OPTIMIZATION MODELING PROJECTS

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September 2021

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1. Electric motorcycle design

As the lead designer on an electric motorcycle racing team, you've been hired to build an electric motorcycle that can complete a racecourse in the fastest time. To get a feel for how each component performs, you've built several sample motorcycle configurations with varying electric motors, batteries, transmission, and brakes and then tested them on the racetrack. Each combination of items has yielded different speed results in minutes, as shown in the table.

Electric motor	Battery	Transmission	Brakes	Race speed [min]
DC	LithiumIon	Manual	Conventional	12
BrushlessDC	LeadAcid	Manual	Conventional	17
BrushlessDC	LithiumIon	Automatic	Regenerative	14
SensorlessBrushless	LeadAcid	Manual	Conventional	13
DC	LeadAcid	Automatic	OnlyOne	15
DC	NickelCadmium	Automatic	Conventional	11
SensorlessBrushless	NickelCadmium	CVT	OnlyOne	16
SensorlessBrushless	LeadAcid	Manual	Regenerative	16
BrushlessDC	LithiumIon	CVT	OnlyOne	18
BrushlessDC	NickelCadmium	CVT	Regenerative	20

There are three types of each component that you can choose from. You must pick precisely one component from each category: Electric motor, Battery, Transmission, and Brakes to decrease the race speed as much as possible.
Due to some design problems, some combinations are not possible: BrushlessDC motor is not compatible with NickelCadmium battery, nor Manual transmission with Regenerative brakes.
Besides, the selection of Automatic transmission requires the use of Conventional brakes, and SensorlessBrushless motor implies the use of LithiumIon battery.
Formulate the model to determine the best configuration for the electric motorcycle.
Suggestion: the model can be set up in two steps. First, determine the impact of each option based on the configurations of the table. Second, make the best selection to minimize the race speed.

2. 5G Spectrum Auction

A country is organizing the first of a series of auctions of the frequency spectrum to accommodate 5G technology. They want to assign blocks of frequency bands in 3 different categories (A - range 700/800 MHz, B – range 4 GHz, C – range 26 GHz). The size of the blocks depends on the category (A – 20 MHz, B – 40 MHz, C – 80 MHz). In this first auction, they will award 4 blocks in category A, 6 blocks in category B and 6 blocks in category C. All these blocks must be assigned.
Four companies have bid in the auction. All of them have placed bids for 3 blocks in each category. The bids are shown in the following table, in M€:

	Bids	Company1	Company2	Company3	Company4
Category A	1st block	160	163	154	156
	2nd block	149	153	150	151
	3rd block	145	147	140	143
Category B	1st block	100	97	95	103
	2nd block	96	93	90	92
	3rd block	88	91	88	85
Category C	1st block	130	125	133	127
	2nd block	124	122	128	121
	3rd block	110	112	115	114

If a company is assigned any block in a category, they must pay an additional fixed price for participating in that category (50 M€ for category A, 60 M€ for category B and 70 M€ for category C).
All the companies have set the condition that if they are assigned at least 2 blocks in category B they must be assigned at least 2 blocks in category A.

a) Formulate a mixed integer linear program to find which is the best way to assign the frequency blocks to the companies to maximize the money collected in the auction.
b) Consider that due to market power concerns, the country wants to consider an additional attribute in the auction, and has set two goals, one for each attribute. The first goal is to collect at least 2650 M€ in the auction. The second goal is that the difference in MHz awarded between the company with more MHz awarded and the company with less MHz awarded is at most 80 MHz. Modify the program from section a) to consider these two goals using goal programming. In the objective function, normalize the units of the attributes dividing by the corresponding target value. Compare these results with those from section a).

3. ORDER-E

ORDER-E is an e-commerce company that sells a wide variety of products: books, clothes, technology, etc. They are planning on expanding into Spain and have selected the Greater Madrid area as their first destination. After an initial assessment of the potential demand that they can expect in the following years, they have decided they need one distribution center (DC) and up to three sortation centers (SC). The distribution of goods always follows the flow DC → SC → end customer.

