OPTIMIZATION MODELING PROJECTS

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1. Souvenir Superstore Strategy

Imagine you're an aspiring entrepreneur planning to launch a souvenir shop in the heart of a bustling tourist city. Your success hinges on smart staffing decisions, demand dynamics, and the ever-evolving world of labor regulations. Let's dive into the exciting challenges of this venture!

Setting the Stage:

You've noticed that demand for souvenirs can skyrocket, sometimes tripling on peak days. This poses a critical question: How many employees should you hire to meet labor regulations and customer demand? Each employee works between 4 and 5 **consecutive** days a week, earning a base rate of €100 daily. Those who work 5 days will receive €105 daily.
Now, let's take a look at your forecasted numbers for the week:

	Min employees	Clients
Monday	8	800
Tuesday	10	1060
Wednesday	10	1150
Thursday	11	1500
Friday	17	2700
Saturday	17	2800
Sunday	12	2100

Challenges Ahead:

1. **Workforce Dilemma:** Calculate the number of employees needed to meet labor regulations and the associated cost (assuming that each employee works the same days weekly). What happens if a labor law changes and the employees who work 5 days a week enjoy a day off after 3 days of working?
2. **Marketing Splash:** With a growing workforce, you're contemplating hiring a social media marketing manager at €800 per week. Anticipate a potential uptick of 50 customers on weekdays and 100 on weekends (Saturday and Sunday) due to their efforts. This also means raising your daily minimum staff by one. How does this impact your planning?
3. **Profit Play:** Your success relies on numbers and profitability. What profit should you expect from each client to make this business profitable? How would profitability change if we reduce the minimum number of employees? How would profitability change if real demand is below forecast (same number of minimum employees)?
4. **Analyzing the Puzzle:** Discuss and dissect the implications of your findings. Please focus on efficiency and use of resources (e.g., employees). How do your decisions impact the overall business strategy?

It's time to put your operations research skills to the test and craft a winning strategy for your Souvenir Superstore!

2. Stadium Schedule

The Santiago Brenabeur, a football stadium in Madrid, is finally available to host events. Floppertino Perez, the president of the football team that owns the stadium, is willing to maximize revenues by hosting different type of events for the next three months (13 weeks of 7 days).
The following table provides the information on constraints and expected revenues for the different types of events that the stadium can host:

Type of event	Event duration	Setup time	Teardown time	Maximum number of events per type	Profit
	(nº of days)	(nº of days)	(nº of days)	(nº of events)	(million €)
Music Festival	4	1	1	3	2
Basketball game	1	1	1	10	0.6
ATP Tennis Masters	14	3	3	1	6
Gaming event	3	2	2	5	2
Music concert	2	2	2	5	1

Whenever the stadium is idle, in a setup mode or teardown mode, the museum, restaurants, and shops are open and provide 0.1 € million per day (they are closed if there is an event or a football match).
The football team also plays at the stadium, so it must be idle on a match day (it cannot hold an event, nor can it be in a setup or teardown mode). The football team plays one out of three Sundays and one out of eight Tuesdays (starting from week number 1).
In addition, basketball games can only be held once per week.

1) Formulate a mixed integer linear problem to obtain the maximum profit the Stadium can yield. Provide a potential calendar for this profit. Is this the unique optimal solution?
2) How much profit should concerts provide to gain more space in the schedule?
3) Is there any event that is so profitable that it would be convenient to expand the maximum number of events of this type?
4) A company is willing to rent the Stadium to organize its own events for one month, for which he offers 10 million € to Floppertino (the Stadium would be unavailable for that month and generate no profits). Should Floppertino rent the Stadium? How would you change the existing mathematical formulation to consider this possibility?

3. A rural energy community

We are providing a rural area (with several villages, a hospital, and a factory) with power. Solar panels, complemented with diesel, will generate this power.
We need to build a distribution network so that all demands in the community can be covered. The cost of building a line is defined by a fixed cost and a variable cost, which depends on the distance between nodes (in m). Once amortization has been calculated, the annual costs are expressed below in USD.

FC = 1000 USD

VC = 500 USD/m

The positions of the nodes are given in the table below (expressed as coordinates in m):

	x	y
Solar Panel 1	1270	80
Solar Panel 2	1269	82
Solar Panel 3	40	710
Battery	470	630
Diesel generator	830	700
Village 1	220	230
Village 2	110	640
Hospital	880	460
Factory	30	450
Village 3	340	450

1. Design the cheapest grid where all nodes have at least one connection.
2. Design the cheapest grid that makes sure all nodes are connected.

3. We must ensure that the distribution lines' capacity is appropriate for the power we need to transport. Please consider that all lines have a capacity of 50 kW, and you can install lines in parallel if necessary. This only increases the variable cost. Please redesign the network taking this into account, knowing you can install up to 4 cables in parallel.

