PROBLEM: R&D

The R&D department of a company is considering developing a new chip that involves an investment of 70000 €. Every time the company has taken any development in the microelectronics field, the company has had success in 70% of the cases.

If the development succeeds, the company must consider two options. The first is to manufacture 4000 units of this device with a global manufacturing cost of 40000 € and a unitary selling price of 100 €. The second one is to manufacture only 3000 units of this device with a global manufacturing cost of 30000 € and a unitary selling price of 110 €.

A market study has established that the demand of this article is a random variable that takes the value of 4000 units with a probability of 60% and 3000 units with a probability of 40%. The manufactured but not sold units are considered net losses.

a) *Set up the decision tree with its probabilities of occurrence.*
b) *Assess the tree and indicate the company's recommended policy.*
c) *How much should the company be willing to pay for the copyright of the chip developed by a Research Institute whose development has succeeded?*
d) *How much should the company be willing to pay for the copyright if the probability of developing it successfully was 20 %?*

PROBLEM: MOBILE SUBSCRIPTION

Eugenio wants to register with a new mobile phone company, MOVICAI, and he has gathered information from a company store and his friends' experiences. The company offers two types of call charges depending on whether he calls to numbers of the same company or numbers of other companies, as well as an additional semi-annual fixed fee. There are no charges for the establishment of the call.

Type of subscription	*Fixed fee*	**Tariff for calls to the same company**	**Tariff for calls to other companies**
Contract	*120 €/semester*	*5 c€/min*	*9 c€/min*
Prepaid card	*0 €/semester*	*8 c€/min*	*12 c€/min*

Contract clients cannot change their type of subscription within a year. Prepaid card clients can switch to a contract six months after having registered, and if they do not change, they have to wait another six months to move to a contract. If the client changes from a prepaid card to a contract after six months, there is a promotional bonus 50€ in consumption. Under no circumstances can one pass from a contract to a prepaid card during the first year.

After asking his friends, Eugenio estimated that the probability of talking for over 5000 minutes for the first six months was 30%, and over 3500 minutes was the remaining 70%. In all cases, calls to the same company's number and other companies' numbers are equally distributed. For the second semester, the probability of each type of call will be different depending on the consumption of the first semester.

		Second semester	
		5000 min	*3500 min*
First semester	*5000 min*	*50 %*	*50 %*
	3500 min	*60 %*	*40 %*

MOVICAI gives a mobile phone either with a contract or prepaid card, forcing the client to stay a year in the company. Therefore, Eugenio wants to know what decision to make on average over the next year.

a) Set up the decision tree
b) Assess the tree and set the optimal decision after the assessment
c) Is there value of perfect information? Justify your answer
d) Would you change the value of the preceding question if there were no bonus when you passed from prepaid card to contract?

PROBLEM: SCARVES

A small salesman has a business that sells commemorative scarves at fairs and festivals usually held in villages in September. This December, he is planning to go or not to Villachotas's festivities in honor of their patron Saint Apapucio, martyr, held next September 18. In case he goes, he has to apply for the appropriate license now because the number of applications is very high, and the City Council gives licenses in order of presentation until the quota is reached, except for two or three irrelevant cases. The cost of the selling license is about 60€.

When June arrives, the weather service can confirm whether the village will have good or bad weather during the festivities. From experience, it is known that the village has good weather for seven out of ten years. The salesman also knows from experience that when the village has bad weather, it is not worth going to the festivities: he would end up making even more losses.

In good weather, the salesman has to order the scarves from a manufacturer. He can make an order of 900 scarves, with a unitary cost of 60 c€ and a selling price per unit of 180 c€, or an order of 600 scarves, with a unitary cost of 75 c€ and a selling price per unit of 210 c€.

The selling price is reported to the three clubs in Villachotas; each of them has agreed to buy 300 scarves, a commitment that each club meets like the Government, that is, whenever they please. This means that the salesperson has to consider this factor and classify years from an economic performance point of view as good, average, and bad. This classification depends on how many clubs had bought the established quantity, all of them, two of them, or only one of them, which has occurred with a probability of 20%, 50%, and 30%, respectively.

If the salesperson orders 600 scarves and the three clubs demand their quota of 300 scarves each, the salesperson, to avoid complaints, gives 200 scarves to each club at a reduced price of 180 c€.

What decision will the merchant take? How much would he pay for perfect information?

ADDITIONAL DECISION THEORY PROBLEM SET

PROBLEM: THE ORCHARD

Don Tomás Verdura has his orchard for his entertainment and to take economic advantage of it during the summer when summer vacationers buy products from the orchard at the village's market. His cousin "el Tachuelas" has a similar orchard but with a greenhouse that protects plants from hail.

Having its greenhouse is not accessible to Don Tomás, but his cousin offers to give his greenhouse up for a fifth part of its income at the market. If the weather and health allow it, Tomas's orchard and his cousin's orchard can produce 200 kg of tomatoes and 200 lettuces in July and again in August. The selling price of each kg of tomato is 2 € and of each lettuce 1 €.

Don Tomás plants the seeds in early May to collect in July. His grandfather told him that every five years, a hailstorm falls in July and every ten years in August. As a good grandson, he believed his grandfather, but he also observed that his grandfather's words were accomplished on average. Don Tomás thinks that all these phenomena of nature occur independently.

If it hails for a month and the orchard has no greenhouse, the harvest decreases by 80% that month. If it hails in July but does not hail in August, the harvest in August will also be affected by 30% if it is not replanted. However, if it hails in July, Don Tomás can replant the whole orchard with 1000 small garlic that can be harvested and grown in August. He bought the small garlic from a neighbor with a greenhouse for 0.3 € per unit. Then he sells the small garlic for 0.6 € per garlic. Garlic will support a hailstorm without damage.

- *What should Don Tomás's decision be?*
- *What is the value of perfect information?*

PROBLEM: THE PROBING

Aunt Severiano is very worried about the in-sight draught ahead. Therefore, he plans to make a single probing sound on his plantation for the following two years to guarantee a water supply. He considers taking this decision only at the beginning of the first year. After these two years, Aunt Severiano plans to retire and leave the profession of farming.

Last year, with the draught, the natural springs of his plantation ran out, and he had to hire a water tank service that cost 3000 €/year. The cost of the tank service is supposed to be the same every year of drought. It is estimated that the probability of a dry year is 0.7. The probability of a year being dry or wet is not influenced by the previous year.

The town's mayor has told him that in the area, there are two probing companies: company A, which can sound until 75 m and costs 3000 € per probing, and company B, which can sound until 200 m and costs 4000€ per probing.

Given the land conditions and the previous experience in probings realized in the area, the probability of success and failure in finding water is known. It depends on the type of probing, as is indicated in the following table:

	Success	*Failure*
Company A	*0.7*	*0.3*
Company B	*0.9*	*0.1*

It must be considered that both companies always find water when it appears at its corresponding depth levels.

Company B could be additionally hired if company A is chosen without success. In this case, taking advantage of the previous work of company A, the probing with company B will additionally cost 1500 €.

Answer:

a) *Considering that Aunt Severiano wants to make the most profitable decision on average for the following two years, establish the decision tree with its occurrence probabilities.*

b) *Assess the tree and determine the optimal decision Aunt Severiano should take.*

c) *Romunaldo, Aunt Severiano's nephew, is a well-known water diviner in the zone that boasts 100% reliability, indicating whether there is water on the plantation and its depth. Is it reasonable for Romunaldo to ask Aunt Severiano for two lambs valued at 100 € each for his prediction?*

PROBLEM: THE GAME SHOW

Mercurio Pérez-López, alias "Corruptelas," has been accepted as a participant in a cultural game show. The dynamic of the game show is that each participant is asked a maximum of three questions: seven out of ten deal with subjects of Science, and the other ones about Humanities.

Each question is randomly chosen between the existing ones in a bank; if the contestant fails, the answer to the first question is eliminated; if he answers correctly, he can aim to win 2000 € and retire from the game show or continue with the second question.

If the contestant fails to answer the second question, he is eliminated, although he earns 1000 €; if he answers the question correctly, he can aim to win 5000 € and retire from the game show or continue with the third question.

If the contestant fails to answer the third question, he is eliminated, although he also earns 1000 €; if he answers correctly, he wins 10000 €, and his participation in the game show ends.

Mercurio has seen the actions of other previous contestants and has taken data about the questions they asked them and his knowledge of the correct answers, creating the following table.

Percentage of correct answers

	Question 1	*Question 2*	*Question 3*
Science	*0.6*	*0.5*	*0.4*
Humanities	*0.85*	*0.75*	*0.65*

a) *What is the expected profit on average for Mercurio?*

b) *After analyzing the table before participating in the game show, Mercurio evaluates the possibility of a speed course on science themes. The course involves some teaching expenditures of 1000 € but increases the percentage of correct answers on Science themes by 0.2 for each of the three possible questions. Is it worth it economically to take the course? How much is it worth?*

c) *Another option different from the speed course is to cheat. This option is done by honoring his alias and with the help of his cousin, who is part of the team that produces the game show, Vespasiano López-Pérez, alias Corruptelas II. His cousin will alert him with a sign if the question is about Science or Humanities. In exchange for his immoral help, Vespasiano will collect 1000 €. Apart from the negative ethical judgment that deserves this practice, is it advantageous from an economic point of view? Justify your answer.*

