate that the failure for any turbine in any plane is a random variable with exponential distribution and the mean time to failure for any turbine is one year. The mean time to repair the turbine failure is only 45 days (one eighth of a year) and follows an exponential distribution. Only a single expert team can repair them, and they provide the service under the policy “first in the workshop, first to be repaired”. In the repair period, the plane doesn't fly. Describe quantitatively the queueing system.

PROBLEM 6

A bank considers the possibility of installing a teller network in one of its branches. Given that they don't know the people inflow demanding the service, they only put a single teller for a month. They collect data daily about the arrival time of the customers, as well as the service time. If the branch is in a district where there is no such service, the customer that arrives waits until he/she can use the teller when it is busy. After the corresponding data analysis, it is estimated that:

1. The arrivals follow a Poisson process
2. The distribution of the service time is exponential
3. The mean time between consecutive arrivals is 7.5 minutes
4. The mean service time is 5 minutes/customer

Determine:

- a) Mean waiting time that each customer must spend in the queue
- b) Average number of waiting customers

- c) Average queue length when it is not empty and probability that when going to the teller there will be somebody in the queue

PROBLEM 7

In a workshop, up to four machines can be repaired by two technicians. Machines arrive at the workshop for being repaired once every three hours on average and the mean repairing time is 45 minutes. If both times follow an exponential distribution, which is the average number of machines broken and waiting for being repaired in steady state?

PROBLEM 8

A bank branch has two office employees equally efficient and capable of attending 60 operations/hour on average, with the true time distributed exponentially. Customers arrive at the bank branch following a Poisson process with a mean rate of 100 customers/hour. Determine:

- a) Probability of strictly more than 3 customers simultaneously in the bank branch  
b) Probability that at least one of the employees will be idle

PROBLEM 9

A self-service car wash has four sections. In each one, the customers can wash and wax their cars. There is only space for a maximum of three additional cars when the washing sections are full. Customers arrive at the service following a Poisson process with a mean rate of 15 cars/hour. If there is no room for waiting, cars must leave the service. Apparently, the time required to service a car is distributed exponentially with a mean of 12 minutes. Determine:

- a) Average number of cars in the service at any moment  
b) Average number of cars per hour that are rejected because the service is full

PROBLEM 10

In the border between countries A and B, the road from B to A is split into five migration and customs inspection booths. Let's assume that car arrivals have a Poisson distribution with 15 arrivals/hour, while the number of services follows an exponential distribution of 8 services/hour.

By Government decree, there is no treatment priority (is that credible?), so the inspection booths provide the service as soon as they are free, attending the cars by order of arrival to the queue.

Check if  $\lambda < \mu$ , and in positive case, interpret this inequality. Moreover, determine:

- a) Compute  $p_0$ ,  $L$ ,  $L_q$ ,  $W_q$ ,  $W$ , defining its meaning  
b) Study if it would be reasonable to suppress three booths

## QUEUEING THEORY

### PROBLEM 11

The workers of a factory must carry their work to the quality control department before the product goes to the final production process. There are many employees and the arrivals are approximately 20 workers/hour, following a Poisson process. The time to inspect a part follows an exponential distribution with a mean of 4 minutes. Determine the average number of workers in the quality control department if there are: a) 2 inspectors, b) 3 inspectors.

### PROBLEM 12

Consider a queue with a single server and Poisson arrivals with 10 arrivals/hour on average. Usually, the server works with an exponential distribution with a mean service time of 5 minutes. The management has a training course that achieves an improvement (decrease) in the service time variance at the expense of a small increase in the mean. After finalizing the study, it is estimated that the mean service time will increase in 0.5 minutes and the standard deviation will decrease from 5 to 4 minutes.

- a) The management would like to know if they should continue with the employee as is or if they should continue with the new training course. Discuss the results
- b) Compute the variance reduction needed to counteract an increase of 0.5 minutes of the mean

### PROBLEM 13

The manufacturer of a specialized electric conductor has a large manufacturing volume. The conductor is subject to a single mechanical manipulation by a specialized machine. Given the sale volume, the company has a lot of equal machines working all the time (let's assume that the number of machines is infinite). Machines break following a Poisson process with a mean 5 machines/hour. The company has a single repair technician and the failures are of two possible types due to the characteristics of the machines. Depending on the failure type, it can be repaired in 9 or 12 minutes.

As the technician is an expert and the machines are identical, the possible variation in repair times are negligible and can be ignored. The type of cause that produces the failure is distributed randomly, but one third of the failures need 12 minutes for being repaired. Determine the average number of failed machines.

### PROBLEM 14

An airplane takes, almost exactly, 4 minutes to land from the time where the control tower gives it the landing signal. If the airplane arrivals are 8/hour on average and follow a Poisson process, how much time can the pilot expect to fly around the airport until receiving the landing signal?

### PROBLEM 15

In the emergency unit of a hospital, emergency cases arrive with a Poisson distribution of once every 5 hours on average. Before any surgical operation or any other

intervention, each patient must follow a medical exam and a diagnosis of his/her disease. This process takes 15 minutes (almost exactly). Compute (assuming the system in steady state):

- a) Average number of patients that have not been operated yet
- b) Average time a patient waits until entering the operating room
- c) Average time a patient waits until being examined
- d) Average number of patients waiting to be examined



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## Additional Queueing Theory Problems

### PROBLEM 16

A computer company has a server and customers can access this server from terminals (of different times) that they rent. A customer needs to determine the terminal optimal speed that should rent. Customer works are generated as a Poisson process with a rate of 50 works per day of 8 hours. The average size of the work is 1000 instructions. It is known that the processing time of the instructions is exponential. The customer estimates in 10 € the cost of delaying a work one day.

The company estimates that a speed of 100 instructions/minute and any similar increase, increases the monthly rent price of the terminal by 100 €. Assume that a month has 22 working days. Determine the optimal terminal speed.

### PROBLEM 17

Consider that a workshop can hire one out of two possible technicians. The first technician earns 3 € an hour and can repair 5 machines/hour. The second one earns 5 € an hour but can repair 8 machines/hour. Both salaries are paid no matter if they are busy or idle. Customers estimate that a broken machine represents a cost of 8 €/hour and the workshop assumes it as a cost per unit of waiting time per customer. Assume that machines arrive at the workshop following a Poisson process of 4 machines/hour and the repair time follows an exponential distribution. Which technician would be more economical for the company?

### PROBLEM 18

A company, which has four machines that are frequently broken, hires a person devoted exclusively to repairing them. The repair time follows an exponential distribution of 2 hours on average. Once the machine has been repaired, the time to failure again follows an exponential distribution of mean 10 hours. When the machine is under repair, the wasted time has a value of 20 €/hour and the repair service costs 45 euro a day. Assume a working day of 8 hours.

- a) Compute the steady-state distribution
- b) Compute the average number of operating machines in a steady state
- c) Which is the average cost per day of the wasted time?
- d) Would it be profitable to have two technicians, each one attending two machines?
- e) Would it be profitable to have two technicians, each one attending the four machines?

### PROBLEM 19

A small company, which manufactures devices for computers, has 10 machines for this task. Machines are frequently broken, but there are 8 persons in charge of operating them. So, two of them are in reserve to be used when some of the active ones fail. Therefore, as soon as there are no more than two broken machines eight will be used.

The time to failure has an exponential distribution with a mean of 20 days. The repair time is distributed with an exponential with a mean of 2 days.

Up to now, the factory has had only a technician that repaired the broken machines with the corresponding effect on the production when there were less than 8 available machines. Therefore, they are thinking about hiring another technician to be able to repair two machines at the same time.

Each technician costs 70 €/day, but the profit loss estimated for productivity reduction per each one of the 8 machines not working is 100 €/day.

PROBLEM 20

A railway company paints its own wagons, as they need, in its own workshops where each wagon is manually and individually painted with a speed that is exponentially distributed with a mean of one every 4 hours and an annual cost of 4 million €. Wagons arrive with a Poisson process of mean one every 5 hours. Besides, the wagon cost when it is not active is 500 euro an hour.

They are thinking about two possibilities. One is to subcontract the process to a painting company that would use aerosol spray with the consequent saving in time. However, the budget for this alternative is 10 million € per year. In this case the process can be approximated by a Poisson process with a rate of one every 3 hours.

The other option is to install a new workshop exactly like the existing one, with equal service rate and annual cost that allows painting simultaneously two wagons.

