OPTIMIZATION MODELING PROJECTS

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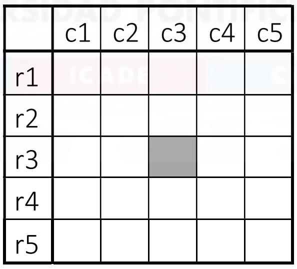
Cell Towers

A long time ago, in a galaxy far, far away… A telecom company plans to install cell towers for wireless communication in a new neighborhood. The cell towers they install can include two types of antennas, A and B. Each cell tower has room for 10 antennas of type A. Antennas of type B are 10% bigger than antennas of type A. Each cell tower can have both antennas if there is room for them. Each antenna of type A supplies 50 Gbps, and each antenna of type B supplies 40 Gbps. Installing a cell tower costs 100 crypto coins, each antenna of type A costs 15 crypto coins, and each antenna of type B costs 10 crypto coins.
The company has divided the neighborhood into a grid of 5x5 areas, each shown here with their corresponding expected demand in Gbps:

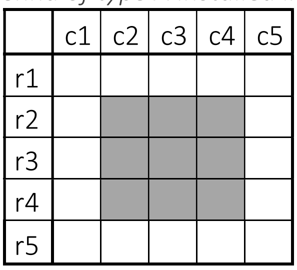
	c1	c2	c3	c4	c5
r1	391	370	363	400	358
r2	313	207	333	250	303
r3	328	308	367	357	332
r4	366	205	245	243	337
r5	393	216	323	356	268

Demand in each area (Gbps)

The company must decide in which areas to install cell towers. At most, they will consider installing one cell tower per area. Furthermore, they must decide how many antennas of each type to include in each installed cell tower. Antennas of type A can only reach customers in the same area, while antennas of type B can reach customers on their own and in neighboring areas. For example, these are the areas reached by antennas included in a cell tower installed in the area (r3, c3):



Reach of an antenna of type A installed in (r3, c3) in grey



Reach of an antenna of type B installed in (r3, c3) in grey

They want to be sure to supply at least all the demand. To achieve this, it is important to check that the broadband provided by antennas is not overused. It is not necessary to check what demand is supplied by each antenna. Still, they need to check what demand is provided by each type of antenna included in each cell tower, as the antennas have different ranges. Consider the following illustrative example with 9 areas, with a cell tower installed in the area (r2, c2) with 6 antennas of type A and 3 antennas of type B:

	c1	c2	c3
r1			
r2		cell tower 6 antennas A 3 antennas B	
r3			

Example

The 6 antennas of type A provide 6*50=300 Gbps, while the 3 antennas of type B provide 3*40=120 Gbps. The 6 antennas of type A can only supply demand in the same area (r2, c2). The broadband provided by the 3 antennas of type B can be distributed among the 9 areas within their reach in any way. For example, the 3 antennas of type B could supply the following demand: 10 Gbps in area (r1, c1), 20 Gbps in area (r1, c2), 15 Gbps in area (r1, c3), 10 Gbps in area (r2, c1), 10 Gbps in area (r2, c2), 25 Gbps in area (r2, c3), 10 Gbps in area (r3, c1), 0 Gbps in area (r3, c2), and 20 Gbps in area (r3, c3), for a total of 120 Gbps supplied by type B antennas.

a. Formulate a mixed integer linear model to help this telecom company decide where to install cell towers and what antennas to use in each cell tower to supply all the demand at minimum cost.
b. Modify the model from section a. to consider that only one type of antenna can be installed in each cell tower. Compare the results with those from section a.
c. Modify the model from section a. to consider that, due to some protests from neighbors, the company has accepted the following condition. If they install 4 or more cell towers in column 5, they must install at least 4 in all the other columns. Compare the results with those from section a.

Spartan Race

Comillas University has been hired to define the sequence of obstacles to set up a spartan race. The available obstacles are listed in this table.

	Obstacle	Type	Difficulty
I	Inverted wall	Climbing	2
R	Rope climbing	Climbing	4
S	Sandbag	Heavy carries	1
A	Atlas stone	Heavy carries	5
L	Low crawl	Crawling	3
B	Box jumps	Cardio	2
F	Fire jump	Skill	0

The sequence conditions are the following:

- The sequence must cover all the obstacles only once.
- The starting obstacle is A, the atlas stone.

