PROBLEM SET - MODELING

PROBLEM: PRODUCTION OF THREE PRODUCTS

A company produces three different products, A, B and C in six different factories. The unitary production cost of each product and the capacities of each factory are given in the table below.

Product	F1	F2	F3	F4	F5	F6
A	25	30	26	34	32	30
B	30	32	34	35	38	40
C	40	46	42	37	40	50
Capacity	550	700	1100	350	400	450

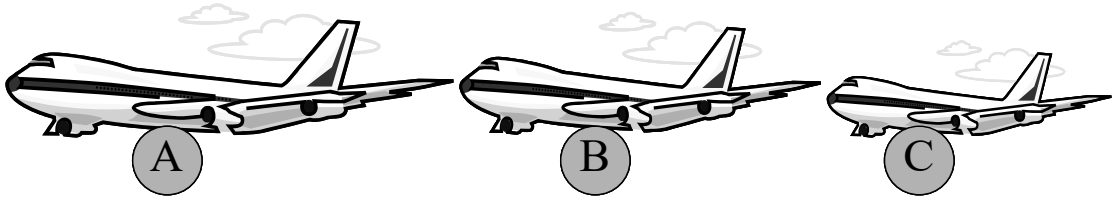
Each quantity of these products can be sold (a continuous quantity), but the marketing department already has already sold 700 units of product A, 500 units of B and 600 units of C via contracts.

The selling Price of each of these products is 60, 82.5 and 108€ respectively (A, B and C).

Propose a model that yields a production plan which maximizes profits.

PROBLEM: CHARTER FLIGHTS

A company owns 15 airplanes: 5 of each type – A, B and C, who in turn have a capacity of 80, 68 and 55 people.



A travel agency makes an inquiry to the company asking how much it would cost to transport 372 people. The company analyzes their costs which depend of the number of airplanes of each type that are used to transport these people. The data is given in the table below:

Type	1	2	3	4	5
A	11	20	30	40	50
B	9	17	24	34	45
C	8	15	21	26	31

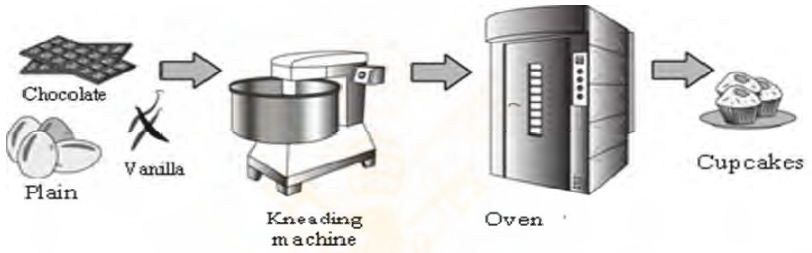
On top of everything the company also has an additional fixed cost of 6000€ for each different type of airplanes that are used in the process.

Propose an optimization model whose objective it is to determine the composition of the optimal fleet used to tackle the transport of 372 people while minimizing overall costs.

PROBLEM: BAKERY

A bakery specialized in homemade cupcakes offers a variety of three different types of cupcakes: plain, vanilla or chocolate cupcakes. Each of these varieties of cupcakes needs to be processed in two types of machines: the kneading machine and the oven. In the table below you find the time in minutes it takes to process a dozen cupcakes of each type in each machine, the weekly total processing availability of each machine in minutes, and the obtained profits in c€ for selling one dozen of each type of cupcakes.

Type of machine	Plain cupcakes	Vanilla cupcakes	Chocolate cupcakes	Availability
Kneading machine	2	5	4	700
Oven	3	4	6	860
Profits (c€)	80	70	95	



a) Formulate a mathematical model to obtain the most profitable weekly production of cupcakes possible.
b) Suppose that the bakery considers increasing the processing availability of each type of machine taking into account the options indicated in the table below. In total, at most one option of investment can be carried out per machine. The bakery disposes of 34€ per week to carry out the investments to increase processing time. Should the bakery invest in order to increase the available processing time and what investment options should be chosen? In case the answer is positive, what should be the new optimal production of cupcakes?
c) If an investment in the oven is made, then the bakery also has to invest in the kneading machine. How do you have to adapt the previous paragraph in order to model this constraint?
d) There is an investment in the oven if and only if there is an investment in the kneading machine. How is the previous solution modified?