In all the cases work is considered uninterrupted, i.e., they work 8760 hours a year. Which of the three procedures is preferable?

PROBLEM 21

A radio repair company receives an average of 10 repair demands per day, which are distributed according to a Poisson process. Assume that  $\mu$  is the repair rate of the repairing machine in radios/day and the repair time is exponential. Each unit of repair rate has a cost of 100 €/week.

Moreover, it has been estimated that the cost of having unrepaired radios is 200 € per radio and week, being this cost proportional to time. Assuming a week has 5 working days, determine:

- a) Optimal repair rate
- b) If it would be more economical to have two machines, each one with half the rate determined in the previous question

PROBLEM 22

A dentist has a consultation room where patients arrive according to a Poisson process with a rate of 5 patents/hour. The waiting room has three chairs and no patients are allowed if there are no seats available. The mean time the dentist pay attention to each patient is exponentially distributed, but its value depends on the number of patients in the consultation room:



Number of patients	1	2	3	4
Mean service time (minutes)	9	10	12	15

Give the steady-state probabilities of the number of patients in the consultation room.

PROBLEM 23

The accounting department of a company has a computer system with several PCs in the same domain, a PC acts as a printing server (which processes the works by first in-first out criterion without any priority) and two equal printers (with an unlimited buffer). Members of the department have raised an application (complaint) to the Management to improve the printing service, claiming that the printers are saturated at the end of the month (when they close the balance sheet).

After making the corresponding observations at the end of the month and analyzing the data obtained, they have extracted the following conclusions:

- Time between printing orders from the server follows an exponential distribution of mean 5 minutes
- Processing time of the server follows an exponential distribution of mean 1 minute
- Printing time follows an exponential distribution of mean 7.5 minutes

Answer the following questions:

1. If a steady state is reached, which would be the arrival rate of the printing works?
2. Let's get rid of the printing server, has the system a steady state distribution? Why?
3. Continue without the server, which is the average number of works pending at the end of the day?
4. In the same conditions that the previous question, if they remain at work doing extra hours until the printers end (without sending additional works), how many hours will need to be paid on average?
5. In the same conditions that the previous question, if we consider that while work is being printed the employee is idle and, therefore, it is wasted time and the employee hour costs 20 €, which cost per hour has this system?
6. In the same conditions that the previous question, if the employee wasted time while waiting for the work to be printed is considered an added cost, which cost will be added to each work if the printed work is invoiced to a customer, if on average each work requires to be printed 10 times before being ready?

PROBLEM 24

Consider a repairing workshop with a robot. Consider the possibility of changing it by another one. Both robots are different, with different costs. The following table shows the data of each robot, understanding that the operation costs correspond to the costs



per unit of time when the robot is doing some repair and amortization costs are costs per unit of time, no matter the robot is repairing or not.

	Amortization costs (€/h)	Operation costs (€/h)
Robot 1	100	200
Robot 2	150	400

The first robot can repair 6 machines/hour and the second one 7 machines/hour.

When a machine is in the repair workshop consider a cost per unit of time due to productivity loss estimated in 80 €/h and the workshop assumes it as cost per unit or time and customer. Assume that the machines arrive at the workshop with a Poisson process with a rate of 5 machines/h and the repair time required by the robots follows an exponential distribution.

Decide analytically which of the two robots is preferable comparing the cost per hour with both robots and give that cost.

PROBLEM 25

Juan Pérez is studying the possibility of opening a car wash workshop. With that purpose, he has received an offer to buy a site priced at 750 €/m<sup>2</sup>. With his experience in this type of business he knows he will need a plot of 30 m<sup>2</sup> to install the washing machine and, besides, 12 m<sup>2</sup> for each vehicle to be admitted in the workshop and waiting the queue to be washed. Cars that don't enter inside the workshop will be forced to go to another service center.

Moreover, and given the knowledge about another workshop of the same type he had in the same area as well as a small statistical study, he has obtained that interarrival times are distributed with an exponential distribution of mean 10 minutes.

Determine:

1. If the time to wash a car is distributed with an exponential of mean 6 minutes and the price is 4 €, determine the additional site is needed to optimize the design of the workshop for an operation period of a year (365x12x60=262800 min), assuming a steady state.
2. Assume there is no site limit, so there is no reserved site required because the cars can wait in front of the workshop door as many as they want, but it is estimated 0.5 €/minute of time the car is in the car workshop of waiting for the access. Juan Pérez wants to buy a slow machine that lasts exactly 6 minutes to wash a car and costs 36000 € and a quick machine costs 60000 € that only requires 4 minutes to wash a car. Moreover, vehicles to be washed can be cars or trucks. The washing time for a truck is double than the one for a car and the percentage of cars is 65 %. Which is the average queue length for each machine? Which machine will be bought thinking on a month of operation (262800 minutes)?

PROBLEM 26

## QUEUEING THEORY

There is a manufacturing plant with two spots, A and B. Products must go through both spots, being processed first in A and then in B. The process in spot A takes 3 minutes on average and in spot B 4 minutes.

Determine:

1. Which is the maximum production that can be achieved per hour in the plant? If the products arrive with an interarrival time distributed exponentially with mean 5 minutes, which is the hourly production assuming a steady state?
2. If besides being exponential the interarrival times are also the service times, which will be the estimated time from the arrival of the product to the plant until it reaches the queue of spot B? And if the service times were deterministic and no exponential?
3. Assuming that the interarrival time is exponential but the products are of two types so in spot A 20% takes 3 minutes to be processed and the remaining ones 4 minutes and the cost of the time a product spends in the system is 0.2 €, which cost of the total hourly cost for the company corresponds to spot A? Which cost attributable to spot A should be assigned to each product?

### PROBLEM 27

At the beginning of the year, a toy manufacturer faces the production planning for the third trimester. From the business point of view, the third trimester has been good 75% of the years, understanding as good the one when the average daily demand is 12 units, and as bad when this demand is only 8 units. In both cases daily demands are random, independent and distributed according to Poisson laws.

Historically, it is known that when the second trimester is good the third one is also good in 80 % of the cases, while if the second one is bad the third one is good in 30% of the cases.

Building a toy requires the use of two machines A and B, and the manufacturer has only one machine of each type. Process in machine A consists of assembling 6 identical components and the machine takes a random time to do it following an exponential distribution of mean 30 minutes. Process in machine B, done after machine A, consists of assembling 9 identical components (although of another type than those assembled in A) and the processing time follows an exponential distribution of mean 48 minutes.

The manufacturer wants to attend all the orders, for this and if needed rents additional machines of any type. Observe that if the third trimester is good, he/she must rent an additional machine of type B to satisfy the demand, given that works 8 hours a day. The rent cost for the third trimester for machine B is the following:

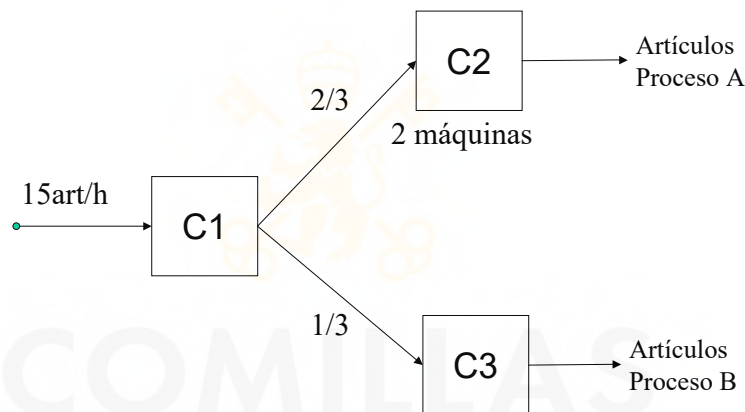
- If the machine is rented at the beginning of the year the cost is 1200 €
- If the machine is rented before the end of June (when it is known if the second trimester has been good or bad) 1500 €
- If the machine is rented at the beginning of the third one and it is known if the third trimester is good or bad, 2000 €

1. Which is the optimal policy for the toy manufacturer regarding the possible renting of the machines?
2. Which is the average time it takes to be sent to his/her requestor since his/her request until the toy is processed by both machines if the trimester is bad and there is only one machine? What if the trimester is good?

Note: if arrivals and services are Poisson-type, departs follow also a Poisson process.

PROBLEM 28

Some parts arrive at a workshop to be processed following a Poisson law with a mean of 15 parts an hour. Parts can go through two different processes, A and B, being  $\frac{2}{3}$  of the processes of type A. The process sequence is shown in the figure, where in spot C2 there are 2 machines for parts type A.