To plan their strategy, they have divided Madrid into a squared cell grid and for each cell they have calculated the expected annual demand (in thousands of orders per year), and the annual cost for opening a DC and a SC (in thousands of euros per year). This data is shown in the table below, where the data in cell (2, C), for example, represents the expected annual demand (30K orders/year), the annual cost for opening a DC (€260K/year), and the annual cost for opening a SC (€110K/year) in the area (2, C).

	A	B	C	D	E
1	70 | 630 | 130	50 | 500 | 200	10 | 120 | 40	50 | 350 | 110	500 | 980 | 380
2	100 | 1060 | 360	300 | 4550 | 1550	30 | 260 | 110	50 | 400 | 100	100 | 1090 | 590
3	50 | 390 | 90	90 | 1270 | 670	100 | 1940 | 440	40 | 1400 | 500	50 | 360 | 160
4	20 | 310 | 110	20 | 150 | 50	400 | 3890 | 1810	50 | 1250 | 250	10 | 90 | 40
5	10 | 160 | 50	50 | 380 | 80	100 | 1100 | 300	10 | 1010 | 410	0 | 170 | 40
6	0 | 110 | 40	30 | 120 | 40	50 | 510 | 210	20 | 990 | 190	10 | 310 | 90

ORDER-E has hired you to help them design their supply chain network. They would like to know where they should open the DC, how many SCs they should open and where, and from which SC/s they should deliver to each area. These are the requirements and extra information they have collected for you:

- All demand must be delivered from a SC except for the location where the DC is opened where delivery from DC to end customer is allowed.
- The DC has a total capacity of 2.5M orders per year.
- SCs have a capacity of 1M orders per year. In addition, for every SC an extra capacity of 0.5M orders/year can be hired at a cost of €350K/year.
- They want to minimize the total annual cost of transportation, plus the cost for opening DCs and SCs, plus SC extra capacity hired.
- The cost of transportation is 1€ per order and unit of distance. The unit of distance is the minimum number of cell changes required to go from one cell to another (e.g., to go from cell (1, A) to (2, C) at least 3 changes of cells are required: (1, A) → (1, B) → (1, C) → (2, C), therefore, the cost is 3€ per order). This can easily be calculated as the difference in rows (2-1=1), plus the difference in columns (C-A=2), i.e., 1+2=3.
- Demand must be satisfied with no excess allowed.
- A DC and a SC cannot be opened in the same location.
- No two SCs can be opened in the same row or column.
- No two SCs can be opened in adjacent cells.
- If extra capacity is hired for any SC, then no more than 2 SCs can be opened.

4. A network for the Moon

We are building a settlement on the Moon, and we need to provide it with power. This power will be generated by orbiting solar panels and sent to the Lunar surface by microwave transmission, which is captured by some rectennae (the combination of an antenna and a rectifier). This energy is complemented by nuclear power.
We need to build a network so that all demands of the base con be covered. The cost of building a line is defined by a fixed cost (FC) and a variable cost (VC), which depends on the distance between nodes (in m). This is very expensive because the materials need to be taken to the Lunar surface and buried underground to protect them. Once amortization has been calculated, the annual costs are expressed in USD: (FC = 1000,000 USD and VC = 500,000 USD/km)
The positions of the nodes are given in the table below (expressed as coordinates in m):

	x	y
Rectenna1	1270	80
Rectenna2	1269	82
Rectenna3	40	710
Storage	470	630
NuclearGen	830	700
HumanBase1	220	230
HumanBase2	110	640
CommunicationsStation	880	460
IceWaterStation	30	450
ResearchLab	340	450

a) Design the cheapest network where all nodes have at least one connection.
b) Design the cheapest network that makes sure all nodes are connected.
c) We need to make sure that the capacity of the distribution lines is appropriate for the power that we need to transport. Please consider that all lines have a capacity of 50 kW, and that you can install lines in parallel if necessary. This only increases the variable cost. Please redesign the network taking this into account, knowing you can install up to 4 cables in parallel.