- After any obstacle, the following one can't have the same difficulty
- The two heavy carries-type obstacles can't be consecutive

Determine the optimal sequence that satisfies all the previous conditions and maximizes the total race distance. The distance among obstacles is proportional to the distance between their letters (e.g., distance between the atlas stone (A) and box jumps (B) is 1).

Green Hydrogen Distribution for Europe's Energy Transition

In 2024, due to the growing pressure to reduce carbon emissions, several European countries have adopted green hydrogen as a sustainable alternative to fossil fuels. However, the production and distribution of green hydrogen remain challenging due to limited infrastructure, high transport costs, and the remote locations of renewable energy production plants.

The European Sustainable Energy Agency (ESEA) aims to optimize the distribution of green hydrogen from three production centers to various distribution points across Europe to minimize transportation, storage, and conversion costs while ensuring that demand at different industrial centers is met.

Context:

- Green hydrogen production centers: Production plants are located in three regions (North Africa, Norway, and Greece), where solar, wind and geothermal energy efficiently produce hydrogen.
- Distribution points: European countries have designated five main distribution centers in Germany, France, Spain, Italy, and Poland, where the hydrogen will be stored before being sent to local industries.

Decisions to be made:

1. How to distribute the hydrogen production (with a maximum of 5 million liters per month) from the plants to the different distribution points.
2. How to minimize transportation, storage, and conversion costs while ensuring that demand is met at each distribution center, according to the energy requirements of each country.

Data:

- Hydrogen demand (in million liters) at the different distribution points is as follows:

Distribution Points	Germany	France	Spain	Italy	Poland
Demand (Mliter)	600	500	400	300	200

- Transport costs from each production center to the distribution points (€/liter of hydrogen):

€/liter of H2	Germany	France	Spain	Italy	Poland
North Africa	0.7	0.6	0.5	0.8	0.9
Norway	0.3	0.4	0.6	0.7	0.2
Greece	0.5	0.3	0.6	0.4	0.5

- Storage and conversion costs at distribution points (€/liter):

€/liter of H2	Germany	France	Spain	Italy	Poland
Storage	0.1	0.2	0.15	0.2	0.25
Conversion	0.05	0.04	0.03	0.06	0.05

Additional Conditions:

1. **Energy conversion**: For every 4 liters of hydrogen produced, only 1 liter is effectively stored and utilized due to energy losses in the transport and conversion process.
2. **Fixed operational costs for production plants**:

Fixed Costs (k€)	North Africa	Norway	Greece
Operational Costs	800	600	700

3. **Maximum transport capacity**: Hydrogen plants can only ship a maximum of 3 million liters per month to each distribution point.

Objective:

Help the ESEA design a transportation and distribution plan for green hydrogen that:

1. **Minimizes total costs**, including transportation, storage, conversion, and production plant operation.
2. **Ensures that hydrogen demand** is met at each distribution point efficiently.

Ticket Distribution

The President of Real Mandril, Perentino Flórez, has devised a strategy to increase members' use of their season tickets, with the dual objective of filling the stadium more and extending the members' support to their families and friends.

This strategy consists of allowing an unspecified number of members with adjacent seats to share their seats so that each can bring some of their family or friends to specific matches using the seats of their neighboring members.

To ensure the attractiveness of the plan, Perentino wants a ticket distribution system that offers the following possibilities:

1. Each member can choose the number of matches they want, such as 1 ticket, 2 tickets, 3 tickets, etc., as long as the total does not exceed 19 (there are 19 matches).
2. Each member can choose one or more matches they want to attend.
3. Each member must evaluate the importance of each of the 19 matches (1: highest interest, 19: lowest interest) so that the system's goal is to minimize the overall dissatisfaction of the group of members involved.
4. It must be guaranteed that each member attends at least one match every two months.