The process in spot C1 is the same for both parts and follows an exponential law of mean 3 minutes. In spot C2, process time follows an exponential distribution of mean 10 minutes and in C3 the processing time follows a normal distribution of mean 7.5 minutes and standard deviation 2 minutes.

- a) Estimate the time required by a part type A and a part type B, since its arrival until its departure<sup>1</sup>.
- b) It is estimated as 0.1 € each minute the time spent by any part in the system, which hourly cost will have the system? Which cost must be assigned to each part by the process?

PROBLEM 29

An entrepreneur wants to install a motorbike repair workshop in a summer zone from June 15 until September 15. The premises that rents have a capacity of 4 motorbikes, either being repaired or waiting to be repaired, so any biker who arrives at the workshop and finds it full must go to another workshop.

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<sup>1</sup> Consider that if the arrivals follow a Poisson process and service times are exponential, then departures also follow a Poisson process.

From June 15 until July 15 the arrivals of failed motorbikes follow a Poisson law with a mean of 12 motorbikes a day. Since July 16, this mean can increase to 24 motorbikes with a 0.75 probability or reduce to 6 with 0.25 probability.

Repairing a motorbike takes a random time governed by an exponential law of mean 2 hours. The workshop Works 24 hours a day in three shifts of 8 hours each.

Regarding the number of technicians to hire for repairing, the entrepreneurship has the following options:

1. Hire on June 15 for all the season a set of three technicians (one for each shift) with a monthly cost of 15000 € or to two sets of three technicians (two for each shift) with a monthly cost of 30000 €.
2. Hire on June 15 for all the season a set of three technicians (one for each shift) with a monthly cost of 15000 €. For the last two months of the season but signing the contract on July 16, before knowing the evolution of the arrival of failed motorbikes, we can hire a second set of three technicians that will cost 25000 € a month.

If the mean profit for each motorbike is 65 € and using the expected value criterion, which hiring policy must be recommended to the entrepreneur? Assume months of 30 days.

How much the entrepreneur would be willing to pay to know before July 15 the average failed motorbikes until the end of the season?

PROBLEM 31

A section of the local Administration has two offices A and B. Users go to office A, where there is one clerk, following an arrival pattern governed by a Poisson law of 12 people/hour and the attention time follows an exponential law of mean 4 minutes.

Users then go to office B, where the attention time also follows an exponential of mean 7.5 minutes for each clerk.

How many clerks will be needed in office B if you want the probability of waiting more than 10 minutes to be lower than 0.05?

Being  $S$  the number of servers in an M/M/S system, the probability of not having people in the system is:

$$\Phi = \frac{\lambda}{S\mu}, \quad p_0 = \frac{1}{\frac{(S\Phi)^S}{S!(1-\Phi)} + \sum_{n=0}^{S-1} \frac{(S\Phi)^n}{n!}}$$

Probability of waiting for  $t_c$  time units greater to  $x$  is given by:

$$P(t_c > 0) = \frac{(S\Phi)^S}{S!(1-\Phi)} p_0$$

$$P(t_c > x) = e^{-S\mu x(1-\Phi)} P(t_c > 0)$$

## Solutions of Queueing Problems

### SOLUTION OF PROBLEM 1

This is a M/M/1 system with infinite population, with  $\lambda = 20$  customers per hour, and  $\mu = 30$  customers per hour.

- $P(\text{idle bank teller}) = p_0 = 1 - \rho = 1/3$ . 33.33 % of the time is idle.
- $W_q = 1/15 = 4$  minutes
- $L = 2, L_q = 4/3$ . Fraction in queue =  $2/3$ . 66.67 % of the system customers are waiting in the queue.

### SOLUTION OF PROBLEM 2

This is a M/M/1 system with infinite population, with  $\lambda = 10$  people per hour, and  $\mu = 15$  people per hour.

- $P(\text{waiting line}) = 1 - p_0 - p_1 = 4/9$
- $L_q = 4/3$  people in the queue.
- $W_q = 2/15$  hours = 8 minutes on average in the queue.

### SOLUTION OF PROBLEM 3

It is an M/M/1/5 system where the arrival rate is constant  $\lambda = 5$  customers/hour and the service rate depends on the number of customers in the hairdresser, where  $\mu_1 = 6.6$  customers/hour or  $\mu_5 = 3$  customers/hour.

$$C_1 = \frac{5}{6.6} = 0.75, \quad C_2 = \frac{5^2}{6.6 \cdot 6} = 0.625, \quad C_3 = \frac{5^3}{6.6 \cdot 6 \cdot 6} = \frac{25}{48},$$

$$C_4 = \frac{5^4}{6.6 \cdot 6 \cdot 6 \cdot 4.62} = \frac{325}{576}, \quad C_5 = \frac{5^5}{6.6 \cdot 6 \cdot 6 \cdot 4.62 \cdot 3} = \frac{1625}{1728}$$

$$p_0 + p_1 + p_2 + p_3 + p_4 + p_5 = 1$$

$$p_0 + C_1 p_0 + C_2 p_0 + C_3 p_0 + C_4 p_0 + C_5 p_0 = 1$$

$$p_0(1 + C_1 + C_2 + C_3 + C_4 + C_5) = 1$$

$$p_0 = \frac{1}{\sum_{n=0}^{\infty} C_n} = \frac{1}{1 + 0.75 + 0.625 + 25/48 + 325/576 + 1625/1728} = 0.2272$$

$$p_1 = C_1 p_0 = 0.1704, \quad p_2 = C_2 p_0 = 0.1420, \quad p_3 = C_3 p_0 = 0.1184,$$

$$p_4 = C_4 p_0 = 0.1282, \quad p_5 = C_5 p_0 = 0.2137$$

- Average number of people in the system  $L = \sum_{n=0}^{\infty} n p_n = 2.3909$  customers. (In this case here:  $L = 0 \cdot p_0 + 1 \cdot p_1 + 2 \cdot p_2 + 3 \cdot p_3 + 4 \cdot p_4 + 5 \cdot p_5$ ).

- Average waiting time  $W_q = \frac{L_q}{\lambda_{EF}} = \frac{1.6182}{3.9315} = 0.4116$  hours being  $\lambda_{EF} = \lambda(1 - p_5) = 5(1 - 0.2137) = 3.9315$  customers/hour and

$$L_q = \sum_{n=2}^5 (n-1)p_n = 1 \cdot 0.1420 + 2 \cdot 0.1184 + 3 \cdot 0.1282 + 4 \cdot 0.2137 = 1.6182$$

customers.

c) Hairstylist is idle with a probability  $P_0 = 0.2272$ .

SOLUTION OF PROBLEM 4

This is a M/M/1 system with infinite population, with  $\lambda = 0.25$  workers per minute (1.25/5), and  $\mu = 0.3$  workers per minute.

a)  $L_q = 4.1\widehat{6}$  workers.

b)  $W_q = 16.6\widehat{6}$  minutes.

c) The worker attention cost is approximately 14000 €/day (7 hours/day\*5 workers\*400 €/hour). The first hour of the shift the inflow is reduced. The L (number of workers in the system) is 5. Office employee cost is 2000 €/day. If another employee is working (model M/M/2) the worker cost is 2823.53 €/day (L=1.0084) and of the employees 4000 €/day. Clearly, it is profitable to hire two employees for the system.

SOLUTION OF PROBLEM 5

This is a M/M/1 system with finite population  $m=4$ , with  $\lambda = 1$  plane per year, and  $\mu = 8$  planes per year.

Se trata de un sistema cerrado de colas de tipo M/M/1 con tamaño de la fuente  $m = 4$  aviones con tasa de llegada individual de cada avión  $\lambda = 1$  avión/año y tasa de salida  $\mu = 8$  aviones/año. El factor de utilización es

$$\rho = \frac{\lambda}{\mu} = \frac{1}{8}$$

Las probabilidades de cada estado se pueden calcular mediante estas expresiones

$$p_0 = \left[ 1 + \sum_{n=1}^m \frac{m! \rho^n}{(m-n)!} \right]^{-1} = \left[ 1 + \frac{4!}{3!8} + \frac{4!}{2!8^2} + \frac{4!}{1!8^3} + \frac{4!}{0!8^4} \right]^{-1} =$$

$$= \left[ 1 + \frac{1}{2} + \frac{3}{16} + \frac{3}{64} + \frac{3}{512} \right]^{-1} = 0.5746$$

$$p_n = (m - n + 1)\rho p_{n-1}$$

$$p_1 = 4\rho p_0 = 0.2873; p_2 = 3\rho p_1 = 0.1077; p_3 = 2\rho p_2 = 0.0269; p_4 = \rho p_3 = 0.0034$$

El número medio de aviones en el sistema es

$$L = m - \frac{1 - p_0}{\rho} = 4 - (1 - 0.5746)8 = 0.5971 \text{ aviones}$$

La tasa media de llegada de los aviones al sistema  $\lambda_{EF} = (m - L)\lambda = 3.4029$ .