	Supply (kW)	Demand (kW)
Rectenna1	50	0
Rectenna2	50	0
Rectenna3	50	0
Storage	0	0
NuclearGen	200	0
HumanBase1	0	50
HumanBase2	0	10
CommunicationsStation	0	5
IceWaterStation	0	50
ResearchLab	0	50

d) Solve the design imposing that all the nodes must be connected

5. Pharma robots

Your company has recently purchased a new workshop where all manufacturing processes will be relocated. The company produces three basic components for pharmaceuticals: capsules, syringes, and vials. The production is automated, and one robot can produce per cycle three capsules, two syringes, or one vial. In the new workshop, there is space for accommodating nine robots, and the plan is to assign three robots to the production of each component.

The CEO has personally hired a team of geomancers, who have divided the working area into 9 cells and, after analyzing the building structure and orientation, have determined the expected robot productivity, in cycles per hour, in the different zones:

	1	2	3
A	6	13	18
B	5	2	15
C	1	3	5

The geomancers have suggested that the maximum productivity will be reached if two components are never simultaneously produced in two adjacent cells. Believe it or not, "this promotes healthy competition between robots" were their words.

The CEO and you are very good mates, and both know your passion for crosswords, puzzles, and Sudoku. Unexpectedly, the task of assigning a component to each robot is on your desk:

- Could you maximize the aggregated number of potentially manufactured components?

Finished your planning, one geomancer has made a phone call to the CEO recommending leveling off the production of components to avoid further stress on the robots. Lucky robots and lucky you:

- To the extent possible, could you level off the total manufactured number of each component?

6. Bank Footprint

The increasing number of neoBanks as well as the higher number of credit card transactions compromise the profitability of retail-banking branches in small municipalities. To cut costs and maximize efficiency, the CEO of a Spanish Bank wants to optimize the bank footprint in a region with 16 municipalities.
The head of the Strategy department has proposed a distribution network that only includes two types of branches (small and large branches) to satisfy existing demand. The decision to set up a branch in a municipality is based on the following principles:

1. The number of employees in each type of branch is fixed: 2 bank tellers and 1 RMs in small branches; 3 bank tellers, 4 RMs and 1 branch manager in large branches. Branch managers have no client interaction.
2. To meet service standards each bank teller can attend 500 customers/month and RMs 250 customers/month. However, in large branches employees spend less time on administrative tasks and they can attend +20% of clients.
3. 70% of clients visit bank branches every month due to cash transactions and 25% of clients meet RMs every month.
4. Branches might not be set up in those municipalities with low demand, but those clients should be assigned to a branch that they can reach in less than 30 minutes (all clients from a municipality with no branch will visit the same municipality, but they might be assigned to different branches).
5. Each branch area will only include one additional municipality with no branches.
6. The bank should assume a cost of 10€/client in those municipalities with no branch. This applies to all bank clients in that municipality, disregarding whether they visit a branch or not.
7. In those municipalities where no branch is set up, the bank will consider the possibility of installing an ATM (2,000 €/month) to provide cash services. If no ATM is installed, the cost of no branch increases by 2.5 €/client in that municipality.
8. Clients in each municipality, distances among municipalities and cost breakdown are shown in the tables attached.

Question 1

- Define the optimal network that minimizes network cost and meet with the previous design principles.
- Discuss if this network is an optimal one in terms of employee allocation, would it be possible to design a cheaper network that meets the required design principles?

The CEO is considering the possibility of introducing a new job position, Area Manager, to help branches to increase profit. Area Managers will be hired only in municipalities with more than 2 small branches and no large branches.

Question 2

- Introduce this new design principle in the model used in Question 1
- What is the revenue per client increase that backs up the decision of hiring Area Managers?

Additional information:

- Rent expense (€/month)

Small branch	2.000
Large branch	4.000

- Employee salary (€/month)

Bank teller	2.500
RM	3.000
Branch manager	4.000
Area Manager	5.000

- Number of clients in each municipality:

m1	7.000
m2	500
m3	200
m4	4.000
m5	300
m6	1.000
m7	1.500
m8	800
m9	400
m10	600
m11	1.300
m12	700
m13	3.100
m14	650
m15	350
m16	250