Type of machine	Kneading machine		Oven	
Increment in availability (min)	100	150	80	120
Weekly investment cost (€)	16	17	17	18

PROBLEM: JOB SHOP

At a job shop they produce two different types of equipment: type XXX and type YYY. One piece of equipment of type XXX allows to be assembled in to different ways:

F1: 3 components of type A, 4 components of type B and 5 components of type C

F2: 4 components of type A, 5 components of type B and 3 components of type C

On the other hand, the equipment of type YYY can only be assembled using 4 components of each type A, B and C.

In total, the job shop has 4000 components of type A, 5000 components of type B and 4500 components of type C in stock.

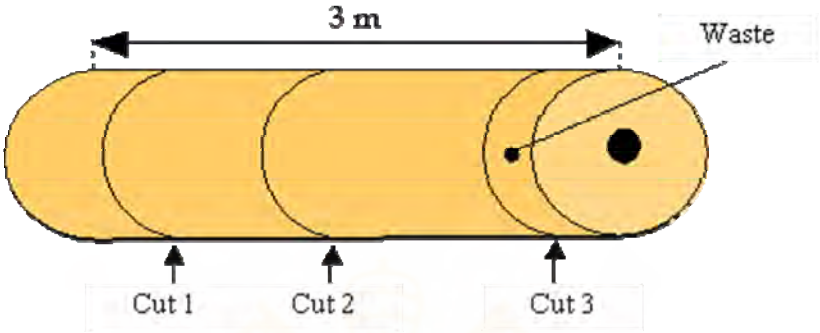
Selling one piece of equipment XXX, which has been assembled following process F1, yields profits of 18€, and 16€ if assembled following process F2. One piece of equipment YYY yields profits of 15€. In the job shop all the equipment of type XXX has to be assembled using the same process. The components A, B, C that have not been used in the assembly processes, generate a loss of 1€ per piece.

The owner of the job shop has signed contracts which state that at least 400 units of each type of equipment have to be manufactured.

Formulate a linear programming model which determines what process should be adopted to produce equipment XXX and how many units of each model should be manufactured to maximize profits of the job shop.

PROBLEM: PAPER ROLL CUT

A company manufactures "master" paper rolls that are 3 meters long. Afterwards this master paper roll is cut transversally into smaller rolls depending on the clients' orders. In the figure below you can see one of these paper rolls which has been cut into 4 smaller rolls, one of which represents waste since it does not correspond to any of the orders' specifications.



Below is a list of all the orders – including the specifications of length and quantity for each roll - accumulated over one week's time:

Type of order	1	2	3	4	5	6	7	8	9	10
Rolls	20	30	15	25	40	25	50	15	10	5
Length [m]	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5	2.75

Due to technical restrictions of the equipment, it has to be taken into account that it is not possible to make more than four cuts in the same master paper roll. Depending on the cuts that are scheduled for each master roll, there can be some part of the roll that does to waste. This waste can only be re-utilized by using it as raw materials in the initial paper production process.

Formulate a mixed integer programming model which establishes the optimal weekly master paper roll production and the corresponding schedule of how these rolls have to be cut in order to meet the clients' orders.

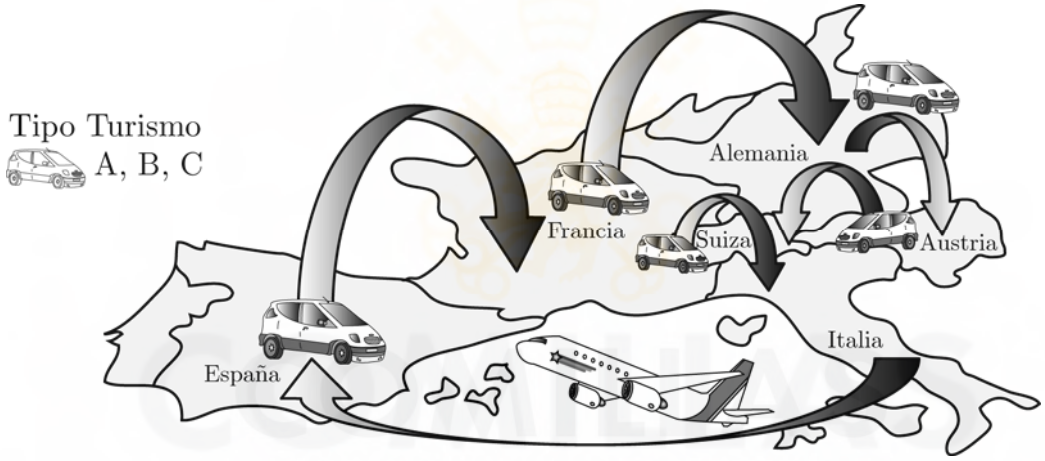
a) Identify the indices, parameters and variables of this model
b) Define the arising objective function in a generic way
c) Define the constraints of the model in a generic way
d) Model the following additional technical constraint: for all the master paper rolls in which an order of type 1 is carried out, also include an order of either type 5 or type 9.