El número medio de aviones en cola en espera para ser reparados es

$$L_q = m - \frac{1 + \rho}{\rho} (1 - p_0) = 4 - \frac{9/8}{1/8} (1 - 0.5746) = 0.1717 \text{ aviones}$$

que también se podía haber calculado como

$$L_q = L - \frac{\lambda_{EF}}{\mu} = 0.5971 - \frac{3.4029}{8} = 0.1717 \text{ aviones}$$

El tiempo medio de aviones en el sistema es

$$W = \frac{L}{(m - L)\lambda} = \frac{0.5971}{(4 - 0.5971)1} = 0.1678 \text{ años} = 61.2 \text{ días}$$

y el tiempo medio de aviones en cola es

$$W_q = \frac{L_q}{(m - L)\lambda} = \frac{0.1717}{(4 - 0.5971)1} = 0.0501 \text{ años} = 18.3 \text{ días}$$

#### SOLUTION OF PROBLEM 6

This is a M/M/1 system with infinite population, with  $\lambda = 8$  customers per hour, and  $\mu = 12$  customers per hour.

- $W_q = 0.1\hat{6}$  hours = 10 min.
- $L_q = 1.\hat{3}$  customers.
- The average queue length when the queue is not empty, is calculated by  $L_q/\text{prob}(\text{queue not empty})$ . This probability =  $1 - p_0 - p_1$ . Finally, the queue length (when it is not empty) is 3 people. And the probability that there are at least 2 people in the system is  $0.\hat{4}$ .

#### SOLUTION OF PROBLEM 7

This is a M/M/2 system with finite population  $m=4$ , with  $\lambda = 1/3$  machine per hour, and  $\mu = 4/3$  machines per hour.

$L=0.2537$  máquinas en el taller.

#### SOLUTION OF PROBLEM 8

This is a M/M/2 system with infinite population, with  $\lambda = 100$  customers per hour, and  $\mu = 60$  customers per hour

- $1 - p_0 - p_1 - p_2 - p_3 = 0.5261$ .
- $P(\text{al menos un cajero ocioso}) = 0.\hat{24}$ .

#### SOLUTION OF PROBLEM 9

- $L=3.285$ .
- 0.96 es el número medio de vehículos que no pueden entrar al servicio.

#### SOLUTION OF PROBLEM 10



## QUEUEING THEORY

- a)  $p_0=0.1526$ ,  $L_q=0.0283$ ,  $L=1.9033$ ,  $W_q=0.0019$  horas=7 s,  $W=0.1269$  horas = 7min.36 s
- c)  $p_0=0.03226$ ,  $L_q=13.61$ ,  $L=15.49$ ,  $W_q=0.90733$  horas=54.44min.,  $W=1.03$  horas = 61.8min.

### SOLUTION OF PROBLEM 11

- a)  $L=2.4$  empleados en el departamento de control de calidad.
- b)  $L=1.4778$  empleados en el departamento de control de calidad.

### SOLUTION OF PROBLEM 12

The first system is a M/M/1 system with infinite population, with  $\lambda = 10$  customers per hour, and  $\mu = 12$  customers per hour. The second system (with the course) corresponds to a M/G/1 system.

- a) Current employee:  $L = 5$  customers,  $W = 30$  minutes  
Employee with the course:  $L = 8.625$  customers,  $W = 51.750$  minutes. The results are more sensitive to the average service time than to the variance.
- b) It is not possible to compensate for the 0.5 minute increase in service time with reducing the variance. Even if the variance were 0, the resulting L would still be higher than 5, so only with a negative variance (which does not exist) could the same  $L=5$  be obtained).

### SOLUTION OF PROBLEM 13

$L=2.9583$  máquinas.

### SOLUTION OF PROBLEM 14

$W_q = 2.28$  minutos.

### SOLUTION OF PROBLEM 15

- a)  $L= 39/760$  pacientes.
- b)  $W= 15$  min. y 23 s
- c)  $W_q= 23$  s
- d)  $L_q= 1/760$  pacientes.

### SOLUTION OF PROBLEM 16

El coste por trabajo por unidad de incremento de  $\mu$  (100 sentencias por minuto) por unidad de tiempo (día) = 0.05 u.m.

El coste por retrasar un trabajo un día es 10 u.m.

$\lambda = 50$  trabajos por día

De aquí se deduce que el  $\mu$  óptimo es 123 trabajos por día, es decir 256 sentencias por minuto.

### SOLUTION OF PROBLEM 17

El coste del primer empleado, suponiendo que estuviera él solo, es de 35 u.m./hora. El coste del segundo empleado es de 13 u.m./hora. Es más económico el segundo.

SOLUTION OF PROBLEM 18

a) Se trata de un sistema cerrado de colas de tipo M/M/1 con tamaño de la fuente  $m = 4$  máquinas con tasa de llegada individual de cada máquina  $\lambda = 0.1$  máquinas/hora y tasa de salida  $\mu = 0.5$  máquinas/hora. El factor de utilización es

$$\rho = \frac{\lambda}{\mu} = 0.2$$

Las probabilidades de cada estado se pueden calcular mediante estas expresiones

$$p_0 = \left[ 1 + \sum_{n=1}^m \frac{m! \rho^n}{(m-n)!} \right]^{-1} = \left[ 1 + \frac{4!}{3!5} + \frac{4!}{2!5^2} + \frac{4!}{1!5^3} + \frac{4!}{0!5^4} \right]^{-1} =$$

$$= \left[ 1 + \frac{4}{5} + \frac{12}{25} + \frac{24}{125} + \frac{24}{625} \right]^{-1} = 0.3983$$

$$p_n = (m - n + 1)\rho p_{n-1}$$

$$p_1 = 4\rho p_0 = 0.3187; p_2 = 3\rho p_1 = 0.1912; p_3 = 2\rho p_2 = 0.0765; p_4 = \rho p_3 = 0.0153$$

b) El número medio de máquinas en el sistema es

$$L = m - \frac{1 - p_0}{\rho} = 4 - (1 - 0.3983)5 = 0.9917 \text{ máquinas}$$

Luego el número de máquinas operativas es aproximadamente 3.

La tasa media de llegada de las máquinas al sistema  $\lambda_{EF} = (m - L)\lambda = 0.3008$ .

El número medio de máquinas en cola en espera para ser reparadas es

$$L_q = L - \frac{\lambda_{EF}}{\mu} = 0.9917 - \frac{0.3008}{0.5} = 0.3900 \text{ máquinas}$$

El tiempo medio de máquinas en el sistema es

$$W = \frac{L}{(m - L)\lambda} = \frac{0.9917}{(4 - 0.9917)0.1} = 0.1678 \text{ años} = 61.2 \text{ días}$$

y el tiempo medio de máquinas en cola es

$$W_q = \frac{L_q}{(m - L)\lambda} = \frac{0.3900}{(4 - 0.9917)0.1} = 0.0501 \text{ años} = 18.3 \text{ días}$$

c) El coste medio total por día es  $20 \cdot 8 \cdot 0.9917 + 45 = 203.67$  u.m./día

- d) En el caso de tener dos mecánicos es equivalente a dos sistemas cerrados de colas M/M/1 de fuente con tamaño  $m = 2$  máquinas y tasa de llegada  $\lambda = 0.1$  máquinas/hora y tasa de salida  $\mu = 0.5$  máquinas/hora. El factor de utilización sigue siendo  $\rho = 0.2$