	Supply (kW)	Demand (kW)
Solar Panel 1	50	0
Solar Panel 2	50	0
Solar Panel 3	50	0
Battery	0	0
Diesel generator	200	0
Village 1	0	50
Village 2	0	10
Hospital	0	5
Factory	0	50
Village 3	0	50

4. Solve the design imposing that all the nodes must be connected

5. VPN

The corporation BC wants to optimize their Virtual Private Network (VPN) services. Considering how employees distribute their work from home during the week, they have estimated their weekday needs. They require at least the bandwidth and, at most, the average latency shown in Table 1.

Weekday	Bandwidth (Gbps)	Latency (ms)
Monday	95	25
Tuesday	80	30
Wednesday	75	32
Thursday	90	27
Friday	110	22

Table 1. Daily requirement of Gbps and latency

They have selected an Internet Service Provider that offers VPN servers. The characteristics of these servers are shown in Table 2:

	Bandwidth (Gbps)	Latency (ms)
Server A	1	20
Server B	1.5	25
Server C	2	30
Server D	2.5	35

Table 2. Server characteristics

They offer BC to lease servers in periods from one to up to five consecutive days. The pricing list, depending on the number of consecutive days and server, is as follows:

Leasing period (consecutive days)	server A	server B	server C	server D
1	30	40	53	60
2	56	74	98	110
3	78	102	135	150
4	96	124	164	180
5	110	140	185	200

Table 3. Cost in € per server and leasing period

The bandwidth and latency needs are different for each day of the week, so it may be more economical to lease the number of servers daily. However, as the additional cost per day when leased in periods of

consecutive days is cheaper, it could be more interesting to lease the maximum number needed for the whole week. Another option is the intermediate and more general approach of leasing servers in packs of different consecutive days. Example: the company can lease 60 servers of type A for the first day (at the cost of 30 €/server), 24 servers of type B for the first two days (at the cost of 74 €/server), 3 servers of type C for the whole week (at the cost of 185 €/server), 25 servers of type C for the second and third day (at the cost of 98 €/server), and so on, until satisfying all their requirements for every weekday.
The total bandwidth available each day is the sum of the bandwidth provided by the servers available that day. Calculate the average latency per day as the average latency of the servers available each day weighted by the bandwidth provided by each server (for example, if a day there are available 20 servers of type B and 15 servers of type C, the average latency that day would be (20 servers * 1.5 Gbps * 25 ms + 15 servers * 2 Gbps * 30 ms) / (20 servers * 1.5 Gbps + 15 servers * 2 Gbps) = 27.5 ms).

a) Formulate a linear programming model to help BC decide how to lease the servers to minimize the total weekly leasing cost. Analyze the results.
b) Consider now that due to some commercial condition, if BC leases a type of server, they must lease at least five of those servers for a consecutive period of 5 days. Analyze and compare the results with those from section a).
c) Starting with the model from section a), consider that due to some technical restriction, servers A, B, and D cannot be used simultaneously on the same day (two of these types of servers can be used on the same day but not the three of them). Analyze and compare the results with those from section a).
d) Starting with the model from section a), consider the following additional commercial condition: For every 10 servers of the same type that BC leases for 5 consecutive days, they get one server of that type free to use for one day the weekday they prefer (for example, if they lease 23 servers of type C for 5 consecutive days they get 2 servers of type C free to use for one weekday each. The free servers can be used any weekday, not necessarily the same day). Analyze and compare the results with those from section a).

6. Retail café

You are the operations manager of a retail café company that operates four branches within a single mall: Bucks, Negro, Taste, and Flower. Each store can receive up to 150 product units daily and must maintain a minimum stock of 100 units. The purchase department provides a reception plan, while the sales department forecasts the expected daily aggregated demand:

	Reception [unit]	Sales [unit]
Monday	550	250
Tuesday	500	250
Wednesday	500	300
Thursday	500	450
Friday	550	550
Saturday	550	700
Sunday		650

The daily sales in each store are directly proportional to the daily deliveries received. The only exception is Sundays, when the sales are determined by the total accumulated stock minus the required minimum stock level. This minimum stock level is the starting point for each store's inventory on Mondays. The profit margin for each store is 0.6€/unit in Bucks, 0.8€/unit in Negro, 0.9€/unit in Taste, and 0.7€/unit in Flower.
In addition, you count on four high-ranked waiters who generate extra earnings through tips and are already assigned to a base store. Their daily wage, how much they increase the profit margin, and the extra cost from transferring to another store are as follows:

	Wage [€/day]	Extra profit [%]	Extra cost [€/day]
Emily – Bucks	100	18	30
John – Negro	150	25	45
Sarah – Taste	120	22	36
Mike – Flower	110	20	33

