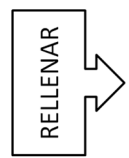




**Midterm Exam MODEL A**

**December 13<sup>th</sup>, 2017**



*STUDENT (Last Name and Name):*

*GROUP:*



<b>Time: 90 minutes</b>	<b>PROBLEM</b>
1.	Cell phones must be <b>turned off</b> and <b>stowed away</b> during the exam
2.	The only materials you are allowed to have are the exam, writing instruments and a non-programmable calculator.
3.	<b>Programmable calculator, books or notes are not allowed.</b>

**TIME AT THE AIRPORT (1.5 p)**

You want to simulate the time that a passenger spends at the airport from arrival until crossing the security control gate. Passengers can arrive by bus or taxi. Each plane can carry up to 180 passengers. A third of the passengers will arrive at the airport by bus. Each bus arrives according to an exponential distribution of average 3 minutes and will carry 12 passengers of our flight. Passengers using a taxi will arrive following a Poisson process of 120 passengers per hour.

The first thing that passengers do when they arrive at the airport is to go to the check-in counters. There are two types of billing counters: Business and Normal. Approximately per each flight there is a 25% of Business travelers and 75% of Normal travelers. There are 2 business desks dedicated for Business clients only, while there are 3 normal desks to serve Normal travelers. The required time at the desk of any passenger depends on the number of suitcases. This process will be based on an exponential distribution with average of the minutes per suitcase. 35% of passengers do not carry suitcases, 30% carry only one suitcase, 20% carry two suitcases and 15% carry three suitcases.

Once this process is finished, all passengers will lead to the security control, where a security check will happen for each traveler in arrival order. There is only one security agent to perform it. The time which they will spend to go through the security control can be modelled with a triangular distribution of the following parameters: 1, 2 and 3 minutes.

In case of finding any inappropriate belonging in the hand-luggage during the security check, this traveler will have to return to the check-in counters again, but this time the process will always take on average 2 minutes (suitcases were already checked). This situation occurs with a probability of 10%.

After passing the security control, the traveler is no longer in the system. The objective of the simulation is to record the time that, for a certain flight, each passenger spends from arriving at the airport to access the boarding area.

Determine:

1. Entities arriving to the system. Attributes associated to these entities and variables (2 p)
2. Resources used (2 p)
3. Pseudocode with explanatory comments (6 p)

### SPRINGFIELD VERSUS HOMER (1.5 p)

The Simpson's TV producers are planning to relaunch the show. For such a purpose, they have created an app for mobiles which is a video game called: "Springfield vs. Homer". These producers, who know a lot about queuing theory, have set the following scheme:

"There is a bag moving around Springfield. It has a capacity of two donuts and one can of beer (Duff, of course). With a rate  $\lambda_{duff}$ , following a Poisson process, one random Homer appears and introduces a beer inside the bag. Also, with a rate of  $\lambda_{donut}$ , following a Poisson process, one random Bart appears and inserts one donut inside the bag.

On the other hand, Krusty "the clown" randomly steals a beer with a Poisson rate  $\mu_{duff}$ , while Nelson appears with a rate (also Poisson)  $\mu_{donut}$  and takes a donut from the bag. The donuts and beers that Homer and Bart are trying to introduce into the bag, when it is full, will be lost in a new space called Futurama and will not come back again to Springfield.

- a) To analyze the system dynamics
  - a. Draw the state transition diagram (3 p)
  - b. Formulate the steady state equations (3 p)
- b) In order to optimize the game dynamics, producers set down a rule: 1/2 of the game has to be concentrated on states in which there are, at least, 2 items in the bag. Indeed, they fix the probability of the state with one beer and one donut to be 1/10. Update the steady state equations and formulate all conditions without finding mathematical solution. (2 p)
- c) Homer wins only if he gets the target of having a full bag with two donuts and one beer (who needs more than that?). The producers want this probability to be 1/5, to achieve an exciting game. If they fix, for instance,  $\mu_{donut} = \mu_{duff} = 2$ , what should be the values of the parameters  $\lambda_{donut}$  and  $\lambda_{duff}$ ? (2 p)