PROBLEM: CAR RENTING

An adventurous tourist is planning his/her upcoming Europe trip in which he/she wishes to travel through Spain, France, Germany, Austria, Switzerland and Italy by car in exactly this order. To get back home to Spain, this tourist is catching a direct flight from Italy to Spain at the end of the journey. In each of the six countries, this tourist can rent one of three different types of cars, which depending on the characteristics of the car and the distances driven, amount to a certain fuel cost, which is indicated in the table below.

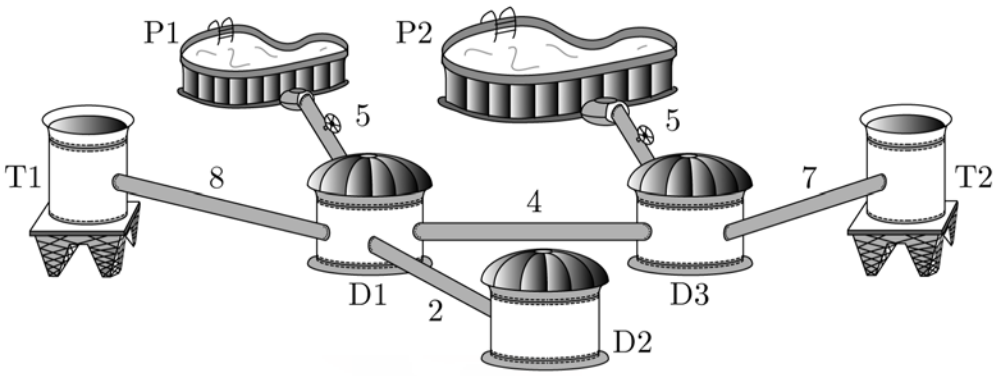
The tourist has the choice to change or not to a new vehicle each time he/she changes the country. However, each change of vehicle supposes an additional cost of 25€ which corresponds to the cost of the arising paper work.

Car	Spain	France	Germany	Austria	Switzerland	Italy
A	160	210	180	110	85	170
B	120	240	165	135	100	160
C	150	200	175	140	115	135



What is the optimal car renting schedule that minimizes the total costs of the tourist?

PROBLEM: SWIMMING POOLS



The town "Go Fish" is known to have the best swimming pools of the entire region, one of which is a medium size pool P1 and the other one P2 has Olympic measures. The weather forecast indicates that the coming summer is going to be extremely hot, which is going to lead to a shortage in water supplies, the local town council is considering to either not open the swimming pools this year, to open only one of them, or to open them both. All of this is considered taking into account that all the other arising consumption of water has to be satisfied.

The town's water supply is coming from two large water tanks, one in the west T1, and one in the east T2, which are connected to three water deposits D1, D2 and D3. From these deposits the water, given in cubic meters, is extracted for different uses as indicated in the table below. It has to be taken into account that in case the minimum daily water supply of each pool is not met, then the water quality would deteriorate and public bathing would be prohibited.

m^3	Domestic consumption	Agricultural consumption	Pool
D1	150	100	50
D2	100	50	
D3	400	0	25

