OPTIMIZATION MODELING PROJECTS

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1. Solar Panels at the building roof

A building community is thinking of installing solar panels on the roof of the building to reduce the cost of the building's electricity consumption. The roof has 10 m^2 useful to place solar panels. In table1, the average hourly consumption of the community in the two semesters of the year (365 days).

Wh	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
Semester 1	155	133	131	130	132	134	130	201	314	225	240	322	693	689	677	662	659	682	685	714	668	690	316	208
Semester 2	168	127	119	122	119	121	124	174	249	216	241	240	273	269	279	239	220	241	244	279	291	295	271	213

Table 1

There are three technologies with different peak generation capacity per m2. Thus, Thin-Film has a capacity of 200 W/m^2, Crystalline 250 W/m^2, and Multijunction 300 W/m^2. The installation company has sent the following offers with different annual costs for each technology and power. Any intermediate power between the values provided will have a proportional cost in each power range.

Peak Power	Thin-Film	Crystalline	Multijunction
1 kW	110 €	150 €	200 €
2 kW	200 €	280 €	390 €
3 kW	280 €	400 €	570 €

Table2 – Annual cost per technology and installed power

Next, the average solar production in Wh per hour and installed kW for each technology and semester.

Semester 1	10	11	12	13	14	15	16	17	18
Thin-Film	50	100	150	200	225	200	150	100	50
Crystalline	60	125	175	250	275	225	175	125	60
Multijunction	75	150	200	300	325	250	200	150	75

Table3 – Wh per hour of semester 1

Semester 2	9	10	11	12	13	14	15	16	17	18	19	20
Thin-Film	75	125	200	300	375	350	350	300	300	250	150	75
Crystalline	100	175	250	350	400	400	400	350	300	275	225	100
Multijunction	110	200	275	400	425	425	425	400	300	300	225	100

Table4 – Wh per hour of semester 2

The community has contracted an hourly energy price in c€/kWh as indicated below:

Hour	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
c€/kWh	25	24	23	20	19	22	29	30	35	30	20	20	20	25	25	25	30	35	35	40	45	40	35	30

Assume that the excess of solar production that is spilled into the grid does not generate benefits for the Community. Answer the following questions after modeling and solving a mathematical programming model:

a) Is it profitable to install solar panels? In this case, what technology and capacity of the installation should be chosen by the Community, and what would be the benefit obtained?
b) Suppose that the excess energy that is spilled to the grid is paid 19 c€/kWh to the Building Community. Does it change the previous decisions of a)? How and why?

c) In addition to question b), the Community wants to install two out of the three possible technologies. Each technology must not exceed 70% of the total installed capacity. How does the final decision change?

d) Assuming the grid payment of b) but not the condition of c) evaluate the possibility of installing a battery with a capacity of 1000 Wh to store the surplus of the energy not consumed or energy bought from the grid. What would be the annual cost that the Community would be willing to pay for this battery? Suppose that the battery is discharged at the beginning of the day and has a smart device that indicates when it is convenient to charge from the grid or from the solar panel and discharge energy to the Community.

2. Fiber-Optic Cable

The first step in manufacturing fiber-optic glass is to make a solid glass rod, known as a preform. Ultra-pure chemicals –primarily silicon tetrachloride ($SiCl_4$) and germanium tetrachloride ($GeCl_4$)– are deposited on the inner wall of a silica tube and converted into glass during preform manufacturing. Preforms are later drawn (pulled) into a thin fiber-optic cable. Currently, there is a high demand for fiber-optic cable and there is a bottleneck in the production of preforms due to limited manufacturing capacity.

A company, Tyndall Inc., is planning to build two new factories to produce preforms in two different locations. There are eight candidate locations, shown in Table1 below where the distance between each city pair is shown in kilometers.