Las probabilidades de cada estado se pueden calcular mediante estas expresiones

$$p_0 = \left[ 1 + \sum_{n=1}^m \frac{m! \rho^n}{(m-n)!} \right]^{-1} = \left[ 1 + \frac{2!}{1!5} + \frac{2!}{0!5^2} \right]^{-1} = 0.\widehat{675}$$

$$p_1 = 2\rho p_0 = 0.\widehat{270}; \quad p_2 = \rho p_1 = 0.\widehat{135}$$

El número medio de máquinas en el sistema es

$$L = m - \frac{1 - p_0}{\rho} = 2 - (1 - 0.\widehat{675})5 = 0.\widehat{378} \text{ máquinas}$$

El coste medio total por día es  $2(20 \cdot 8 \cdot 0.\widehat{378} + 45) = 211.08$  u.m./día

- e) En el caso de tener dos mecánicos para las cuatro máquinas es equivalente a un sistema cerrado de colas M/M/2 de fuente con tamaño  $m = 4$  máquinas y tasa de llegada  $\lambda = 0.1$  máquinas/hora y tasa de salida  $\mu = 0.5$  máquinas/hora. El factor de utilización ahora es  $\rho = \lambda/c\mu = 0.1$ . Las probabilidades se pueden calcular a partir de la expresión

$$p_n = \begin{cases} \binom{m}{n} \left( \frac{\lambda}{\mu} \right)^n p_0 & 1 \leq n \leq c \\ \binom{m}{n} \frac{n! (\lambda/\mu)^n}{c! c^{n-c}} p_0 & c \leq n \leq m \end{cases}$$

$$p_1 = 0.8p_0; \quad p_2 = 0.24p_0; \quad p_3 = 0.048p_0; \quad p_4 = 0.0048p_0$$

de donde resulta  $p_0 = 0.4778$ ,  $p_1 = 0.3823$ ,  $p_2 = 0.1147$ ,  $p_3 = 0.0229$ ,  $p_4 = 0.0002$  y el número medio de máquinas en el sistema es

$$L = 0 \cdot 0.4778 + 1 \cdot 0.3823 + 2 \cdot 0.1147 + 3 \cdot 0.0229 + 4 \cdot 0.0002 = 0.6896 \text{ máquinas}$$

El coste medio total por día es  $20 \cdot 8 \cdot 0.6896 + 2 \cdot 45 = 200.34$  u.m./día

Resulta ser el sistema más económico.

#### SOLUTION OF PROBLEM 19

Con un mecánico se obtiene un coste de 140 u.m. y con dos de 152 u.m.

#### SOLUTION OF PROBLEM 20

Coste actual por hora = 2456.62. Coste con aerosol = 1891.55. Coste con dos servidores = 1389.43. Resulta preferible poner dos servidores.

SOLUTION OF PROBLEM 21

- a) La tasa óptima es de 14.47 radios por día, con un coste diario de 378.43225 €.
- b) En este caso se obtiene un coste diario de 395.184, que es peor.

SOLUTION OF PROBLEM 22

Aplicando el modelo general ya que las distribuciones son exponenciales, pero la tasa de servicio varía según el número de clientes que haya en el sistema, se obtiene:

$$p_0 = 0.264, p_1 = 0.1983, p_2 = 0.1653, p_3 = 0.1653, p_4 = 0.2066$$

SOLUTION OF PROBLEM 23

1.  $\lambda = 1/5$  trabajos/minuto
2. Sí, pues  $\rho = 3/4 < 1$
3.  $p_0 = 1/7$  y por lo tanto  $L = 24/7 = 3.4286$  trabajos
4. Tasa de atención  $c\mu = 4/15$  trabajos por minuto, entonces  $3.4286 \frac{15}{4} = 12.857$  minutos
5. Coste/h =  $20L = 68.572$  €/h
6. Coste total por trabajo =  $10 \frac{20}{60} W = \frac{10}{3} \frac{L}{\lambda} = \frac{10}{3} \frac{3.4286}{0.2} = 57.1433$  €

SOLUTION OF PROBLEM 24

Coste/h = Coste amortización + Coste operativo  $(1 - p_0)$  + Coste clientes  $L$

$$\text{Coste robot 1} = 100 + 200 \frac{5}{6} + 80 \frac{5/6}{1/6} = 666.66 \text{ €/h}$$

$$\text{Coste robot 2} = 150 + 400 \frac{5}{7} + 80 \frac{5/7}{2/7} = 635.7143 \text{ €/h}$$

Es más barato el segundo robot.

SOLUTION OF PROBLEM 27

- a) Se trata de un Problem que mezcla teoría de la decisión y de colas.  
 $P(3^\circ B) = 0.75$ ;  $P(3^\circ R) = 0.25$ ;  $P(3^\circ B/2^\circ B) = 0.8$ ;  $P(3^\circ B/2^\circ R) = 0.3$   
 De estos valores se puede deducir  $P(3^\circ R/2^\circ B) = 0.2$ ;  $P(3^\circ R/2^\circ R) = 0.7$   
 $P(3^\circ B) = 0.75 = P(3^\circ B/2^\circ B) P(2^\circ B) + P(3^\circ B/2^\circ R) P(2^\circ R) =$   
 $= 0.8 P(2^\circ B) + 0.3 (1 - P(2^\circ B))$   
 $P(2^\circ B) = 0.45/0.5 = 0.9$ ;  $P(2^\circ R) = 1 - 0.9 = 0.1$   
 La opción más económica es alquilar la máquina en el primer trimestre.
- b) El proceso de llegada a las máquinas puede tener  $\lambda = 8$  piezas/día para la demanda regular o  $\lambda = 12$  piezas/día para la demanda buena. Las tasas de servicio son  $\mu_A = 16$  piezas/día y  $\mu_B = 10$  piezas/día. El sistema de colas es M/M/1 para cada máquina.

Para la demanda regular se tiene  $\rho_1 = \frac{8}{16} = 0.5$  y  $\rho_2 = \frac{8}{10} = 0.8$ .  
 $W_1 = \frac{1}{\mu_1(1-\rho_1)} = \frac{1}{16(1-0.5)} = 0.125$  día y  $W_2 = \frac{1}{\mu_2(1-\rho_2)} = \frac{1}{10(1-0.8)} = 0.5$  día.

Luego en este sistema el tiempo total en el sistema es de 0.625 días.

Para el caso de la demanda buena es necesario instalar dos máquinas B en paralelo para poder satisfacer la demanda. Entonces es un sistema de colas M/M/1 para la máquina A y M/M/2 para la B. Para la demanda buena se tiene  $\rho_1 = \frac{12}{16} = 0.75$  y

$\rho_2 = \frac{12}{2 \cdot 10} = 0.6$ . Para la máquina A  $W_1 = \frac{1}{\mu_1(1-\rho_1)} = \frac{1}{16(1-0.75)} = 0.25$  día.

Para la máquina B  $p_0 = \frac{1}{\frac{(2 \cdot 0.6)^2}{2!(1-0.6)} + 1 + 2 \cdot 0.6} = 0.25$ ,

$L_2 = \frac{(1.2)^2 \cdot 0.6}{2(1-0.6)^2} \cdot 0.25 + 1.2 = 1.875$   $W_2 = \frac{L_2}{\lambda_2} = \frac{1.875}{12} = 0.15625$  día.

Luego en este sistema el tiempo total en el sistema es de 0.40625 días.

#### SOLUTION OF PROBLEM 28

a) Tiempo medio de un artículo en C1:

$$\lambda_{C1} = 15 \text{art/h} \quad \mu_{C1} = \frac{1 \text{art}}{3 \text{min}} = \frac{60}{3} \text{art/h} = 20 \text{art/h}$$

$$\text{Modelo M/M/1: } \rho = \frac{15}{20} = 0.75 < 1$$

$$W_{C1} = \frac{1}{\mu(1-\rho)} = \frac{1}{20 \cdot 0.25} = 1/5 \text{h} = 12 \text{min} \quad L_{C1} = \lambda_{C1} W_{C1} = 15 \cdot 1/5 = 3$$

Tiempo medio de un artículo en C2:

$$\lambda_{C2} = 15 \frac{2}{3} = 10 \text{art/h} \quad \mu_{C2} = \frac{1 \text{art}}{10 \text{min}} = \frac{60}{10} \text{art/h} = 6 \text{art/h}$$

$$\text{Modelo M/M/2: } \rho = \frac{10}{12} = 5/6 < 1$$

$$p_0 = \frac{1}{\frac{(5/3)^2}{2 \cdot 1/6} + 1 + 5/3} = \frac{1}{25/3 + 3/3 + 5/3} = 1/11$$

$$L_{C2} = \frac{(5/3)^2 \cdot 5/6}{2 \cdot 1/6^2} \cdot 1/11 + 5/3 = \frac{180}{33} = 5.46$$

$$W_{C2} = \frac{L_{C2}}{\lambda_{C2}} = \frac{180/33}{10} = 18/33 \text{h} = 32.72 \text{min}$$