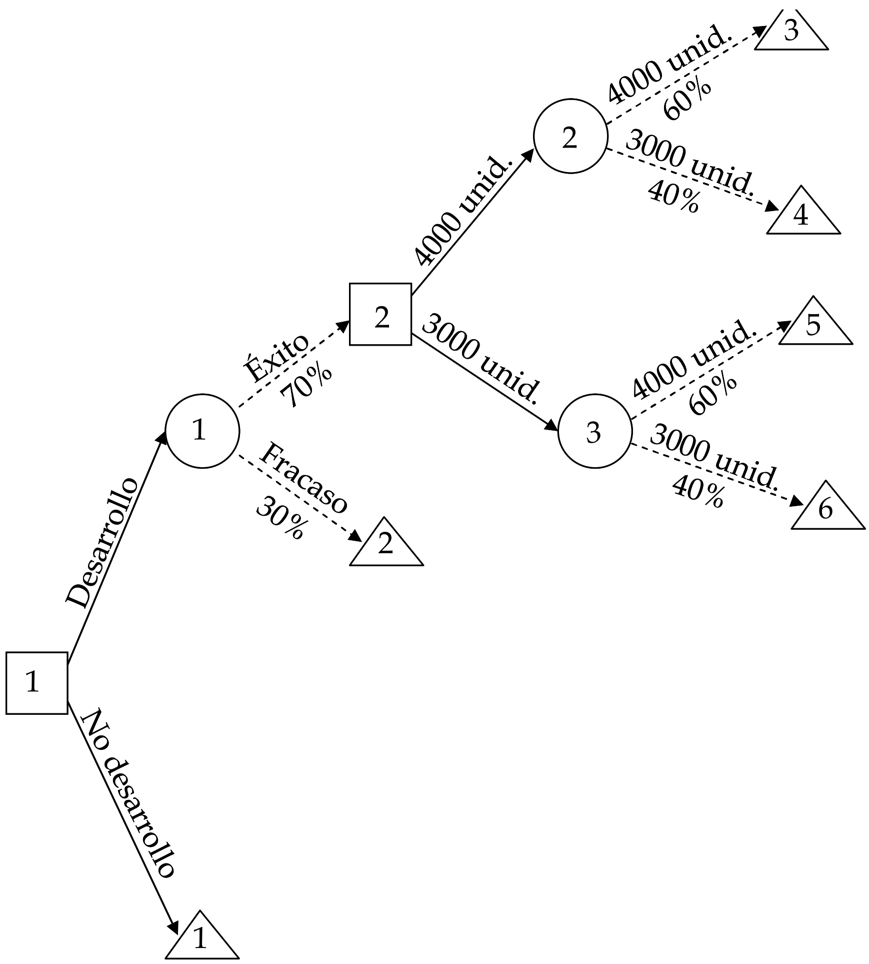
Problem: Scarves (revisited)

Consider the "Scarves" problem, and add the following information to the original problem.

On Christmas Eve, the salesperson can also consult Uncle Emerenciano, a pastor who, thanks to very particular rheumatism, predicts weather in advance and has a reliability of 80%. Uncle Emerenciano, in exchange for the information, will settle for austere food: a good bottle of wine, a special dish of Iberian products, a fabada, four trouts with ham, half a cheese, puddings and pastries, coffee, a drink, and a cigar (total: 59 €, including taxes) and 30€ more for other eventualities.

SOLUTION: R&D

a) Set the decision tree with its probabilities of occurrence



The decision to develop or not is taken first in the decision tree. After seeing the success or failure of this development, a second decision on the number of units is taken: 4000 or 3000 units to finally consider the probability of demanding 4000 or 3000 units.

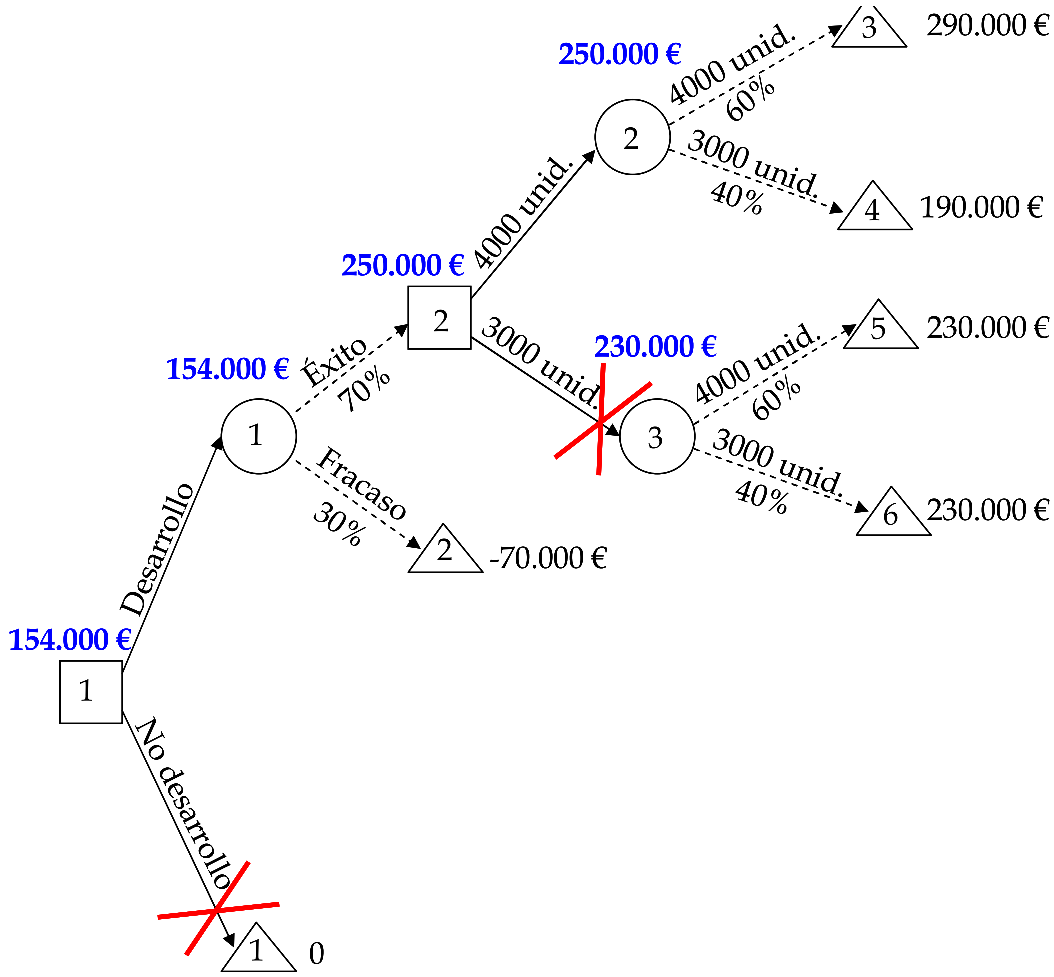
b) Asses the tree and indicate what is the recommended policy for the company

To assess each end leaf of this tree, consider the income from each unit sold, the investment cost in the development, and the cost of manufactured units.

Leaf	*Income of units sold*	*Cost of development*	*Manufacturing cost*	*Total income*
1	*0 €*	*0 €*	*0 €*	*0*

2	*0 €*	*-70000 €*	*0 €*	*-70000 €*
3	*4000 x 100 = 400000 €*	*-70000 €*	*-40000 €*	*290000 €*
4	*3000 x 100 = 300000 €*	*-70000 €*	*-40000 €*	*190000 €*
5	*3000 x 110 = 330000 €*	*-70000 €*	*-30000 €*	*230000 €*
6	*3000 x 110 = 330000 €*	*-70000 €*	*-30000 €*	*230000 €*

Assessing the tree, the optimal decision on average (Laplace criterion) is obtained:



The optimal solution is to develop the device and then manufacture 4000 units with an expected profit of 154000 €

c) How much should the company be willing to pay for the copyright of the chip developed by the Research Institute, whose development has succeeded?

A chip already developed must be assessed as the difference between the earnings when the company has it and what it would earn on average if it develops the chip.

If the company has the Research Institute chip, earnings are summed to the successful expected profit, the value of the development cost, 250000 + 70000 € = 320000 €.

On average, the profit is calculated for part b), 154000 €. So, the price the company would be willing to pay for the copyright of the chip is:

Value of the chip's copyright = 320000-154000=166000 €

This value is similar to the expected profit with uncertainty, so this product has a risk of about 100%. Investing in developing this product is highly risky, knowing that someone else has already developed it.

d) How much should the company be willing to pay for the copyright if the probability of developing it successfully was 20 %?

The expected profit is recalculated if the probability of success goes down to 20%. For that reason, random node 1 is assessed again:

Node 1 = 0.2x250000+0.8x(-70000)=-6000 €

As it is a negative value, it turns out to be more beneficial on average not to develop any chip, so the optimal value on average will be 0.

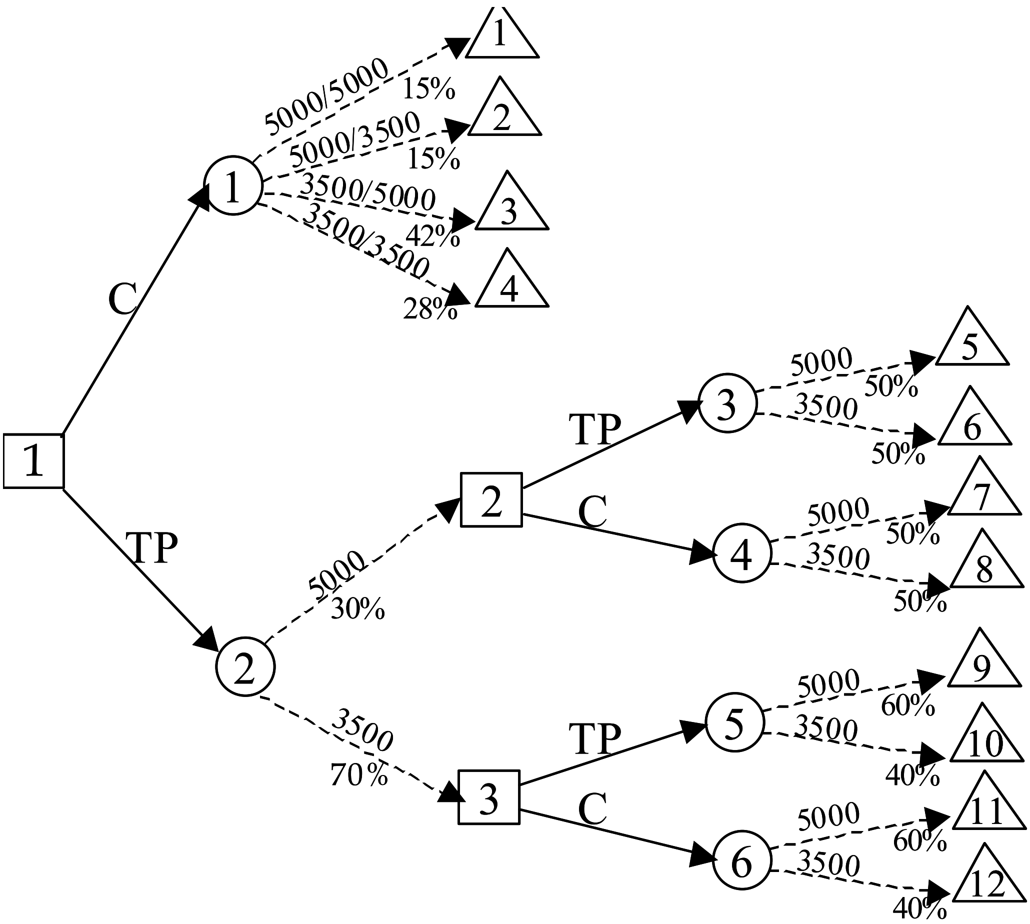
The company will be willing to pay 320000 €, which is the value of the expected profit with success without any development cost.