For example, consider the group of members formed by Santi, Fernandito, Toño, Salva, Iñigo, and Mike. The following table shows the ratings these members give to each of the nineteen matches:

	Santi	Fernandito	Toño	Salva	Iñigo	Mike
1	7	8	9	11	9	11
2	15	14	16	16	12	14
3	8	15	15	12	13	15
4	14	13	17	15	16	16
5	10	7	14	9	2	12
6	16	12	8	13	14	13
7	13	16	18	19	19	17
8	3	2	4	2	4	1
9	11	5	6	6	8	4
10	4	4	5	5	7	7
11	17	19	10	18	15	18
12	18	18	19	17	18	19

13	2	3	3	3	3	3
14	12	11	13	10	10	8
15	1	1	1	1	1	2
16	6	9	7	4	5	10
17	19	17	11	14	17	6
18	5	10	12	8	6	9
19	9	6	2	7	11	5

The following table indicates the number of matches each member wants to attend with the specified number of tickets:

	Santi	Fernandito	Toño	Salva	Iñigo	Mike
1 ticket	9	3	3	1	17	5
2 tickets	5	8	4	7	1	7
4 tickets	0	0	2	1	0	0

The following table indicates the date of each match to ensure that each member attends the stadium at least once every two months:

Match	Date	Match	Date
1	01-sep	11	23-feb
2	22-sep	12	09-mar
3	06-oct	13	23-mar
4	27-oct	14	06-apr
5	17-nov	15	20-apr
6	01-dec	16	04-may
7	15-dec	17	18-may
8	05-jan	18	01-jun
9	19-jan	19	22-jun
10	02-feb		

Additionally:

- All members want to attend match No. 15 (Mandril vs Parchelona)
- Fernandito does not want to go to the stadium in January.

Formulate and solve the optimization problem that arises from this illustrative example of Perentino’s strategy.

Additionals

- How would the problem change if the dissatisfaction of the most dissatisfied member has to be minimized?

How would the problem change if an additional match of old glories could be considered? In this case, each seat reduces by 3 points the dissatisfaction, and up to 5 seats can be assigned to a member (and, consequently, members will not attend the tribute match).

Bike Sharing Services

A medium-sized city has decided to offer its residents a bike-sharing service, like BiciMAD. The aim is to provide an eco-friendly transportation alternative, especially during peak commuting hours. To better understand the demand and to ensure optimal placement of bike stations, the mayor hired a market analyst. After a thorough analysis, the analyst estimated the potential morning and evening departures at various locations across the city.

The city is divided into an 8x8 grid, representing key locations, including both suburban areas and the city center. The demand is primarily influenced by work-related commuting patterns:

- There is a significant number of bike rides starting in suburban areas (the outer edges of the grid) as people commute to work in the city center. The demand peaks near the center, where most office buildings and business districts are located.
- In the evening, the trend reverses. Workers return home, leading to high demand for bikes in the city center, while suburbs experience an influx of arrivals.

Here is the estimated potential demand for morning commutes:

40	36	31	29	20	25	32	37
35	30	25	22	16	20	27	32
33	28	20	17	14	15	24	29
27	24	17	13	5	13	17	24
24	15	10	5	3	5	14	17
27	22	19	10	6	10	15	24
30	29	20	19	11	17	22	28
39	34	27	20	15	21	25	31

And evening commutes:

2	9	10	18	24	18	11	5
5	14	19	21	28	22	16	13
10	17	24	27	31	26	24	16
18	24	26	33	39	33	26	21
21	27	34	38	41	37	33	28
19	20	26	32	36	32	25	21
11	16	21	26	33	27	24	17
8	12	17	21	27	23	15	10

While the potential demand is clear, the city faces several logistical and financial challenges. The cost of setting up a bike dock at any grid square is 2000€. Each dock has a maximum capacity of 10 bikes, ensuring that the dock can handle peak periods of demand. However, not all demands may be met. The city estimates that each completed ride will generate €15 in profit. The city has a total budget of €50,000 allocated for the setup of the bike docks. The profits from the rides cannot be used for installing new docks.
The mayor has added clauses to the contract to ensure high-quality service. Up to three docks can be installed at any given location. However, docks cannot be placed in three consecutive locations in either a row or a column. For example, stations can be installed in a 2x2 square, but not in a 1x3 or 3x1 configuration.
Additionally, the population in areas where the sum of x and y coordinates is less than 8 (with x numbered left to right and y numbered bottom to top) is economically disadvantaged and has expressed concerns about being excluded from new services. In response, and to avoid risking the next election, the mayor has mandated that one out of every four stations must be placed in the poorest neighborhoods.
This set of constraints forces the operations research team of any interested company to carefully decide where to place the docks to maximize the benefits, disregarding the origin and destination of each ride. That is, the number of positions in each dock must be sufficient to cover either the morning or the evening demand.