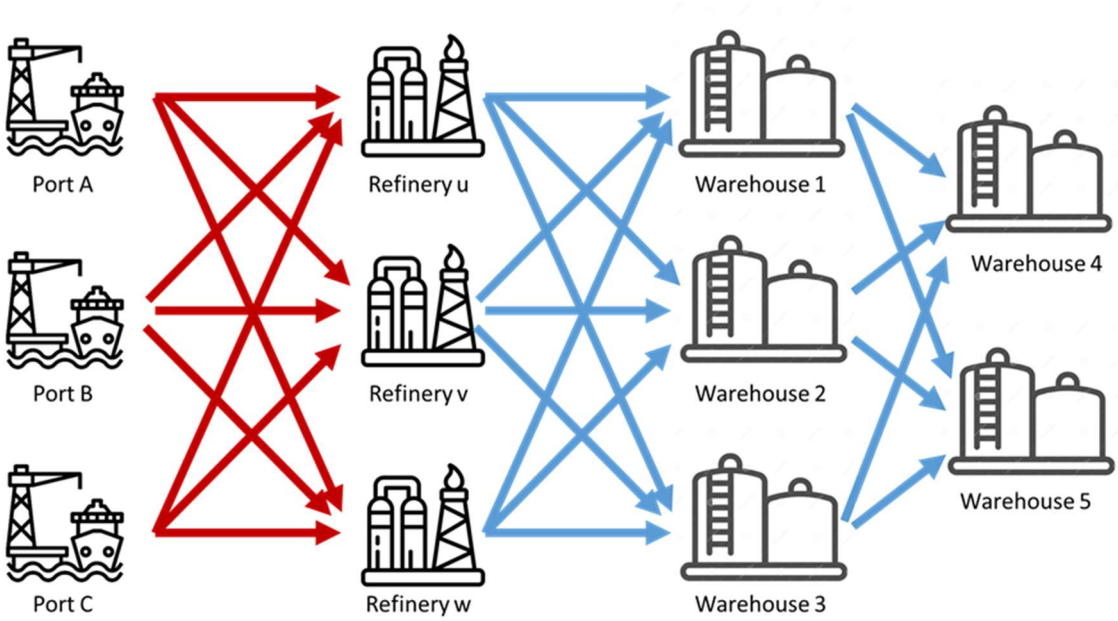
The union mandates that no waiter can work more than three days per week in the same store. Each waiter must also take one day off but must be hired for at least four days.
Your objective is to optimize the allocation of units across these stores and efficiently route your waiters throughout the stores, including when they take days off. To facilitate your decisions, the number of units of product does not need to be an integer.
After elaborating your plan, the human resources department, which is highly attentive to employee well-being and has conducted a survey to optimize staff satisfaction, communicates the following conditions:

- John and Mike are roommates and insist on working on the same days.
- If Emily stays at Bucks, Sarah insists on staying at Taste.

How do both conditions modify the schedule? How much does it cost?

7. Fuel distribution in Zamunda

The petroleum refining process starts with crude oil, and after refining, it is transformed into gasoline. Zamunda, a developing country, lacks the pipelines developed countries have. Therefore, we need to help the Prince of Zamunda solve the transshipment problem that will allow him to optimize the cost of the overall process.



The decisions we need to make are:

- how to distribute the agreed-upon supply from the neighboring producer Trinidad to distribute among the different ports, with a maximum of 3.6 million liters of crude oil per month.
- how to distribute this crude oil to the refineries for refining and subsequently from the refineries to the intermediate warehouses and final gasoline warehouses, taking into account the demand to be met in each warehouse in thousands of liters of gasoline:

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kliters	1	2	3	4	5
Dem	400	300	150	250	100

The costs to consider in the process are:

- the cost of unloading at any of the three ports is the same and therefore is not taken into account
- transporting crude oil from the ports to the refineries incurs a unit cost per liter of crude oil:

€/liter of crude oil	u	v	w
A	0,4	0,7	1,1
B	0,2	0,3	0,5
C	1,3	0,6	0,1

- Refining process costs: if a refinery is operational, it incurs fixed costs (CF) and a refining cost per liter of crude oil (Ref):

k€	u	v	v
CF	400	300	600

€/liter of crude oil	u	v	w
Ref	0,2	0,4	0,1

- Additionally, we must transport the finished product (gasoline) from the refinery to the intermediate warehouses and from the intermediate warehouses to the final warehouses. The transportation costs per kiloliter of gasoline:

€/liter of gasoline	1	2	3
u	0,6	2,4	4,5
v	2,7	1,8	2,7
w	5,4	3	1,5

€/liter of gasoline	4	5
1	2,1	3
2	3,6	0,6
3	1,5	1,5

Data: for every 3 liters of crude oil, 1 liter of gasoline is produced.