This product will benefit the company depending on the final purchase price of the chip's copyright.

a) Draw the decision tree

Decisions in the first semester are contract (C) or prepaid card (TP). The first four tree leaves are possible scenarios when he has decided to choose a contract.

If he chooses a prepaid card in the first semester, there are two possible scenarios depending on the length of conversations. After both uncertainty scenarios during the first semester appear, deciding to stay with a prepaid card (TP) or move to a contract (C) is possible, depending on the length of conversations during the second semester.

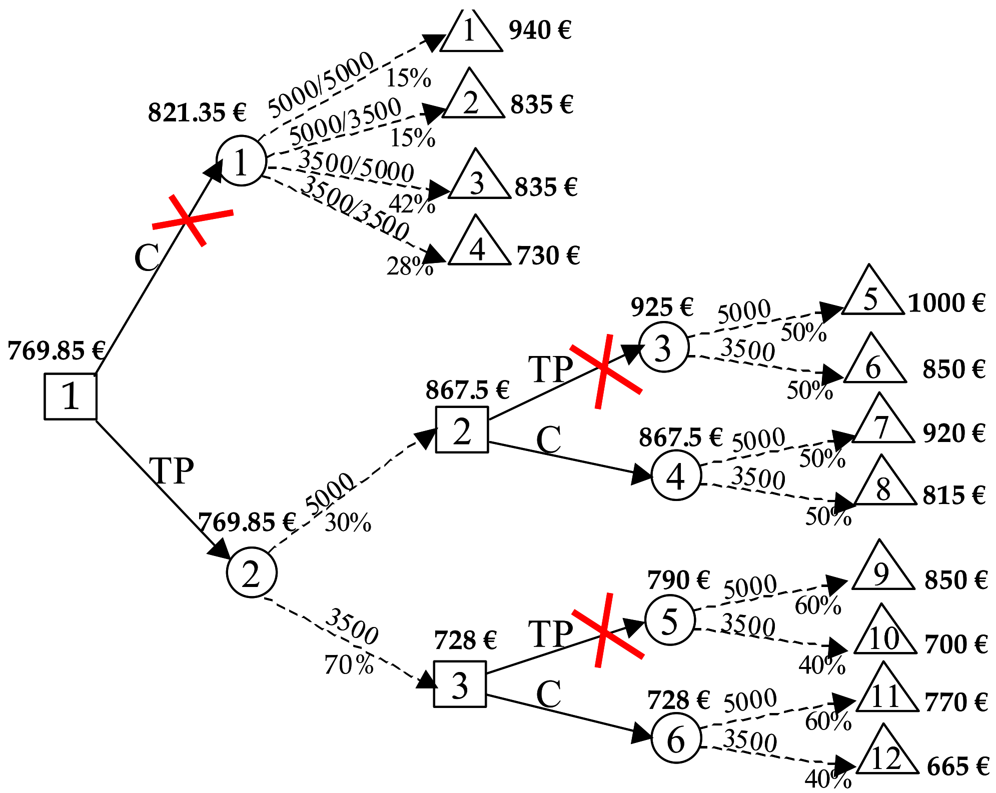


b) Assess the tree and set the optimal decision after the assessment

The values of the annual cost of the different scenarios that can result from the consumption of the first semester and the second semester are calculated. Moreover, the semester contact fee is the bonus for moving from a prepaid card to a contract.

Leaf	**First-semester consumption**	**Second-semester consumption**	**Semester fee**	**Discount**	**Annual cost**
1	*2500x5+2500x9*	*2500x5+2500x9*	*24 000*		**940 €**
2	*2500x5+2500x9*	*1750x5+1750x9*	*24 000*		**835 €**
3	*1750x5+1750x9*	*2500x5+2500x9*	*24 000*		**835 €**
4	*1750x5+1750x9*	*1750x5+1750x9*	*24 000*		**730 €**
5	*2500x8+2500x12*	*2500x8+2500x12*			**1000 €**
6	*2500x8+2500x12*	*1750x8+1750x12*			**850 €**
7	*2500x8+2500x12*	*2500x5+2500x9*	*12 000*	*-5000*	**920 €**
8	*2500x8+2500x12*	*1750x5+1750x9*	*12 000*	*-5000*	**815 €**
9	*1750x8+1750x12*	*2500x8+2500x12*			**850 €**
10	*1750x8+1750x12*	*1750x8+1750x12*			**700 €**
11	*1750x8+1750x12*	*2500x5+2500x9*	*12 000*	*-5000*	**770 €**
12	*1750x8+1750x12*	*1750x5+1750x9*	*12 000*	*-5000*	**665 €**

The assessed tree is represented below:



The decision to take, on average, is to start buying a prepaid card six months after moving to a contract.

c) Is there value of perfect information? If so, calculate its value.

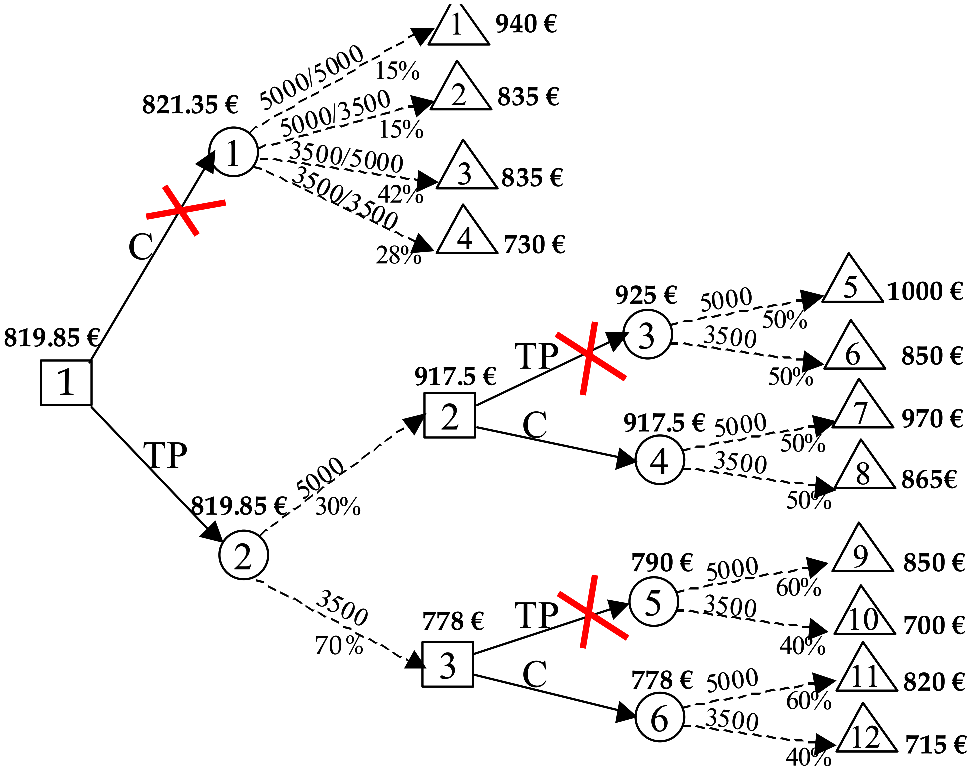
The value of perfect information considers that the length of phone calls is known with accuracy during both semesters.

First semester	*Second semester*	*Probability*	*Decision*	*Cost*	*Weighted cost*
5000	*5000*	*15 %*	*TP(1er sem)* *C (2^{0} sem)*	*920 €*	*138 €*
5000	*3500*	*15 %*	*TP(1er sem)* *C (2^{0} sem)*	*815 €*	*122.25 €*
3500	*5000*	*42 %*	*TP(1er sem)* *C (2^{0} sem)*	*770 €*	*323.4 €*
3500	*3500*	*28 %*	*TP(1er sem)* *C (2^{0} sem)*	*665 €*	*186.2 €*
Total					*769.85 €*

Therefore, as the decision regarding the phone bill during both semesters has not been changed, the value of this information is null.

d) Would you change the value of the preceding question if there was no bonus when you passed from prepaid card to contract?

Values of leaves 7, 8, 11, and 12 must be increased by 50€ (the bonus). The tree assessing changes is shown below:



Perfect information allows changing decisions to find the lowest cost at each consumption scenario between semesters.