Tiempo medio de un artículo en C3:

$$\lambda_{C3} = 15 \frac{1}{3} = 5 \text{art/h} \quad \mu_{C3} = \frac{1 \text{art}}{7.5 \text{min}} = \frac{60}{7.5} \text{art/h} = 8 \text{art/h}$$

$$\text{Modelo M/G/1: } \rho = 5/8 < 1$$

$$\sigma_{C_3}^2 = 4/60^2$$

$$L_{C_3} = 5/8 + \frac{(5/8)^2 + 5^2 \cdot 4/60^2}{2 \cdot 3/8} = \frac{511}{432}$$

$$W_{C_3} = \frac{L_{C_3}}{\lambda_{C_3}} = \frac{511/432}{5} = 0.2366\text{h} = 14.19\text{min}$$

Tiempo proceso A: 12 min + 32.72 min = 44.72 min

Tiempo proceso B: 12 min + 14.19 min = 26.19 min

b) El coste horario del sistema es

$$0.1\text{€}/\text{min} \cdot 60\text{min}/\text{h}(L_{C_1} + L_{C_2} + L_{C_3}) = 6(3 + 180/33 + 511/432) = 57.82\text{€}/\text{h}$$

El coste imputable a los productos procesados según A:

$$0.1\text{€}/\text{min}(W_{C_1} + W_{C_2}) = 0.1 \cdot 44.72 = 4.47\text{€}/\text{art}$$

El coste imputable a los productos procesados según B:

$$0.1\text{€}/\text{min}(W_{C_1} + W_{C_3}) = 0.1 \cdot 26.19 = 2.619\text{€}/\text{art}$$

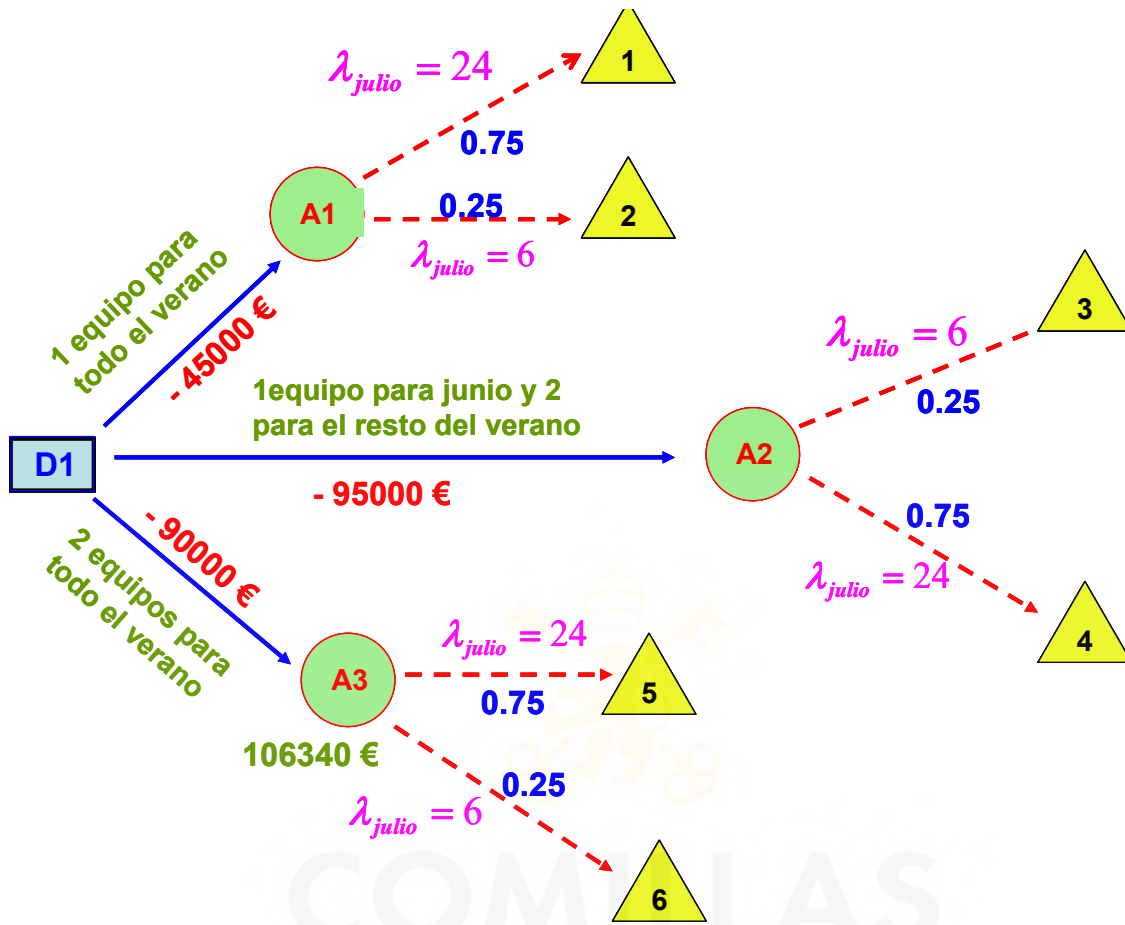
#### SOLUTION OF PROBLEM 29

El empresario se encuentra a comienzos de junio ante tres posibles alternativas:

1. Contratar un equipo de tres mecánicos para los tres meses de verano,
2. Contratar dos equipos de tres mecánicos para todo el verano, o
3. Contratar en junio un equipo y otro más en julio.

Los costes asociados a las tres decisiones son respectivamente de 45000, 90000 y 95000 €. Posteriormente, en el mes de julio, el azar juega su papel modificando al alza,  $\lambda = 24$ , o a la baja,  $\lambda = 6$ , la tasa diaria de llegadas de motos averiadas, con probabilidades respectivas de 0.75 y 0.25.

Todo ello se refleja en el siguiente árbol de decisión.



Dependiendo de la decisión tomada y de la evolución de las tasas se obtienen diferentes ingresos. Para calcularlos, hay que considerar que nos encontramos con diferentes modelos de colas, todos ellos del tipo  $M/M/s/4$ , donde  $s$  toma los valores 1 ó 2, con  $\mu = 12$ , y  $\lambda = 12, 24$  ó 6.

El punto clave es determinar la proporción de motos que no podrán ser atendidas, proporción que viene dada por la probabilidad de que el taller se encuentre lleno, es decir, la probabilidad  $p_4$  de que la cola se encuentre en el estado 4. De acuerdo con las fórmulas correspondientes a este modelo tenemos que

a)  $s = 1$

$$p_0 = \frac{1}{\sum_{n=0}^4 \left(\frac{\lambda}{\mu}\right)^n}, \quad p_n = \left(\frac{\lambda}{\mu}\right)^n p_0$$

Por tanto:

1. Si  $\lambda = 12$ ,  $\frac{\lambda}{\mu} = 1 \Rightarrow p_0 = 0.2 \Rightarrow p_4 = 0.2 \quad \lambda_{EF} = 9.6$
2. Si  $\lambda = 24$ ,  $\frac{\lambda}{\mu} = 2 \Rightarrow p_0 = \frac{1}{31} \Rightarrow p_4 = \frac{16}{31} \quad \lambda_{EF} = 11.613$



3. Si  $\lambda = 6$ ,  $\frac{\lambda}{\mu} = 0.5 \Rightarrow p_0 = \frac{16}{31} \Rightarrow p_4 = \frac{1}{31} \quad \lambda_{EF} = 5.807$

b)  $s = 2$

$$p_0 = \frac{1}{1 + \frac{\lambda}{\mu} \sum_{n=0}^3 \left(\frac{\lambda}{2\mu}\right)^n}, \quad p_n = 2 \left(\frac{\lambda}{2\mu}\right)^n p_0 \quad (n > 1)$$

Por tanto:

4. Si  $\lambda = 12$ ,  $\frac{\lambda}{\mu} = 1 \Rightarrow p_0 = \frac{8}{23} \Rightarrow p_4 = \frac{1}{23} \quad \lambda_{EF} = 11.478$