### Arena Basic Elements

- ENTITY (entityName)
- attributeName1
  - attributeName2
- ATTRIBUTE (attributeName)
- number of rows
  - number of columns
  - Initial value
- VARIABLE (variableName)
- number of rows
  - number of columns
  - Initial value
- RESOURCE (resourceName)
- Fixed capacity or based on Schedule
  - Capacity value or scheduleName
- SCHEDULE (scheduleName)
- Capacity type or Arrival type

### Arena Modules Glossary

- CREATE (entityName, InterArrDist, entPerArr, maxArrival, firstCreation)
- *entityName*: Entity that is created by the module
  - *InterArrDist*: Interarrival time distribution with its parameters (e.g.: Expo(10))
  - *entPerArr*: Entities per arrival (a number or a distribution)
  - *maxArrival*: maximum number of arrivals
  - *firstCreation*: distribution for the first arrival time
- PROCESS (action, resource, quantity, delayDist)
- *action*: Logic of the process (e.g., Seize Delay Release, SD, DR, D)
  - *resource*: Resource type seized by the entity
  - *quantity*: Number of seized resources by the entity
  - *delayDist*: Delay time distribution with its parameters
- ASSIGN
- List of assignments for attributes or variables
  - READWRITE (type, fileName)
  - *type*: Read or Write
  - *fileName*: File to read or write
  - List of assignments: inputs or outputs to read or write
- DECIDE (type) %description
- *type*: 2-way or n-way by Chance, 2 or n-way by Condition
  - List of Percentages or Conditions and destination labels
- RECORD (type, value, attribute, counterName) %description
- *type*: Count, Expression, Entity Statistics, Time Interval, Time Between
  - *value*: value or the expression to increment the counter
  - *attribute*: attribute to compute the Time Interval
  - *counterName*: output to record the value
- BATCH (size, rule, save) %description
- *size*: number of entities to be batched
  - *rule*: any entity or by attribute
  - *save*: first, last, sum or product of attributes for the entity batched
- SEPARATE (type, number) %description
- *type*: duplicate or split batch
  - *number*: Number of duplicates
- DISPOSE %description

**TIME AT THE AIRPORT**

1) ENTITIES:

Passengers

ATTRIBUTES:

my\_suitcase (0,1,2,3)

my\_arrivaltime

my\_processtime

my\_class (0 business, 1 normal)

VARIABLES

2) RESOURCES

Business (Fixed Capacity=2)

Normal (Fixed Capacity=3)

Security (Fixed Capacity=1)

3) CREATE Bus (Passenger, Expo(3), 12, 60,Expo(3)) →A

CREATE Taxi (Passenger, Expo(0.5), 1, 120,Expo(0.5)) →A

A:

ASSIGN (my\_arrivaltime = TNOW

my\_suitcase = DISC(0.35,0,0.65,1,0.85,2,1,3)

my\_class = DISC(0.25,0,1,1)

my\_processtime=my\_suitcase

)

B:

DECIDE (2 way by condition)

if my\_class=Business →C

else (my\_class=Normal → D

C:

PROCESS Desk\_Business (SDR,Business,1,3\*EXPO(my\_processtime))→E

D:

PROCESS Desk\_Normal (SDR,Normal,1,3\*EXPO(my\_processtime))→E

E:

PROCESS Control (SDR, Control,2,TRI(1,2,3))

DECIDE (2 way by chance)

10% → F

90% → G

F:

ASSIGN (my\_processtime=2) →B

G:

READWRITE (Data)