First semester	*Second semester*	*Probability*	*Decision*	*Cost*	*Weighted cost*
5000	*5000*	*15 %*	*C (1er sem)* *C (2º sem)*	*940 €*	*141 €*
5000	*3500*	*15 %*	*C (1er sem)* *C (2º sem)*	*835 €*	*125.25 €*
3500	*5000*	*42 %*	*TP(1er sem)* *C (2º sem)*	*820 €*	*344.4 €*
3500	*3500*	*28 %*	*TP(1er sem)* *TP (2º sem)*	*700 €*	*196 €*
Total					*806.65 €*

The value of perfect information is calculated as the difference between the valuation of the tree and the value obtained with perfect information. Therefore, such perfect information applies:

Value of perfect information = *819.95 € – 806.65 €* = **13.2 €**

SOLUTION: PROBLEM SCARVES

Figure 1 shows the decision tree for the problem of the seller of scarves at Villachotas's festivities. In this tree, **NO** *represents the scenario of not going to the festivity, and* **IR** *the scenario of going to the festivity. On the other hand,* **B** *is the possible scenario of having good weather, and M is the one of having bad weather.* **GR** *symbolizes the alternative of making a big order (900 scarves), and PEQ is the alternative of making a small order (600 scarves). Finally, a good year when every club claims AB represents its orders, a regular year in which two of the three clubs claim its orders is represented by* **AR** *and a bad year in which only one claims its scarves is represented by* **AM***.*

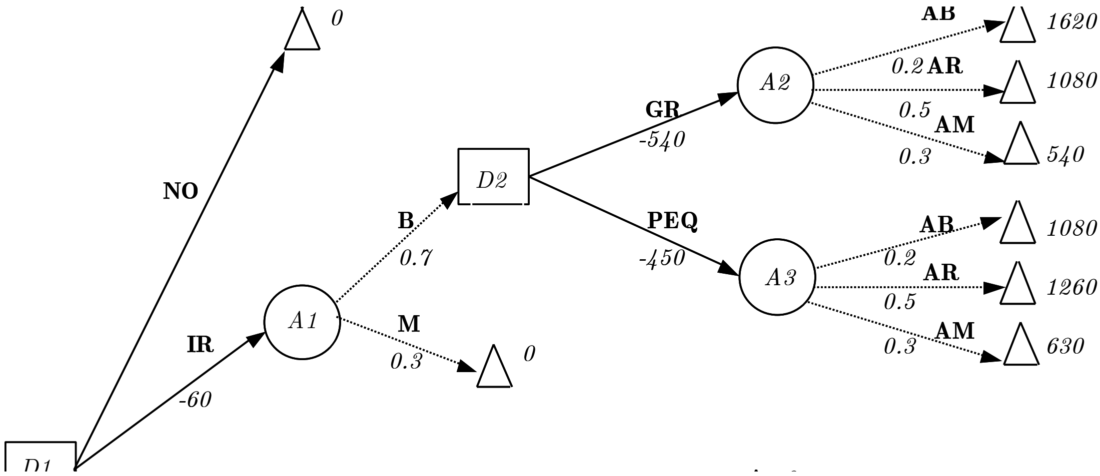


Figure 1

*At first, the seller must choose between not going to the festivity or the festivity. The decision not to go to the festivity leads to an ending vertex with an economic consequence of zero, as the seller rejects the business. If the decision is to go to the festivity, he will arrive by the appropriate branch to an uncertainty vertex. Going through this branch has an associated cost of 60€ as the seller has to pay the license. The two branches of the uncertainty vertex are good weather (***B***) and bad weather (***M***). The choice of the following branch is a random process. If the Weather Center reveals that it will have a good time, then the branch to be followed leads to a decision vertex that also has two branches: making a big order of 900 scarves (***GR***) with an associated cost of 540€ for the purchase of the scarves; or making a small order of 600 scarves (***PEQ***), that has an associated cost of 450€. The seller must choose one of these branches to continue. After selecting one, he reaches the appropriate branch to an uncertainty vertex that reflects the possible scenarios related to the sale of scarves: a good year when the three clubs claim their orders (***AB***), two of them claim their orders (***AR***), or one of them claiming its order (***AM***). Again, the resulting branch at this point is uncertain, and the final leaf to arrive depends on the outcome of this event. The economic consequences are easily calculated from the unitary selling price data and the number of sold scarves.*

Following the method of working backward, all vertexes and branches of the decision tree are measured. The optimal path of decisions to be taken by the seller is obtained from non-eliminated branches, and the final profit to be achieved is a function of the random events that depend on the consequences of their decisions.

Figure 2 shows the decision tree evaluated for the problem of the scarf seller. It can be appreciated that the policy of the seller, after eliminating the least profitable decision, is:

- *Apply for the sales license.*
- *If finally, September has bad weather, not going to the festivity and renouncing the business.*
- *If the weather is good, make a small order (600 scarves).*

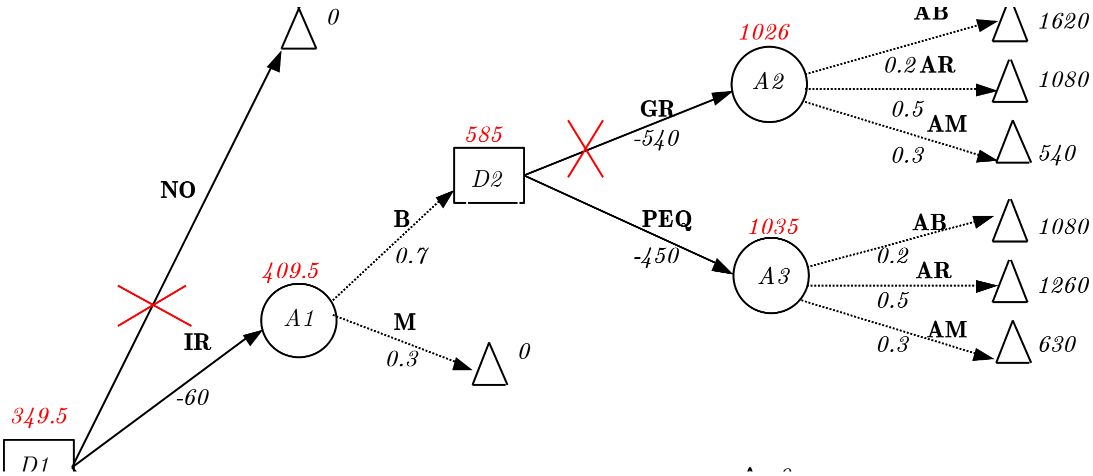


Figure 2

Finally, how much would the seller be willing to pay for having perfect information, or what is the same maximum value that would be reasonable to pay to know the future? It is noticed that perfect information affects two random factors: what the weather will be and how the demand will be during the festivity.

In order to calculate the value of perfect information, it is initially supposed that in September, the weather will be good, and also, how many clubs will buy scarves is known. If the information received is that the demand will be good and all clubs will claim their order, which happens in 20% of the cases, the decision that reports a greater benefit is to make a large order, obtaining a profit of 1020 €. If the information received is that the demand will be regular, which happens with a probability of 0.5, the best decision is to make a small order, which means a profit of 750 €. If the information is that demand will be bad, and only one club will buy scarves, which happens with a probability of 0.3, the optimum solution is to make a small order, which means a profit of 120€. To calculate these benefits, the cost associated with taking the corresponding decision has been taken into account (540 € if it is a large order and 450 € if it is a small order, plus 60 € to pay for the license in both cases). However, good weather during the festivities occurs with a probability of 70%. In the remaining 30%, years with bad weather, *the optimal decision will be to give up the business and not going to the festivity, with a null profit. The following table summarizes these decisions:*

Scenarios				
Time	**Demand**	**Probability**	**Decision**	**Profit**
B	*AB*	*0.7 x 0.2*	*IR+GR*	*1020 €*
B	*AR*	*0.7 x 0.5*	*IR+PEQ*	*750 €*
B	*AM*	*0.7 x 0.3*	*IR+PEQ*	*120 €*
M	-	*0.3*	*NO IR*	*0*

Therefore, the expected profit with perfect information is:

$$GECIP = (1020 \cdot 0.2 + 750 \cdot 0.5 + 120 \cdot 0.3) \cdot 0.7 + 0 \cdot 0.3 = 430.5€$$

It is estimated that the expected benefit with uncertainty is the corresponding value (which coincides in this case with the vertex assessment of the tree):

$$GECI = 349.5€$$

The expected value of perfect information (VEIP) is calculated as the difference between the expected profit with perfect information and the expected profit with uncertainty. Therefore, it will be:

$$VEIP = GECIP - GECI = 430.5 - 349.5 = 81€$$

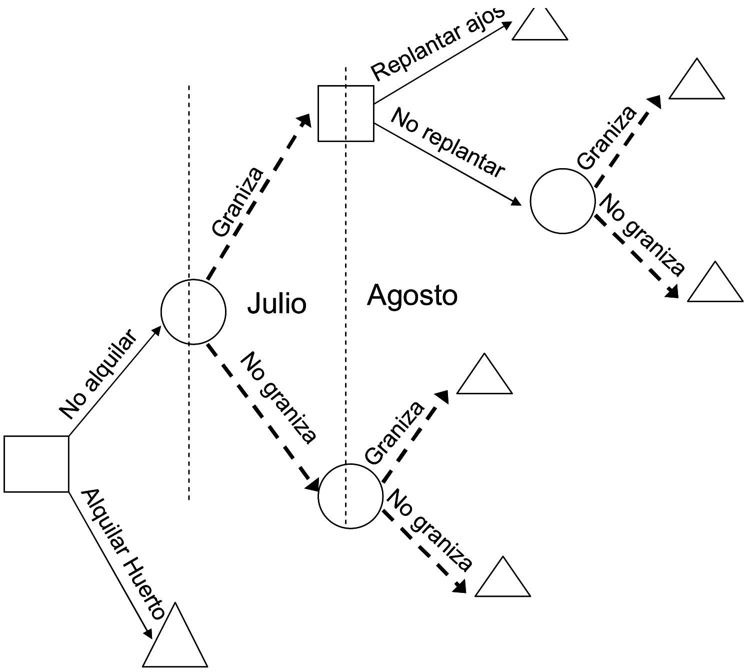
$$VEIP = 81€$$

The value of perfect information can be interpreted as a measure of risk in the seller's decisions. If you are willing to pay a very high price to know the future, your expected benefit with uncertainty is very different from the profit obtained by making the optimal decision in each situation; then their decisions will not be the most beneficial in most of the cases, and the risk of the process is high. On the contrary, if the price you will pay to know the future is not very high, their decisions will involve a smaller risk. In this case, the value of perfect information is 18.82%, which is not very high considering that decisions should always be made under uncertainty. Therefore, the scarf seller's policy does not appear to involve great risk.