	City 2	City 3	City 4	City 5	City 6	City 7	City 8
City 1	1484	1037	385	871	1290	2489	2165
City 2		1948	1095	1177	391	1271	2123
City 3			1244	862	1596	2013	1216
City 4				836	908	2272	2130
City 5					792	1621	1299
City 6						1301	1734
City 7							1616

Table1 - Distance in km between candidate locations

To produce the preforms they need supplies of SiCl4 and GeCl4 as well as silica tubes. For each unit of preform 1,3 liters of SiCl4, 1 liter of GeCl4, and one silica tube are required. The densities of SiCl4 and GeCl4 are 1.48 kg/liter and 1.88 kg/liter respectively. Each silica tube weighs 4,6 kg. Each preform, once manufactured, weighs 7 kg.

Suppliers of SiCl4 and GeCl4 and their annual production in liters can be found in the cities shown below in Table2:

	SiCl4	GeCl4
City 5	150000	80000
City 6	200000	70000

Table2 – Supplies in liters of SiCl4 and GeCl4

Suppliers of silica tubes and their annual production in units can be found in the cities shown below in Table3:

	Silica tubes
City 1	100000
City 3	130000

Table3 – Supplies in units of silica tubes

Tyndall Inc. wants to supply preform tubes to four locations where these preforms will be drawn to be transformed into fiber-optic cables. These locations and their annual demand for preform tubes are shown in Table4. The company wants to supply the total preform demand for these locations.

	Preforms
City 2	50000
City 4	50000
City 7	50000
City 8	50000

Table4 – Demand in units of preforms

Thus, Tyndall Inc. will have to transport SiCl4, GeCl4, and silica tubes from the locations where these materials are available to the locations where they decide to build the new factories of preforms, and they will have to transport preforms from the new factories to the locations where the preforms are demanded. Each factory can manufacture a different number of preforms, but each factory must receive enough materials (SiCl4, GeCl4, and silica tubes) to produce its preform output.
Tyndall Inc. estimates that the transport cost, per each ton and each kilometer, is 0.15 €/(t·km)

a) Formulate a mixed integer linear program to help Tyndall Inc. decide where to build their two factories to minimize the total transport costs.
b) What would be the solution if they build only one factory to supply all the demand? Compare the results with section a).
c) Modify the model in section a) to include the following condition: City 6 has a strong influence on the central government and forces Tyndall Inc. to buy all its annual supply of SiCl4 and GeCl4 in case Tyndall Inc. does not build a factory in their city. Compare the results with section a).

3. Fire Management

We are the Fire Department responsible for using the available resources to fight a fire recently started in a wooded area. The fire has been identified using the gray cells in this simplified map.

	C1	C2	C3	C4	C5	C6	C7	C8	C9
R1	Res								
R2									
R3				SP				3	
R4							7	9	9
R5						3			
R6					5	6			

R7				9	8	1			
R8			6	10	5				SP
R9			5	9					

The resources available for fighting the fire are:

- Two hydroplanes that can take water from a close reservoir Res (in blue and located in cell (R1, C1))
- Three helicopters that will take the water from the municipal swimming pools (SP) of two close villages (in blue in the figure, located in (R3, C4) and (R8, C9))
- Five human brigades, each one driving a truck

To organize the firefighting, the following conditions must be met:

- A helicopter can only attack a single cell
- The hydroplane can attack two consecutive cells, but the organization of their flights requires deciding whether to attack horizontally (W-E) or vertically (N-S) for both hydroplanes. No restriction on attacking contiguous rows or columns.
- A human brigade can only attack a single cell
- No helicopter can work in a cell contiguous to the working cells of the hydroplanes in the attack direction.
- At most, two helicopters can take the water from the same swimming pool

The ecological value of the area under fire has been assessed depending on the tree and plant composition and assigned a number depicted in the figure for each cell. The higher the most ecologically valuable the cell.

a) Determine the optimal decisions on deploying the different available resources to maximize the value of the areas attacked by the resources or minimize the lost value of the cells. We are assuming that a cell attacked by any resource will stop the fire and, consequently, has a lost value of zero.
    a. Determine the use of each swimming pool by each helicopter.
b) Now, change the objective function to minimize the flight distances of the helicopters and hydroplanes. Distances will be measured as Euclidean ones.
c) Now, determine the best solution to minimize the area lost subject to moving hydroplanes and helicopters the lower the better