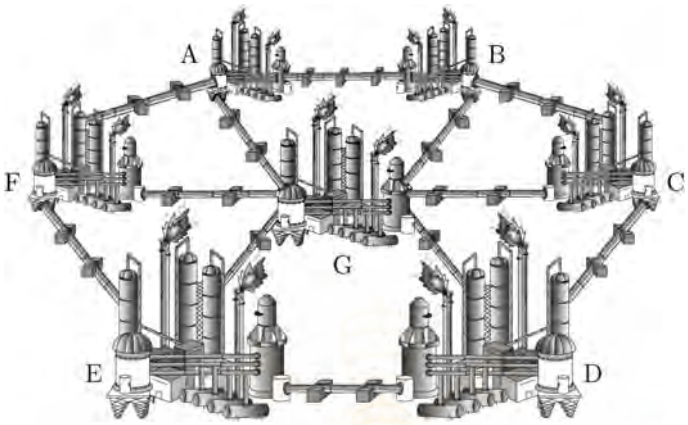
The city councilor of public services has estimated that a daily shortage of water for domestic use implies a social cost of 3€/cubic meter and a cost of 1€/cubic meter for the shortage of water for agricultural use. Not opening the medium-sized pool would lead to bad publicity for the city council valued in 100€ per day and not opening the Olympic pool would imply a cost of 50€ per day. If none of the pools were to be opened, the costs would amount to 200€ per day.

Each of the pipes which are connecting the tanks, deposits and swimming pools infer a transport cost in c€/cubic meter, as indicated in the figure. The daily available water volume of tank T1 is 400 cubic meters and T2 disposes of 300 cubic meters per day.

What should be the optimal decisions of the town council regarding the water management in order to minimize overall cost?

PROBLEM: CONNECTING TWO PIPELINES

An oil company is trying to decide which should be the refineries to which to connect each of the two new pipelines. The oil company disposes of seven refineries {A, B, C, D, E, F, G} which are already connected to one another via the pipeline network indicated in the figure below.



The following table contains the maximum capacity of each existing pipeline between refineries in liters per second.

Connected	A-B	B-C	C-D	D-E	E-F	F-A	A-G	B-G	C-G	D-G	E-G	F-G
[l/s]	600	700	700	650	550	700	850	950	600	550	700	650

The table below represents the total demand in liters per second that has been contracted by each refinery and which has to be met using the two new pipelines. This table furthermore contains the penalty that the oil company has to pay for contracted demand that is not met.

	A	B	C	D	E	F	G
Contracted capacity [l/s]	700	650	450	570	490	630	810
Penalty [€/l/s]	300	300	400	300	450	340	350

Pipeline 1 has a supply capacity of 1500 l/s and pipeline 2 has a supply capacity of 2500 l/s. Both of these numbers apply to the refinery to which the pipeline is connected to, and afterwards the existing pipeline network distributes the oil corresponding to its capacity limits as indicated in the first table.

Due to technical reasons, the two new pipelines cannot be connected to two adjacent refineries.

How should the two new pipelines be connected to the refineries such that the possible shortage of supply has the least possible economic impact?

PROBLEM: SEQUENCING TASKS IN TWO MACHINES

Five tasks need to be processed in two machines, M1 and M2, and in exactly that order, since M1 corresponds to the pre-processing and M2 corresponds to the finishing touches. The table below contains the processing time in hours of each task on each machine, and the delivery time (deadline) of each task, also given in hours.

	Machine 1	Machine 2	Delivery time
Task 1	9	12	40
Task 2	6	8	25
Task 3	10	7	33
Task 4	8	10	45
Task 5	7	9	45

Due to a technical constraint, task 3 needs to be processed before task 2.

What should be the optimal sequencing of the tasks such that the following condition is satisfied?

a) The maximum delay of all the different tasks with respect to the delivery time should be minimal.
b) The total time at which all tasks are finished should be minimal without taking into account individual delivery times.

PROBLEM: EXAMS

A student has seven days in order to prepare for four different exams corresponding to four different subjects {A,B,C,D}. The exams are of very short duration and are carried out one right after the other, without the option of studying in between exams. The student wants to dedicate at least one day to each of the four subjects, and the student cannot study for more than one subject per day. Moreover, the student can dedicate at most four days to the same subject.

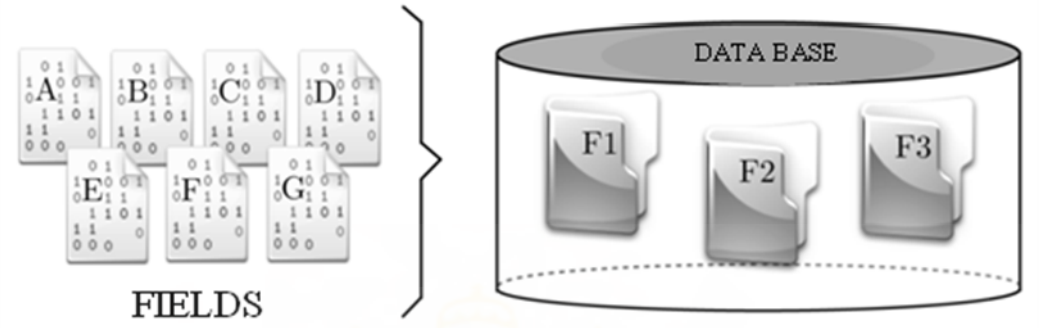
The following table contains the grade the students can expect after having dedicated a certain amount of days preparing for a certain subject. The student passes each exam if his/her grade is equal or higher than 5.