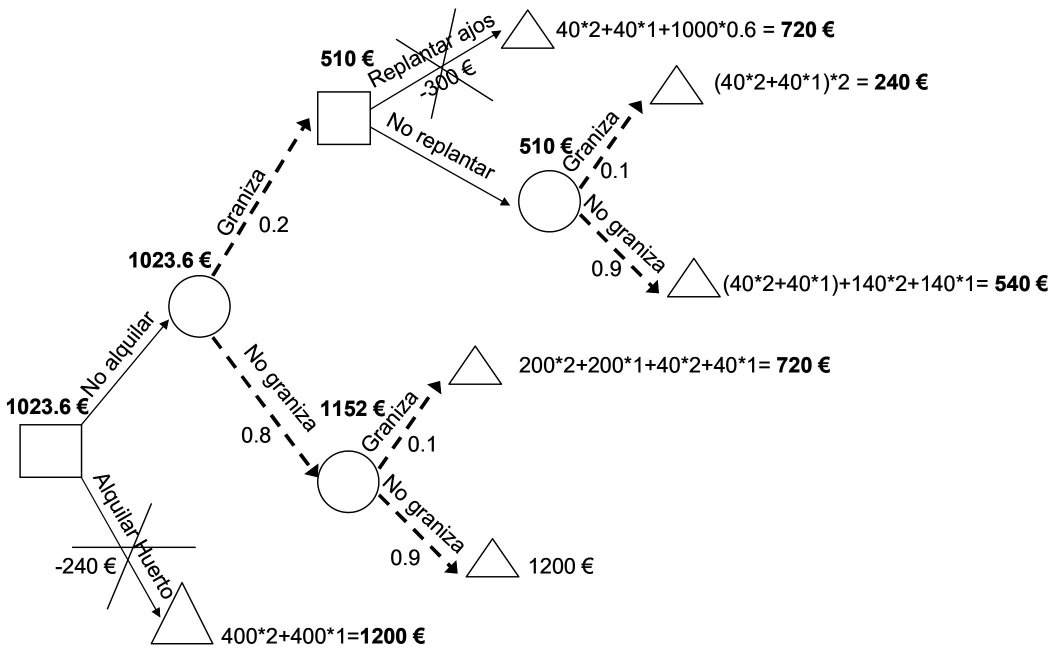
SOLUTION: THE ORCHARD

a) What should Don Tomás's decision be?

The decision tree is:



When assessing tree leaves, the decision policy to be adopted by Don Tomás is set.



Therefore, Don Tomás should not accept his cousin's offer considering the expected obtained value. If it hails in July, he should not replant garlic and continue with the tomatoes and lettuce that have already been planted.

b) What is the value of perfect information?

The expected benefit of the orchard with perfect information is calculated as the sum of the profit from the different scenarios.

Hailstorm in July	Hailstorm in August	Optimal decision	Probability	Profit	Weighted profit
SI	SI	Rent	0.2*0.1=0.02	960 €	19.2 €
SI	NO	Rent	0.2*0.9=0.18	960 €	172.8 €
NO	SI	Rent	0.8*0.1=0.08	960 €	76.8 €
NO	NO	Not to rent	0.8*0.9=0.72	1200 €	864 €
TOTAL					1132.8 €

The expected value of perfect information is the difference between the expected profit with uncertainty and the expected profit with perfect information:
1132.8 € – 1023.6 € = 109.2 €

__SOLUTION: THE PROBING__

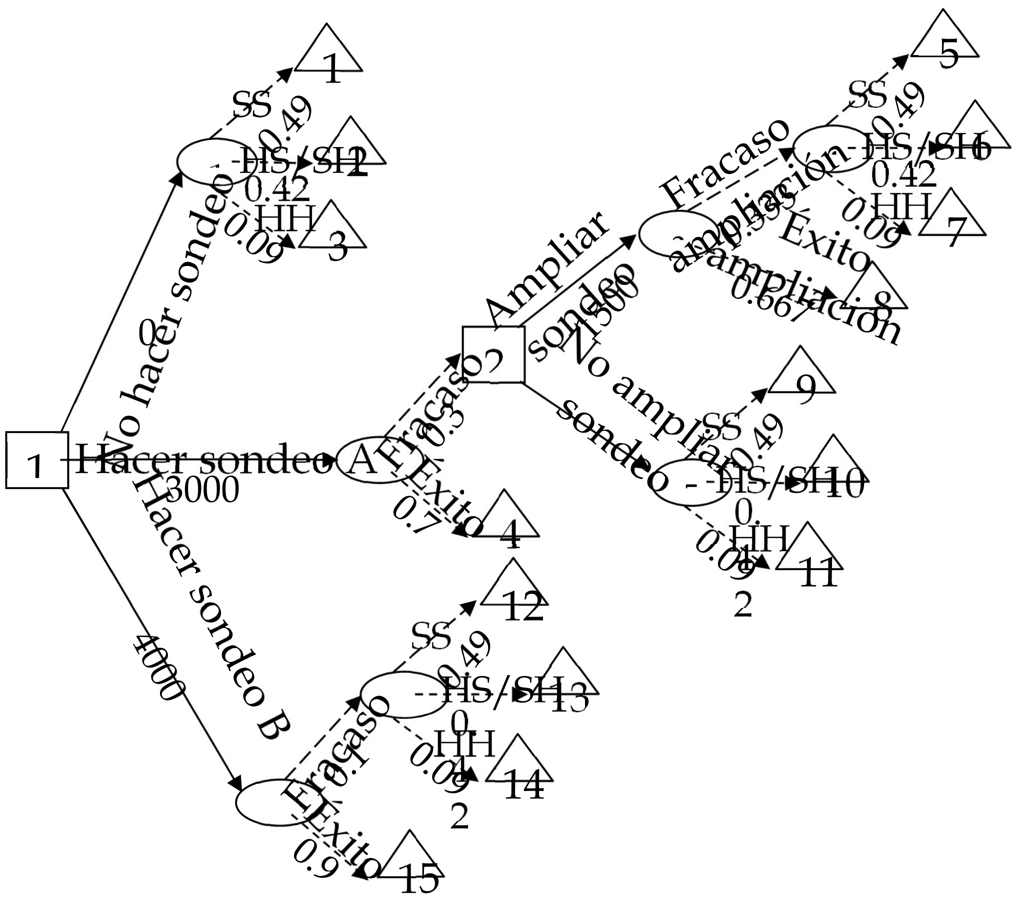
a) Considering that Aunt Severiano wants to make the most profitable decision on average for the following two years, establish the decision tree with its occurrence probabilities.

The notation used to describe all the possible scenarios of drought is:

- *SS: Two consecutively years of drought*
- *SH: The first year is dry, and the second one is wet*
- *HS: The first year is wet, and the second is dry*
- *HH: Both years are wet*

The two decisions that can be taken after deciding to do the probing with company A and have not been successful in finding water are:

- *To expand the probing: Company B continues to dig deeply using the previous probing*
- *Do not expand the probing: The probing made by company A is not extended, although no water is extracted*

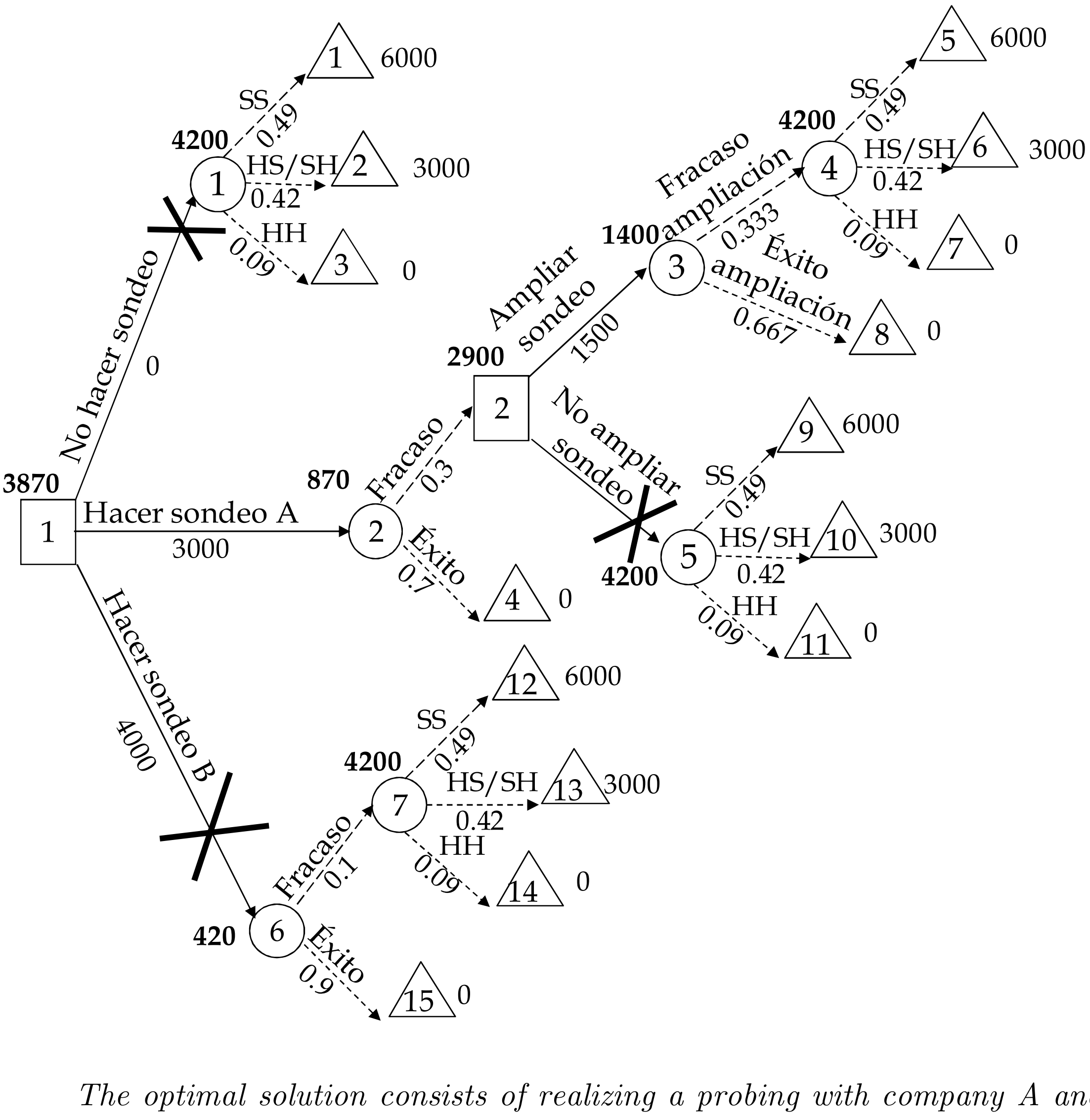


The probability of company B finding water when company A has failed has been obtained as follows, where X is the depth at which water is found, A is the event "company A finds water", and B is "company B finds water":

$$P\left(B/\overline{A}\right)=P\left(X\leq 200/X>75\right)=\frac{P\left(75<X\leq 200\right)}{P\left(X>75\right)}=\frac{P\left(X\leq 200\right)-P\left(X\leq 75\right)}{1-P\left(X\leq 75\right)}=$$

$$=\frac{P\left(B\right)-P\left(A\right)}{1-P\left(A\right)}=\frac{0.9-0.7}{1-0.7}=\frac{2}{3};\quad 0.667$$

b) Assess the tree and determine the optimal decision Aunt Severiano should take.