4. Computer Manufacturing Company

A computer manufacturing company is analyzing the impact of producing computers directly in Europe and the USA to mitigate risk linked to the supply chain. Nowadays, this company only produces computers in China. They are willing to open two new factories, one in Europe and one in the States.
Their catalog includes 1 desktop computer and 2 laptops (1 premium and 1 standard). For each computer type, they include 2 memory options (regular, and extra).
The COO would like to understand how production would be allocated if they run this project.
To allocate production you should consider that:

1. Premium computers cannot be produced in Europe and desktop computers cannot be produced in America; production capacity is defined in the table below (thousands of units)

	USA	Europe	China
Desktop	-	150	200
Standard Laptop	100	200	200
Premium Laptop	25	-	5

2. For each type of computer (desktop, standard, or premium) that is produced in a factory, the company should assume 50 k€ costs that are not included in the margin.
3. If the production in a factory is less than 90% of its maximum capacity, it has a negative impact of 50 k€ on the company's profit
4. If the same factory produces both desktops and laptops, they have 100 k€ extra costs in that factory.
5. Expected minimum and maximum demand for each type of computer are shown in the table below (thousands of units).

	Desktop Regular	Desktop Extra	Standard Regular	Standard Extra	Premium Regular	Premium Extra
Minimum	100	150	75	100	2	2
Maximum	300	350	250	250	30	10

6. Expected margin for each type of computer is shown in the table below.

	Desktop Regular	Desktop Extra	Standard Regular	Standard Extra	Premium Regular	Premium Extra
Margin	5	4	5	6	20	30

Questions:

A) What is the expected production in each factory that maximizes profit?

B) In Europe, employees have requested an extra bonus related to production. Basically, they propose that if the European factory reaches 80% of capacity, the company must pay a bonus. The bonus has an estimated cost of 100 k€. How would this condition production?

C) The COO has fixed that the production of each type of computer (desktop, standard, or premium) should represent at least 20% of the total capacity of each factory. How does it condition the production? (To solve this section, you should include the employee bonus described in section B)

5. Movie Shoot

The very well-known American media company Metro-Óscar-Mayer (MOM for short) will start its next production in Spain. But as COVID has depleted the company's coffers, instead of going to big cities they are looking for villages and small towns. You, as the Operations Research Scientist behind the company, are planning the scheduling of scenes given that each scene is shot in a specific location and needs a particular subset of actor/actresses.

You can see in the following table a matrix with tuples of costs in the form of <number of moving days, fixed cost in k€> of moving from one location to another one:

	Malcocinado	Guarromán	Casasbuenas	O Quinto Pino	Guasa
Malcocinado	(0, 0)	(1, 9)	(1, 7)	(3, 12)	(3, 13)
Guarromán	(1, 9)	(0, 0)	(1, 4)	(3, 10)	(3, 11)
Casasbuenas	(1, 7)	(1, 4)	(0, 0)	(2, 8)	(2, 6)
O Quinto Pino	(3, 12)	(3, 10)	(2, 8)	(0, 0)	(3, 15)

Guasa	(3, 13)	(3, 11)	(2, 6)	(3, 15)	(0, 0)

(*) Just to be clearer: if we go from Malcocinado to Guarromán it will take us 1 moving day and 9k€, while moving from Casasbuenas to O Quinto Pino will take us 2 moving days and 8k€.

Having said that, a movie consists of a set of scenes, and each scene takes place for a determined duration in a given location with a set of actors/actresses. The order of shooting is not influenced by the order in the final version of the movie, but by economic reasons related to costs of actresses/actors and locations. Each actor/actress has a daily cost given in the following table, as well as a set of scenes in which they appear:

	Angelina Cruz	Morgan Dicaprio	Charlize Pataki	Matthew Schwarzenegger	Thandie Hathaway	Brendan Cruise
Cost in k€/day	8	2	9	9	20	17
Scenes	1, 7, 8, 9	3, 5, 6, 7, 8, 9	1, 3, 4, 5, 6, 9	3, 6, 7, 8	1, 4, 5, 6, 7, 8	1, 2, 3, 4, 5, 6, 7, 8, 10