5. Si  $\lambda = 24$ ,  $\frac{\lambda}{\mu} = 2 \Rightarrow p_0 = \frac{1}{9} \Rightarrow p_4 = \frac{2}{9} \quad \lambda_{EF} = 18.6$

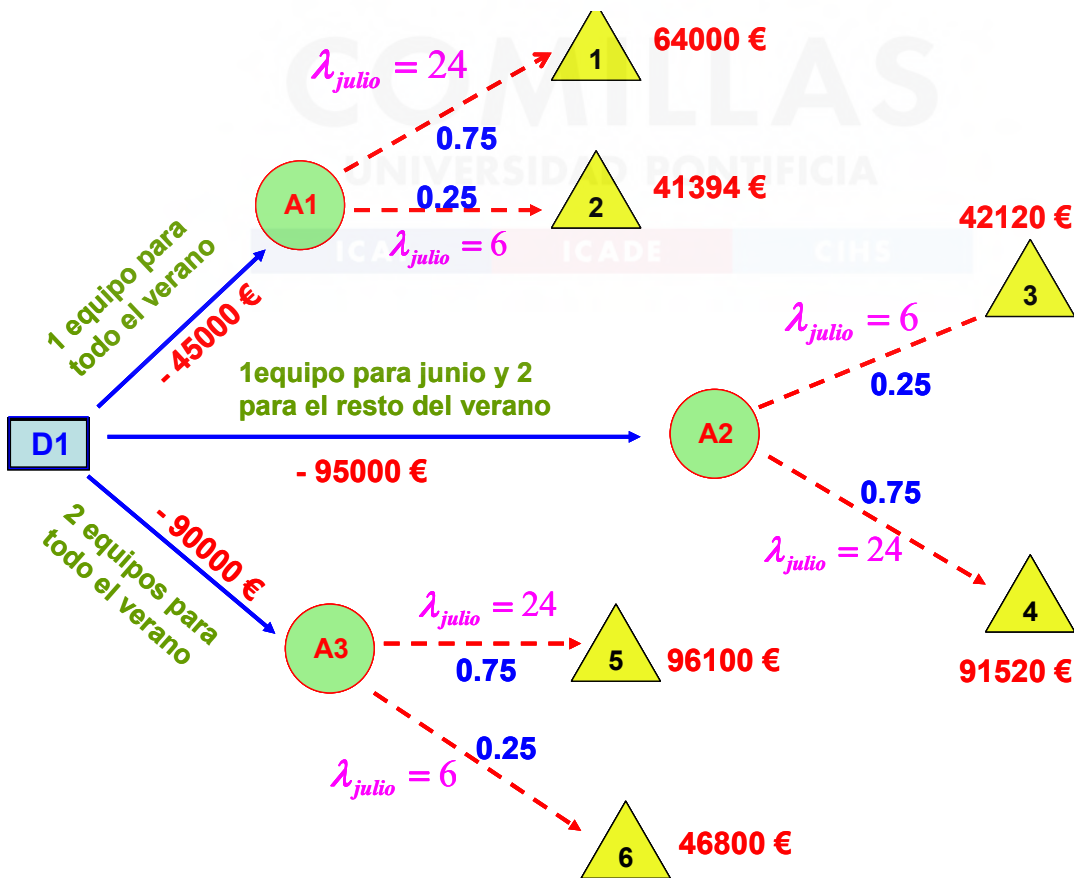
6. Si  $\lambda = 6$ ,  $\frac{\lambda}{\mu} = 0.5 \Rightarrow p_0 = \frac{128}{213} \Rightarrow p_4 = \frac{1}{213} \quad \lambda_{EF} = 5.972$

Con estos datos, los ingresos esperados son los siguientes:

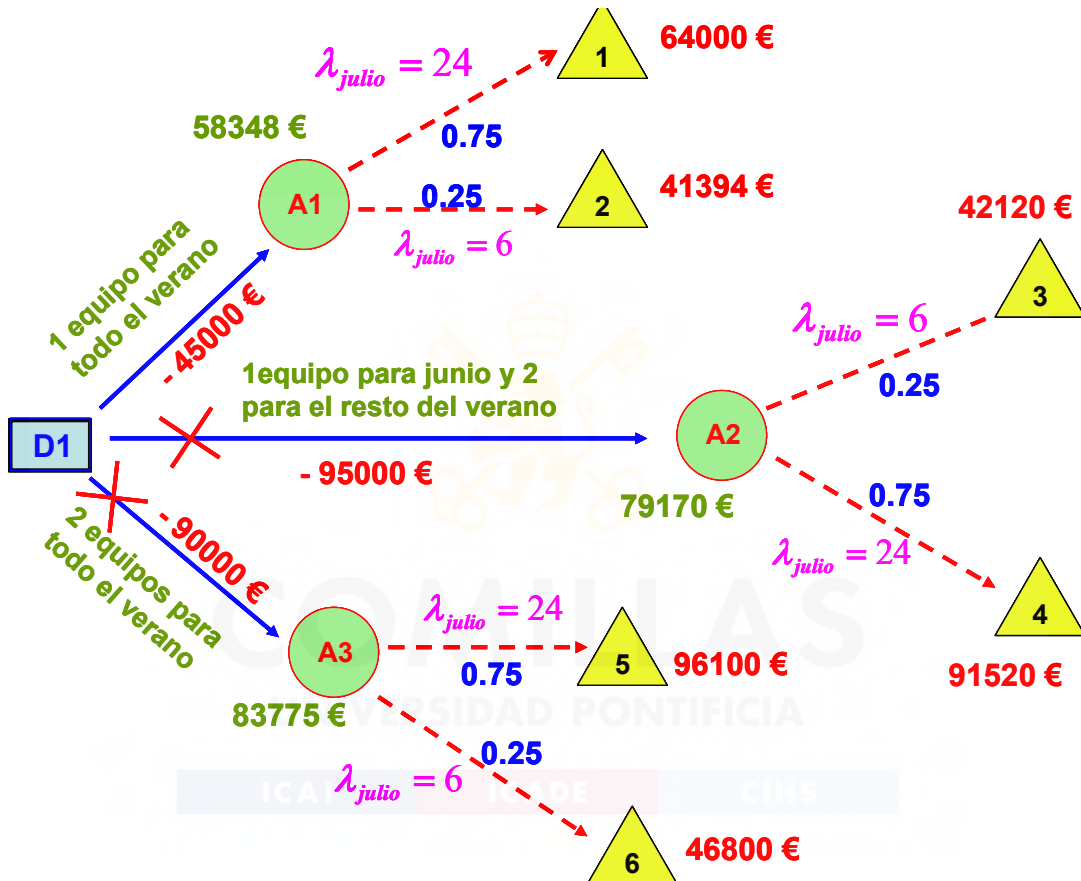
1. si decide contratar un solo equipo para todo el verano son:
  - a. Durante el primer mes del verano llegan en promedio 360 motos, de las que son atendidas el 80 %. Los ingresos durante este mes son de  $360 \cdot 0.8 \cdot 65 = 18720 \text{ €}$ .
  - b. Durante los otros dos meses, si la tasa de motos sube a 24, sólo es atendida una proporción del  $15/31$  de las  $60 \cdot 24 = 1440$  motos que llegan en promedio, lo que supone unos ingresos adicionales para esos 2 meses de  $1440 \cdot (15/31) \cdot 65 = 45290 \text{ €}$ . Esto da unos ingresos totales para toda la campaña de 64010 €.
  - c. Si la media de motos desciende a 6, es atendida una proporción de  $30/31$ , lo que se traduce en unos ingresos durante estos dos meses de  $60 \cdot 6 \cdot (30/31) \cdot 65 = 22645 \text{ €}$  y unos ingresos globales de 41365 €.
2. si decide contratar dos equipos para todo el verano son:
  - a. Durante el primer mes del verano llegan en promedio 360 motos, de las que son atendidas la proporción  $22/23$ . Los ingresos durante este mes son de  $360 \cdot (22/23) \cdot 65 = 22383 \text{ €}$ .
  - b. Durante los otros dos meses, si la tasa de motos sube a 24, sólo es atendida una proporción del  $7/9$  de las  $60 \cdot 24 = 1440$  motos que llegan en promedio, lo que supone unos ingresos adicionales para esos 2 meses de  $1440 \cdot (7/9) \cdot 65 = 72800 \text{ €}$ . Esto da unos ingresos totales para toda la campaña de 95183 €.