- Distance among municipalities

	m1	m2	m3	m4	m5	m6	m7	m8	m9	m10	m11	m12	m13	m14	m15	m16
m1	0	5	40	67	27	34	45	78	52	33	43	80	49	73	66	35
m2	5	0	12	45	23	7	13	67	29	16	10	34	56	87	26	44
m3	40	12	0	35	28	5	16	56	23	3	19	67	36	54	87	45
m4	67	45	35	0	43	33	67	23	56	67	87	31	55	87	34	90
m5	27	23	28	43	0	17	23	56	3	27	19	43	65	42	74	53
m6	34	7	5	33	17	0	12	35	21	19	8	54	67	72	54	31
m7	45	13	16	67	23	12	0	45	6	16	25	65	41	33	57	86
m8	78	67	56	23	56	35	45	0	31	56	76	98	37	39	67	43
m9	52	29	23	56	3	21	6	31	0	13	16	78	3	23	56	42
m10	33	16	3	67	27	19	16	56	13	0	1	34	29	24	37	45
m11	43	10	19	87	19	8	25	76	16	1	0	76	33	56	13	49
m12	80	34	67	31	43	54	65	98	78	34	76	0	54	33	9	9
m13	49	56	36	55	65	67	41	37	3	29	33	54	0	10	56	33
m14	73	87	54	87	42	72	33	39	23	24	56	33	10	0	67	89
m15	66	26	87	34	74	54	57	67	56	37	13	9	56	67	0	78
m16	35	44	45	90	53	31	86	43	42	45	49	9	33	89	78	0

7. Urgency Medical Guards

Puerta de Hierro, a famous hospital in Madrid, is trying to increase the satisfaction of recently hired residents after they have passed the MIR exam. One of the main complaints they usually get is that there is not an efficient way of assigning the huge amount of urgency guards that first-year resident doctors must do.
To solve this problem, Puerta de Hierro has hired you to help them distribute the medical guards for first-year resident doctors in the best possible way. The conditions to do such a distribution are the following:

- Each night, at least 1 first-year resident must be present at the 4 levels of the urgency
- Each resident doctor must not do more than 2 guards per week

Your boss has helped you get started in this project by distributing a survey where first year doctors can assign up to 100 points at each day of the week. To ensure they all choose more than 1, your boss didn't allow for more than 50 points given to any day. Since the payment received is higher on weekends, each doctor has different preferences. You have the resulting table below:

Specialty	Doctor	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Internal Medicine	Resident 1	50	28	10	12	0	0	0
Internal Medicine	Resident 2	0	0	0	35	38	15	12
Internal Medicine	Resident 3	0	0	0	25	38	10	27
Cardiology	Resident 4	38	10	12	40	0	0	0
Cardiology	Resident 5	38	0	28	34	0	0	0
Cardiology	Resident 6	0	0	0	28	10	38	24
Traumatology	Resident 7	10	0	0	0	40	50	0
Traumatology	Resident 8	0	50	40	10	0	0	0
Traumatology	Resident 9	0	35	40	25	0	0	0
Oncology	Resident 10	10	0	0	0	40	25	25
Oncology	Resident 11	40	0	0	0	38	10	12
Oncology	Resident 12	38	0	28	34	0	0	0
Anesthesia	Resident 13	0	0	0	50	40	10	0
Anesthesia	Resident 14	0	0	0	34	0	38	28

a) Formulate and solve a mixed integer linear program to distribute the guards of the week amongst the 14 first year resident doctors
b) The hospital director, seeing the result of your first model and to ensure a better education of future teams, asks you now to also ensure that:
    - At least one doctor of each specialty does one guard at each of the 4 levels.
    - Not 2 residents of the same specialty share a guard

    Formulate and solve a new mixed integer linear program to distribute the guards of the week amongst the 14 first year resident doctors including these constraints

c) The hospital director is now happy, but seeing the results of your last model, the HR responsible reaches out to you asking for a last change. Residents 1 and 4, who share a guard in your last model, are married. The bylaws of the hospital forbid the assignment of guards to marriages to ensure at least one of them can take care of their children. Thus, if doctor 1 has a guard, doctor 4 cannot have it and vice versa. Formulate and solve a mixed integer linear program to distribute the guards of the week amongst the 14 first year resident doctors including these constraints