The optimal solution consists of realizing a probing with company A and extending the probing with company B if no water is found. The expected cost with this sequence of decisions is 3870 €.

c) Romunaldo, Aunt Severiano's nephew, is a well-known water diviner in the zone that boasts of having 100% reliability, indicating whether there is water on the plantation and its depth. Is it reasonable for Romunaldo

to ask Aunt Severiano for two lambs valued at 100 € each for his prediction?

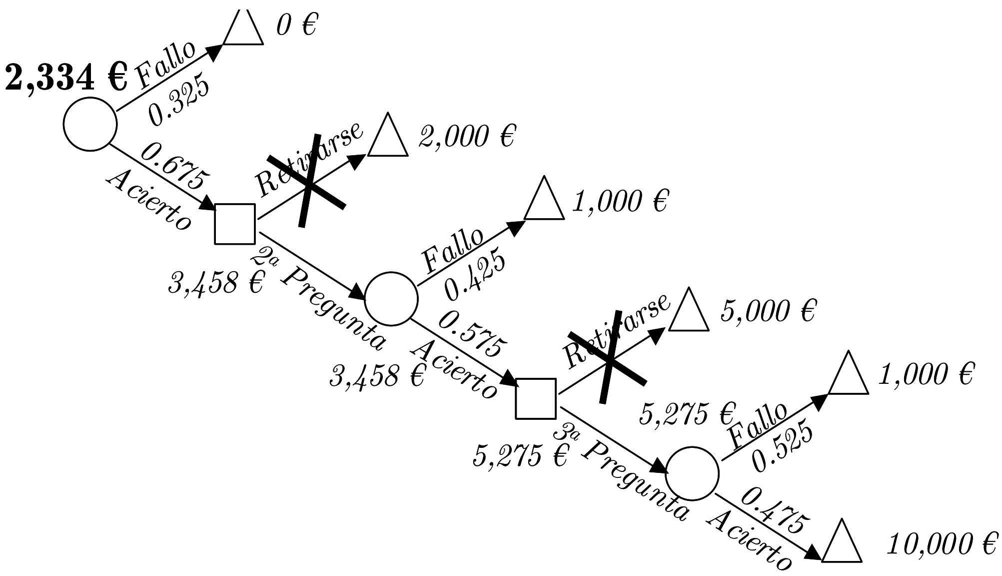
If your nephew is as good as he says, and taking into account the probabilities indicated in the wording. In 70% of cases, water will be found within 75 meters or less; in 20% of cases, water will be above 75 meters and 200 meters; and in the remaining 10%, the probing at that location finds no water, or it is found over 200 meters. Assessing the cost incurred in each of these three possible scenarios and its associated probability by multiplying the cost incurred.

Scenario	Probability	Decision	Cost	Weighted cost
Distance 75 m	*0.7*	*Company A probing*	*3000 €*	*2100 €*
Distance > 75 m and 200 m	*0.2*	*Company B probing*	*4000 €*	*800 €*
Distance > 200 m or without water	*0.1*	*Without probing*	*4200 €*	*420 €*
Total				*3320 €*

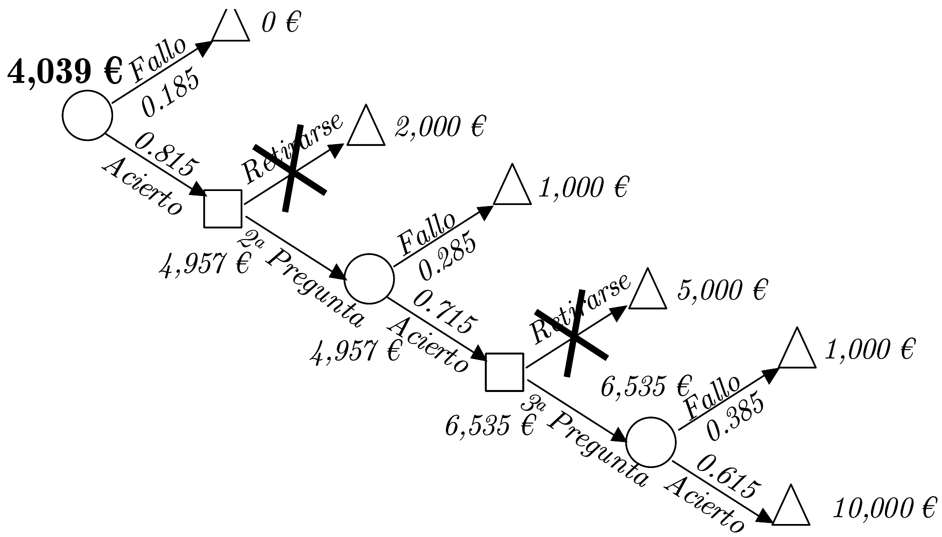
The value of the information provided by Romunaldo is calculated as the difference between the average optimal value of 3870 € and the obtained value with perfect information of 3320 €. Therefore, the value of that information is 550 €. With this value, Aunt Severiano should accept the proposal to give him two lambs; he could also give him eight lambs, and the information about his nephew will still be beneficial. However, it must be considered that the wording assumes that the nephew is always as good a water diviner as he claims to be.

<u>SOLUTION: THE GAME SHOW</u>

a) What is the expected profit on average for Mercurio?



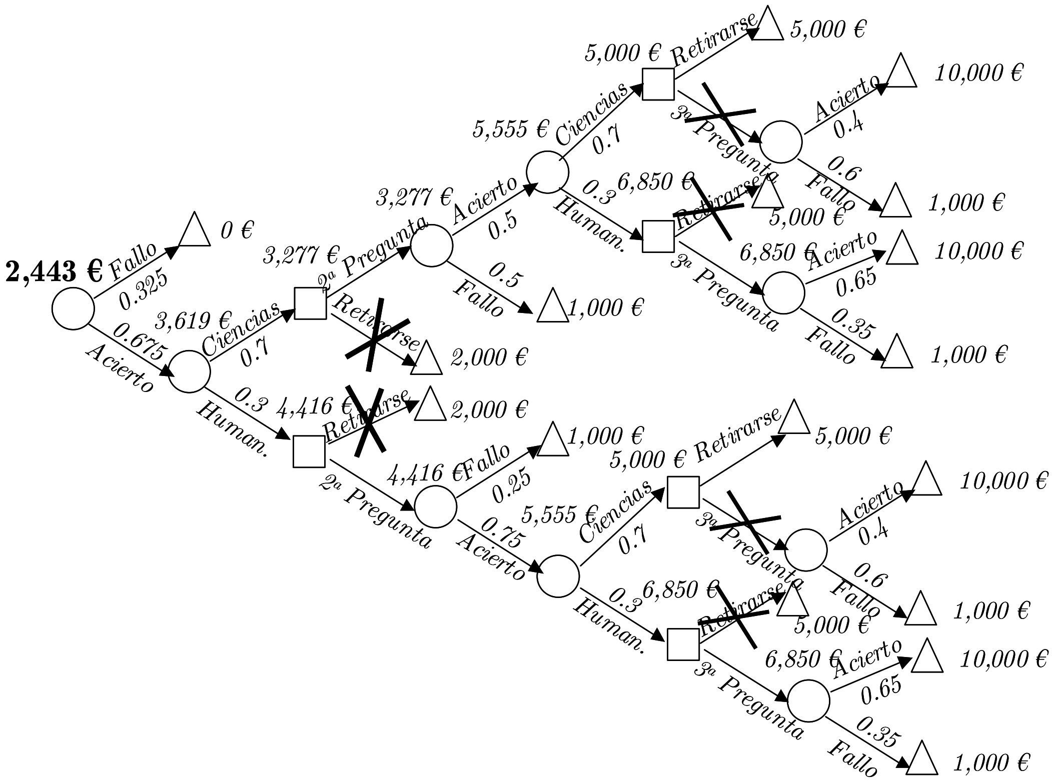
b) After analyzing the table before participating in the game show, Mercurio evaluates the possibility of a speed course on science themes. The course involves some teaching expenditures of 1000 € but increases the percentage of correct answers on Science themes by 0.2 for each of the three possible questions. Is it worth it economically to take the course? How much it is worth?



The value of the increasing profit obtained when the speed course of Science is made is calculated by subtracting the assessment of this tree from the assessment of the previous tree, 4039-2334=1705 €. Therefore, you are interested in taking the course of 1000 € as the net expected profit is 705€.

c) Another option different from the speed course is to cheat. This option is done by honoring his alias and with the help of his cousin, who is part of the

team that produces the game show, Vespasiano López-Pérez, alias Corruptelas II. His cousin will alert him with a sign if the question is about Science or Humanities. In exchange for his immoral help, Vespasiano will collect 1000 €. Apart from the negative ethical judgment that deserves this practice, is it advantageous from an economic point of view? Justify your answer.



The difference between the expected value with the "help" of Vespasiano (2443 Euros) and the expected profit without any help (2334 Euros) is how much this help is valued, 109 Euros compared with 1000 Euros demanded by Vespasiano. Therefore, Vespasiano's help is not economically interesting.