IDENT,my\_arrivaltime,my\_class,my\_suitcase,TNOW

DISPOSE

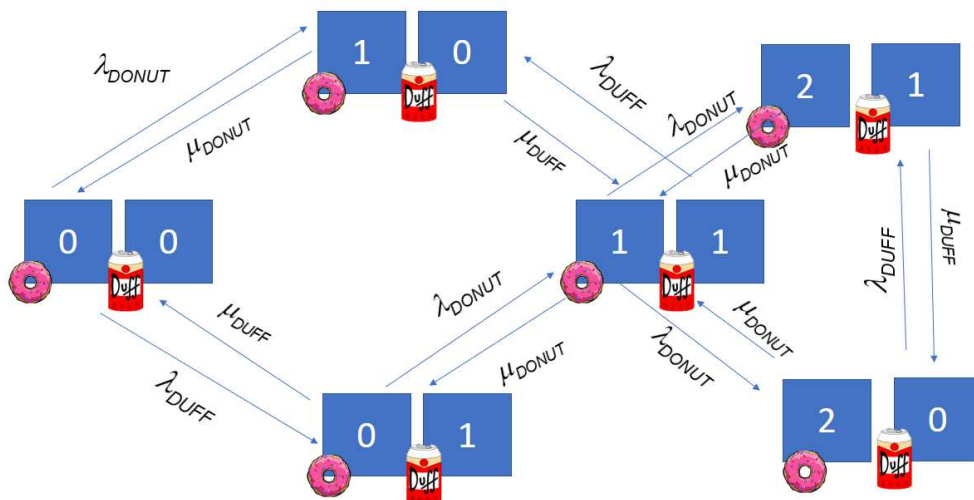
### SPRINGFIELD VERSUS HOMER

To analyze the system dynamics

- Draw the state transition diagram (3 p)
- Formulate the steady state equations (3 p)

**Remark 1:** In the sentence from the statement: "The donuts and beers that Homer and Bart are trying to introduce into the bag, when it is full, will be lost in a new space called Futurama and will not come back again to Springfield". Means:

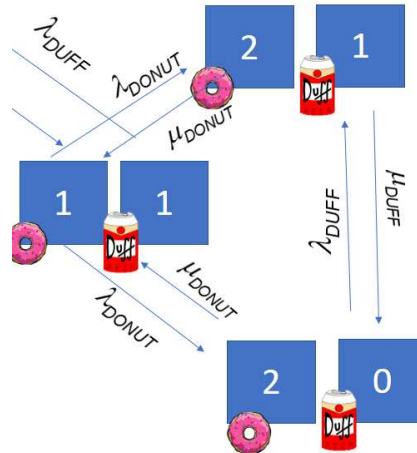
- The system is stable.** If the bag is full (So, the system has arrived to the state 2,1) it is impossible to introduce nothing more into the bag and any additional item will be "processed". Nevertheless, the system can go back again either to 1,1 or to 2,0 according to Poisson laws.
- Nobody says, so far, that if Homer wins (state 2,1) the bag becomes empty: **Since it is a Poisson process, it is impossible to pass from 2,1 to 0,0 in one step.** Remember from the very beginning of the course the main items about Poisson processes.
- By the way, the additional name of "Homer wins" to state 2.1 it is given on part c).



$$\begin{aligned}
 (\lambda_{Donut} + \lambda_{Duff})p_{00} &= \mu_{Duff}p_{01} + \mu_{Donut}p_{10} \\
 (\mu_{Donut} + \mu_{Duff})p_{10} &= \lambda_{Duff}p_{11} + \lambda_{Donut}p_{00} \\
 (\lambda_{Donut} + \mu_{Duff})p_{01} &= \mu_{Donut}p_{11} + \lambda_{Duff}p_{00} \\
 (2\lambda_{Donut} + \mu_{Donut} + \lambda_{Duff})p_{11} &= \mu_{Duff}p_{10} + \lambda_{Donut}p_{01} + \mu_{Donut}p_{21} + \mu_{Donut}p_{20} \\
 (\mu_{Donut} + \mu_{Duff})p_{21} &= \lambda_{Duff}p_{20} + \lambda_{Donut}p_{11} \\
 (\lambda_{DUFF} + \mu_{donut})p_{20} &= \lambda_{Donut}p_{11} + \mu_{Duff}p_{21} \\
 p_{00} + p_{10} + p_{01} + p_{11} + p_{21} + p_{20} &= 1
 \end{aligned}$$

b) In order to optimize the game dynamics, producers set down a rule: 1/2 of the game has to be concentrated on states in which there are, at least, 2 items in the bag. Indeed, they fix the probability of the state with one beer and one donut to be 1/10. Update the steady state equations and formulate all conditions without finding mathematical solution. (2 p)

The more straightforward idea was to focus only in the states with "at least" 2 items:



Note that fixing  $p_{11} = 1/10$  we can decouple the system, and “forget” the rest of the equations to focus only in this part of the game

$$\begin{aligned}(\mu_{Donut} + \mu_{Duff})p_{21} &= \lambda_{Duff}p_{20} + \lambda_{Donut}0.10 \\ (\lambda_{DUFF} + \mu_{donut})p_{20} &= \lambda_{Donut}0.10 + \mu_{Duff}p_{21} \\ p_{21} + p_{20} &= 0.40\end{aligned}$$

However, you could also have interpreted that you were asked to do the following:

$$\begin{aligned}(\lambda_{Donut} + \lambda_{Duff})p_{00} &= \mu_{Duff}p_{01} + \mu_{Donut}p_{10} \\ (\mu_{Donut} + \mu_{Duff})p_{10} &= \lambda_{Duff}0.10 + \lambda_{Donut}p_{00} \\ (\lambda_{Donut} + \mu_{Duff})p_{01} &= \mu_{Donut}0.10 + \lambda_{Duff}p_{00} \\ (2\lambda_{Donut} + \mu_{Donut} + \lambda_{Duff})0.10 &= \mu_{Duff}p_{10} + \lambda_{Donut}p_{01} + \mu_{Donut}p_{21} + \mu_{Donut}p_{20} \\ (\mu_{Donut} + \mu_{Duff})p_{21} &= \lambda_{Duff}p_{20} + \lambda_{Donut}0.10 \\ (\lambda_{DUFF} + \mu_{donut})p_{20} &= \lambda_{Donut}0.10 + \mu_{Duff}p_{21} \\ p_{00} + p_{10} + p_{01} + p_{21} + p_{20} &= 0.90 \\ p_{21} + p_{20} &= 0.40\end{aligned}$$

More boring (and repetitive) but valid too.

Homer wins only if he gets the target of having a full bag with two donuts and one beer (who needs more than that?). The producers want this probability to be  $1/5$ , to achieve an exciting game. If they fix, for instance,  $\mu_{donut} = \mu_{duff} = 2$ , what should be the values of the parameters  $\lambda_{donut}$  and  $\lambda_{duff}$ ? (2 p)

The idea on here was:

- To use the previous conditions
- “wins” has not any sense for the system (it doesn’t mean nothing than the state 2,1 has a “label”). We know that Homer wins with probability  $1/5$

According to the previous FIRST system:

$$\begin{aligned}2\lambda_{donut}0.10 &= 2[0.20 + 0.20] \\ \lambda_{donut} &= 4\end{aligned}$$

$$\begin{aligned}4 \times 0.20 &= \lambda_{DUFF}0.20 + 4 \times 0.10 \\ \lambda_{duff} &= 2\end{aligned}$$

However, if you deduce that your system is the SECOND one:

$$\begin{aligned}(\lambda_{Donut} + \lambda_{Duff})p_{00} &= 2p_{01} + 2p_{10} \\ 4p_{10} &= \lambda_{Duff}0.10 + \lambda_{Donut}p_{00}\end{aligned}$$

$$\begin{aligned}(\lambda_{Donut} + 2)p_{01} &= 0.20 + \lambda_{Duff}p_{00} \\(2\lambda_{Donut} + 2 + \lambda_{Duff})0.10 &= 2(p_{10}) + \lambda_{Donut}p_{01} + 0.80 \\0.80 &= \lambda_{Duff}0.20 + \lambda_{Donut}0.10 \\(\lambda_{DUFF} + 2)0.20 &= \lambda_{Donut}0.10 + 0.40 \\p_{00} + p_{10} + p_{01} &= 0.50\end{aligned}$$

With unknowns:  $(\lambda_{Donut}, \lambda_{Duff}, p_{00}, p_{10}, p_{01})$  and 7 equations.

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$$\int_{\text{donut}}^{\text{beer}} \text{beer}(x) dx \approx \frac{\text{beer} - \text{donut}}{6} \left[ \text{beer}(\text{donut}) + 4 \text{beer}\left(\frac{\text{donut} + \text{beer}}{2}\right) + \text{beer}(\text{beer}) \right]$$