a) What should be the student's optimal studying schedule in order to pass the maximum number of exams?
b) What should be the student's optimal studying schedule in order to maximize the average grade received in all exams?

Number of days	A	B	C	D
1	3	4	2	5
2	5	5	5	6
3	6	6	7	7
4	7	9	8	9

PROBLEM: DATA BASE

The information contained in the fields A, B, C, D, E, F and G needs to be distributed among three files F1, F2 and F3 such that the time it takes to consult the data base, which consists of these three files, is minimal. Each field cannot be contained in more than one file.



The following table indicates the probability that the following fields are being requested when consulting the data base.

Consultation C1	Consultation C2	Consultation C3	Consultation C4
A-D	C-E-F	B-D-G	A-B-G
35	25	20	20

What should be the optimal assignment of fields to files in order to satisfy the following technical conditions?

- Each file cannot contain more than four fields
- The file that contains field A cannot contain file F
- If a file contains both fields A and B, then fields D and E have to be in the same file
- The inquiries that search two files take 25% longer than the inquiries where all the fields are contained in one file
- The inquiries that search three files take 50% longer

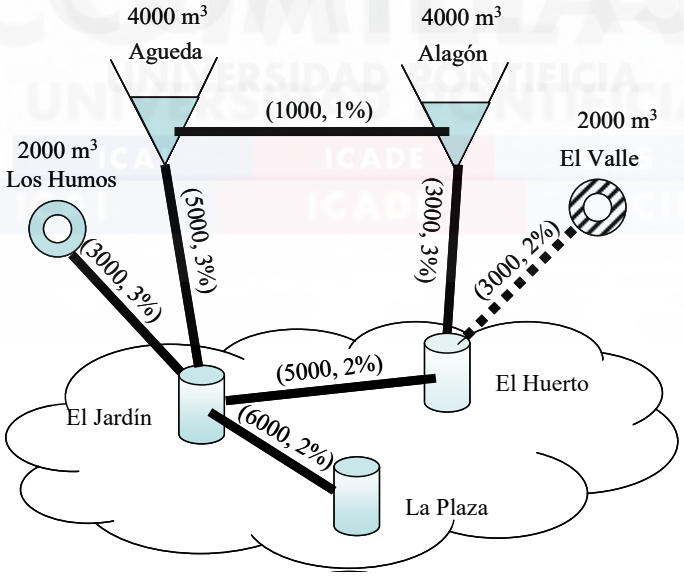
ADDITIONAL PROBLEM SET-MODELING

PROBLEM. THE DROUGHT

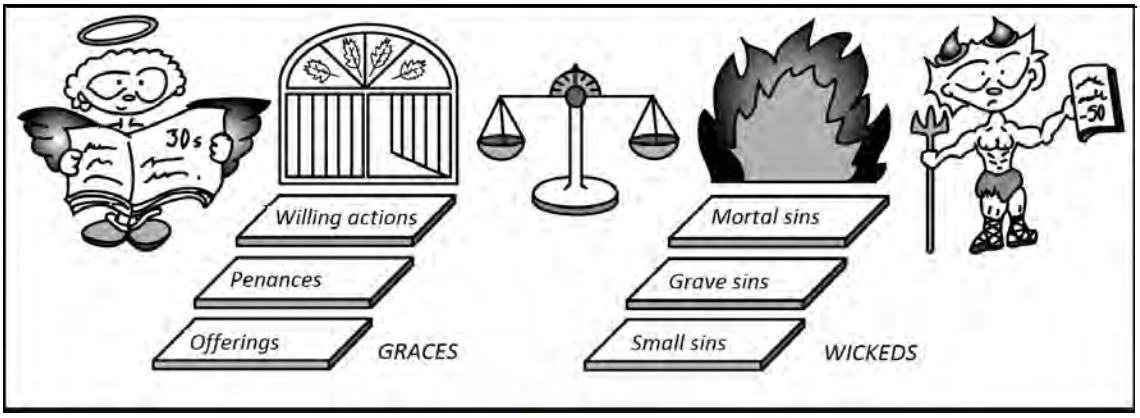
Ciudad Rodrigo is an agricultural population that has a water supply from the reservoirs, Agueda and Alagón, and the groundwater from the well "well of Los Humos". The infrastructure and urban planning city councilwoman is evaluating different possibilities of limiting this summer's water consumption. This water is distributed through three tanks named el Huerto, el Jardín and la Plaza. To achieve this goal, she has classified daily consumption in domestic and agricultural consumption, and penalizes with a different cost the lack of water in both types of consumption as is indicated on the following table.