Each scene is shot in a different location given in the following table, and has a specific duration given in number of days needed for the shot:

Scene	1	2	3	4	5	6	7	8	9	10
Location	Guasa	Malcocinado	O Quinto Pino	Guasa	Guasa	Malcocinado	Malcocinado	Casasbuenas	Guarromán	O Quinto Pino
Duration (days)	5	4	2	2	4	5	1	5	5	5

There are also precedence constraints between some scenes:

- Scene 9 needs to be shot before scene 10.
- Scene 1 needs to be shot before scene 7.
- Scene 6 needs to be shot before scene 4.

Therefore, the problem consists in finding the optimal sequence of scenes that satisfies the precedence constraints and minimizes the following costs:

- The cost of changing the location. Every time we need to move all the sets from one location to another one, we need to pay a fixed cost given in the first table.
- The cost of an actress/actor. You can consider that:
    - For every scene they play, they get paid every day of the scene. So, if the scene takes 5 days, they get paid for those 5 days.
    - They must be present for their scenes and must stay on the set in between. Even when not playing, the actor/actress is paid for these extra days of presence (days of moving from one location to another one plus the days of the scenes they do not play in between their scheduled scenes). This is the real actress/actor cost since they do not have to be on the set until their first scheduled scene and they can leave the set after their last scheduled scene.

6. Music Festivals

COVID is ending and music lovers are eager to attend to music festivals again. You, as responsible of an entertainment company, are planning the organization of four festivals in four cities: Madrid, Badajoz, Las Palmas, and Sevilla. The traveling cost, in hours, between cities is shown in the following table:

	Madrid	Badajoz	Las Palmas	Sevilla
Madrid	1	4	3.5	3.5
Badajoz	3.5	0.2	7	2

Las Palmas	3	7	0.7	3
Sevilla	3	2.5	3.5	1

(*) Note that traffic may increase traveling time. For example, Madrid to Badajoz takes 4 hours, while Badajoz to Madrid takes 3.5 hours. Spending the night in the same city has a cost too.

Each festival will last one day. You know the sequence of festivals will happen from June 11 to June 14, but you have not decided on the location for each date. The number of groups or musicians that will participate in each festival is 5 in Madrid, 2 in Badajoz, 4 in Las Palmas, and 3 in Sevilla. You have already pre-arranged the venues and know that the maximum capacity of each is 50,000 in Madrid, 7,000 in Badajoz, 13,000 in Las Palmas, and 31,000 in Sevilla. The profit per ticket amounts to 25€ in Madrid, 20€ in Badajoz, 30€ in Las Palmas, and 25€ in Sevilla.

The potential cast of groups and musicians include Rui Diaz, Red Chilies, CT, Rossa, DAOS, Fighters, Bizz, and Paul Alba. The preliminary conversations oblige to accommodate each group or musician, at least, in one festival. The following table shows the attractiveness (i.e., what percentage of attendants will purchase a ticket), the hiring cost per participant, the traveling cost per equivalent hour of the trip, and where they are located before starting the tour:

	Attractiveness [%]	Hiring cost [€]	Traveling cost [€/h]	City
Rui Diaz	10	10,000	500	Badajoz
Red Chilies	40	100,000	3,000	Madrid
CT	50	80,000	2,000	Sevilla
Rossa	70	150,000	4,000	Sevilla
DAOS	30	80,000	2,500	Madrid
Fighters	40	120,000	3,500	Madrid
Bizz	20	30,000	500	Las Palmas
Paul Alba	20	50,000	800	Sevilla

You also receive a report from the marketing department with the following information:

- CT and Rossa are incompatible with DAOS.
- Rui Diaz and Paul Alba playing together can attract an additional 30%.
- People without ticket cause a reputational loss of 5€ per unavailable ticket.
- All artists must finish their tour in the departing city.

Your duty is to decide the location and lineup for each date that maximizes the profit.

HINT: formulate first the problem, and then establish the boundary conditions—the starting and finishing cities of the tour.