- c. Si la media de motos descende a 6, son atendidas prácticamente todas 212/213, lo que se traduce en unos ingresos durante estos dos meses de  $60 \cdot 6 \cdot 65 = 23400 \text{ €}$  y unos ingresos globales de 45783 €.
3. si decide contratar un equipo al comienzo y otro más en julio
- a. Durante el primer mes del verano llegan en promedio 360 motos, de las que son atendidas el 80 %. Los ingresos durante este mes son de  $360 \cdot 0.8 \cdot 65 = 18720 \text{ €}$ .
- b. Durante los otros dos meses, si la tasa de motos sube a 24, sólo es atendida una proporción del 7/9 de las  $60 \cdot 24 = 1440$  motos que llegan en promedio, lo que supone unos ingresos adicionales para esos 2 meses de  $1440 \cdot (7/9) \cdot 65 = 72800 \text{ €}$ . Esto da unos ingresos totales para toda la campaña de 91520 €.
- c. Si la media de motos descende a 6, son atendidas prácticamente todas, lo que se traduce en unos ingresos durante estos dos meses de  $60 \cdot 6 \cdot 65 = 23400 \text{ €}$  y unos ingresos globales de 42120 €.

Con estos datos la valoración de las distintas hojas del árbol queda reflejada en la siguiente figura



Pasamos a valorar los vértices del árbol. Los de azar con el valor medio ponderado de los valores asociados a los vértices extremos de arcos que salen de ellos y ya estén valorados. Los de decisión, eligiendo la alternativa de mayor valor esperado. Esto nos lleva a que la decisión óptima es la de contratar un solo equipo para toda la campaña con una ganancia esperada de 13349 €.

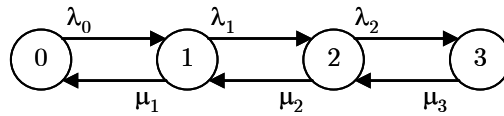


El último apartado pide calcular el VEIP, es decir, la diferencia entre la ganancia esperada si se supiera el futuro y la ganancia esperada bajo riesgo.

Si se superiera que la tasa de llegadas para los dos últimos meses va a ser  $\lambda = 24$  y se toma la decisión óptima, la ganancia sería el máximo entre 19010, -3480 y 6100, es decir, 19010 €. Si se supiera que la tasa es  $\lambda = 6$  entonces la decisión óptima es el máximo entre -3635, -52880, -43200, es decir, -3635 €. La ganancia esperada con información perfecta es  $19010 \cdot 0.75 - 3635 \cdot 0.25 = 13349$  €. La ganancia bajo riesgo es de 13349 €. Luego el VEIP es la diferencia entre ambos que, en este caso, es 0. Este resultado se produce por la coincidencia en la decisión óptima, la primera, tanto cuando la tasa es  $\lambda = 24$ , como cuando es  $\lambda = 6$  como bajo riesgo.

SOLUTION OF PROBLEM 30

- a) La variable de estado son los vehículos en el taller principal y en el anexo.
- b)



$$p_3 = \frac{\lambda_2 \lambda_1 \lambda_0}{\mu_3 \mu_2 \mu_1} p_0, \quad p_2 = \frac{\lambda_1 \lambda_0}{\mu_2 \mu_1} p_0, \quad p_1 = \frac{\lambda_0}{\mu_1} p_0, \quad p_3 + p_2 + p_1 + p_0 = 1$$

$$\lambda_2 = \lambda_1 = \lambda_0 = \lambda, \quad \lambda_n = 0 \quad n \geq 3, \quad \mu_3 = v_2 + v_1, \quad \mu_2 = \mu_1 = v_1,$$

- c)  $p_0 = p_1 = p_2 = 2/7 \simeq 0.286$ ,  $p_3 = 1/7 \simeq 0.143$   
 d) La probabilidad de no poder ser reparado en el taller principal ni en el anexo es  $p_3 \simeq 0.143$ .  
 e)  $d = \lambda(1 - p_3) = 6(6/7) \simeq 5.143$  piezas/hora,  $c_p = 500$  €,  $c_a = 4/(9 \cdot 250)$  €/pieza y hora,  $l = 16$  días.

$$Q^* = \sqrt{\frac{2 \cdot 5.143 \cdot 500}{4}} \cdot 9 \cdot 250 \simeq 1700.86 \quad \text{piezas.}$$

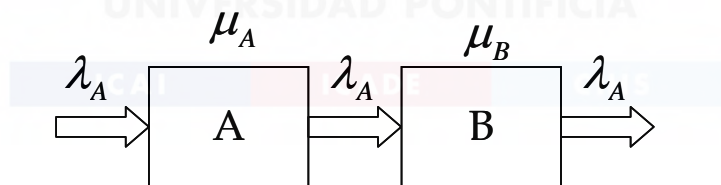
$$T_0 = 1700.86 / 5.143 = 330.71h / 9 \simeq 36.7 \quad \text{días} \quad \text{y punto de reorden}$$

$$ld = 16 \cdot 5.143 \cdot 9 = 740.592 \quad \text{piezas.}$$

SOLUTION OF PROBLEM 31

**¿Cuántos administrativos serán necesarios que estén en la oficina B si se desea que la probabilidad de una espera superior a los 10 minutos sea inferior a 0.05?**

Dados los sistemas A y B enlazados en serie, si el primer sistema A recibe los usuarios según una ley poissoniana con tasa de llegada  $\lambda_A$  y dicho sistema es estable, el sistema B que recibe usuarios del sistema A los recibe según un proceso poissoniano de tasa de llegada  $\lambda_A$ . En la siguiente figura se representa los dos sistemas enlazados con sus tasas de entrada y salida suponiendo que ambos sistemas, A y B, son estables.



El valor de la tasa de entrada al sistema A es de 12 personas/hora. Al ser la tasa de servicio de dicho sistema de 15 personas/hora (60/4) el sistema A resulta ser estable. El sistema B con un solo administrativo no sería estable ya que la tasa de servicio es de 8 personas/hora frente a una tasa de llegada de 12 personas/hora. Por lo tanto, con dos administrativos el sistema B sí sería estable.

El cálculo de la probabilidad de que exista un tiempo de espera superior a 10 minutos para **dos administrativos** en el sistema B se calcula como:

$$\Phi = \frac{\lambda}{S\mu} = \frac{12}{2 \cdot 8} = 0.75$$

$$p_0 = \frac{1}{\frac{(S\Phi)^S}{S!(1-\Phi)} + \sum_{n=0}^{S-1} \frac{(S\Phi)^n}{n!}} = \frac{1}{\frac{(2 \cdot 0.75)^2}{2!(0.25)} + 1 + \frac{2 \cdot 0.75}{1}} = 0.142857$$

$$P(t_c > 0) = \frac{(S\Phi)^S}{S!(1-\Phi)} p_0 = \frac{(2 \cdot 0.75)^2}{2! \cdot 0.25} 0.142857 = 0.64285$$

$$P\left(t_c > \frac{10}{60}\right) = e^{-S\mu x(1-\Phi)} P(t_c > 0) = e^{-2 \cdot 8 \cdot \frac{1}{6} \cdot 0.25} 0.64285 = 0.33$$

Al ser la probabilidad mayor de 0.05 el número de administrativos ha de incrementarse para evaluar la probabilidad de que el tiempo de espera supere los 10 minutos con **tres administrativos**.

$$\Phi = \frac{\lambda}{S\mu} = \frac{12}{3 \cdot 8} = 0.5$$

$$p_0 = \frac{1}{\frac{(S\Phi)^S}{S!(1-\Phi)} + \sum_{n=0}^{S-1} \frac{(S\Phi)^n}{n!}} = \frac{1}{\frac{(3 \cdot 0.75)^3}{3!(0.5)} + 1 + \frac{3 \cdot 0.5}{1} + \frac{(3 \cdot 0.5)^2}{2!}} = 0.21$$

$$P(t_c > 0) = \frac{(S\Phi)^S}{S!(1-\Phi)} p_0 = \frac{(3 \cdot 0.5)^2}{3! \cdot 0.5} 0.21 = 0.23625$$

$$P\left(t_c > \frac{10}{60}\right) = e^{-S\mu x(1-\Phi)} P(t_c > 0) = e^{-3 \cdot 8 \cdot \frac{1}{6} \cdot 0.5} 0.23625 = 0.03197$$

Con tres administrativos se obtiene que sólo el 3 % de las personas que acceden al sistema B van a tener un tiempo de espera superior a 10 minutos.