SOLUTION: SCARVES (REVISITED)

Figure 1 shows the decision tree for the problem of the seller of scarves at Villachotas's festivities. In this tree, NO represents the scenario of not going to the festivity, IR represents the scenario of going to the festivity, and TIO represents the scenario of consulting Aunt Emerenciano about how the weather will be in September. On the other hand, **B** *is the possible scenario of having good weather, and* **M is** *the one of having bad weather, while* **PB** *is the possible scenario where Aunt Emerenciano predicts good weather in September, and* **PM** *represents the scenario where he predicts bad weather.* **GR** *symbolizes the alternative of making a big order (900 scarves), and* **PEQ** *is the alternative of making a small order (600 scarves). Finally, a good year when every club claims AB represents its orders, a regular year in which two of the three clubs claim AR represents its* **orders,** *and a bad year in which only one claims AM represents its* **scarves***.*

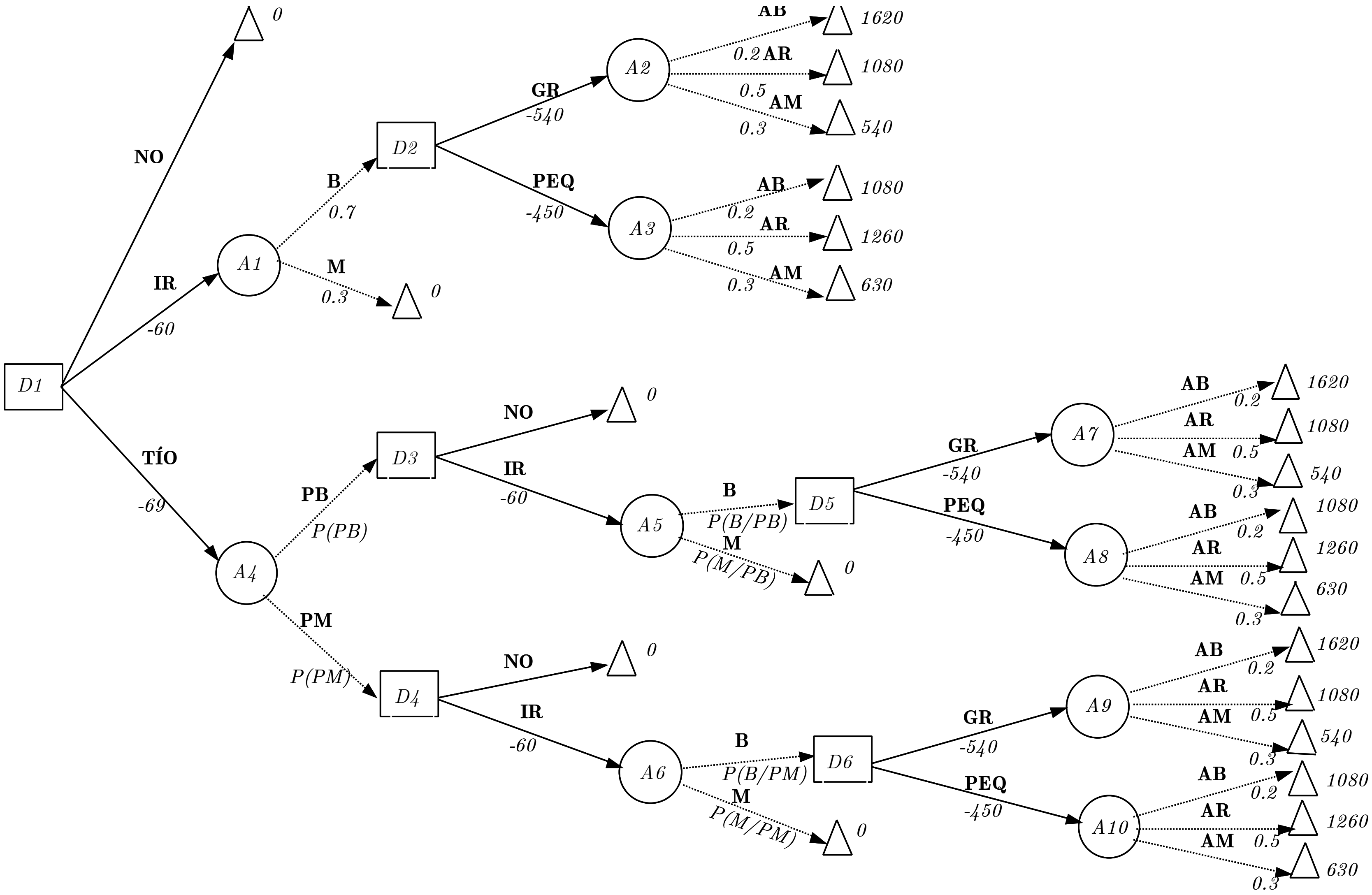

D1
NO
0
IR
-60
A1
B
0.7
M
0.3
0
D2
GR
-540
PEQ
-450
A2
AB
1620
0.2
AR
1080
0.5
AM
540
0.3
A3
AB
1080
0.2
AR
1260
0.5
AM
630
0.3
TÍO
-69
A4
PB
P(PB)
PM
P(PM)
D3
NO
0
IR
-60
A5
B
P(B/PB)
M
P(M/PB)
0
D5
GR
-540
PEQ
-450
A7
AB
1620
0.2
AR
1080
0.5
AM
540
0.3
A8
AB
1080
0.2
AR
1260
0.5
AM
630
0.3
D4
NO
0
IR
-60
A6
B
P(B/PM)
M
P(M/PM)
0
D6
GR
-540
PEQ
-450
A9
AB
1620
0.2
AR
1080
0.5
AM
540
0.3
A10
AB
1080
0.2
AR
1260
0.5
AM
630
0.3

Figura 1

*At first, the seller has to choose between not going to the festivity, going to the festivity, or consulting Aunt Emerenciano about how the weather will be during Villachotas's festivities. The decision not to go to the festivity leads to an ending vertex with an economic consequence of zero, as the seller rejects the business. Any of the other two actions have consequences. If he decides to go to the festivity without predicting the weather, he will arrive by the appropriate branch to an uncertain vertex. Going through this branch has an associated cost of 60€ as the seller has to pay the license. The two branches of the uncertainty vertex are good weather (***B***) and bad weather (***M***). The choice of the following branch is a random process. If the Weather Center reveals that it will have a good time, then the branch to be followed leads to a decision vertex that also has two branches: making a big order of 900 scarves (***GR***) with an associated cost of 540€ for the purchase of the scarves; or making a small order of 600 scarves (***PEQ***), that has an associated cost of 450€. The seller must choose one of these branches to continue. After selecting one, he reaches the appropriate branch to an uncertainty vertex that reflects the possible scenarios related to the sale of scarves: a good year when the three clubs claim their orders (***AB***), two of them claim their orders (***AR***), or one of them claiming its order (***AM***). Again, the resulting branch at this point is uncertain, and the final leave to arrive depends on the outcome of this event. The economic consequences are easily calculated from the unitary selling price data and the number of sold scarves.*

If the seller's initial decision is to consult about the weather during Villachotas's festivities with Aunt Emerenciano, an additional stage will be added to the process described above. The fact of taking this decision has an associated cost of 69€, payment that Uncle Emerenciano demands for the information, and that leads the seller to an uncertainty vertex with two branches named as PB (Aunt Emerenciano's prediction of good weather for September) and PM (Aunt Emerenciano's prediction of bad weather for September). The election of the following branch is a random event that, in any case, leads to a decision vertex with two alternatives: going or not to the festivity. From this point, the tree continues to build analogously.

It can be observed that the uncertainty branches carry the probability of occurrence of each scenario. All probabilities associated with uncertainty branches resulting from deciding to go to the festivity without any consultation about the weather are simple probabilities of occurrence not conditioned by any previous event as there is no additional information. Therefore, its values are obtained directly from the wording information. As the wording tells that seven out of ten years there is good weather, we know the probabilities of good and bad weather are, respectively:

$$P(B) = 0.7$$
$$P(M) = 0.3$$

The probabilities that the demand for scarves is good (three clubs buy scarves), regular (two clubs buy scarves), and bad (one club buys scarves) are, respectively:

$$P(AB) = 0.2$$
$$P(AR) = 0.5$$
$$P(AM) = 0.3$$

However, the alternative of consulting Aunt Emerenciano involves a case of partial information that allows for an increase, with additional costs, and partially, the available information about the weather in September. Thanks to Bayesian analysis, which is based on the Bayes theorem and total probability theorem, we can incorporate the partial information that Aunt Emerenciano offers about the weather in September. It is, therefore, a matter of updating probabilities of the occurrence of good and bad weather due to the newly obtained information.

It is said that Uncle Emerenciano can predict the weather in September in January with 80% success. Therefore, the conditional probabilities of the weather prediction are:

$$P\left(\frac{PB}{B}\right) = 0.8 \quad P\left(\frac{PM}{M}\right) = 0.8$$
$$P\left(\frac{PB}{M}\right) = 0.2 \quad P\left(\frac{PM}{B}\right) = 0.2$$

*The probabilities associated with uncertainty branches represent the possible scenarios of a good weather prediction (**PB**) and a bad weather prediction (**PM**). These probabilities are calculated from its conditional probability with complementary scenarios **B** (good weather) and **M** (bad weather) by the theorem of total probability:*

$$P(PB) = P\left(\frac{PB}{B}\right) \cdot P(B) + P\left(\frac{PB}{M}\right) \cdot P(M) = 0.8 \cdot 0.7 + 0.2 \cdot 0.3 \rightarrow P(PB) = 0.62$$

$$P(PM) = P\left(\frac{PM}{M}\right) \cdot P(M) + P\left(\frac{PM}{B}\right) \cdot P(B) = 0.8 \cdot 0.3 + 0.2 \cdot 0.7 \rightarrow P(PM) = 0.38$$

The probabilities associated with the arcs that represent the random events of good weather and bad weather are conditional probabilities to the prediction on Christmas of how the weather would be in each case. The Bayes theorem calculates these conditional probabilities:

$$P\left(\frac{B}{PB}\right) = \frac{P\left(\frac{PB}{B}\right) \cdot P(B)}{P(PB)} = \frac{0.8 \cdot 0.7}{0.62} \rightarrow P\left(\frac{B}{PB}\right) = 0.9032$$

$$P\left(\frac{M}{PB}\right) = \frac{P\left(\frac{PB}{M}\right) \cdot P(M)}{P(PB)} = \frac{0.2 \cdot 0.3}{0.62} \rightarrow P\left(\frac{M}{PB}\right) = 0.0968$$

$$P\left(\frac{B}{PM}\right)=\frac{P\left(\frac{PM}{B}\right)\cdot P(B)}{P(PM)}=\frac{0.2\cdot 0.7}{0.38}\rightarrow P\left(\frac{B}{PM}\right)=0.3684$$

$$P\left(\frac{M}{PM}\right)=\frac{P\left(\frac{PM}{M}\right)\cdot P(M)}{P(PM)}=\frac{0.8\cdot 0.3}{0.38}\rightarrow P\left(\frac{M}{PM}\right)=0.6316$$