	Domestic Consumption [m^3]	Agricultural Consumption [m^3]	Lack of Domestic Consumption cost [€/m^3]	Lack of Agricultural Consumption cost [€/m^3]
El Huerto	1500	3000	1	0.6
El Jardín	1000	2000	1.1	0.6
La Plaza	4000	0	1.2	-

The pipelines that connect the reservoirs and the wells allow a maximum daily water flow and has also some loss percentages that are indicated both in brackets in each pipeline of the picture. It also shows the diary available volume of each reservoir and well. On the right side the well "El Valle" is shown, its implementation involves a cost of 1000€ per day. Technically it is necessary that before supplying "La Plaza" tank, both tanks "El huerto" and "El Jardín" have to be fully supplied. What should be the optimal management of water in Ciudad Rodrigo?



PROBLEM. ST PETER



St. Peter is sophisticating the access system to heaven and for this he has decided to classify good actions in 3 categories: Offerings, Penances and Willing actions and sins in 3 categories: Mortal Sins, Grave and small sins.

Mortal sins without being forgiven by offerings involve that the soul goes directly to hell. One mortal sin is forgiven if the sinner makes three offerings. For the rest of the actions the criteria followed is to value with positive "graces" good actions and negative "graces" for sins. The minimum level required for the soul to go directly to heaven is of 100 graces, in case of having between 50 (included) and 100 graces the soul spends some time in the Purgatory and then goes to heaven, and in case of having less than 50 graces the soul goes to hell unless divine intervention. The following table indicates the value of St. Peter for good actions and sins expressed in "graces".

Good actions		**Sins**	
Penances	Willing	Graves	Small sins
4 graces	2 graces	-10 graces	-3 graces

Lucifer ("the demon"), given his evil trades, has known this table of valuation of St. Peter. He contacts certain humans that are the worst of society because they enjoy acting badly. Lucifer has offered them to know the valuation system, taking them off 50 graces and they are glad to accept.

With Divine authority, St. Peter assigns everyone 50 graces when born, and if he or she is baptized, he assigns 50 more graces, a total of 100 graces, some of which, the amount agreed with Lucifer, have to be taken off.

These evil humans tend to realize on average 200 actions that punctuate along their lives (between good actions and sins). From the wicked point of view the valuation of actions changes completely from St. Peter's valuation such as is indicated in the following table expressed in "wickeds".

Good actions			**Sins**		
Offerings	Penances	Willing	Mortal	Graves	Small sins
-30 wickeds	-10 wickeds	-5 Wickeds	60 wickeds	30 wickeds	10 wickeds

a) For a wicked, which should be the distribution between good actions and sins that maximizes his/her evil valuation of "wickeds" and still allow him/her to go directly to heaven according to St. Peter's valuation on "graces"?. *Note: Wickeds are baptized.*

b) And if they don't mind spending some time in Purgatory unless they lose 25 "wickeds"?

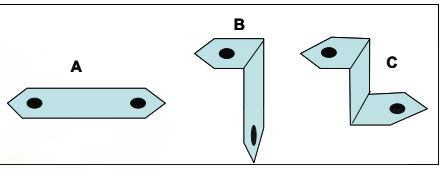
PROBLEM. JOINTS

A garage manufactures three components A, B and C, in one elaborating process. Component A requires the use of a cutting-drilling-angled machine (CDA) and components B and C require the additional use of a press. The processing time in minutes per component is indicated in the following table:

	A	B	C
CDA	1.0	1.0	1.0
Press	0.0	0.25	0.75

The garage supplying contracts during the first trimester, the initial stock and its manufacturing cost are indicated in the following table.