Fuel Procurement Strategy for SpacePP

Company Background

SpacePP is a company specializing in launching satellites into space. To enhance operational efficiency and reduce costs, SpacePP is developing a comprehensive fuel procurement strategy for its rockets. The goal is to minimize the total cost associated with purchasing and transporting fuel while meeting all operational requirements and environmental regulations.

Problem Statement

SpacePP needs to determine the optimal quantities of rocket fuel to purchase from various suppliers. The decision must consider multiple factors, including fuel properties, supplier characteristics, operational needs, and environmental considerations.

Objective

Develop a strategy to determine the quantities of fuel to purchase from each supplier in a manner that minimizes the total cost while satisfying all operational and environmental requirements.

Requirements and Associated Data

Meet the Energy Requirement

Constraint: The total energy provided by the purchased fuel must meet or exceed the rocket's required energy for the mission, including the additional energy needed to carry the fuel itself.

Data:

Required Energy (A): 1,000,000 MJ.

Energy Transportation Factor (H): 5 MJ/kg.

Fuel Properties:

Table 1: Fuel Properties per Supplier

Supplier (sup)	Energy Capacity A_i (MJ/kg)	Density C_i (kg/L)	Emissions E_i (kg CO_2/kg)
sup1 (Alpha Fuels)	42	0.80	2.5
sup2 (Beta Energies)	40	0.85	2.8
sup3 (Gamma Propellants)	44	0.78	2.3
sup4 (Delta Combustibles)	41	0.82	2.6
sup5 (Epsilon Resources)	43	0.80	2.4

Respect Fuel Tank Capacity

Constraint: The total volume of the purchased fuel must not exceed the rocket's fuel tank capacity.

Data:

- Maximum Fuel Tank Capacity (C): 1,500,000 L.
- Fuel Properties: (Refer to the Fuel Properties table above for density C_i.)

Stay Within Supplier Capacities

Constraint: The quantity purchased from each supplier must not exceed their maximum supply capacity.

Data:

- Supplier Capacities (S_i):

Table 2: Supplier Capacities

Supplier (sup)	Supply Capacity S_i (kg)
sup1 (Alpha Fuels)	5,000
sup2 (Beta Energies)	7,000
sup3 (Gamma Propellants)	6,000
sup4 (Delta Combustibles)	8,000
sup5 (Epsilon Resources)	5,500

Manage Ordering Costs

Constraint: Fixed ordering costs should be considered when deciding whether to purchase from a supplier.

Data:

- **Fixed Ordering Costs (F_i):**

Table 3: Fixed Ordering Costs

Supplier (sup)	Fixed Ordering Cost F_i ($)
sup1 (Alpha Fuels)	1,000
sup2 (Beta Energies)	1,200
sup3 (Gamma Propellants)	1,100
sup4 (Delta Combustibles)	1,300

sup5 (Epsilon Resources)	1,150

Utilize Fixed Discounts Appropriately

Constraints:

- Fixed discounts are applied as a constant amount in USD if an order is placed with a supplier.
- At most two fixed discounts can be applied among all suppliers.

Data:

- Fixed Discounts (D_i):

Table 4: Fixed Discounts

Supplier (sup)	Fixed Discount D_i ($)
sup1 (Alpha Fuels)	500
sup2 (Beta Energies)	400
sup3 (Gamma Propellants)	600
sup4 (Delta Combustibles)	550
sup5 (Epsilon Resources)	450

Comply with Environmental Regulations

Constraint: The total emissions resulting from the purchased fuel must not exceed the maximum allowable emissions.

Data:

- **Maximum Allowable Emissions (E_{max}):** 250,000 kg CO_2.
- **Fuel Properties:** (Refer to the Fuel Properties table above for emissions E_i.)

Avoid Fuel Incompatibilities

Constraint: Incompatible fuels cannot be used together. Only fuels from compatible types or within the same compatibility group can be used in combination.

Data:

- Fuel Incompatibility Indicator (M_{ik}):

Table 5: Fuel Incompatibility Indicator M_{ik}

Supplier (sup)	Type 1 (*Cryogenic*)	Type 2 (*Hypergolic*)
sup1 (Alpha Fuels)	0	0
sup2 (Beta Energies)	1	0
sup3 (Gamma Propellants)	0	0
sup4 (Delta Combustibles)	0	0
sup5 (Epsilon Resources)	0	0