*The tree's vertex is assessed from back to front, using the Laplace chance criterion for the uncertainty vertex. For example, the expected average earning of vertexes of chance A2 and A*3 are:

$$1620\cdot 0.2+1080\cdot 0.5+540\cdot 0.3=1026\,€\ para\ A2$$

$$1080\cdot 0.2+1260\cdot 0.5+630\cdot 0.3=1035\,€\ para\ A3$$

By continuing backward, we find the next vertex, A2, and A3 is a decision vertex. The earnings associated with this vertex are the maximum earnings on all the branches that come out of it, considering the cost associated with decision branches (cost of taking that decision), in case there is such cost. For example, in the case of vertex D2, we have:

$$1026-540=486\,€\ para\ GR$$

$$1035-450=585\,€\ para\ PEQ$$

585€ is the maximum between 486€ and 585€, corresponding to making a small order of scarves. This action is the best one when the seller is on that vertex. Therefore, the other branch is crossed out to indicate that the scenario of making a big order in that path is eliminated.

Following this method of working backward, all vertexes and branches of the decision tree are measured. The optimal patch of decisions to be taken by the seller is obtained from non-eliminated branches, and the final profit to be achieved is a function of the random events that depend on the consequences of their decisions.

Figure 2 shows the decision tree evaluated for the problem of the scarf seller. It can be appreciated that the policy of the seller, after eliminating the least profitable decision, is:

- *Apply for the sales license without consulting Aunt Emerenciano about the weather in September.*
- *If finally, September has bad weather, not going to the festivity and renouncing the business.*
- *If the weather is good, make a small order (600 scarves).*

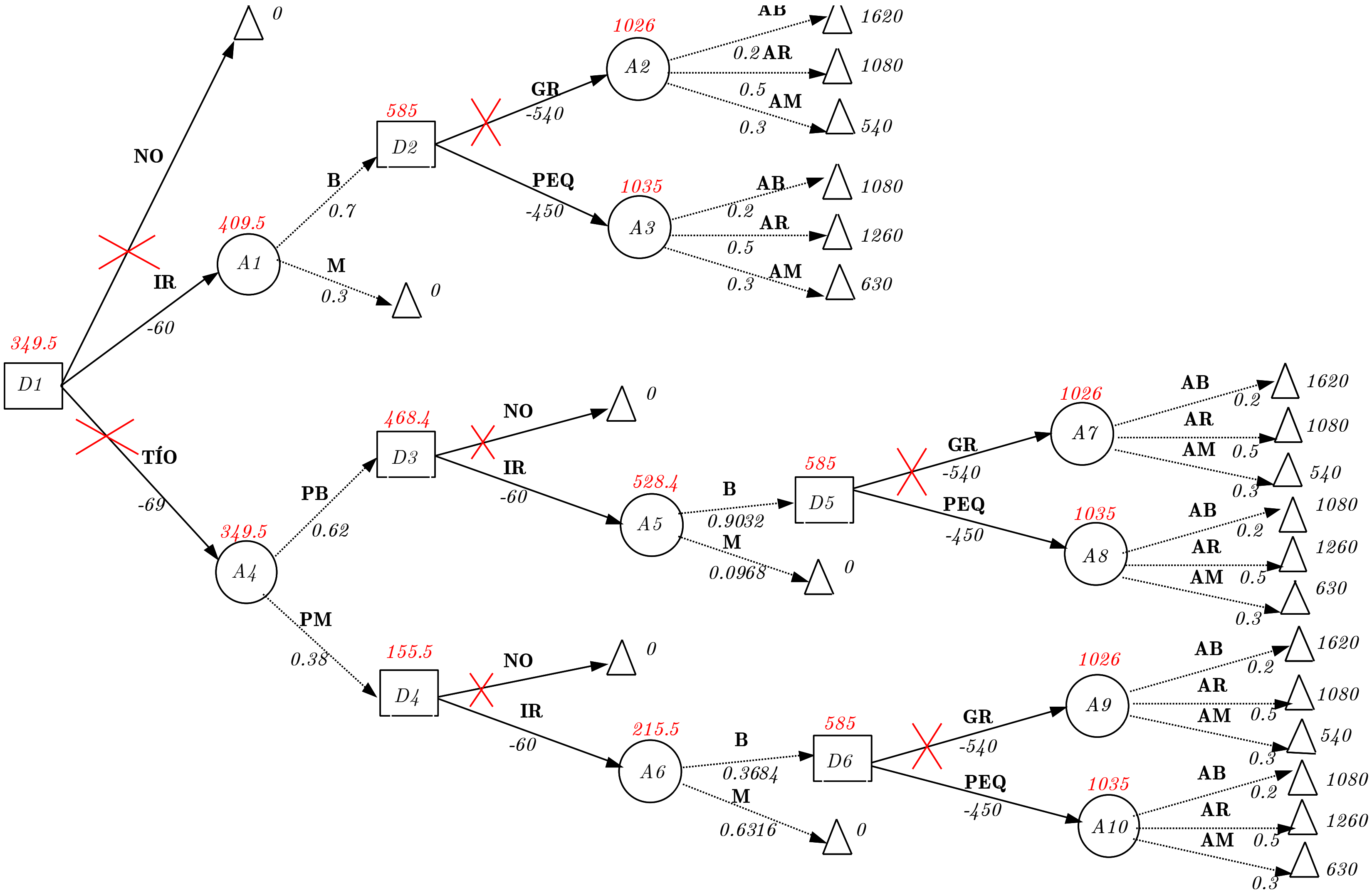

349.5
D1
NO
0
IR
-60
409.5
A1
B
0.7
M
0.3
0
585
D2
GR
-540
PEQ
-450
1026
A2
AB
0.2
AR
0.5
AM
0.3
1620
1080
540
1035
A3
AB
0.2
AR
0.5
AM
0.3
1080
1260
630
TÍO
-69
349.5
A4
PB
0.62
PM
0.38
468.4
D3
NO
0
IR
-60
528.4
A5
B
0.9032
M
0.0968
0
585
D5
GR
-540
PEQ
-450
1026
A7
1620
1080
540
1035
A8
1080
1260
630
155.5
D4
NO
0
IR
-60
215.5
A6
B
0.3684
M
0.6316
0
585
D6
GR
-540
PEQ
-450
1026
A9
1620
1080
540
1035
A10
1080
1260
630

Figura 2

The value of the partial information that Aunt Emerenciano provides is calculated as the difference between the expected earnings with uncertainty (value of the vertex A1 minus 60€ of the City Council license) and the expected earnings with the partial information without taking into account the payment for the prediction, vertex A4. This difference is null in this problem and, therefore, is not taken. It is noticed that the partial information has no value in this problem because regardless of the prediction, the decision to take is the same as the one that would be brought under uncertainty. Therefore, this information does not provide added value that allows a change in the decision.

Finally, he wondered how much the seller would be willing to pay for having perfect information or what the same maximum value would be reasonable to pay to know the future. It is noticed that perfect information, unlike Aunt Emerenciano's forecast, affects two random factors: what the weather will be and how the demand will be during the festivity.

To calculate the value of perfect information, it is initially supposed that the weather will be good in September, and also, how many clubs will buy scarves is known. If the information received is that the demand will be good and all clubs will claim their order, which happens in 20% of the cases, the decision that reports a greater benefit is to make a large order, obtaining a profit of 1020 €. If the information received is that the demand will be regular, which happens with a probability of 0.5, the best decision is to make a small order, which means a profit of 750 €. If the information is that demand will be bad, and only one club will buy scarves, which happens with a probability of 0.3, the optimum solution is to make a small order, which means a profit of 120€. To calculate these benefits, the cost associated with taking the corresponding decision has been taken into account (540 € if it is a large order and 450 € if it is a small order, plus 60 € to pay for the license in both cases). However, good weather during the festivities occurs with a probability of 70%. In the remaining 30%, years with bad weather, the optimal decision will be to give up the business and not go to the festivity, with a null profit. The following table summarizes these decisions:

Scenarios				
Time	**Demand**	**Probability**	**Decision**	**Profit**
B	*AB*	*0.7 x 0.2*	*IR+GR*	*1020 €*
B	*AR*	*0.7 x 0.5*	*IR+PEQ*	*750 €*
B	*AM*	*0.7 x 0.3*	*IR+PEQ*	*120 €*
M	-	*0.3*	*NO IR*	*0*

Therefore, the expected profit with perfect information is:

$$GECIP = (1020 \cdot 0.2 + 750 \cdot 0.5 + 120 \cdot 0.3) \cdot 0.7 + 0 \cdot 0.3 = 430.5\,€$$

It is estimated that the expected benefit with uncertainty is the corresponding value of not taking the possibility of Aunt Emerenciano's prediction (which coincides in this case with the vertex assessment of the tree):

$$GECI = 349.5\,€$$

The expected value of perfect information (VEIP) is calculated as the difference between the expected profit with perfect information and the expected profit with uncertainty. Therefore, it will be:

$$VEIP = GECIP - GECI = 430.5 - 349.5 = 81\,€$$

$$VEIP = 81\,€$$

The value of perfect information can be interpreted as a measure of risk in the seller's decisions. Suppose you are willing to pay a very high price to know the future. In that case, your expected benefit with uncertainty differs greatly from the profit obtained by making the optimal decision in each situation. Their decisions will not be the most beneficial in most cases, and the risk of the process is high. On the contrary, if the price you will pay to know the future is not very high, their decisions will involve a smaller risk. In this case, the value of perfect information is 18.82%, which is not very high considering that decisions should always be made under uncertainty. Therefore, the scarf seller's policy does not involve great risk.