Component	January	February	March	Initial stock	Cost [€]
A	5000	2500	7500	2000	1
B	3000	4000	5000	1000	1.5
C	2000	1500	2500	500	2



For technical reasons, the garage's staff gets every month 200 hours to use the CDA and 50 to use the press. The penalty defined by contract for not supplying the demand completely is estimated at 2.2€ for component A, 2.6€ for component B and 2.8€ for component C. It has to be taken into account that not supplied components in a month cannot be supplied in the following months as the clients will have bought them in another garage. However, components can be manufactured in advance.

There is a warehouse at the factory that allows manufactured components during a month to be stored so they can be sold in the following months. When component A is totally manufactured components B or C cannot be made from them. The storage cost of component in a month is established in 0.05€. The maximum amount of storage components is 6000 components regardless of the type of component.

The future profit from the sale of those components manufactured and not sold during the first trimester is 1.2€ for component A, 1.6€ for component B and 2.2€ for component C.

a) What should be the production plan that minimizes the overall cost of the garage?
b) How the above planning changes if it is possible to manufacture components B and C from components A that are stored?
   - Take into account that the months that exceed the number of 10000 components the manufacturing cost of component A reduces to 0.95€ per unit.
   - Take into account that manufacturing components B and C from component A entail the additional cost indicated in the heading table.

PROBLEM. DIGITAL TV

A company manufactures signal decoders that give TVs the possibility to surf the net. There are three types of decoders: basic, transmitter and wifi. There are 10 permanent workers at the factory who work 160 hours a month each. The workers have been classified by their working efficiency in four groups. The following table shows the number of workers in each group, the average time used for manufacturing each decoder and the profit margin of each decoder.

	Workers' groups				Profit margin
	A	B	C	D	
Number of workers	3	2	2	3	
Basic	35 min	30 min	40 min	45 min	5 €
Transmitter	45 min	40 min	50 min	70 min	10 €
Wifi	60 min	55 min	70 min	75 min	20 €

To increase productive capacity temporary workers can be subcontracted with an additional salary cost of 2000€ per month. Only 5 temporary workers can be subcontracted. The temporary worker efficiency can be resembled to permanent worker efficiency of group D. The company has a supply contract that requires the company to provide monthly 300 basic decoders, 400 transmitters and 550 wifi to malls. If these quantities are not satisfied, the penalty is 25€ for each not supplied decoder.

The production director establishes that each type of decoder must not exceed the 50% of the manufactured units of each month. For human resources reasons, workers of group A cannot assemble the same type of decoder as workers of group C.

What should be the optimal production plan and the optimal workers' management from an economic point of view?

PROBLEM. THE WINE-PRODUCER

This year, a wine-producer has had a good harvest estimated at 10000 liters of grape juice (must). He has to decide how much of the harvest he wants to dedicate to produce "must", how much of it he wants to keep in barrel for one year and sell it bottling as "young wine", how much of it he wants to keep in barrel for two years and sell it bottling as "crianza wine", how much of it he wants to keep in barrel for three years and sell it bottling as "reserve wine", and how much of it he wants to keep in barrel for four years and sell it bottling as "premium reserve wine".

The evaporation in barrel percentage of the different types of wine, such as the selling price and the cost of each wine including bottling is shown in the following table. For example, the reserve wine that results of the quantity of the juice grape destined to this wine is about 77% (100-23). Every year every bottle that remains in storage without being sold generates an additional cost of storage for each bottle of 20% of its cost.

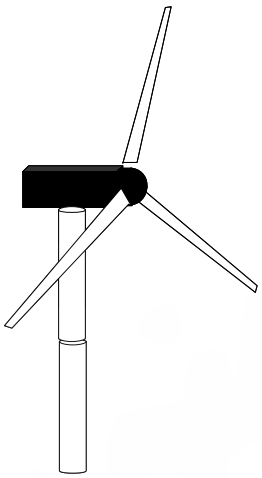
	Must	Young wine	Crianza wine	Reserve wine	Premium reserve
Price [€/l]	1.5	1.9	2.8	3.8	5.5
Evaporation [%]	0	15	20	23	25
Cost [€/l]	0.2	0.6	1.5	2.2	2.9