Part One

1) What is the optimal distribution with these data?
2) How does this situation change if, due to an accident, refinery 3 is closed?
3) How does the initial scenario change if port C is limited to receiving 1,500 thousand liters of crude oil?

Part Two

The Prince of Zamunda has purchased a soccer club in the Premier League and has had to privatize gasoline transport. Two companies have offered their services:

- Rates offered by Company 1

€/liter of gasoline	1	2	3
u	1,8	3,3	4,05
v	3,15	2,85	3,15
w	4,65	3,15	2,85

€/liter of gasoline	4	5
1	3	3,15
2	2,95	2,25
3	2,85	2,85

- Rates offered by Company 2

€/liter of gasoline	1	2	3
u	2,16	2,7	3,48
v	3,15	2,16	2,64
w	2,94	2,4	2,7

€/liter of gasoline	4	5
1	3,15	2,49
2	2,7	2,76
3	2,40	2,7

Due to ecological reasons, each route must be covered by only one Company (it cannot be shared or split)

4) Evaluate the new scenario (with all refineries available and no limitations at the ports) if the transport of all gasoline routes must be assigned to one of the two companies:
5) Evaluate how much the state saves if the allocation of routes is done one by one. Which routes does each company take?
6) The new Marketing Director of Company A offers the state a marketing campaign in which he offers a 100K€ bonus for each route assigned (100K€ if it's only one, 200K€ if two are assigned, and so on). Remember the ecological condition that routes cannot be shared.
7) Open question: how would you optimize the system from the state's point of view? You can consider risk reduction, social implications, cost reduction, ecological concerns…

8. NGO

After recently suffering three separate natural disasters in three countries (MO, LY, GR), an NGO based in Madrid has committed to assisting. The humanitarian effort involves transporting essential goods from the NGO's central headquarters to these countries to meet the population's basic needs. There are three types of goods: food, clean water, and medical supplies. The aid workers deployed in these countries have estimated the required quantities of each kind of goods.

	Food	Clean water	Medical supplies
	[t]	[hm^3]	[t]
MO	200	0.150	32
LY	2000	0.500	120
GR	150	0.025	10
	2350	0.675	162

The NGO already has specific quantities of food and medical supplies stored in its premises, received through anonymous donations, but these are insufficient.

Food	Medical supplies
[t]	[t]
190	15

The remaining merchandise must be purchased at the origin prices.

Food	Clean water	Medical supplies
[€/t]	[€/l]	[€/t]
1000	0.30	10

To carry out the transportation, the NGO has access to five different aircrafts (A400M, C-212, CN-235, C-295, C-130) with varying characteristics provided by the military.

	Maximum Load [t]	Roundtrip cost to MO [k€]	Roundtrip cost to LY [k€]	Roundtrip cost to GR [k€]
A400M	37	30	50	60
C-212	3	2	4	4
CN-235	6	4	7	8
C-295	10	8	13	16
C-130	20	15	25	30
	76			

Each aircraft must be sent to any country. Due to the disasters, each respective airport limits the aircraft landings to 50 operations. If there is any available space, it will be used for volunteer personnel and logistical materials at no additional cost. The costs (in k€) of sending each type of aircraft to each country

(including the round trip to the corresponding country) are known, as well as the maximum load (in t) available per aircraft.

Furthermore, due to rampant corruption, it is known that certain quantities of the goods sent may get 'lost' along the way. Based on past experiences, the NGO estimates that 20% of food, 1% of water, and 40% of medical supplies will be lost, regardless of the destination country.

1) After initial calculations, the NGO has determined that it can assist only two countries. Develop an optimization model that allows the selection of these two countries and making the necessary decisions to supply their populations with the committed quantities, minimizing the total operation cost. Establish indices, parameters, variables, the objective function, and constraints.
2) Due to urgent airport improvements, the number of airport landing operations in these countries has been increased to 75. Consequently, the NGO decided to supply the requested merchandise in all three countries. Which are the new optimal decisions made?
3) The army has required the A400M aircraft for other operations and can no longer attend these natural disasters. Alternatively, the NGO can rent an Embraer C-390 aircraft (with a maximum load of 26 t and double roundtrip costs to each country than the C-130). The rental price (10 k€ daily) must be added to the transportation cost. It is assumed that, at most, 4 roundtrips are possible daily. Modify the model from part a) to account for the new situation presented.
4) What would need to be added to the model from part 2) if the small planes (C-212, CN-235) could only carry a maximum of two types of merchandise?
5) Extend the problem to consider under multicriteria decision-making the total operation cost and the food requirement, assuming that in the case of MO the total food required was also 2000 t.