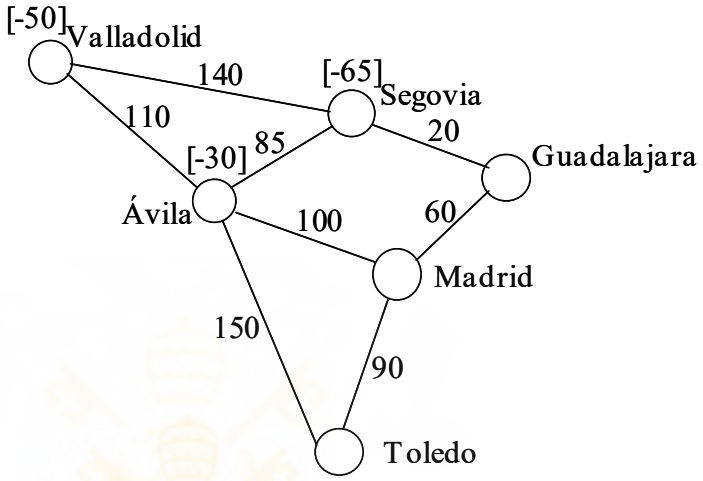
The different product demand for the next six years (including the harvest year) that the wine-producer estimates is shown in table 2. The wine-producer covers these estimated demands wanting to maximize economical profit. Which should be the juice grape assignation for each of the products and the scheduled sale of them?

Year of sale	1	2	3	4	5	6
Must	1000	200				
Young wine		2000	1000			
Crianza wine			3000	2000	1000	
Reserve wine				2500	500	
Premium reserve					1000	250

TRANSPORT. WIND TURBINES

WINDPOWER is specialized in manufacturing and supplying blades and circular sections of wind turbines. Each wind turbine consists of three blades and two sections as is shown in the figure below.





The blades are manufactured in Guadalajara's factory and the columns' sections in Toledo's factory. WINDPOWER wants to supply 50 wind turbines for a wind farm in the province of Ávila, 50 wind turbines to another wind farm in Valladolid's area and 65 more to another wind farm located in Segovia's area. The figure shows the possible special transport routes required for these components. A truck can use two types of platforms to transport two blades or one circular section.

The figure shows over the arcs the distances in km and in the nodes the wind turbine necessities in the wind farms with a negative singe (in square brackets). The special transport costs of each truck are about 500€ per route (arc in the figure), and additionally 10€ for each km that the truck goes over. For example, if a special transport makes the Guadalajara-Madrid-Ávila-Valladolid route it will cost for the three stretches 900€ and 2700€ for the 270 km the truck goes over, in total it will cost 5600€. Special transports allow some modifications in the load's configuration in each real node.

a) Which should be the number and management of the special transports for supplying the components to the different installation zones?
b) Assume that the trucks have an additionally third platform available that can carry a circular section and a blade at the same time. However, the use of this platform increases in a 25% the cost of stretch and km. What would be the new transport management that minimizes costs?

PROBLEM. THE OIL TANKER

An oil tanker leaves Dubai's harbor placed next to the main oil wells of the United Arab Emirates. This oil tanker has the capacity to house 2 million crude oil barrels. The oil tanker holds can house two types of crude oil: light and heavy. The oil tanker can leave carrying less crude oil than the maximum capacity the oil tanker can house and ends the route the moment its holds are empty, returning to Dubai's harbor.

The route to follow after leaving Dubai is the harbors of Damman (Saudi Arabia), Suez (Egypt), Cartagena (Spain) and Calais (France). In each harbor the oil has obtain buying offers expressed in millions of barrels for both types of crude oil shown in the table such as its buying price per barrel.

	Damman	Suez	Cartagena	Calais
Light oil barrels [millions]	0.4	0.6	0.7	0.9
Heavy crude oil [millions]	0.4	0.5	0.6	0.9
Light oil price [€/barrel]	60	60.5	61	61.5
Heavy oil price [€/barrel]	60.2	60.6	60.5	61

The transportation costs of oil between two consecutive harbors are given by the combination of two terms, one fixed for each stretch and one variable depending on the load carried by the tanker. Both types of cost are included in the following table:

	Dubai Damman	Damman Suez	Suez Cartagena	Cartagena Calais
Fixed cost [€]	70 000	140 000	230 000	500 000
Variable cost [€/barrel]	0.15	0.2	0.2	0.3

It has to be taken into account that although crude oil is not sold in a harbor the maritime route always passes near the mentioned harbors. What should be the economically optimal management for this oil tanker?